For each of the problems below, transform the input into a graph and apply a standard graph algorithm that you've seen in class. Whenever you use a standard graph algorithm, you ***must*** provide the following information. (I recommend actually using a bulleted list.)

- What are the vertices? What does each vertex represent?
- What are the edges? Are they directed or undirected?
- If the vertices and/or edges have associated values, what are they?
- What problem do you need to solve on this graph?
- What standard algorithm are you using to solve that problem?
- What is the running time of your entire algorithm, *including* the time to build the graph, *as a function of the original input parameters*?

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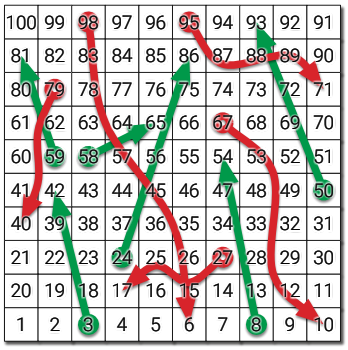
1. ***Snakes and Ladders*** is a classic board game, originating in India no later than the 16th century. The board consists of an $n \times n$ grid of squares, numbered consecutively from 1 to n^2, starting in the bottom left corner and proceeding row by row from bottom to top, with rows alternating to the left and right. Certain pairs of squares, always in different rows, are connected by either "snakes" (leading down) or "ladders" (leading up). ***Each square can be an endpoint of at most one snake or ladder.***



A typical Snakes and Ladders board.
Upward straight arrows are ladders; downward wavy arrows are snakes.

You start with a token in cell 1, in the bottom left corner. In each move, you advance your token up to k positions, for some fixed constant k (typically 6). Then if the token is at the *top* of a snake, you ***must*** slide the token down to the bottom of that snake, and if the token is at the *bottom* of a ladder, you ***may*** move the token up to the top of that ladder.

Describe and analyze an efficient algorithm to compute the smallest number of moves required for the token to reach the last square of the Snakes and Ladders board. The input to your algorithm consists of an $n \times n$ grid where each cell either contains a snake or ladder pointing towards another cell, or nothing at all.

2. Let G be an undirected graph. Suppose we start with two coins on two arbitrarily chosen vertices of G. At every step, each coin ***must*** move to an adjacent vertex. Describe and analyze an efficient algorithm to compute the minimum number of steps to reach a configuration where both coins are on the same vertex, or to report correctly that no such configuration is reachable. The input to your algorithm consists of a graph $G = (V, E)$ and two vertices $u, v \in V$ (which may or may not be distinct).

Think about later:

3. Let G be an undirected graph. Suppose we start with 374 coins on 374 arbitrarily chosen vertices of G. At every step, each coin ***must*** move to an adjacent vertex. Describe and analyze an efficient algorithm to compute the minimum number of steps to reach a configuration where all 374 coins are on the same vertex, or to report correctly that no such configuration is reachable. The input to your algorithm consists of a graph $G = (V, E)$ and starting vertices $s_1, s_2, \ldots, s_{374}$ (which may or may not be distinct).