Programming Languages and Compilers (CS 421)

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Based on slides by Elsa Gunter, which were inspired by earlier slides by Mattox Beckman, Vikram Adve, and Gul Agha

Data type in Ocaml: lists

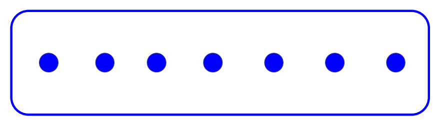
- Frequently used lists in recursive program
- Matched over two structural cases
  - [ ] - the empty list
  - (x :: xs) a non-empty list
- Covers all possible lists
- type 'a list = [ ] | (::) of 'a * 'a list
  - Not quite legitimate declaration because of special syntax

Variants - Syntax (slightly simplified)

- type *name* = C_1 [of ty_1] | . . . | C_n [of ty_n]
- Introduce a type called *name*
- (fun x -> C_i x) : ty_1 -> *name*
- C_i is called a *constructor*; if the optional type argument is omitted, it is called a *constant*
- Constructors are the basis of almost all pattern matching

Enumeration Types as Variants

An enumeration type is a collection of distinct values



In C and Ocaml they have an order structure; order by order of input

Enumeration Types as Variants

```
# type weekday = Monday | Tuesday | Wednesday
  | Thursday | Friday | Saturday | Sunday;;
type weekday =
   Monday
  | Tuesday
  | Wednesday
  | Thursday
  | Friday
  | Saturday
  | Sunday
```


Functions over Enumerations

```
# let day_after day = match day with
    Monday -> Tuesday
  | Tuesday -> Wednesday
  | Wednesday -> Thursday
  | Thursday -> Friday
  | Friday -> Saturday
  | Saturday -> Sunday
  | Sunday -> Monday;;
 val day_after : weekday -> weekday = <fun>
```

Functions over Enumerations

type weekday = Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday;;

Write a function days_later n day that computes a day which is n days away from the day. Note that n can be greater than 7 (more than one week) and also negative (meaning a day before

```
# let rec days_later n day =
    match n with
      0 -> day
    | _ -> if n > 0
          then day_after (days_later (n - 1) day)
         else days_later (n + 7) day;;
val days_later : int -> weekday -> weekday=<fun>
```


Functions over Enumerations

```
# days_later 2 Tuesday;;
- : weekday = Thursday

# days_later (-1) Wednesday;;
- : weekday = Tuesday

# days_later (-4) Monday;;
- : weekday = Thursday
```


Problem:

```
# type weekday = Monday | Tuesday | Wednesday
  | Thursday | Friday | Saturday | Sunday;;
```

- Write function is_weekend : weekday -> bool

```
let is_weekend day =
```


Problem:

```
# type weekday = Monday | Tuesday | Wednesday
  | Thursday | Friday | Saturday | Sunday;;
```

- Write function is_weekend : weekday -> bool

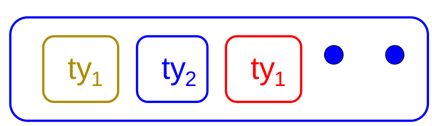
```
let is_weekend day =
   match day with
     Saturday -> true
   | Sunday -> true
   | _ -> false
```


Example Enumeration Types

```
# type bin_op = IntPlusOp | IntMinusOp
        | EqOp | CommaOp | ConsOp

# type mon_op = HdOp | TlOp | FstOp
        | SndOp
```


Disjoint Union Types

- **Disjoint union of types**, with some possibly occurring more than once



- We can also add in some new singleton elements

Disjoint Union Types

```
# type id = DriversLicense of int
  | SocialSecurity of int | Name of string;;
type id = DriversLicense of int |
  SocialSecurity of int | Name of string

# let check_id id =
    match id with
      DriversLicense num ->
       not (List.mem num [13570; 99999])
    | SocialSecurity num -> num < 900000000
    | Name str -> not (str = "John Doe");;
 val check_id : id -> bool = <fun>
```


Problem

- Create a type to represent the currencies for US, UK, Europe and Japan
  - Hint: Dollar, Pound, Euro, Yen

Problem

- Create a type to represent the currencies for US, UK, Europe and Japan

```
type currency =
  Dollar of int
 | Pound of int
 | Euro of int
 | Yen of int
```


Example Disjoint Union Type

```
# type const =
    BoolConst of bool
  | IntConst of int
  | FloatConst of float
  | StringConst of string
  | NilConst
  | UnitConst
```


Example Disjoint Union Type

```
# type const = BoolConst of bool
   | IntConst of int | FloatConst of float
   | StringConst of string  | NilConst
   | UnitConst
```

- How to represent 7 as a const?
- Answer: IntConst 7

Polymorphism in Variants

- The type 'a option gives us something to represent non-existence or failure

```
# type 'a option = Some of 'a | None;;
type 'a option = Some of 'a | None
```

- Used to encode partial functions
- Often can replace the raising of an exception

Functions producing option

```
# type 'a option =
    Some of 'a
  | None;;
```

```
# let rec first p list =
    match list with [ ] -> None
    | (x::xs) -> if p x then Some x else first p xs;;
val first : ('a -> bool) -> 'a list -> 'a option =
  <fun>

# first (fun x -> x > 3) [1;3;4;2;5];;
- : int option = Some 4

# first (fun x -> x > 5) [1;3;4;2;5];;
- : int option = None
```


Functions over option

```
# type 'a option =
    Some of 'a
  | None;;
```

```
# let result_ok r =
   match r with None -> false
    | Some _ -> true;;
val result_ok : 'a option -> bool = <fun>

# result_ok (first (fun x -> x > 3) [1;3;4;2;5]);;
- : bool = true
# result_ok (first (fun x -> x > 5) [1;3;4;2;5]);;
- : bool = false
```


Problem

```
# type 'a option =
    Some of 'a
  | None;;
```

- Write a hd and tl on lists that doesn't raise an exception and works at all types of lists.

Problem

```
# type 'a option =
    Some of 'a
  | None;;
```

- Write a hd and tl on lists that doesn't raise an exception and works at all types of lists.
- ```
  let hd list =
        match list with
            [] -> None
          | (x::xs) -> Some x
  ```
- ```
  let tl list =
        match list with
            [] -> None
          | (x::xs) -> Some xs
  ```


Mapping over Variants

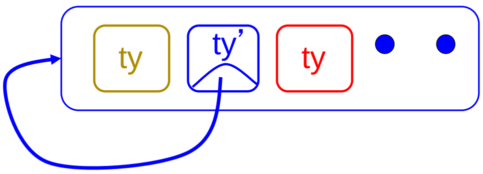
```
# let optionMap f opt =
    match opt with
      None -> None
    | Some x -> Some (f x);;
val optionMap : ('a -> 'b) -> 'a option -> 'b
  option = <fun>

# optionMap
  (fun x -> x - 2)
  (first (fun x -> x > 3) [1;3;4;2;5]);;
- : int option = Some 2
```


Folding over Variants

```
# let optionFold someFun noneVal opt =
    match opt with
      None -> noneVal
    | Some x -> someFun x;;
val optionFold : ('a -> 'b) -> 'b -> 'a option
  -> 'b = <fun>

# let optionMap f opt =
   optionFold (fun x -> Some (f x)) None opt;;
val optionMap : ('a -> 'b) -> 'a option -> 'b
  option = <fun>
```

Recursive Types

- The type being defined may be a component of itself



Recursive Data Types

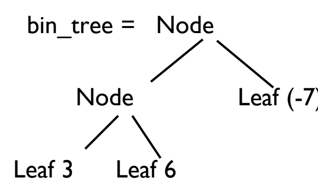
```
# type int_Bin_Tree =
   Leaf of int
 | Node of (int_Bin_Tree * int_Bin_Tree);;

type int_Bin_Tree = Leaf of int | Node of
  (int_Bin_Tree * int_Bin_Tree)
```


Recursive Data Type Values

\# let bin_tree =
 Node(Node(Leaf 3, Leaf 6),Leaf (-7));;

val bin_tree : int_Bin_Tree = Node (Node (Leaf 3, Leaf 6), Leaf (-7))

Recursive Data Type Values



Recursive Data Types

```
# type exp =
     VarExp of string
   | ConstExp of const
   | MonOpAppExp of mon_op * exp
   | BinOpAppExp of bin_op * exp * exp
   | IfExp of exp* exp * exp
   | AppExp of exp * exp
   | FunExp of string * exp
```


Recursive Data Types

\# type bin_op = IntPlusOp | IntMinusOp
  | EqOp | CommaOp | ConsOp | …
\# type const = BoolConst of bool | IntConst of int | …
\# type exp = VarExp of string | ConstExp of const
  | BinOpAppExp of bin_op * exp * exp | …

- How to represent 6 as an exp?

Recursive Data Types

type bin_op = IntPlusOp | IntMinusOp
| EqOp | CommaOp | ConsOp | …
type const = BoolConst of bool | IntConst of int | …
type exp = VarExp of string | ConstExp of const
| BinOpAppExp of bin_op * exp * exp | …

- How to represent 6 as an exp?
- Answer: ConstExp (IntConst 6)

Recursive Data Types

type bin_op = IntPlusOp | IntMinusOp
| EqOp | CommaOp | ConsOp | …
type const = BoolConst of bool | IntConst of int | …
type exp = VarExp of string | ConstExp of const
| BinOpAppExp of bin_op * exp * exp | …

- How to represent (6, 3) as an exp?

Recursive Data Types

type bin_op = IntPlusOp | IntMinusOp
| EqOp | CommaOp | ConsOp | …
type const = BoolConst of bool | IntConst of int | …
type exp = VarExp of string | ConstExp of const
| BinOpAppExp of bin_op * exp * exp | …

- How to represent (6, 3) as an exp?
- BinOpAppExp (CommaOp,
ConstExp (IntConst 6),
ConstExp (IntConst 3)
)

Recursive Data Types

type bin_op = IntPlusOp | IntMinusOp
| EqOp | CommaOp | ConsOp | …
type const = BoolConst of bool | IntConst of int | …
type exp = VarExp of string | ConstExp of const
| BinOpAppExp of bin_op * exp * exp | …
- How to represent [(6, 3)] as an exp?
- BinOpAppExp (ConsOp,
BinOpAppExp (CommaOp, ConstExp (IntConst 6),
ConstExp (IntConst 3)),
ConstExp NilConst))));;

Recursive Functions

```
# let rec first_leaf_value tree =
    match tree
      with (Leaf n) -> n
    |   Node (left_tree, right_tree) ->
             first_leaf_value left_tree;;
val first_leaf_value : int_Bin_Tree -> int
  = <fun>
# let left = first_leaf_value bin_tree;;
val left : int = 3
```


Problem

```
type int_Bin_Tree =
   Leaf of int
| Node of (int_Bin_Tree * int_Bin_Tree);;
```

- Write sum_tree : int_Bin_Tree -> int
- Adds all ints in tree

```
let rec sum_tree t =
```

Problem

type int_Bin_Tree =Leaf of int
| Node of (int_Bin_Tree * int_Bin_Tree);;

- Write sum_tree : int_Bin_Tree -> int
- Adds all ints in tree

```
let rec sum_tree t =
  match t with Leaf n -> n
  | Node(t1,t2) -> sum_tree t1 + sum_tree t2
```


Recursion over Recursive Data Types

```
# type exp = VarExp of string
    | ConstExp of const
    | BinOpAppExp of bin_op * exp * exp
    | FunExp of string * exp
    | AppExp of exp * exp
```

- How to count the number of variables in an exp?

Recursion over Recursive Data Types

```
# type exp = VarExp of string | ConstExp of const
   | BinOpAppExp of bin_op * exp * exp
   | FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?

```
# let rec varCnt exp =
   match exp with
      VarExp x ->
    | ConstExp c ->
    | BinOpAppExp (b, e1, e2) ->
    | FunExp (x,e) ->
    | AppExp (e1, e2) ->
```


Recursion over Recursive Data Types

```
# type exp = VarExp of string | ConstExp of const
   | BinOpAppExp of bin_op * exp * exp
   | FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?

```
# let rec varCnt exp =
   match exp with
      VarExp x -> 1
    | ConstExp c -> 0
    | BinOpAppExp (b, e1, e2) -> varCnt e1 +varCnt e2
    | FunExp (x,e) -> 1 + varCnt e
    | AppExp (e1, e2) -> varCnt e1 + varCnt e2
```


Mapping over Recursive Types

```
# let rec ibtreeMap f tree =
    match tree with
      (Leaf n) ->
    | Node (left_tree, right_tree) ->
```


Mapping over Recursive Types

```
# let rec ibtreeMap f tree =
    match tree with
      (Leaf n) -> Leaf (f n)
    | Node (left_tree, right_tree) ->
          Node (ibtreeMap f left_tree,
                IbtreeMap f right_tree);;
```

val ibtreeMap : (int -> int) -> int_Bin_Tree ->
  int_Bin_Tree = <fun>

Mapping over Recursive Types

let bin_tree =
 Node(Node(Leaf 3, Leaf 6),Leaf (-7));;

ibtreeMap ((+) 2) bin_tree;;

- : int_Bin_Tree = Node (Node (Leaf 5, Leaf 8),
  Leaf (-5))

Summing up Elements of a Tree

```
# let rec tree_sum_0 tree =
   match tree with
     Leaf n ->

   | Node (left_tree, right_tree) ->
```


Folding over Recursive Types

```
# let rec ibtreeFoldRight leafFun nodeFun tree =
   match tree with
     Leaf n ->
   | Node (left_tree, right_tree) ->
```

val ibtreeFoldRight : (int -> 'a) -> ('a -> 'a -> 'a) -> int_Bin_Tre
  -> 'a = <fun>

Folding over Recursive Types

```
# let rec ibtreeFoldRight leafFun nodeFun tree =
   match tree with
     Leaf n -> leafFun n
   | Node (left_tree, right_tree) ->
     nodeFun
     (ibtreeFoldRight leafFun nodeFun left_tree)
     (ibtreeFoldRight leafFun nodeFun right_tree);
```

val ibtreeFoldRight : (int -> 'a) -> ('a -> 'a -> 'a) -> int_Bin_Tre
  -> 'a = <fun>

Folding over Recursive Types

```
# let tree_sum =
    ibtreeFoldRight (fun x -> x) (+);;
val tree_sum : int_Bin_Tree -> int = <fun>

# tree_sum bin_tree;;
- : int = 2
```


Mutually Recursive Types

```
# type 'a tree =
       TreeLeaf of 'a
     | TreeNode of 'a treeList
and
    'a treeList =
       Last of 'a tree
     | More of ('a tree * 'a treeList);;
```

type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a
  treeList)

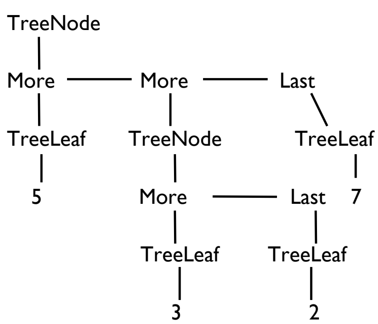
Mutually Recursive Types - Values

```
# let tree =
   TreeNode
    (More (TreeLeaf 5,
           (More (TreeNode
                  (More (TreeLeaf 3,
                         Last (TreeLeaf 2))),
                  Last (TreeLeaf 7)))));;
```


Mutually Recursive Types - Values

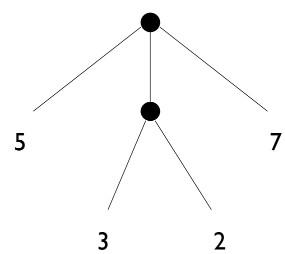
val tree : int tree =
TreeNode
(More
(TreeLeaf 5,
More
(TreeNode (More (TreeLeaf 3, Last (TreeLeaf 2))), Last (TreeLeaf 7))))

Mutually Recursive Types - Values



Mutually Recursive Types - Values

A more conventional picture



Mutually Recursive Functions

```
# let rec fringe tree =
    match tree with
       (TreeLeaf x) -> [x]
  | (TreeNode list) -> list_fringe list
and list_fringe tree_list =
    match tree_list with
       (Last tree) -> fringe tree
  | (More (tree,list)) ->
       (fringe tree) @ (list_fringe list);;
```

val fringe : 'a tree -> 'a list = <fun>
val list_fringe : 'a treeList -> 'a list = <fun>

Mutually Recursive Functions

fringe tree;;
- : int list = [5; 3; 2; 7]

Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size

Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size

```
let rec tree_size t =
    match t with TreeLeaf _ ->
    | TreeNode ts ->
```


Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size

```
let rec tree_size t =
    match t with TreeLeaf _ -> 1
    | TreeNode ts -> treeList_size  ts
```


Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size and treeList_size

```
let rec tree_size t =
    match t with TreeLeaf _ -> 1
    | TreeNode ts -> treeList_size  ts
and treeList_size ts =
```


Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size and treeList_size

```
let rec tree_size t =
    match t with TreeLeaf _ -> 1
    | TreeNode ts -> treeList_size  ts
and treeList_size ts =
    match ts with Last t ->
    | More t ts' ->
```


Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size and treeList_size

```
let rec tree_size t =
    match t with TreeLeaf _ -> 1
    | TreeNode ts -> treeList_size  ts
and treeList_size ts =
    match ts with Last t -> tree_size t
    | More t ts' -> tree_size t + treeList_size ts'
```

Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size and treeList_size

```
let rec tree_size t =
    match t with TreeLeaf _ -> 1
    | TreeNode ts -> treeList_size  ts
and treeList_size ts =
    match ts with Last t -> tree_size t
    | More t ts' -> tree_size t + treeList_size ts'
```


Nested Recursive Types

```
# type intlist =
        Nil | Cons of (int * intlist)

# type 'a mylist =
        Nil | Cons of ('a * 'a mylist)
```

If only we had control over extra syntax:

“ type ‘a list = [ ] | (::) of ‘a * ‘a list ”

Nested Recursive Types

```
# type 'a labeled_tree =
 TreeNode of ('a * 'a labeled_tree list);;
```

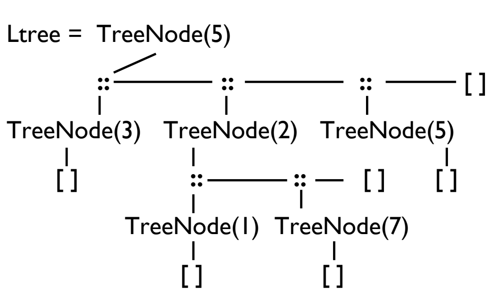
type 'a labeled_tree = TreeNode of ('a * 'a labeled_tree list)

```
Compare:
# type 'a tree =
        TreeLeaf of 'a
      | TreeNode of 'a treeList
and 'a treeList =
        Last of 'a tree
      | More of ('a tree * 'a treeList);;
```

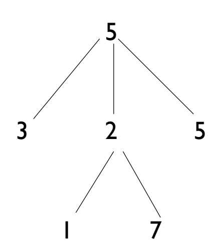

Nested Recursive Type Values

```
# let ltree =
  TreeNode(5,
    [TreeNode (3, []);
     TreeNode (2, [TreeNode (1, []);
                   TreeNode (7, [])]);
     TreeNode (5, [])]);;
```


Nested Recursive Type Values



Nested Recursive Type Values



Mutually Recursive Functions

```
# let rec flatten_tree labtree =
    match labtree with
       TreeNode (x,treelist) ->
            x::flatten_tree_list treelist

  and flatten_tree_list treelist =
     match treelist with
       [] -> []
     | labtree::labtrees ->
         flatten_tree labtree
              @ (flatten_tree_list labtrees);;
```


Mutually Recursive Functions

```
val flatten_tree : 'a labeled_tree -> 'a list = <fun>
val flatten_tree_list : 'a labeled_tree list -> 'a list =
  <fun>

# flatten_tree ltree;;
- : int list = [5; 3; 2; 1; 7; 5]
```

- **Nested recursive types lead to mutually recursive functions**

Records

- Records serve the same programming purpose as tuples
- Provide better documentation, more readable code
- Allow components to be **accessed by label instead of position**
  - Labels (aka *field names)* must be **unique**
  - Fields accessed by **suffix dot notation**

Record Types

- Record types must be declared before they can be used in OCaml

```
# type person = {name : string;
                 ss : (int * int * int);
                 age : int};;
type person = { name : string; ss :
  int * int * int; age : int; }
```

- person is the type being introduced
- name, ss and age are the labels, or fields

Record Values

- Records built with labels; order does not matter

```
# let teacher = {name = "Elsa L. Gunter"; age
  = 102; ss = (119,73,6244)};;
val teacher : person =
  {name = "Elsa L. Gunter"; ss = (119, 73,
  6244); age = 102}

# teacher.name;;
- : string = "Elsa L. Gunter"
```


Record Pattern Matching

```
# let {name = elsa; age = age; ss =
  (_,_,s3)} = teacher;;

val elsa : string = "Elsa L. Gunter"
val age : int = 102
val s3 : int = 6244
```

Record Field Access

```
# let soc_sec = teacher.ss;;

val soc_sec : int * int * int = (119, 73,
  6244)
```


Record Values

```
# let student = {
     ss=(325,40,1276);
     name="Usain Bolt";
     age=22};;
val student : person =
  {name = "Usain Bolt"; ss = (325, 40,
  1276); age = 22}

# student = teacher;;
- : bool = false
```


New Records from Old

```
# let birthday person =
   {person with age = person.age + 1};;
val birthday : person -> person = <fun>

# birthday teacher;;
- : person = {name = "Elsa L. Gunter"; ss =
  (119, 73, 6244); age = 103}
```


New Records from Old

```
# let new_id name soc_sec person =
 {person with name = name; ss = soc_sec};;

val new_id : string -> int * int * int -> person -
  > person = <fun>

# new_id "Lionel Messi" (523,04,6712) student;;
- : person = {name = "Lionel Messi";
-              ss = (523, 4, 6712); age = 22}
```