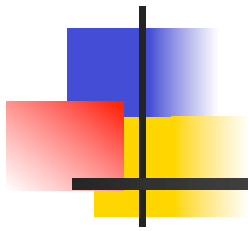


Programming Languages and Compilers (CS 421)



Elsa L Gunter

2112 SC, UIUC

<http://courses.engr.illinois.edu/cs421>

Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha



Parser Code

- `<grammar>.ml` defines one parsing function per entry point
- Parsing function takes a lexing function (lexer buffer to token) and a lexer buffer as arguments
- Returns semantic attribute of corresponding entry point



Ocamlyacc Input

- File format:

```
%{
```

```
  <header>
```

```
%}
```

```
  <declarations>
```

```
%%
```

```
  <rules>
```

```
%%
```

```
  <trailer>
```



Ocamlyacc *<header>*

- Contains arbitrary Ocaml code
- Typically used to give types and functions needed for the semantic actions of rules and to give specialized error recovery
- May be omitted
- *<footer>* similar. Possibly used to call parser



Ocamlyacc <declarations>

- **%token** *symbol ... symbol*
- Declare given symbols as tokens
- **%token** *<type> symbol ... symbol*
- Declare given symbols as token constructors, taking an argument of type *<type>*
- **%start** *symbol ... symbol*
- Declare given symbols as entry points; functions of same names in *<grammar>.ml*



Ocamlyacc <declarations>

- **%type** <type> *symbol ... symbol*

Specify type of attributes for given symbols.
Mandatory for start symbols

- **%left** *symbol ... symbol*

- **%right** *symbol ... symbol*

- **%nonassoc** *symbol ... symbol*

Associate precedence and associativity to given symbols. Same line, same precedence; earlier line, lower precedence (broadest scope)



Ocamlyacc <rules>

- *nonterminal* :

symbol ... symbol { semantic_action }

| ...

| *symbol ... symbol { semantic_action }*

;

- Semantic actions are arbitrary Ocaml expressions
- Must be of same type as declared (or inferred) for *nonterminal*
- Access semantic attributes (values) of symbols by position: \$1 for first symbol, \$2 to second ...



Example - Base types

```
(* File: expr.ml *)
type expr =
  Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
and term =
  Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
and factor =
  Id_as_Factor of string
  | Parenthesized_Expr_as_Factor of expr
```




Example - Lexer (exprlex.mll)

```
{ (*open Exprparse*) }
let numeric = ['0' - '9']
let letter = ['a' - 'z' 'A' - 'Z']
rule token = parse
  | "+" {Plus_token}
  | "-" {Minus_token}
  | "*" {Times_token}
  | "/" {Divide_token}
  | "(" {Left_parenthesis}
  | ")" {Right_parenthesis}
  | letter (letter|numeric|"_" )* as id {Id_token id}
  | [' ' '\t' '\n'] {token lexbuf}
  | eof {EOL}
```



Example - Parser (exprparse.mly)

```
%{ open Expr
```

```
%}
```

```
%token <string> Id_token
```

```
%token Left_parenthesis Right_parenthesis
```

```
%token Times_token Divide_token
```

```
%token Plus_token Minus_token
```

```
%token EOL
```

```
%start main
```

```
%type <expr> main
```

```
%%
```



Example - Parser (exprparse.mly)

expr:

term

{ Term_as_Expr \$1 }

| term Plus_token expr

{ Plus_Expr (\$1, \$3) }

| term Minus_token expr

{ Minus_Expr (\$1, \$3) }



Example - Parser (exprparse.mly)

term:

factor

{ Factor_as_Term \$1 }

| factor Times_token term

{ Mult_Term (\$1, \$3) }

| factor Divide_token term

{ Div_Term (\$1, \$3) }



Example - Parser (exprparse.mly)

factor:

Id_token

{ Id_as_Factor \$1 }

| Left_parenthesis expr Right_parenthesis

{ Parenthesized_Expr_as_Factor \$2 }

main:

| expr EOL

{ \$1 }



Example - Using Parser

```
# #use "expr.ml";;
```

```
...
```

```
# #use "exprparse.ml";;
```

```
...
```

```
# #use "exprlex.ml";;
```

```
...
```

```
# let test s =
```

```
  let lexbuf = Lexing.from_string (s^"\n") in  
    main token lexbuf;;
```



Example - Using Parser

```
# test "a + b";;
```

```
- : expr =
```

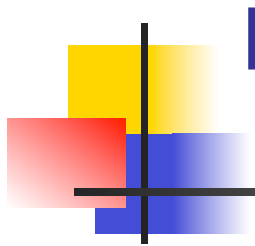
```
Plus_Expr
```

```
(Factor_as_Term (Id_as_Factor "a"),  
Term_as_Expr (Factor_as_Term  
  (Id_as_Factor "b")))
```



LR Parsing

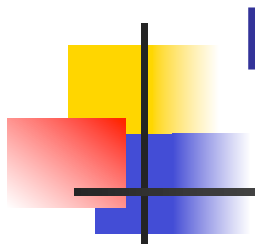
- Read tokens left to right (L)
- Create a rightmost derivation (R)
- How is this possible?
- Start at the bottom (left) and work your way up
- Last step has only one non-terminal to be replaced so is right-most
- Working backwards, replace mixed strings by non-terminals
- Always proceed so that there are no non-terminals to the right of the string to be replaced



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

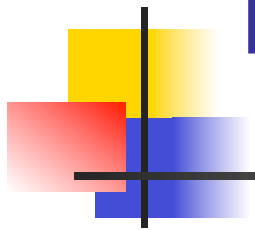
$$= \bullet (0 + 1) + 0 \quad \text{shift}$$



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$$\begin{aligned} &= (\bullet 0 + 1) + 0 && \text{shift} \\ &= \bullet (0 + 1) + 0 && \text{shift} \end{aligned}$$

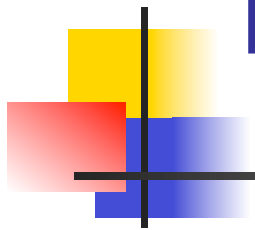


Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$$\begin{aligned} &\Rightarrow (0 \bullet + 1) + 0 \\ &= (\bullet 0 + 1) + 0 \\ &= \bullet (0 + 1) + 0 \end{aligned}$$

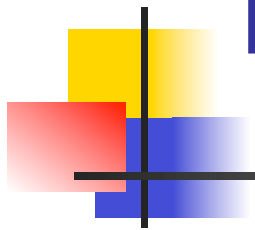
reduce
shift
shift



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

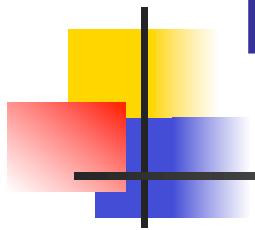
$= (\langle \text{Sum} \rangle \bullet + 1) + 0$	shift
$\Rightarrow (0 \bullet + 1) + 0$	reduce
$= (\bullet 0 + 1) + 0$	shift
$= \bullet (0 + 1) + 0$	shift



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= (\langle \text{Sum} \rangle + \bullet 1) + 0$	shift
$= (\langle \text{Sum} \rangle \bullet + 1) + 0$	shift
$\Rightarrow (0 \bullet + 1) + 0$	reduce
$= (\bullet 0 + 1) + 0$	shift
$= \bullet (0 + 1) + 0$	shift



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$	reduce
$= (\langle \text{Sum} \rangle + \bullet 1) + 0$	shift
$= (\langle \text{Sum} \rangle \bullet + 1) + 0$	shift
$\Rightarrow (0 \bullet + 1) + 0$	reduce
$= (\bullet 0 + 1) + 0$	shift
$= \bullet (0 + 1) + 0$	shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= (\langle \text{Sum} \rangle \bullet) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$ reduce
 $= (\langle \text{Sum} \rangle \bullet) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= \langle \text{Sum} \rangle \bullet + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$ reduce
 $= (\langle \text{Sum} \rangle \bullet) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= \langle \text{Sum} \rangle + \bullet 0$ shift
 $= \langle \text{Sum} \rangle \bullet + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$ reduce
 $= (\langle \text{Sum} \rangle \bullet) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

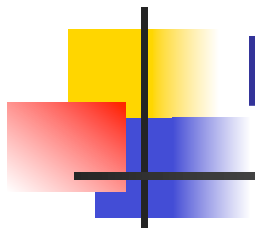
$\langle \text{Sum} \rangle \Rightarrow$
 $\Rightarrow \langle \text{Sum} \rangle + 0 \bullet$ reduce
 $= \langle \text{Sum} \rangle + \bullet 0$ shift
 $= \langle \text{Sum} \rangle \bullet + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$ reduce
 $= (\langle \text{Sum} \rangle \bullet) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle$	$\Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet$	reduce
	$\Rightarrow \langle \text{Sum} \rangle + 0 \bullet$	reduce
	$= \langle \text{Sum} \rangle + \bullet 0$	shift
	$= \langle \text{Sum} \rangle \bullet + 0$	shift
	$\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$	reduce
	$= (\langle \text{Sum} \rangle \bullet) + 0$	shift
	$\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$	reduce
	$\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$	reduce
	$= (\langle \text{Sum} \rangle + \bullet 1) + 0$	shift
	$= (\langle \text{Sum} \rangle \bullet + 1) + 0$	shift
	$\Rightarrow (0 \bullet + 1) + 0$	reduce
	$= (\bullet 0 + 1) + 0$	shift
	$= \bullet (0 + 1) + 0$	shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \bullet$	\Rightarrow	$\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet$	reduce
	\Rightarrow	$\langle \text{Sum} \rangle + 0 \bullet$	reduce
	$=$	$\langle \text{Sum} \rangle + \bullet 0$	shift
	$=$	$\langle \text{Sum} \rangle \bullet + 0$	shift
	\Rightarrow	$(\langle \text{Sum} \rangle) \bullet + 0$	reduce
	$=$	$(\langle \text{Sum} \rangle \bullet) + 0$	shift
	\Rightarrow	$(\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$	reduce
	\Rightarrow	$(\langle \text{Sum} \rangle + 1 \bullet) + 0$	reduce
	$=$	$(\langle \text{Sum} \rangle + \bullet 1) + 0$	shift
	$=$	$(\langle \text{Sum} \rangle \bullet + 1) + 0$	shift
	\Rightarrow	$(0 \bullet + 1) + 0$	reduce
	$=$	$(\bullet 0 + 1) + 0$	shift
	$=$	$\bullet (0 + 1) + 0$	shift



Example

$$(0 + 1) + 0$$





Example

$$(\quad 0 \quad + \quad 1 \quad) \quad + \quad 0$$





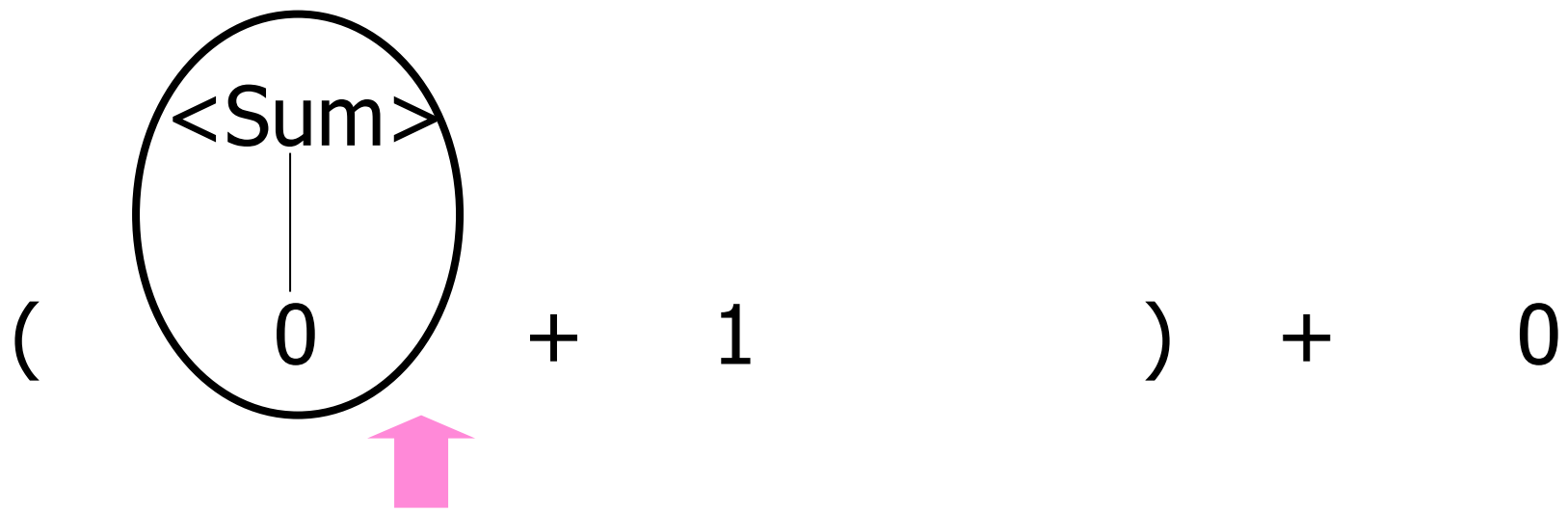
Example

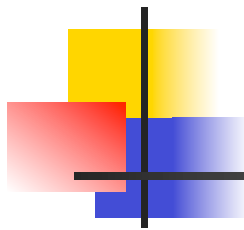
(0 + 1) + 0



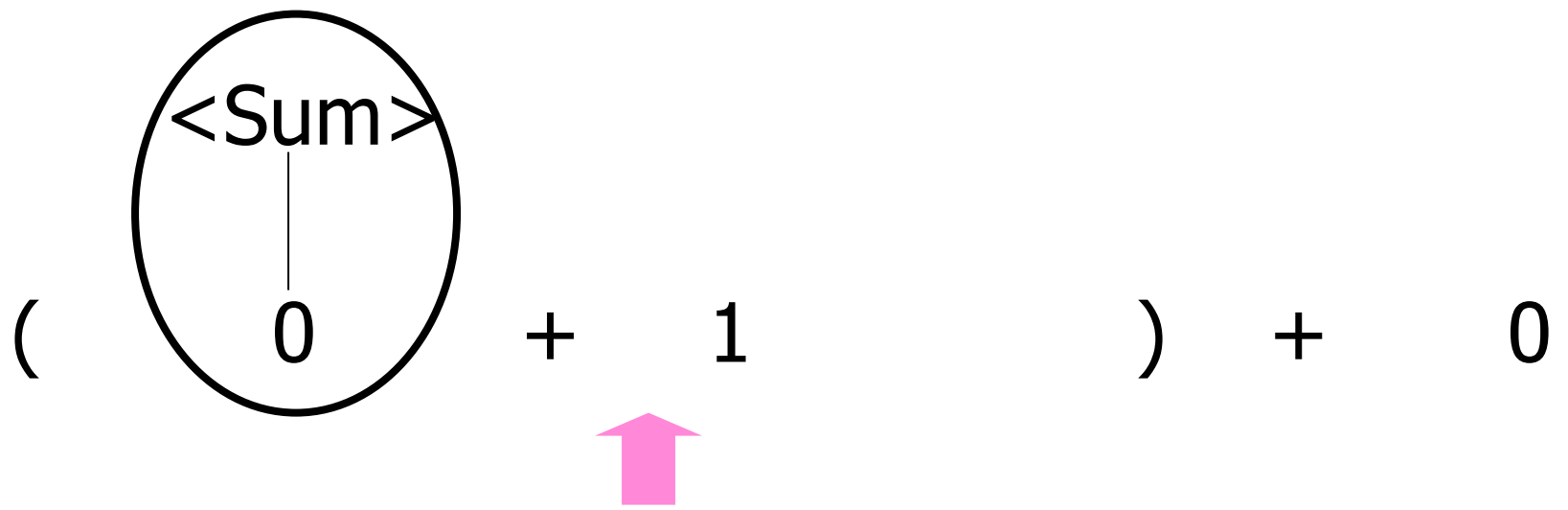


Example



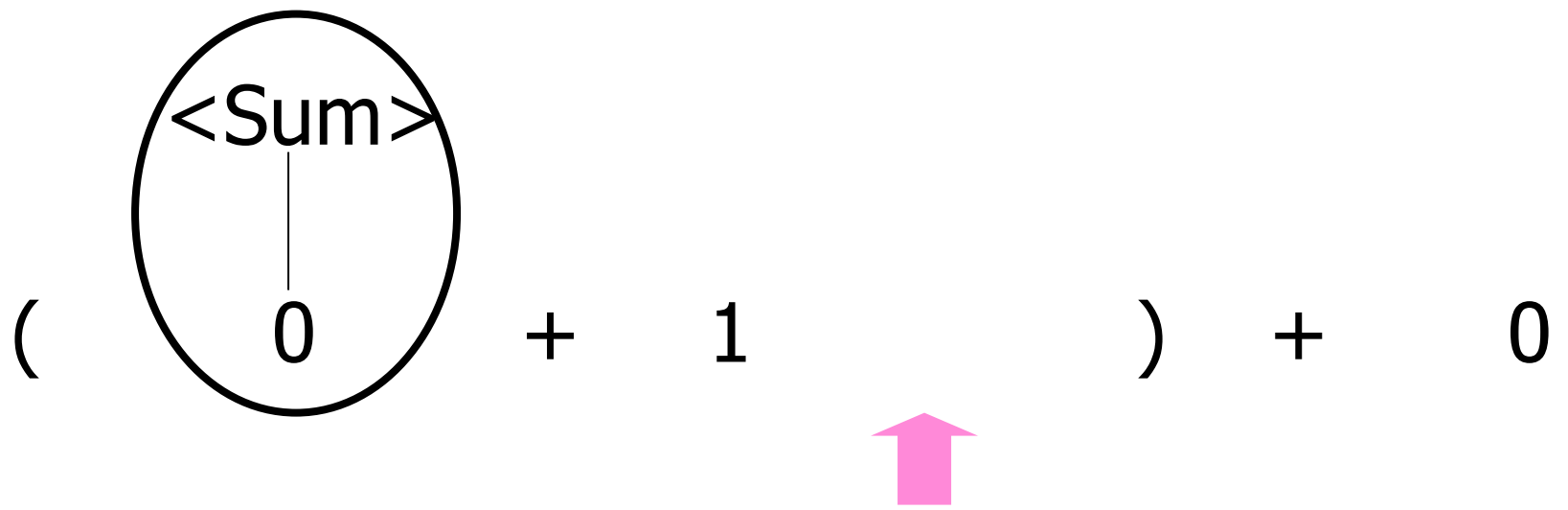


Example



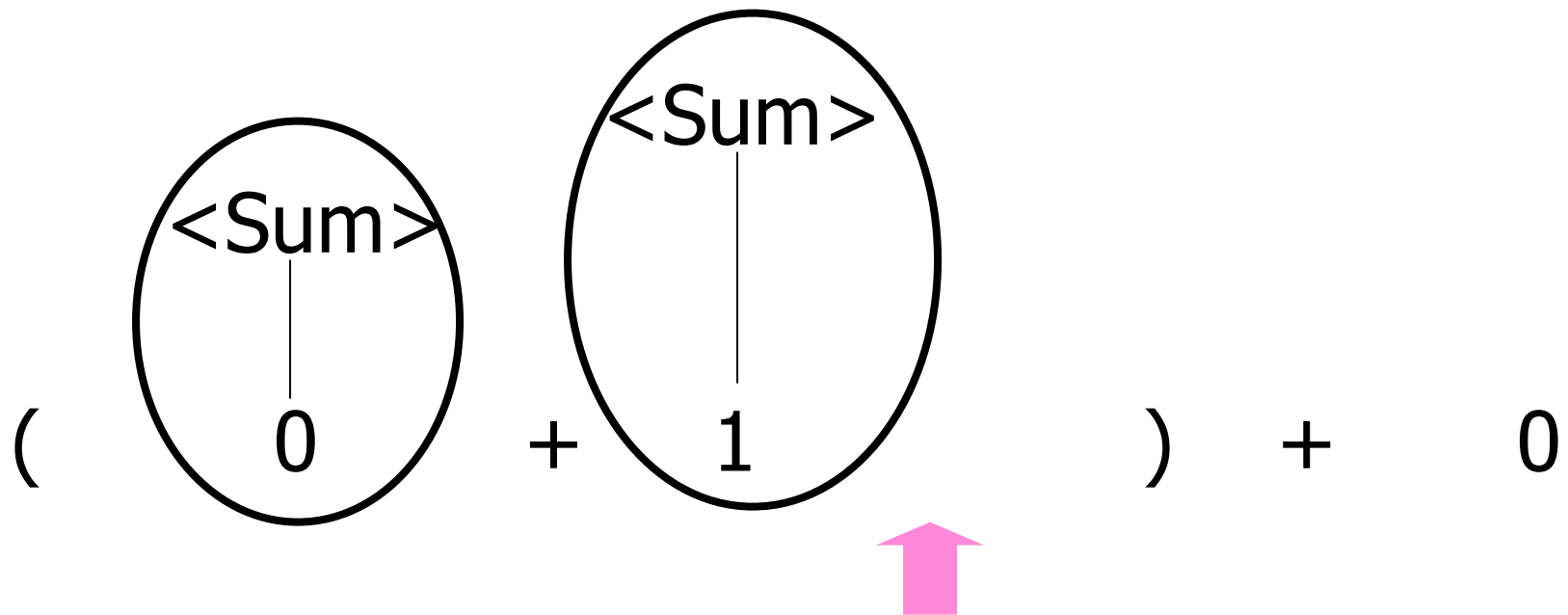


Example



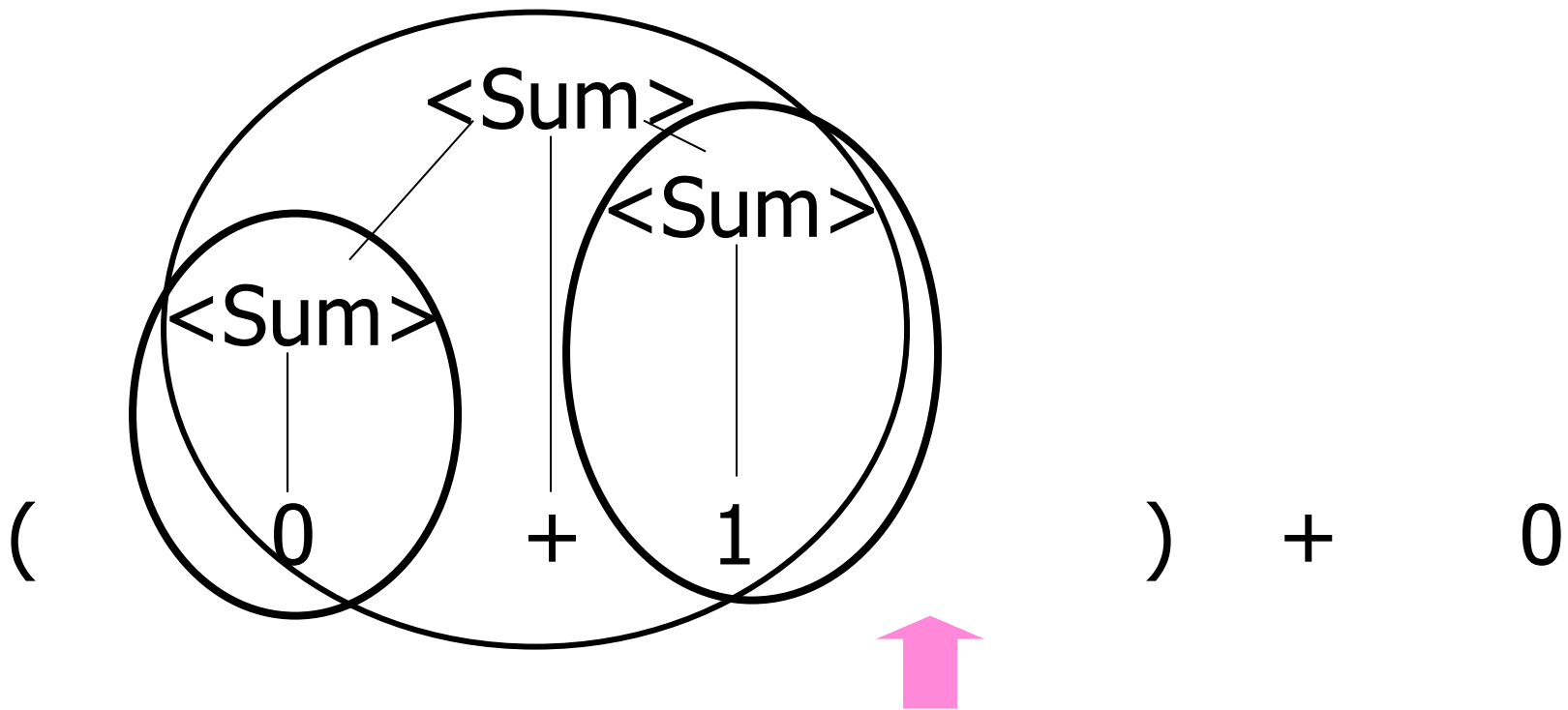


Example



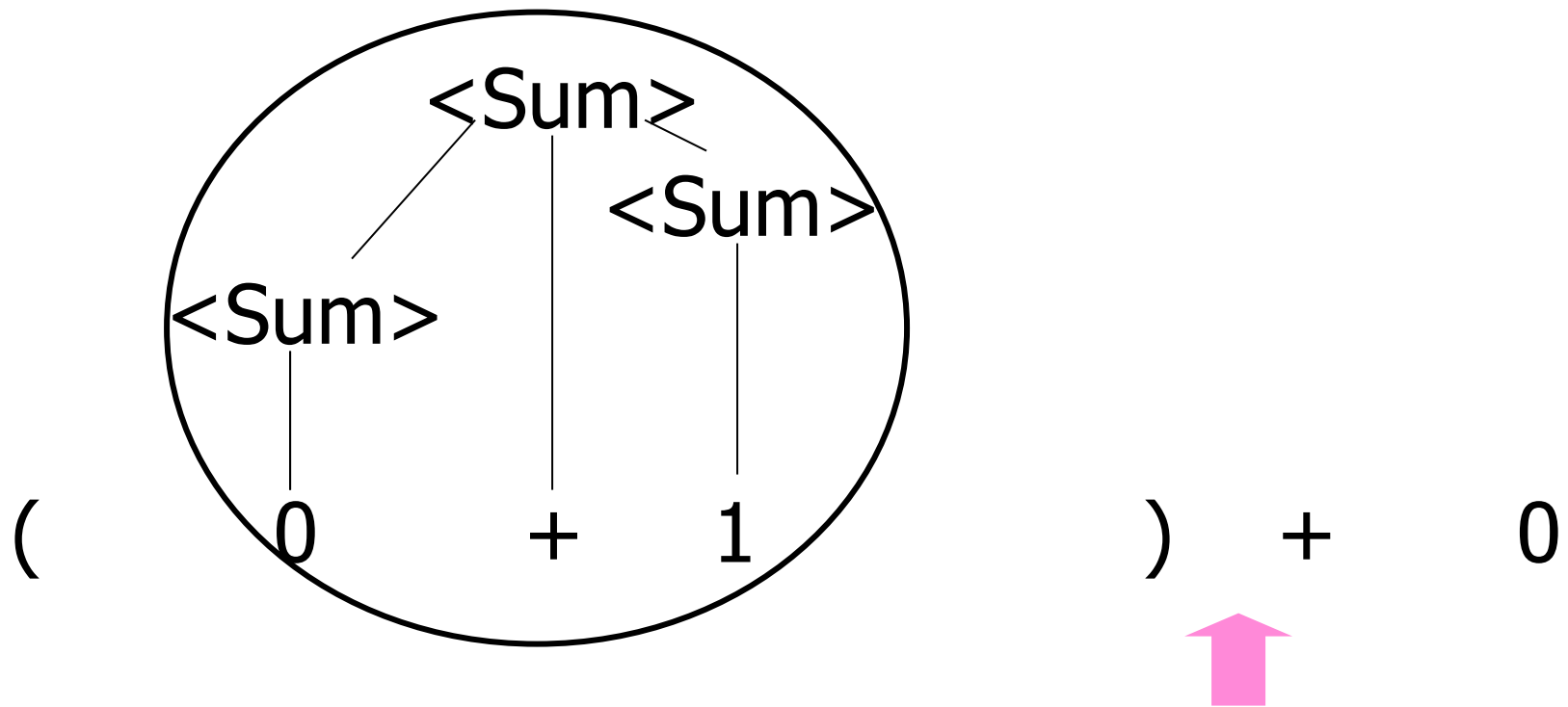


Example



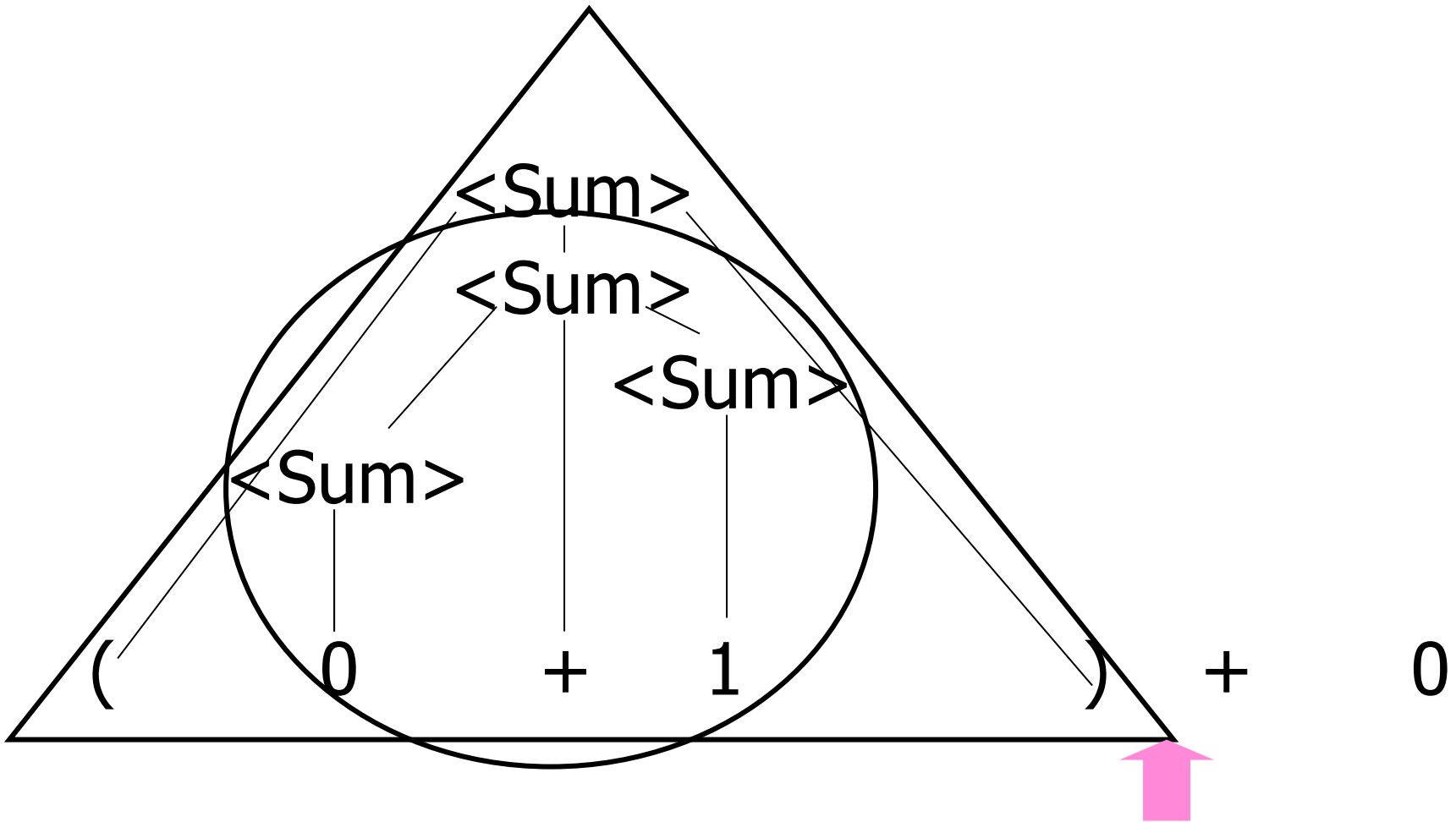


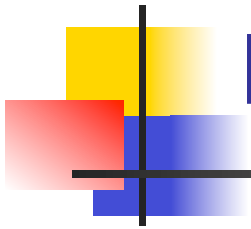
Example



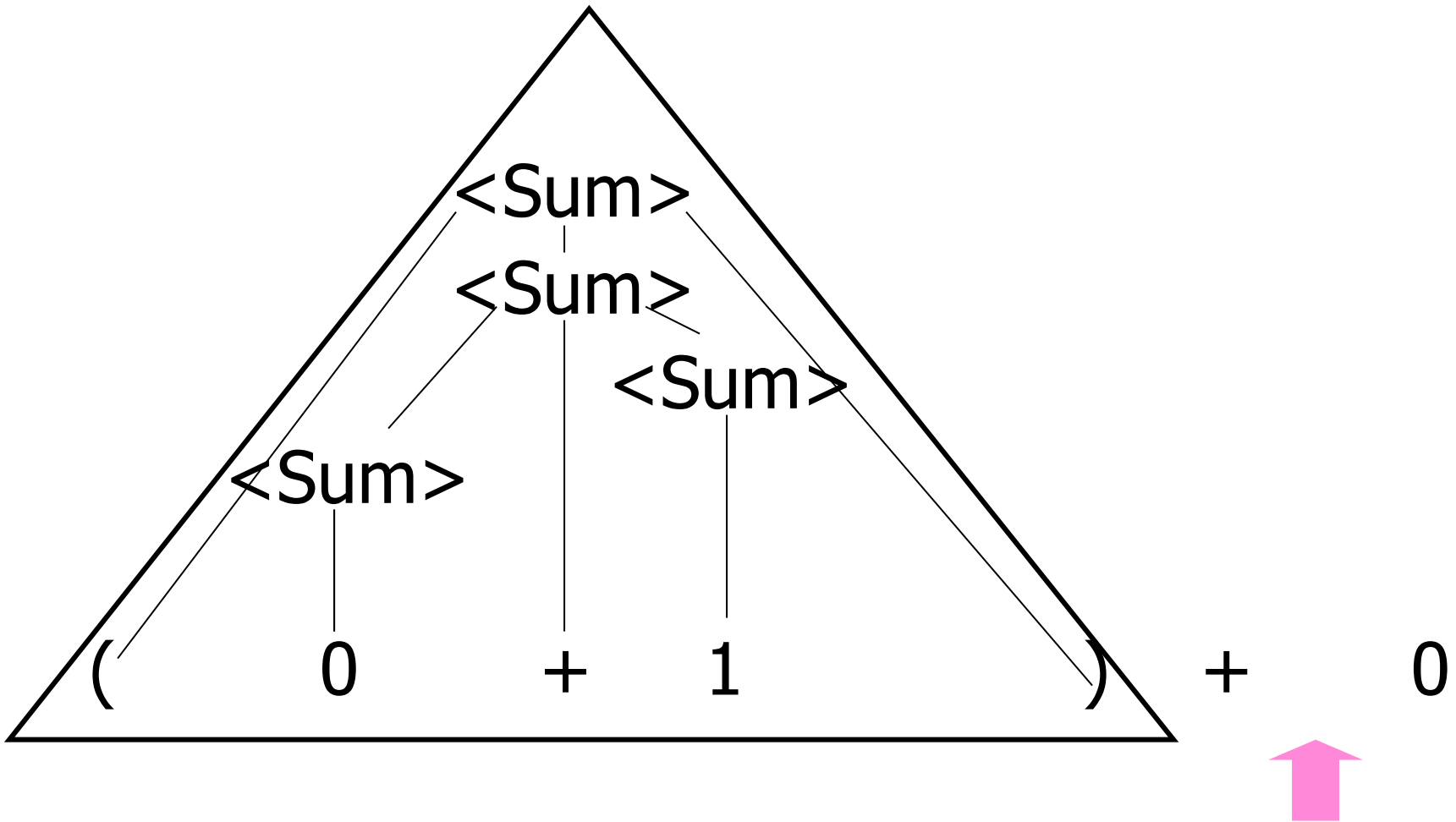


Example



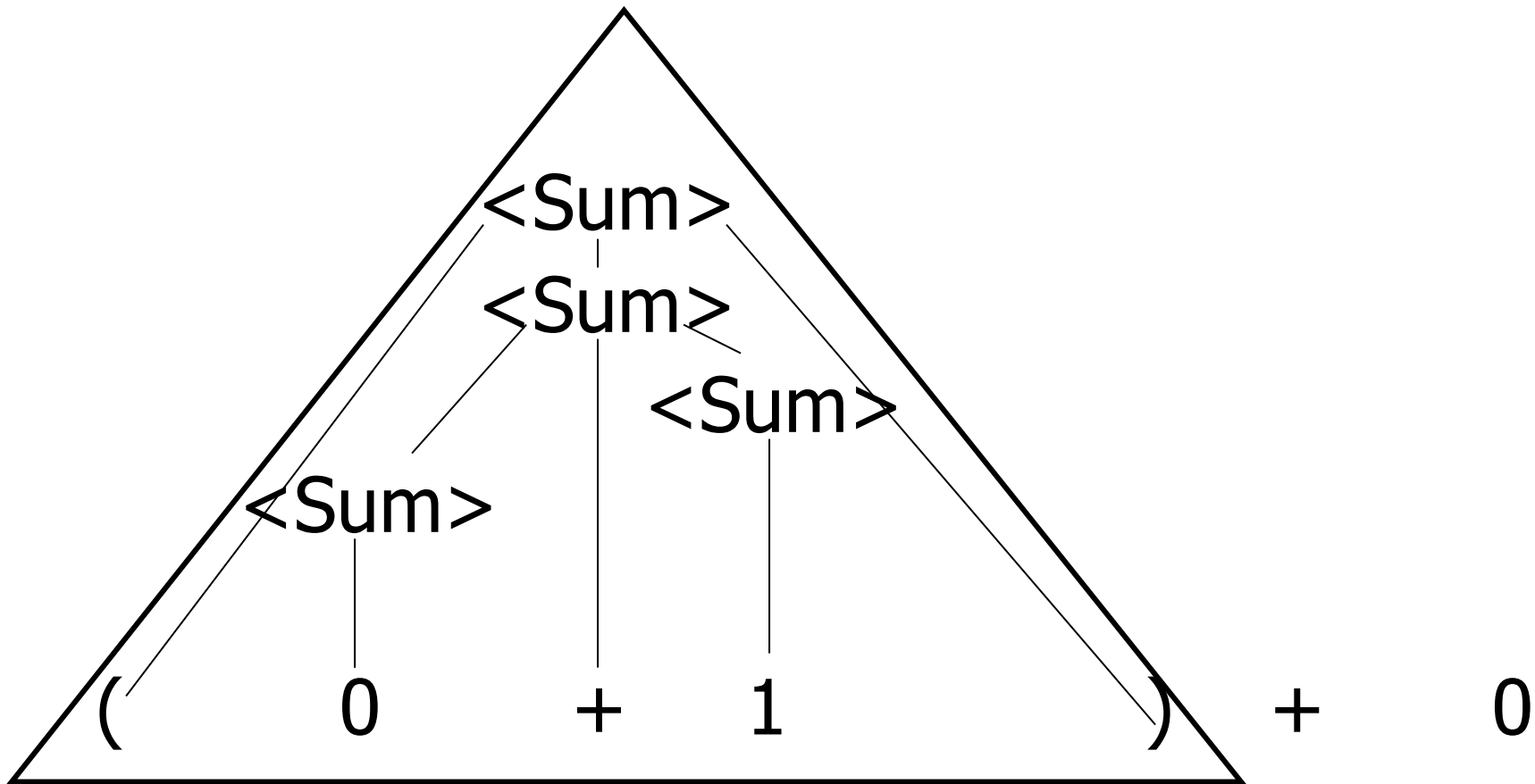


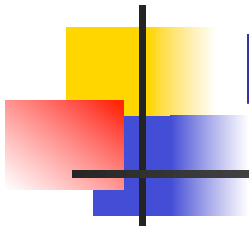
Example



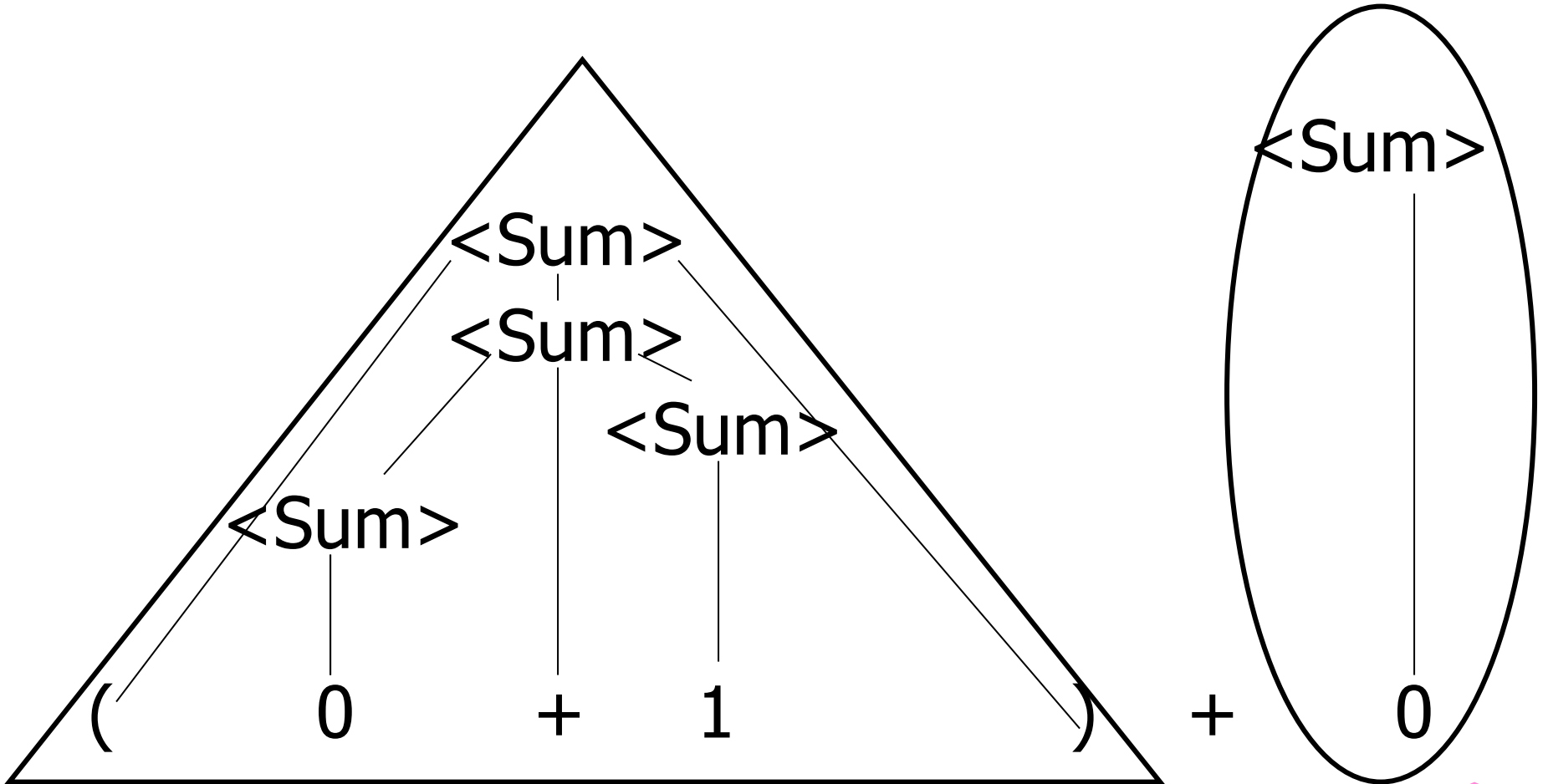


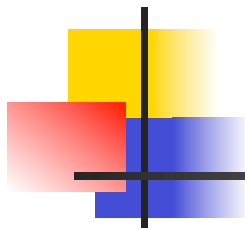
Example



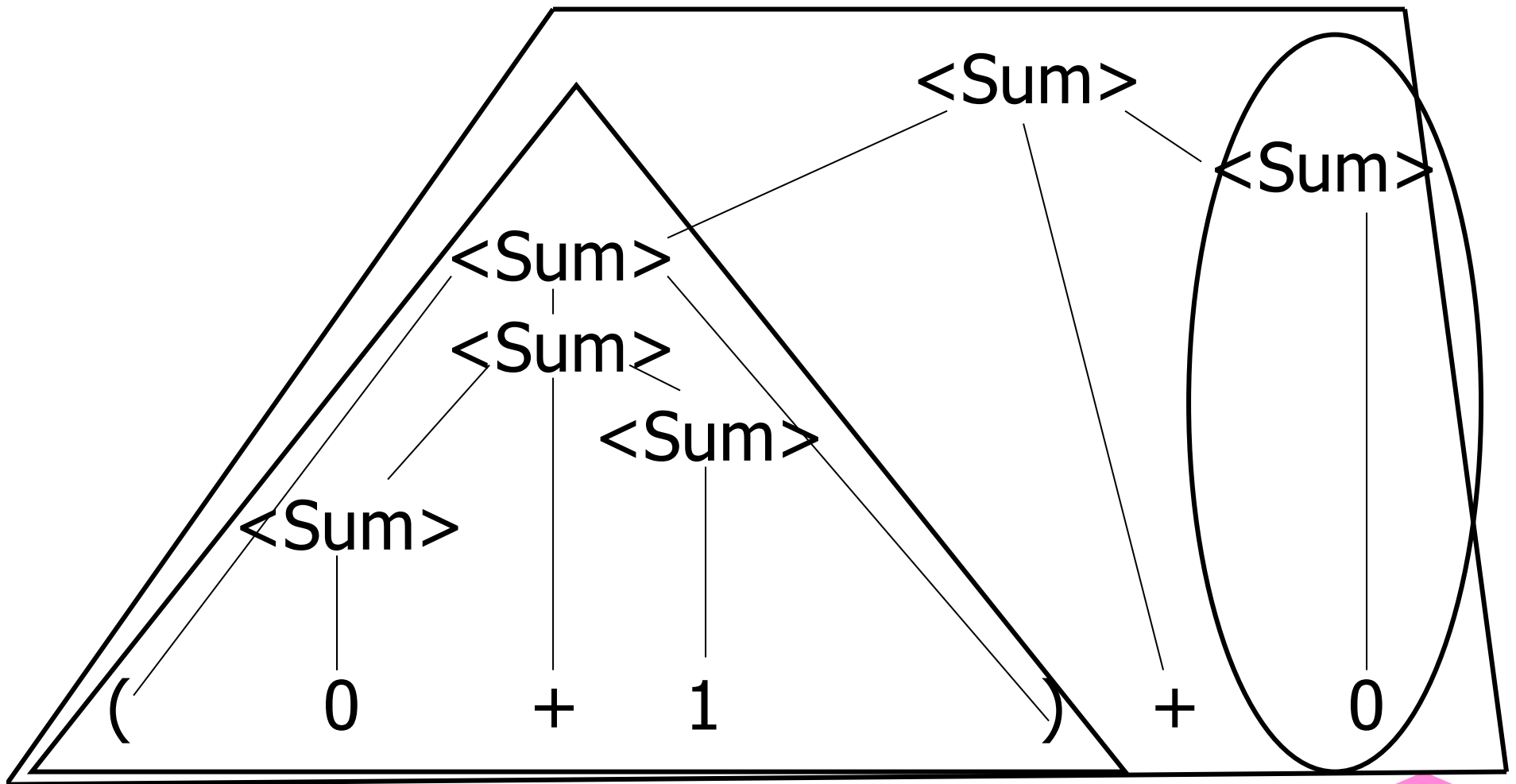


Example





Example





LR Parsing Tables

- Build a pair of tables, Action and Goto, from the grammar
 - This is the hardest part, we omit here
 - Rows labeled by states
 - For Action, columns labeled by terminals and “end-of-tokens” marker
 - (more generally strings of terminals of fixed length)
 - For Goto, columns labeled by non-terminals



Action and Goto Tables

- Given a state and the next input, Action table says either
 - **shift** and go to state n , or
 - **reduce** by production k (explained in a bit)
 - **accept** or **error**
- Given a state and a non-terminal, Goto table says
 - go to state m



LR(i) Parsing Algorithm

- Based on push-down automata
- Uses states and transitions (as recorded in Action and Goto tables)
- Uses a stack containing states, terminals and non-terminals



LR(i) Parsing Algorithm

0. Insure token stream ends in special “end-of-tokens” symbol
1. Start in state 1 with an empty stack
2. Push **state**(1) onto stack
- 3. Look at next i tokens from token stream ($toks$) (don't remove yet)
4. If top symbol on stack is **state**(n), look up action in Action table at $(n, toks)$



LR(i) Parsing Algorithm

5. If action = **shift** m ,
- a) Remove the top token from token stream and push it onto the stack
 - b) Push **state**(m) onto stack
 - c) Go to step 3



LR(i) Parsing Algorithm

6. If action = **reduce** k where production k is
 $E ::= u$
- a) Remove $2 * \text{length}(u)$ symbols from stack (u and all the interleaved states)
 - b) If new top symbol on stack is **state**(m), look up new state p in $\text{Goto}(m, E)$
 - c) Push E onto the stack, then push **state**(p) onto the stack
 - d) Go to step 3



LR(i) Parsing Algorithm

7. If action = **accept**

- Stop parsing, return success

8. If action = **error**,

- Stop parsing, return failure



Adding Synthesized Attributes

- Add to each **reduce** a rule for calculating the new synthesized attribute from the component attributes
- Add to each non-terminal pushed onto the stack, the attribute calculated for it
- When performing a **reduce**,
 - gather the recorded attributes from each non-terminal popped from stack
 - Compute new attribute for non-terminal pushed onto stack



Shift-Reduce Conflicts

- **Problem:** can't decide whether the action for a state and input character should be **shift** or **reduce**
- Caused by ambiguity in grammar
- Usually caused by lack of associativity or precedence information in grammar

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\bullet 0 + 1 + 0$ shift
 $\rightarrow 0 \bullet + 1 + 0$ reduce
 $\rightarrow \langle \text{Sum} \rangle \bullet + 1 + 0$ shift
 $\rightarrow \langle \text{Sum} \rangle + \bullet 1 + 0$ shift
 $\rightarrow \langle \text{Sum} \rangle + 1 \bullet + 0$ reduce
 $\rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet + 0$



Example - cont

- **Problem:** shift or reduce?
- You can shift-shift-reduce-reduce or reduce-shift-shift-reduce
- Shift first - right associative
- Reduce first- left associative



Reduce - Reduce Conflicts

- **Problem:** can't decide between two different rules to reduce by
- Again caused by ambiguity in grammar
- **Symptom:** RHS of one production suffix of another
- Requires examining grammar and rewriting it
- Harder to solve than shift-reduce errors



Example

■ $S ::= A \mid aB$ $A ::= abc$ $B ::= bc$

● abc shift

a ● bc shift

ab ● c shift

abc ●

■ Problem: reduce by $B ::= bc$ then by $S ::= aB$, or by $A ::= abc$ then $S ::= A$?



Recursive Descent Parsing

- Recursive descent parsers are a class of parsers derived fairly directly from BNF grammars
- A recursive descent parser traces out a parse tree in top-down order, corresponding to a left-most derivation (LL - left-to-right scanning, leftmost derivation)



Recursive Descent Parsing

- Each nonterminal in the grammar has a subprogram associated with it; the subprogram parses all phrases that the nonterminal can generate
- Each nonterminal in right-hand side of a rule corresponds to a recursive call to the associated subprogram



Recursive Descent Parsing

- Each subprogram must be able to decide how to begin parsing by looking at the left-most character in the string to be parsed
 - May do so directly, or indirectly by calling another parsing subprogram
- Recursive descent parsers, like other top-down parsers, cannot be built from left-recursive grammars
 - Sometimes can modify grammar to suit



Sample Grammar

$\langle \text{expr} \rangle ::= \langle \text{term} \rangle \mid \langle \text{term} \rangle + \langle \text{expr} \rangle$
 $\mid \langle \text{term} \rangle - \langle \text{expr} \rangle$

$\langle \text{term} \rangle ::= \langle \text{factor} \rangle \mid \langle \text{factor} \rangle * \langle \text{term} \rangle$
 $\mid \langle \text{factor} \rangle / \langle \text{term} \rangle$

$\langle \text{factor} \rangle ::= \langle \text{id} \rangle \mid (\langle \text{expr} \rangle)$



Tokens as OCaml Types

- + - * / () <id>

- Becomes an OCaml datatype

type token =

 Id_token of string

 | Left_parenthesis | Right_parenthesis

 | Times_token | Divide_token

 | Plus_token | Minus_token



Parse Trees as Datatypes

$\langle \text{expr} \rangle ::= \langle \text{term} \rangle \mid \langle \text{term} \rangle + \langle \text{expr} \rangle$
 $\mid \langle \text{term} \rangle - \langle \text{expr} \rangle$

type expr =

Term_as_Expr of term

| Plus_Expr of (term * expr)

| Minus_Expr of (term * expr)



Parse Trees as Datatypes

$$\langle \text{term} \rangle ::= \langle \text{factor} \rangle \mid \langle \text{factor} \rangle * \langle \text{term} \rangle$$
$$\mid \langle \text{factor} \rangle / \langle \text{term} \rangle$$

and term =

- Factor_as_Term of factor
- | Mult_Term of (factor * term)
- | Div_Term of (factor / term)



Parse Trees as Datatypes

$\langle \text{factor} \rangle ::= \langle \text{id} \rangle \mid (\langle \text{expr} \rangle)$

and factor =

Id_as_Factor of string

| Parenthesized_Expr_as_Factor of expr



Parsing Lists of Tokens

- Will create three mutually recursive functions:
 - $\text{expr} : \text{token list} \rightarrow (\text{expr} * \text{token list})$
 - $\text{term} : \text{token list} \rightarrow (\text{term} * \text{token list})$
 - $\text{factor} : \text{token list} \rightarrow (\text{factor} * \text{token list})$
- Each parses what it can and gives back parse and remaining tokens



Parsing an Expression

$\langle \text{expr} \rangle ::= \langle \text{term} \rangle [(+ | -) \langle \text{expr} \rangle]$

let rec expr tokens =

(match term tokens

with (term_parse , tokens_after_term) ->

(match tokens_after_term

with(Plus_token :: tokens_after_plus) ->



Parsing an Expression

$\langle \text{expr} \rangle ::= \langle \text{term} \rangle [(+ | -) \langle \text{expr} \rangle]$

let rec expr tokens =

(match **term tokens**

with (term_parse , tokens_after_term) ->

(match tokens_after_term

with (Plus_token :: tokens_after_plus) ->



Parsing a Plus Expression

`<expr> ::= <term> [(+ | -) <expr>]`

let rec expr tokens =

(match term tokens

with (**term_parse** , tokens_after_term) ->

(match tokens_after_term

with (Plus_token :: tokens_after_plus) ->

Parsing a Plus Expression

$\langle \text{expr} \rangle ::= \langle \text{term} \rangle [(+ | -) \langle \text{expr} \rangle]$

let rec expr tokens =

(match term tokens

with (**term_parse** , tokens_after_term) ->

(match **tokens_after_term**

with (Plus_token :: tokens_after_plus) ->



Parsing a Plus Expression

$\langle \text{expr} \rangle ::= \langle \text{term} \rangle [(+ | -) \langle \text{expr} \rangle]$

let rec expr tokens =

(match term tokens

with (term_parse , tokens_after_term) ->

(match tokens_after_term

with (**Plus_token** :: tokens_after_plus) ->



Parsing a Plus Expression

$\langle \text{expr} \rangle ::= \langle \text{term} \rangle + \langle \text{expr} \rangle$

(match **expr tokens_after_plus**
with (expr_parse , tokens_after_expr) ->
(Plus_Expr (term_parse , expr_parse),
tokens_after_expr))



Parsing a Plus Expression

$\langle \text{expr} \rangle ::= \langle \text{term} \rangle + \langle \text{expr} \rangle$

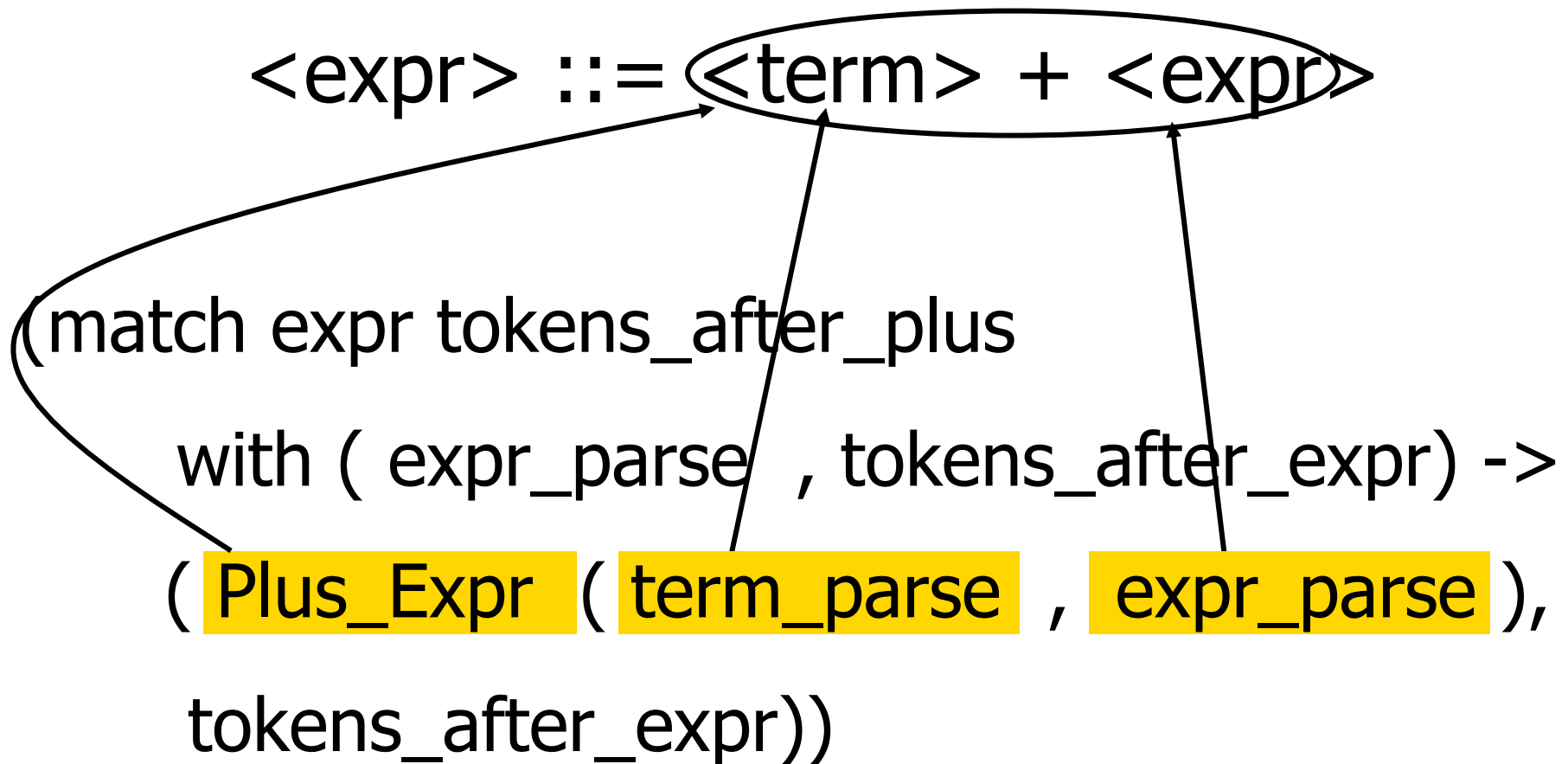
(match expr tokens_after_plus

with (**expr_parse** , tokens_after_expr) ->

(Plus_Expr (term_parse , expr_parse),

tokens_after_expr))

Building Plus Expression Parse Tree





Parsing a Minus Expression

$\langle \text{expr} \rangle ::= \langle \text{term} \rangle - \langle \text{expr} \rangle$

```
| ( Minus_token :: tokens_after_minus) ->  
  (match expr tokens_after_minus  
   with ( expr_parse , tokens_after_expr) ->  
        ( Minus_Expr ( term_parse , expr_parse ),  
          tokens_after_expr))
```

Parsing a Minus Expression

$\langle \text{expr} \rangle ::= \langle \text{term} \rangle - \langle \text{expr} \rangle$

| (**Minus_token** :: tokens_after_minus) ->
(match expr tokens_after_minus
with (expr_parse , tokens_after_expr) ->
(**Minus_Expr** (**term_parse** , **expr_parse**),
tokens_after_expr))



Parsing an Expression as a Term

`<expr> ::= <term>`

| `_ -> (Term_as_Expr term_parse , tokens_after_term))`



- Code for **term** is same except for replacing addition with multiplication and subtraction with division



Parsing Factor as Id

`<factor> ::= <id>`

and factor tokens =

(match tokens

with (Id_token id_name :: tokens_after_id) =

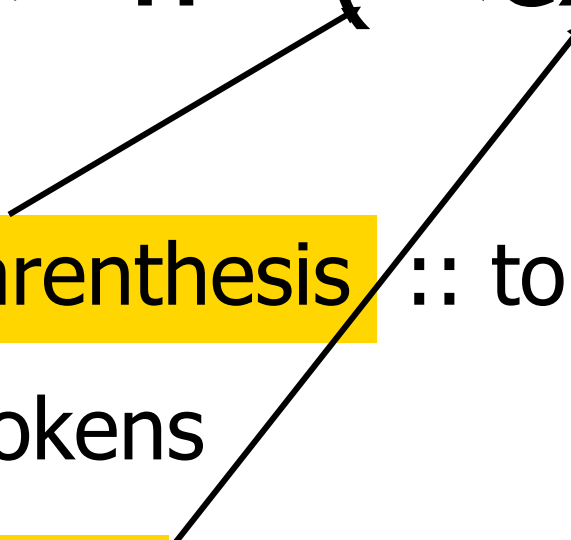
(**Id_as_Factor** id_name, tokens_after_id)



Parsing Factor as Parenthesized Expression

$\langle \text{factor} \rangle ::= (\langle \text{expr} \rangle)$

| factor (**Left_parenthesis** :: tokens) =
(match expr tokens
with (**expr_parse** , tokens_after_expr) ->



Parsing Factor as Parenthesized Expression

$\langle \text{factor} \rangle ::= (\langle \text{expr} \rangle)$

(match tokens_after_expr

with **Right_parenthesis** :: tokens_after_rparen ->

(**Parenthesized_Expr_as_Factor** **expr_parse** ,
tokens_after_rparen)



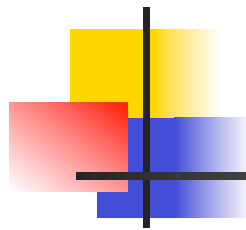
Error Cases

- What if no matching right parenthesis?

```
| _ -> raise (Failure "No matching  
rparen" ) )
```

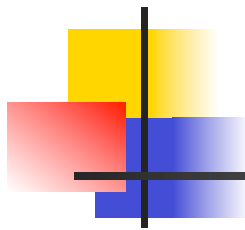
- What if no leading id or left parenthesis?

```
| _ -> raise (Failure "No id or lparen" ) );;
```

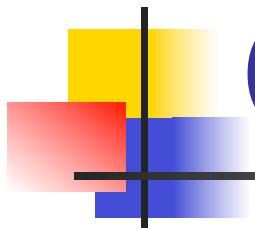


$(a + b) * c - d$

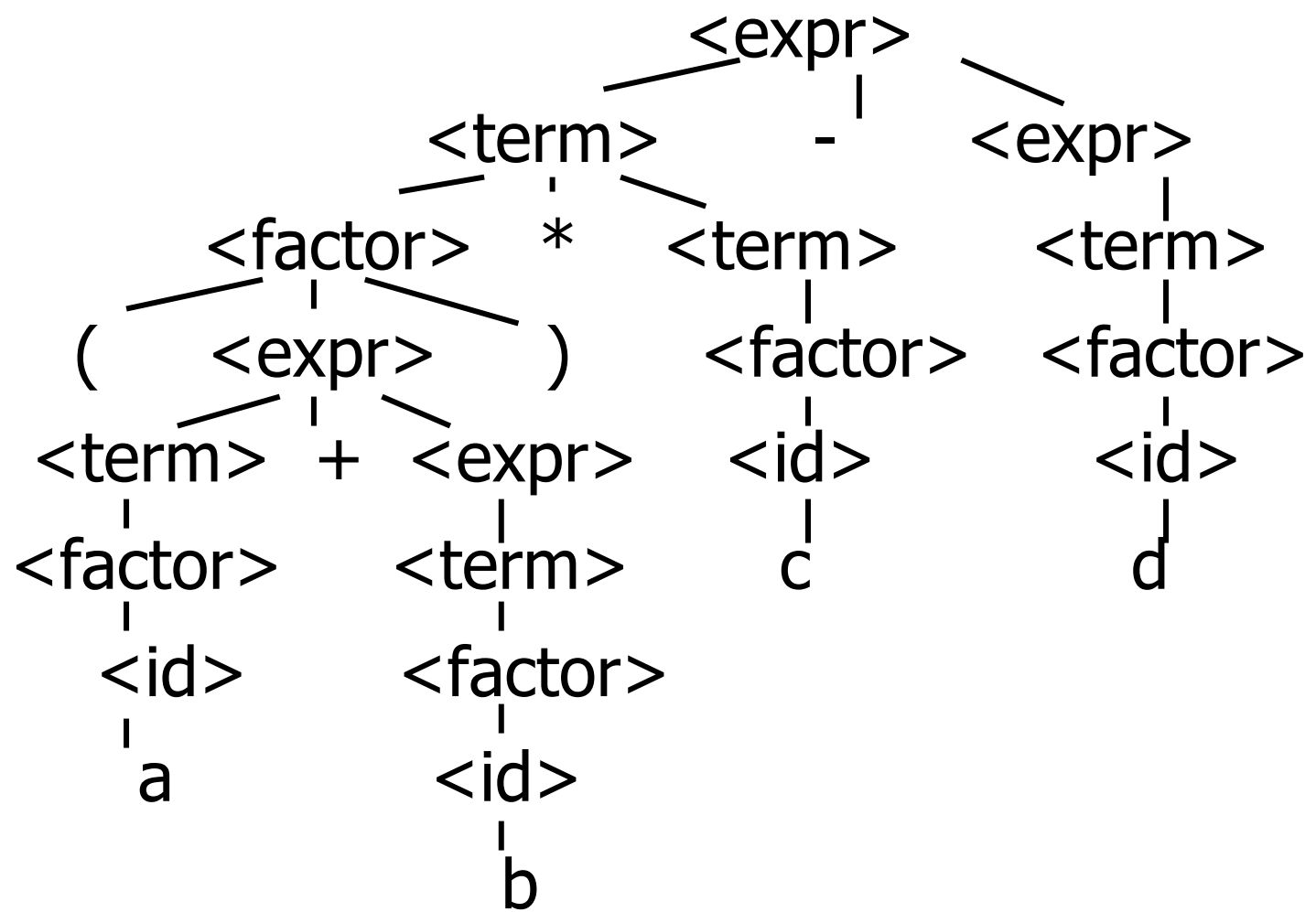
```
expr [Left_parenthesis; Id_token "a";  
      Plus_token; Id_token "b";  
      Right_parenthesis; Times_token;  
      Id_token "c"; Minus_token;  
      Id_token "d"];;
```

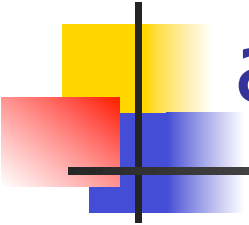

$$(a + b) * c - d$$

```
- : expr * token list =  
(Minus_Expr  
  (Mult_Term  
    (Parenthesized_Expr_as_Factor  
      (Plus_Expr  
        (Factor_as_Term (Id_as_Factor "a"),  
          Term_as_Expr (Factor_as_Term  
            (Id_as_Factor "b")))),  
        Factor_as_Term (Id_as_Factor "c")),  
      Term_as_Expr (Factor_as_Term (Id_as_Factor  
        "d")))),  
  [])
```



$(a + b) * c - d$





a + b * c - d

```
# expr [Id_token "a"; Plus_token; Id_token "b";  
Times_token; Id_token "c"; Minus_token;  
Id_token "d"];;
```

```
- : expr * token list =
```

```
(Plus_Expr
```

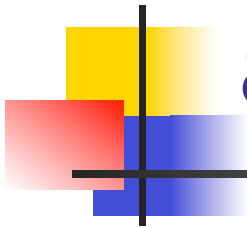
```
(Factor_as_Term (Id_as_Factor "a"),
```

```
Minus_Expr
```

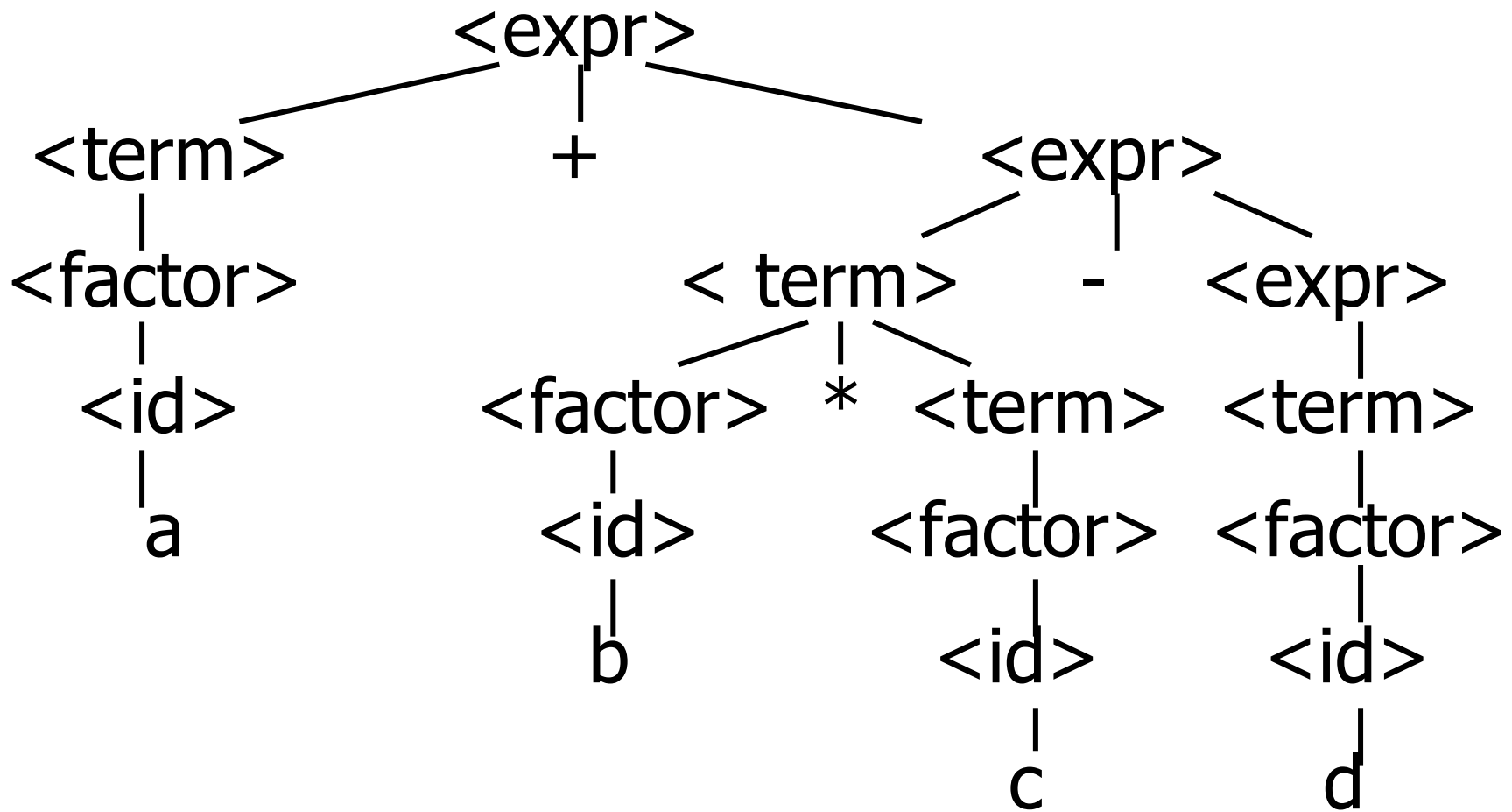
```
(Mult_Term (Id_as_Factor "b", Factor_as_Term  
(Id_as_Factor "c")),
```

```
Term_as_Expr (Factor_as_Term (Id_as_Factor  
"d")))),
```

```
[])
```



a + b * c - d





(a + b * c - d

```
# expr [Left_parenthesis; Id_token "a";  
Plus_token; Id_token "b"; Times_token;  
Id_token "c"; Minus_token; Id_token "d"];;
```

Exception: Failure "No matching rparen".

Can't parse because it was expecting a right parenthesis but it got to the end without finding one



$(a + b) * c - d *$

```
expr [Id_token "a"; Plus_token; Id_token "b";  
      Right_parenthesis; Times_token; Id_token "c";  
      Minus_token; Id_token "d"];;
```

- : expr * token list =

(Plus_Expr

(Factor_as_Term (Id_as_Factor "a"),

Term_as_Expr (Factor_as_Term (Id_as_Factor
"b"))),

[Right_parenthesis; Times_token; Id_token "c";
Minus_token; Id_token "d"])



Parsing Whole String

- Q: How to guarantee whole string parses?
- A: Check returned tokens empty

let parse tokens =

match **expr** tokens

with (expr_parse, []) -> expr_parse

| _ -> raise (Failure "No parse");;

- Fixes <expr> as start symbol



Streams in Place of Lists

- More realistically, we don't want to create the entire list of tokens before we can start parsing
- We want to generate one token at a time and use it to make one step in parsing
- Will use $(\text{token} * (\text{unit} \rightarrow \text{token}))$ or $(\text{token} * (\text{unit} \rightarrow \text{token option}))$
in place of token list



Problems for Recursive-Descent Parsing

- Left Recursion:

$A ::= Aw$

translates to a subroutine that loops forever

- Indirect Left Recursion:

$A ::= Bw$

$B ::= Av$

causes the same problem



Problems for Recursive-Descent Parsing

- Parser must always be able to choose the next action based only on the very next token
- Pairwise Disjointedness Test: Can we always determine which rule (in the non-extended BNF) to choose based on just the first token



Pairwise Disjointedness Test

- For each rule

$$A ::= y$$

Calculate

$$\text{FIRST}(y) =$$

$$\{a \mid y \Rightarrow^* aw\} \cup \{\varepsilon \mid \text{if } y \Rightarrow^* \varepsilon\}$$

- For each pair of rules $A ::= y$ and $A ::= z$, require $\text{FIRST}(y) \cap \text{FIRST}(z) = \{\}$



Example

Grammar:

$$\langle S \rangle ::= \langle A \rangle a \langle B \rangle b$$
$$\langle A \rangle ::= \langle A \rangle b \mid b$$
$$\langle B \rangle ::= a \langle B \rangle \mid a$$
$$\text{FIRST}(\langle A \rangle b) = \{b\}$$
$$\text{FIRST}(b) = \{b\}$$

Rules for $\langle A \rangle$ not pairwise disjoint



Eliminating Left Recursion

- Rewrite grammar to shift left recursion to right recursion
 - Changes associativity

- Given

$\langle \text{expr} \rangle ::= \langle \text{expr} \rangle + \langle \text{term} \rangle$ and

$\langle \text{expr} \rangle ::= \langle \text{term} \rangle$

- Add new non-terminal $\langle e \rangle$ and replace above rules with

$\langle \text{expr} \rangle ::= \langle \text{term} \rangle \langle e \rangle$

$\langle e \rangle ::= + \langle \text{term} \rangle \langle e \rangle \mid \varepsilon$



Factoring Grammar

- Test too strong: Can't handle

$\langle \text{expr} \rangle ::= \langle \text{term} \rangle [(+ | -) \langle \text{expr} \rangle]$

- Answer: Add new non-terminal and replace above rules by

$\langle \text{expr} \rangle ::= \langle \text{term} \rangle \langle e \rangle$

$\langle e \rangle ::= + \langle \text{term} \rangle \langle e \rangle$

$\langle e \rangle ::= - \langle \text{term} \rangle \langle e \rangle$

$\langle e \rangle ::= \varepsilon$

- You are delaying the decision point



Example

Both $\langle A \rangle$ and $\langle B \rangle$
have problems:

Transform grammar
to:

$\langle S \rangle ::= \langle A \rangle a \langle B \rangle b$	$\langle S \rangle ::= \langle A \rangle a \langle B \rangle b$
$\langle A \rangle ::= \langle A \rangle b \mid b$	$\langle A \rangle ::= b \langle A1 \rangle$
$\langle B \rangle ::= a \langle B \rangle \mid a$	$\langle A1 \rangle ::= b \langle A1 \rangle \mid \varepsilon$
	$\langle B \rangle ::= a \langle B1 \rangle$
	$\langle B1 \rangle ::= a \langle B1 \rangle \mid \varepsilon$



Ocamlyacc Input

- File format:

```
%{
```

```
  <header>
```

```
%}
```

```
  <declarations>
```

```
%%
```

```
  <rules>
```

```
%%
```

```
  <trailer>
```



Ocamlyacc *<header>*

- Contains arbitrary Ocaml code
- Typically used to give types and functions needed for the semantic actions of rules and to give specialized error recovery
- May be omitted
- *<footer>* similar. Possibly used to call parser



Ocamlyacc <declarations>

- **%token** *symbol ... symbol*
- Declare given symbols as tokens
- **%token** <*type*> *symbol ... symbol*
- Declare given symbols as token constructors, taking an argument of type <*type*>
- **%start** *symbol ... symbol*
- Declare given symbols as entry points; functions of same names in <*grammar*>.ml



Ocamlyacc *<declarations>*

- **%type** *<type> symbol ... symbol*

Specify type of attributes for given symbols.
Mandatory for start symbols

- **%left** *symbol ... symbol*

- **%right** *symbol ... symbol*

- **%nonassoc** *symbol ... symbol*

Associate precedence and associativity to given symbols. Same line, same precedence; earlier line, lower precedence (broadest scope)



Ocamlyacc <rules>

- *nonterminal* :

symbol ... symbol { semantic_action }

| ...

| *symbol ... symbol { semantic_action }*

;

- Semantic actions are arbitrary Ocaml expressions
- Must be of same type as declared (or inferred) for *nonterminal*
- Access semantic attributes (values) of symbols by position: \$1 for first symbol, \$2 to second ...



Example - Base types

```
(* File: expr.ml *)
type expr =
  Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
and term =
  Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
and factor =
  Id_as_Factor of string
  | Parenthesized_Expr_as_Factor of expr
```




Example - Lexer (exprlex.mll)

```
{ (*open Exprparse*) }
let numeric = ['0' - '9']
let letter = ['a' - 'z' 'A' - 'Z']
rule token = parse
  | "+" {Plus_token}
  | "-" {Minus_token}
  | "*" {Times_token}
  | "/" {Divide_token}
  | "(" {Left_parenthesis}
  | ")" {Right_parenthesis}
  | letter (letter|numeric|"_"|"_")* as id {Id_token id}
  | [' ' '\t' '\n'] {token lexbuf}
  | eof {EOL}
```



Example - Parser (exprparse.mly)

```
%{ open Expr
```

```
%}
```

```
%token <string> Id_token
```

```
%token Left_parenthesis Right_parenthesis
```

```
%token Times_token Divide_token
```

```
%token Plus_token Minus_token
```

```
%token EOL
```

```
%start main
```

```
%type <expr> main
```

```
%%
```



Example - Parser (exprparse.mly)

expr:

term

{ Term_as_Expr \$1 }

| term Plus_token expr

{ Plus_Expr (\$1, \$3) }

| term Minus_token expr

{ Minus_Expr (\$1, \$3) }



Example - Parser (exprparse.mly)

term:

factor

{ Factor_as_Term \$1 }

| factor Times_token term

{ Mult_Term (\$1, \$3) }

| factor Divide_token term

{ Div_Term (\$1, \$3) }



Example - Parser (exprparse.mly)

factor:

Id_token

{ Id_as_Factor \$1 }

| Left_parenthesis expr Right_parenthesis

{ Parenthesized_Expr_as_Factor \$2 }

main:

| expr EOL

{ \$1 }



Example - Using Parser

```
# #use "expr.ml";;
```

```
...
```

```
# #use "exprparse.ml";;
```

```
...
```

```
# #use "exprlex.ml";;
```

```
...
```

```
# let test s =
```

```
  let lexbuf = Lexing.from_string (s^"\n") in  
    main token lexbuf;;
```



Example - Using Parser

```
# test "a + b";;
```

```
- : expr =
```

```
Plus_Expr
```

```
(Factor_as_Term (Id_as_Factor "a"),  
Term_as_Expr (Factor_as_Term  
  (Id_as_Factor "b")))
```