Some of your colleagues have decided to enter a sledding competition. In this competition, the contestants start at the top of a hill and race down on their sleds. This hill is filled with a number of ramps. Whenever a contestant reaches a ramp *while on the ground*, they can either use that ramp to jump through the air, possibly flying over one or more ramps, or sled past that ramp and stay on the ground. Obviously, if someone flies over a ramp, they cannot use that ramp to extend their jump.

The rules state that whoever spends the most time in the air wins the competition.

After gathering details about the hills and ramps and doing some practice, your colleagues have turned to you to help them figure out how to get the first prize trophies in a few different categories.

1. Suppose you are given a pair of arrays $Ramp[1..n]$ and $Length[1..n]$, where $Ramp[i]$ is the distance from the top of the hill to the ith ramp, and $Length[i]$ is the distance that any sledder who takes the ith ramp will travel through the air.

   For example, consider an instance where $n = 8$, $Ramp = [1, 5, 10, 15, 20, 25, 40, 50]$ and $Length = [16, 6, 1, 30, 2, 3, 2, 0]$. Then taking the first ramp will result in landing at distance 17 from the top of the hill, which is after the fourth ramp, and then taking all the subsequent ramps will give a total air time of $16+2+3+2 = 23$. On the other hand, if one skips the first one and takes the second ramp and the fourth one, the total air time is $6+30 = 36$.

   Describe and analyze an algorithm to determine the maximum total distance that a contestant can spend in the air.

2. Your colleagues have decided to try the expert version of the competition, which states that each contestant can use at most k jumps.

   Describe and analyze an algorithm to determine the maximum total distance that a contestant can spend in the air *with at most k jumps*, given the original arrays $Ramp[1..n]$ and $Length[1..n]$ and the integer k as input.

3. **To think about later:** In the *team-based* expert version of the competition, sledders compete in teams of two. Each team member is still limited to k jumps, but each team can only use each ramp *once*, i.e., if a team member uses a ramp, then their teammate cannot use it.

   Describe and analyze an algorithm to determine the maximum total distance that a team of two contestants can spend in the air, with each of the two contestants taking at most k jumps (so at most $2k$ jumps total), and with each ramp used at most once.