



Real-time Scheduling

Introduction to Real-Time



A Robotic Design Example (Revisited)

- A robot has a camera that detects obstacles with probability 70%, a bump sensor that detects imminent collisions with a probability of 99.9% (when an obstacle is 1 inch away), and a cliff sensor that detects imminent falls off a cliff with a probability of 99.9% (when the cliff is 1 inch away). The robot has brakes that can stop it within 0.1 second. The mission is to deliver supplies from point A to point B, safely.
 - What are safety-critical requirements?
 - What are mission-critical (i.e., performance) requirements?
 - What is a safe state?
 - How to ensure well-formed dependencies?
 - **What is a safe speed for the robot?**
 - Is the algorithm that computes speed based on preferred arrival time and route safety-critical or mission-critical?

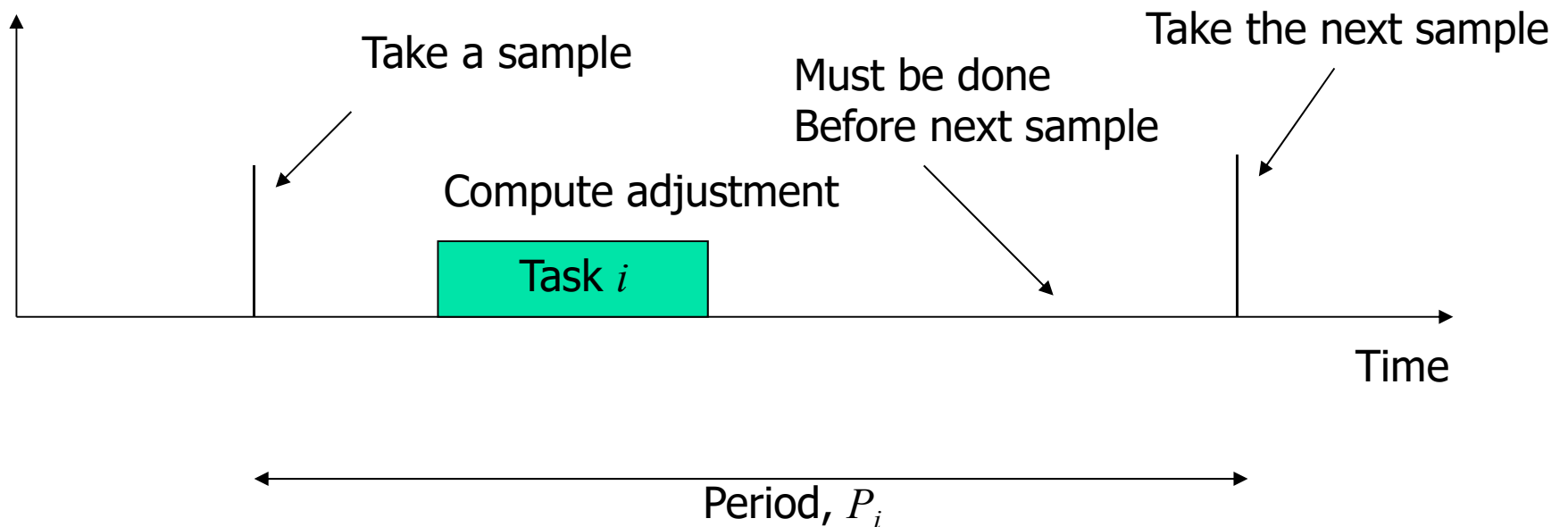


The Schedulability Question: Drive-by-Wire Example

- Consider a control system in an autonomous robot
 - Navigation guidance is computed every 10 ms – wheel positions adjusted accordingly (computing the adjustment takes 4.5 ms of CPU time)
 - Threats and obstacles are reassessed every 4 ms – breaks adjusted accordingly (computing the adjustment takes 2ms of CPU time)
 - Optimal speed is computed every 15 ms – robot speed is adjusted accordingly (computing the adjustment takes 0.45 ms)
 - For safe operation, adjustments must always be computed before the next sample is taken
- Is it possible to always compute all adjustments in time?

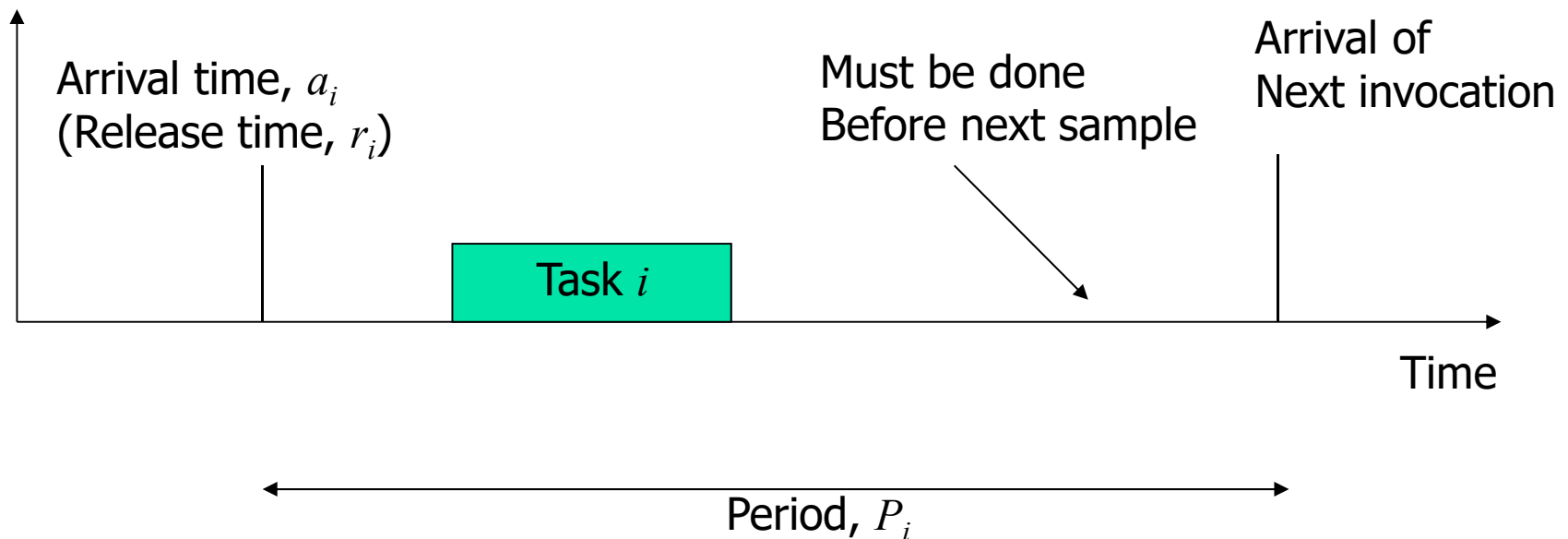
Some Terminology

- Tasks, periods, arrival-time, deadline, execution time, etc.



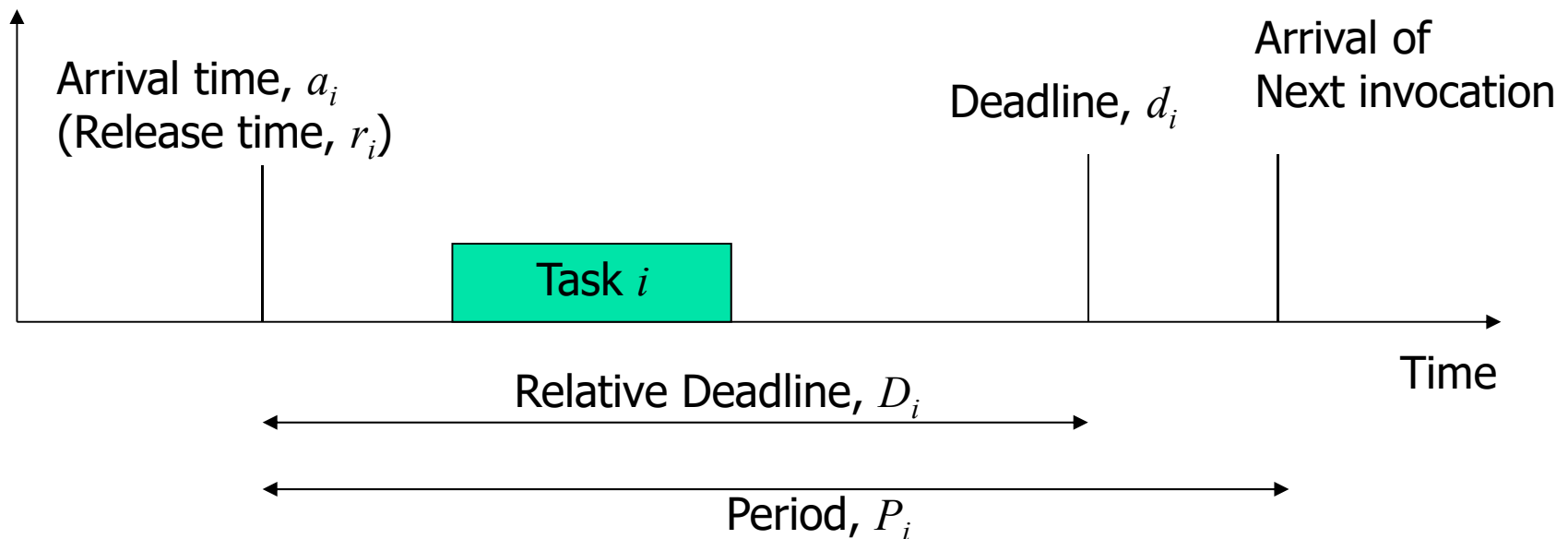
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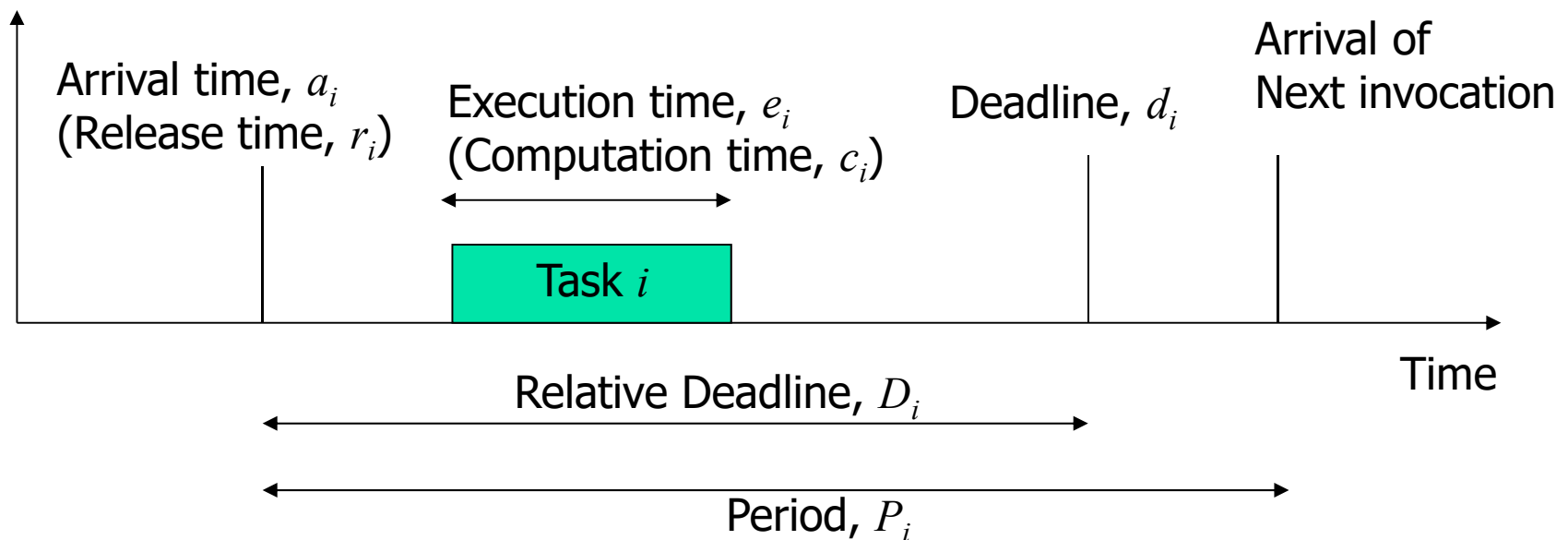
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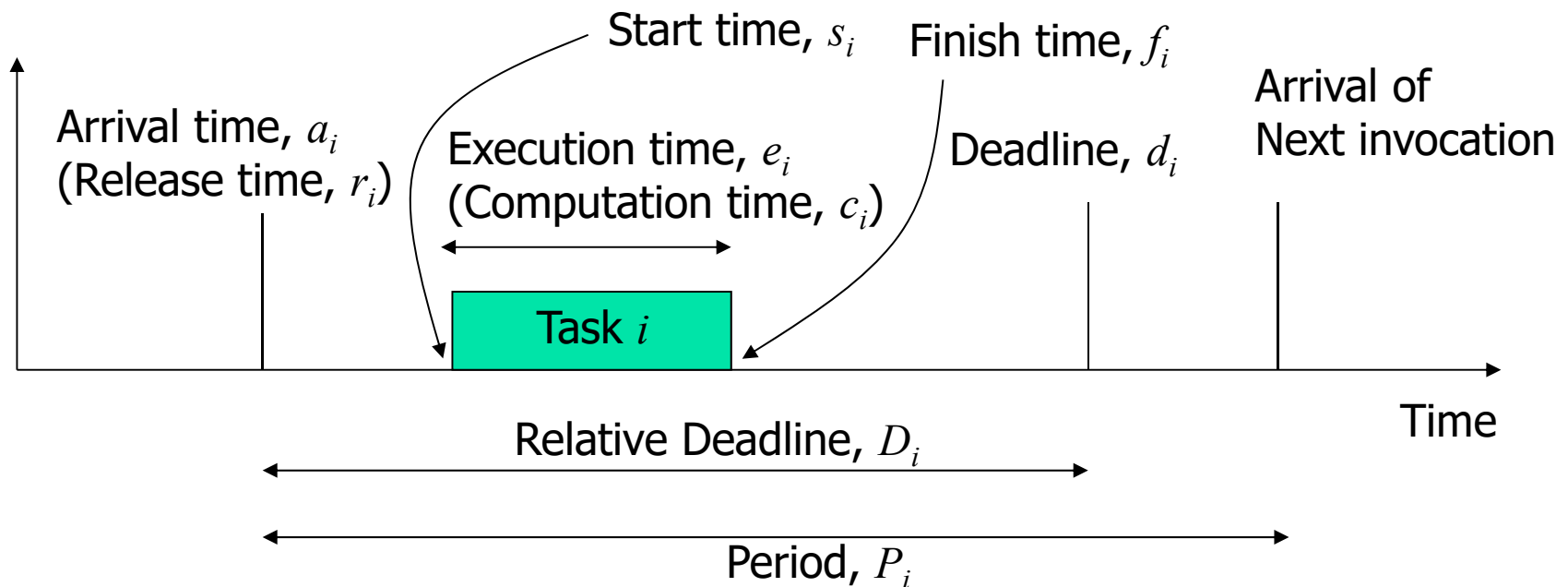
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Some Terminology

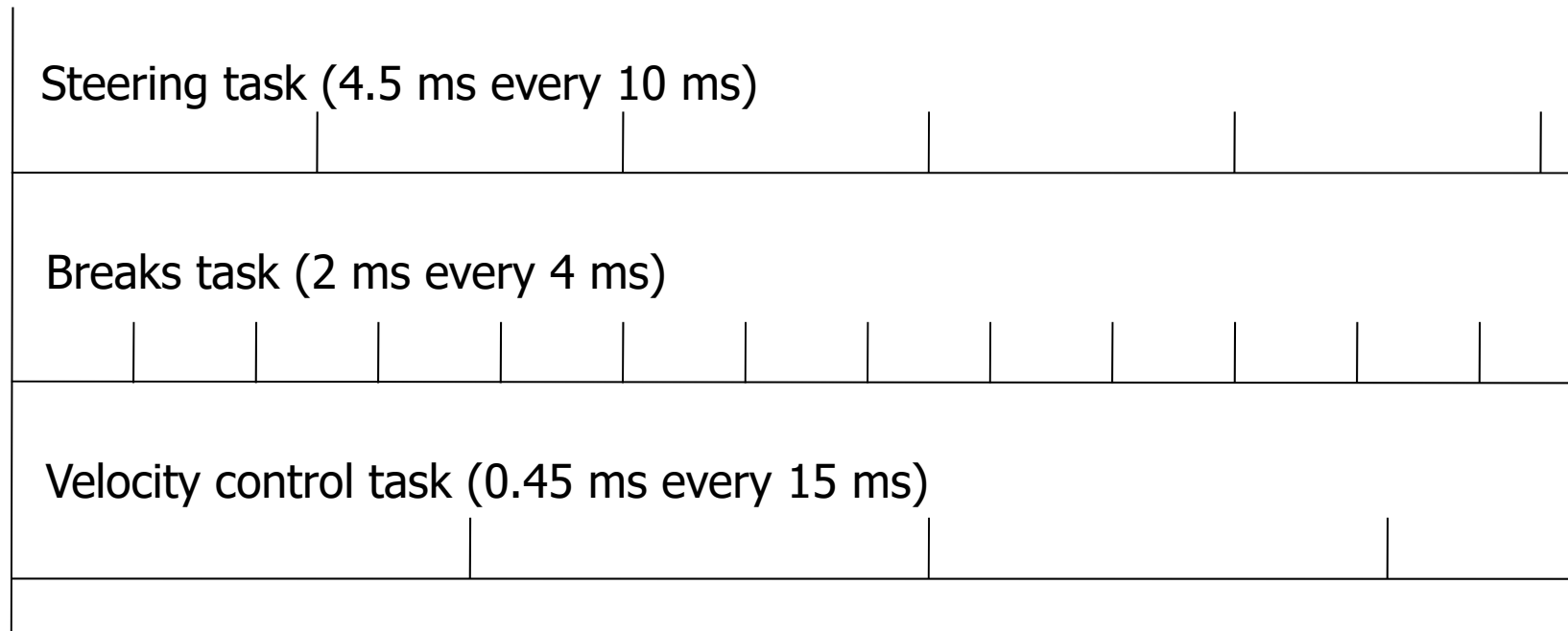
- Tasks, periods, arrival-time, deadline, execution time, etc.



Back to

Drive-by-Wire Example

- Find a schedule that makes sure all task invocations meet their deadlines



Back to

Drive-by-Wire Example

- Sanity check #1: Is the processor over-utilized? (e.g., if you have 5 homeworks due this time tomorrow, each takes 6 hours, then $5 \times 6 = 30 > 24 \rightarrow$ you are overutilized)

Steering task (4.5 ms every 10 ms)

Breaks task (2 ms every 4 ms)

Velocity control task (0.45 ms every 15 ms)

Back to

Drive-by-Wire Example

- Sanity check #1: Is the processor over-utilized? (e.g., if you have 5 homeworks due this time tomorrow, each takes 6 hours, then $5 \times 6 = 30 > 24 \rightarrow$ you are overutilized)
 - Hint: Check if processor utilization $> 100\%$

Steering task (4.5 ms every 10 ms)

Breaks task (2 ms every 4 ms)

Velocity control task (0.45 ms every 15 ms)



Task Scheduling

- Decision #1: In what order should tasks be executed?
 - Hand-crafted schedule (fill timeline by hand)
 - Priority based schedule (assign priorities → schedule is implied)

Steering task (4.5 ms every 10 ms)

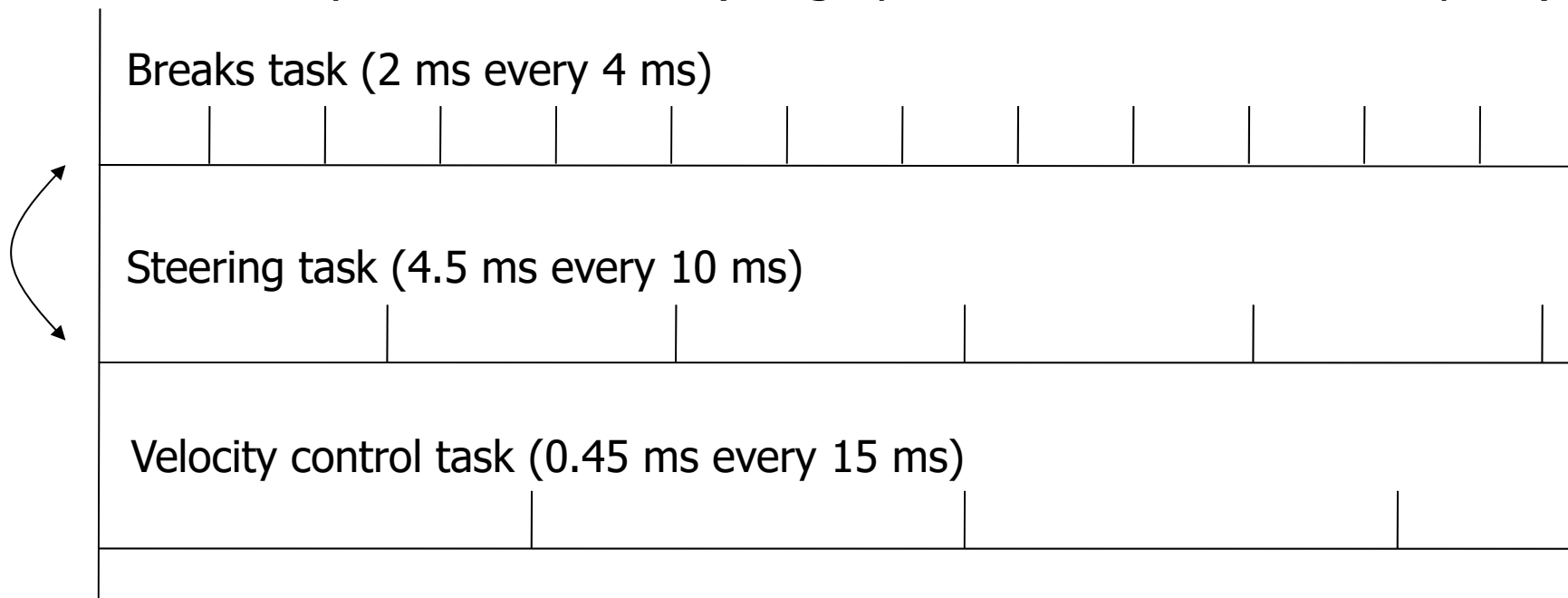
Breaks task (2 ms every 4 ms)

Velocity control task (0.45 ms every 15 ms)

How to assign priorities to tasks?

Task Scheduling

- Decision #1: In what order should tasks be executed?
 - Hand-crafted schedule (fill timeline by hand)
 - Priority based schedule (assign priorities → schedule is implied)



Intuition: Urgent tasks should be higher in priority

Task Scheduling

- Decision #2: Preemptive versus non-preemptive?
 - Preemptive: Higher-priority tasks can interrupt lower-priority ones
 - Non-preemptive: They can't

Breaks task (2 ms every 4 ms)

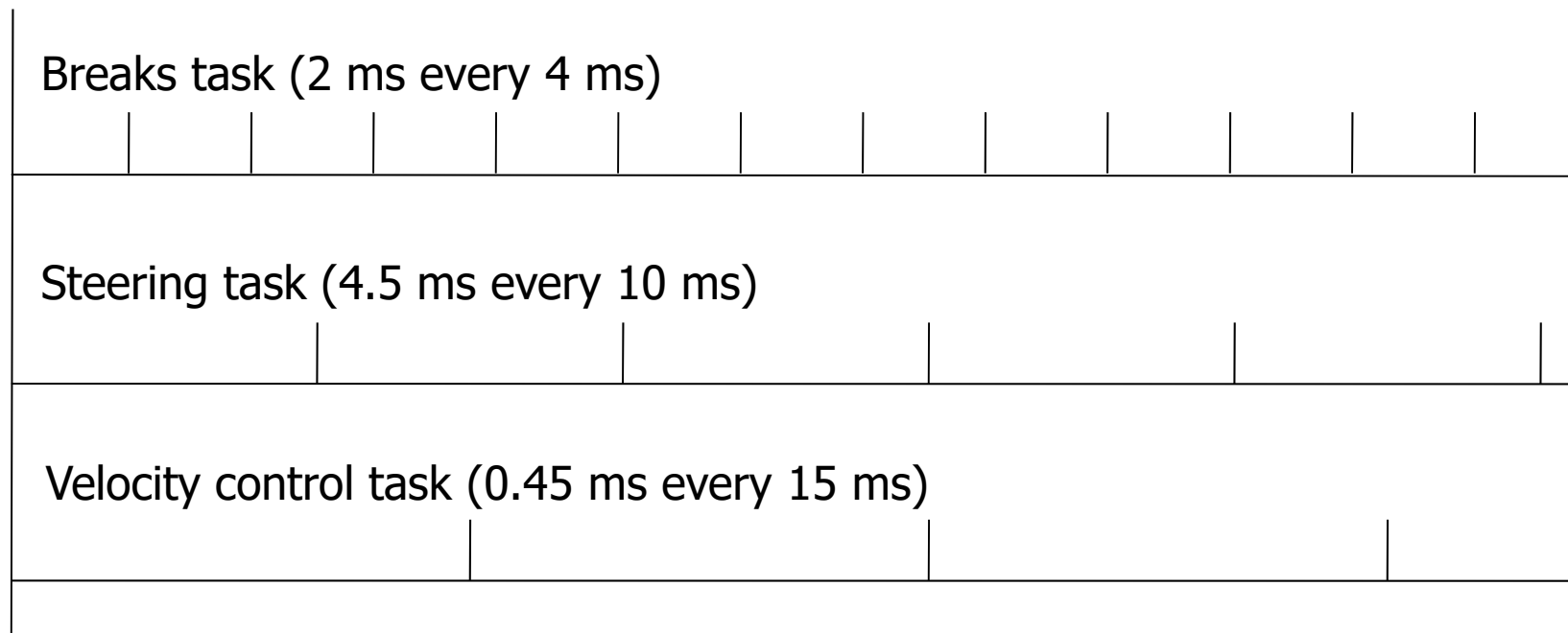
Steering task (4.5 ms every 10 ms)

Velocity control task (0.45 ms every 15 ms)

In this example, will non-preemptive scheduling work?

Task Scheduling

- Decision #2: Preemptive versus non-preemptive
 - Preemptive: Higher-priority tasks can interrupt lower-priority ones
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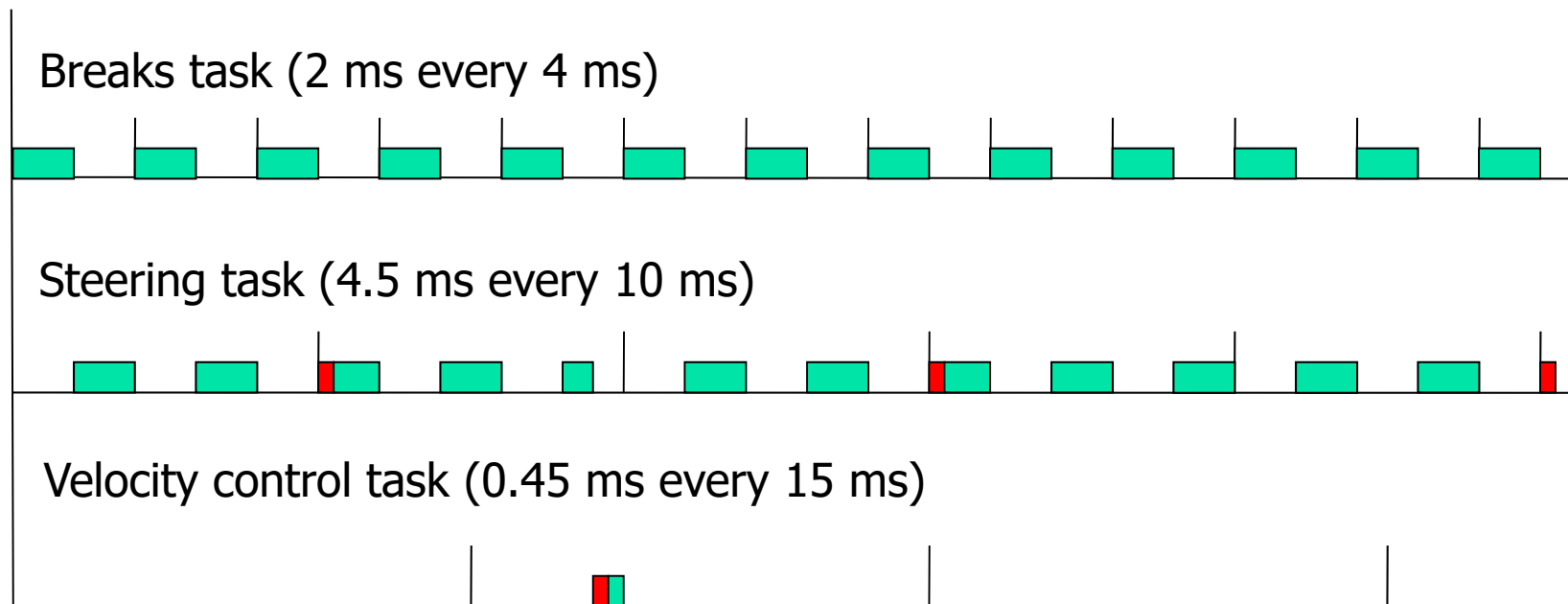


In this example, will non-preemptive scheduling work?

- Hint: Compare relative deadlines of tasks to execution times of others

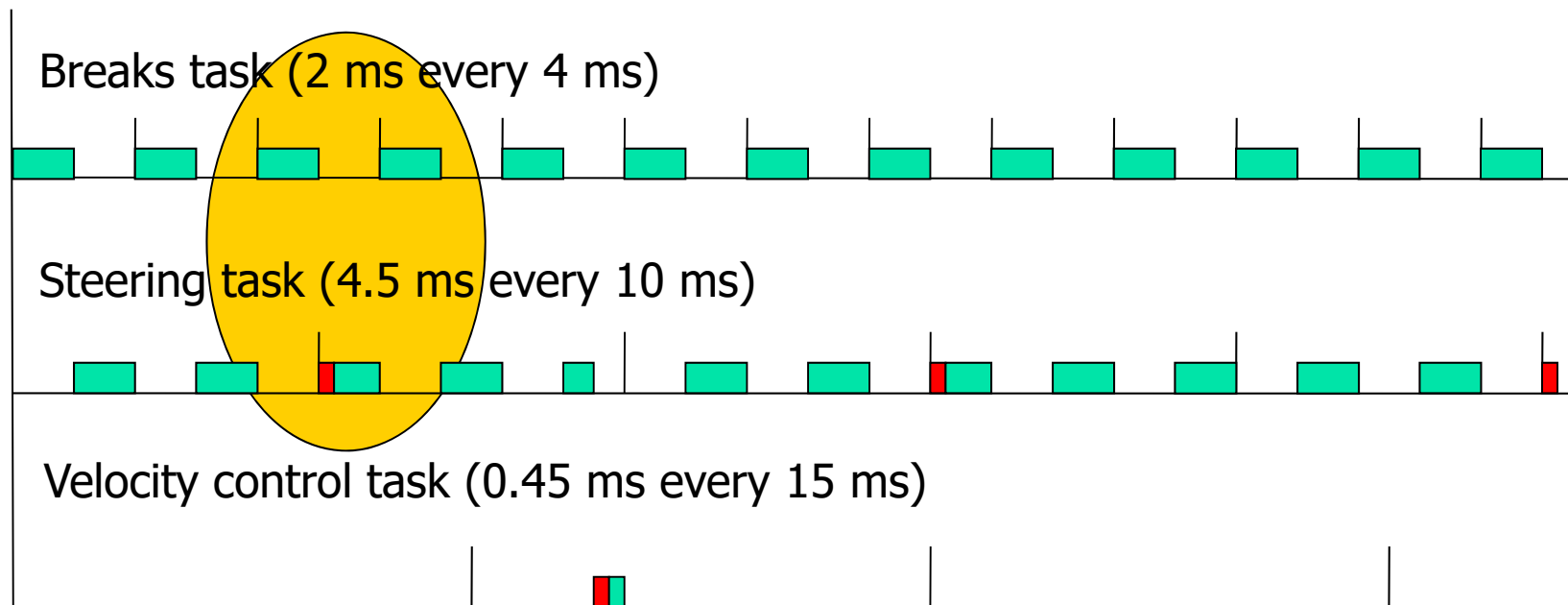
Timeline

- Deadlines are missed!
- Average Utilization < 100%



Timeline

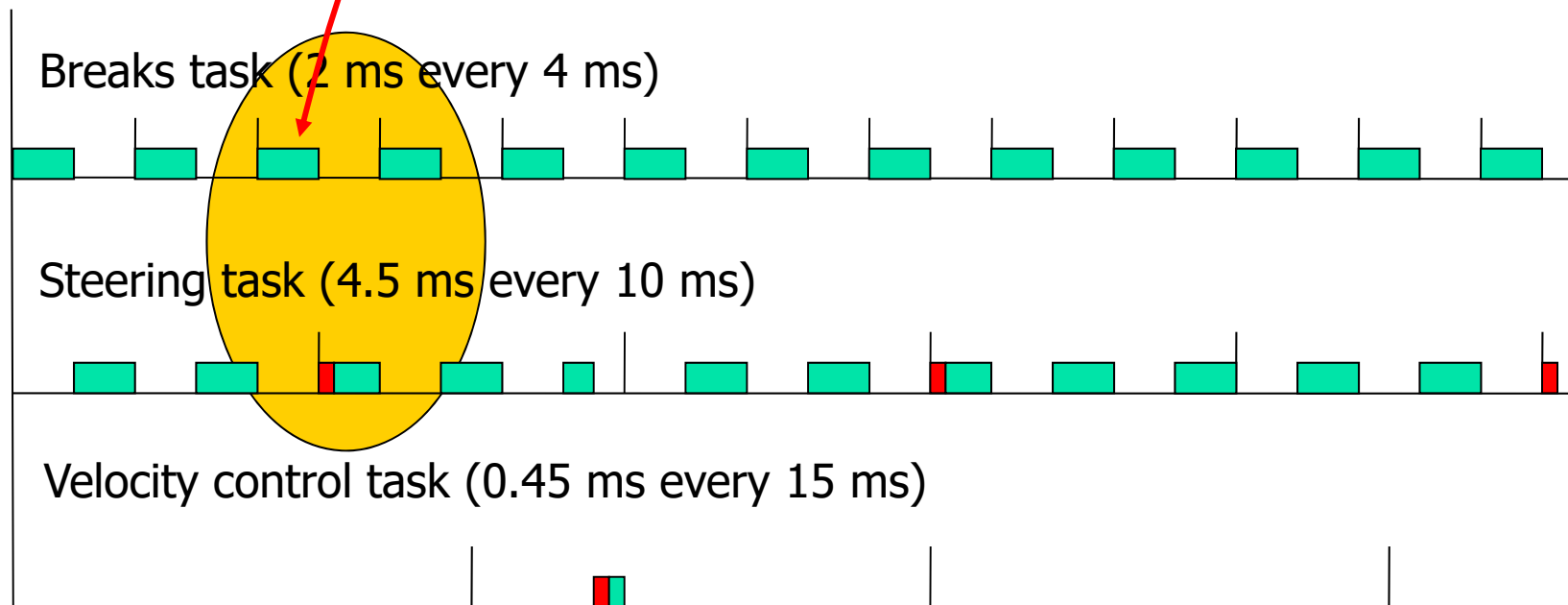
- Deadlines are missed!
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Timeline

Fix:
Give this task invocation
a lower priority

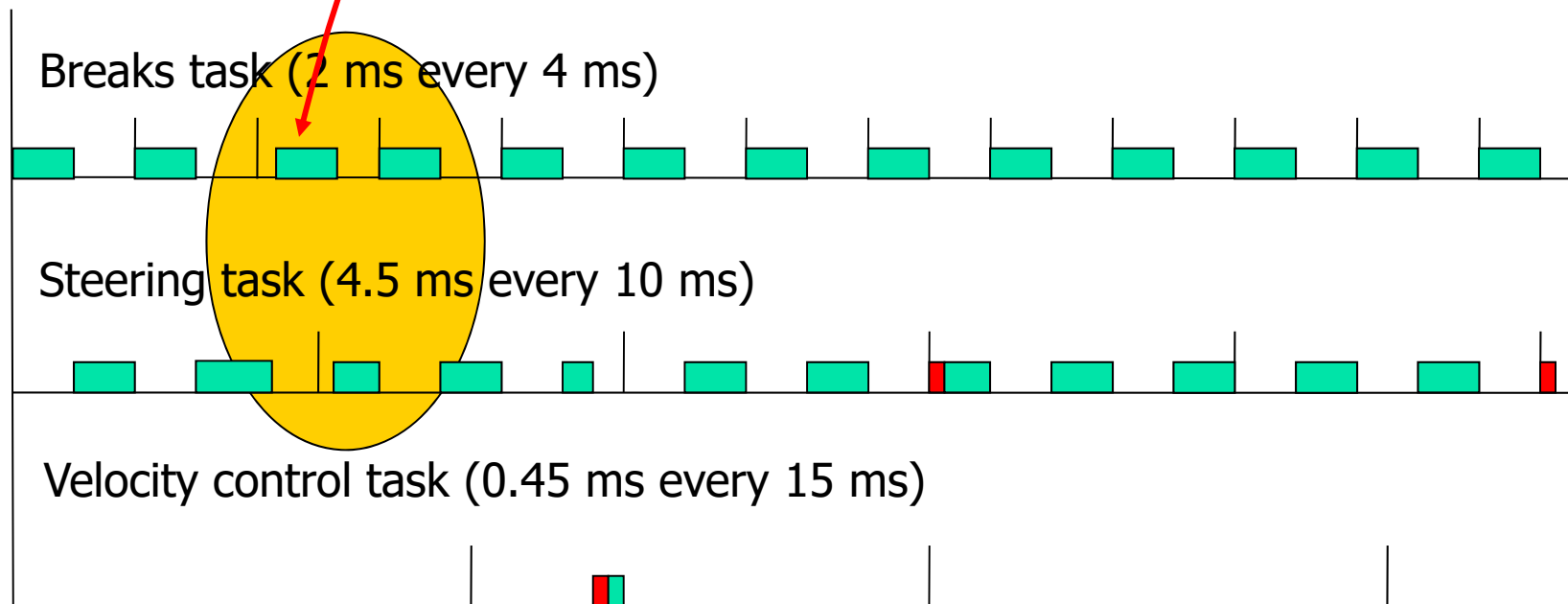
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Timeline

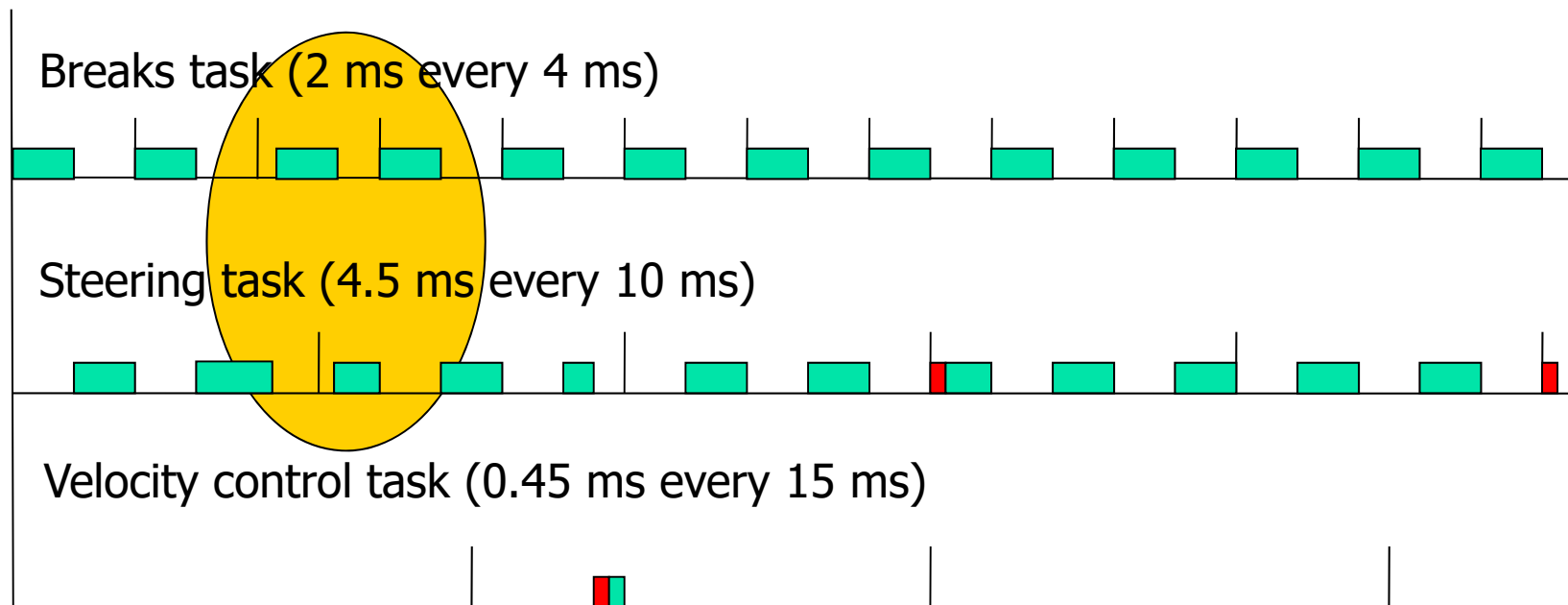
Fix:
Give this task invocation
a lower priority

- Deadlines are missed!
- Average Utilization < 100%



Task Scheduling

- Decision #3: Static versus Dynamic priorities?
 - Static: Instances of the same task have the same priority
 - Dynamic: Instances of same task may have different priorities



Intuition: Dynamic priorities offer the designer more flexibility and hence are more capable to meet deadlines



Interesting Questions

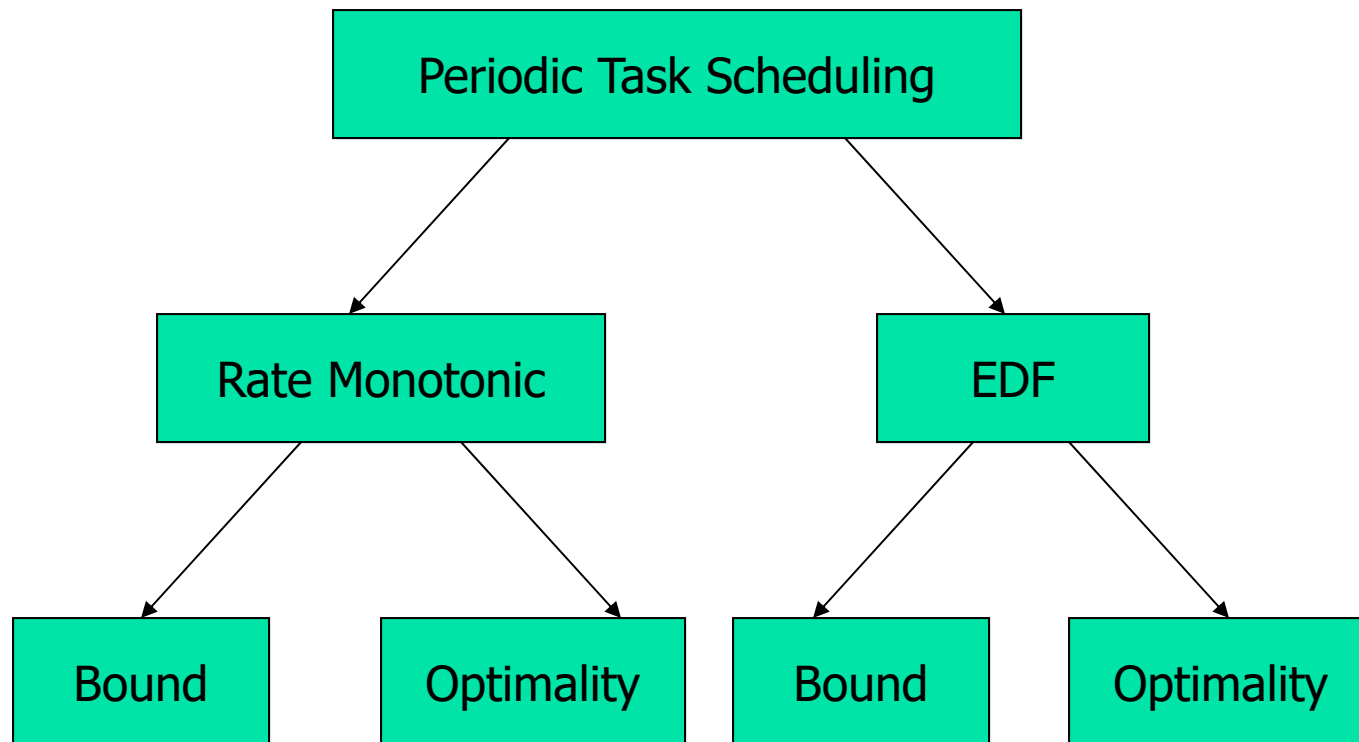
- What is the optimal dynamic priority scheduling policy? (Optimal: meets all deadlines as long as any other policy in its class can)
 - Can it meet all deadlines as long as the processor is not over-utilized?
- What is the optimal static priority scheduling policy?
 - When can it meet all deadlines?
 - Can it meet all deadline as long as the processor is not over-utilized?



Interesting Questions

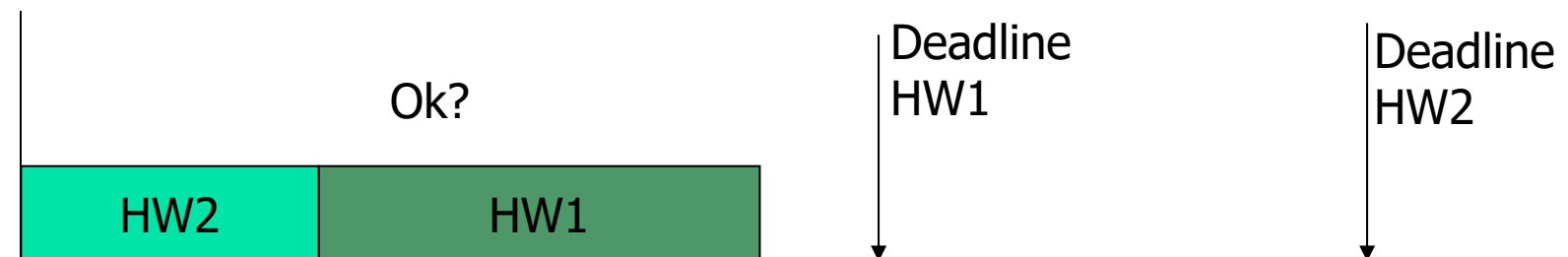
- What is the optimal dynamic priority scheduling policy? (Optimal: meets all deadlines as long as any other policy in its class can)
 - Can it meet all deadlines as long as the processor is not over-utilized?
- What is the optimal static priority scheduling policy?
 - When can it meet all deadlines? → **Utilization Bounds**
 - Can it meet ~~all~~ deadline as long as the processor is not over-utilized?

Main Results in Real-time Scheduling of Periodic Tasks



Advanced: Earliest Deadline First (EDF) Optimality Result

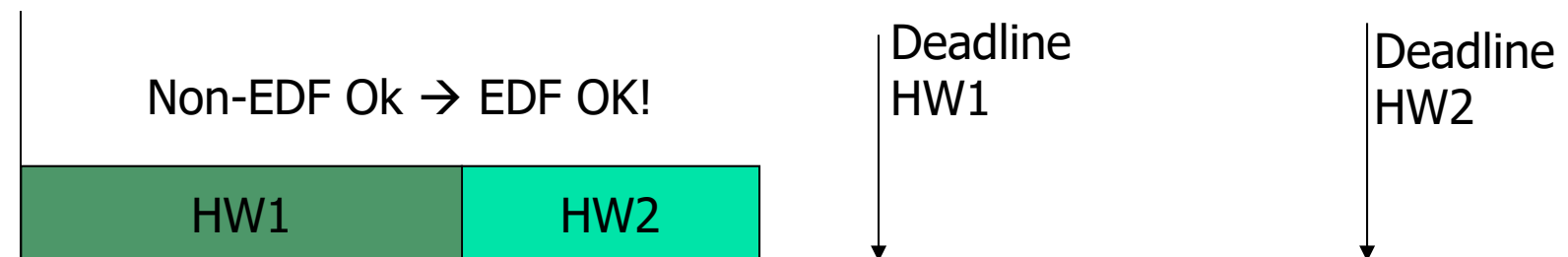
- EDF is the optimal dynamic priority scheduling policy
 - It can meet all deadlines whenever the processor utilization is less than 100%
 - Intuition:
 - You have HW1 due tomorrow and HW2 due the day after, which one do you do first?
 - If you started with HW2 and met both deadlines you could have started with HW1 (in EDF order) and still met both deadlines
 - EDF can meet deadlines whenever anyone else can



Earliest Deadline First (EDF)

Optimality Result

- EDF is the optimal dynamic priority scheduling policy
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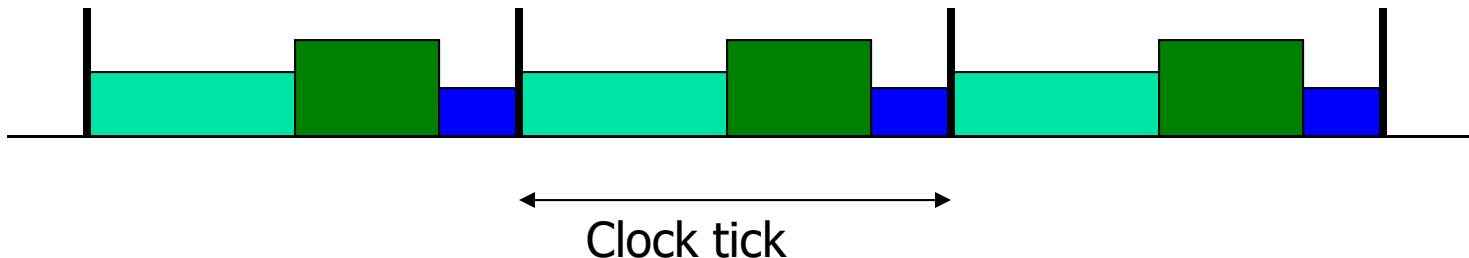


When can EDF Meet Deadlines?

- Consider a task set where:

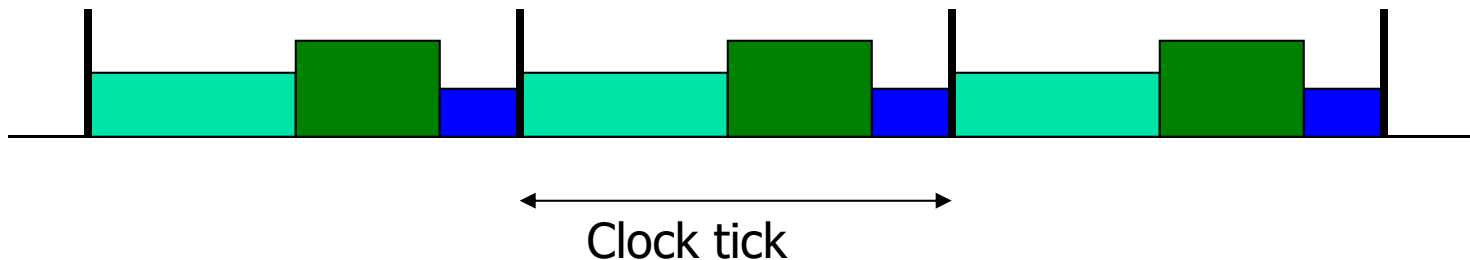
$$\sum_i \frac{C_i}{P_i} = 1$$

- Imagine a policy that reserves for each task i a fraction f_i of each clock tick, where $f_i = C_i / P_i$



Utilization Bound of EDF

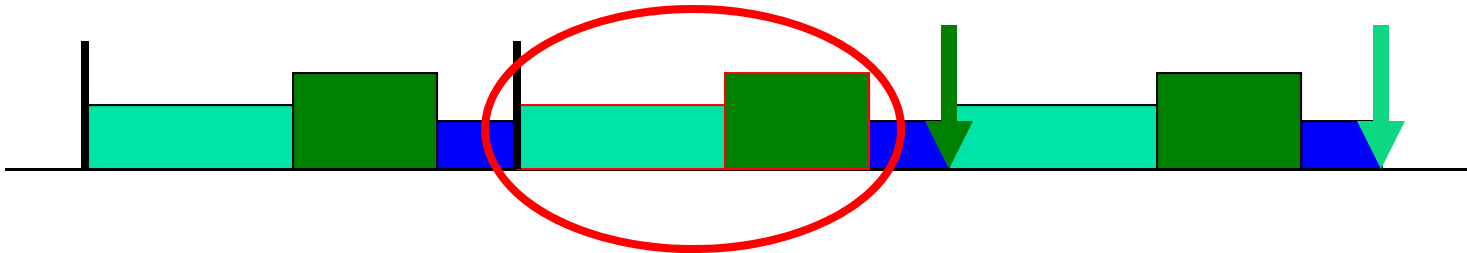
- Imagine a policy that reserves for each task i a fraction f_i of each time unit, where $f_i = C_i/P_i$



- This policy meets all deadlines, because within each period P_i it reserves for task i a total time
 - Time = $f_i P_i = (C_i/P_i) P_i = C_i$ (i.e., enough to finish)

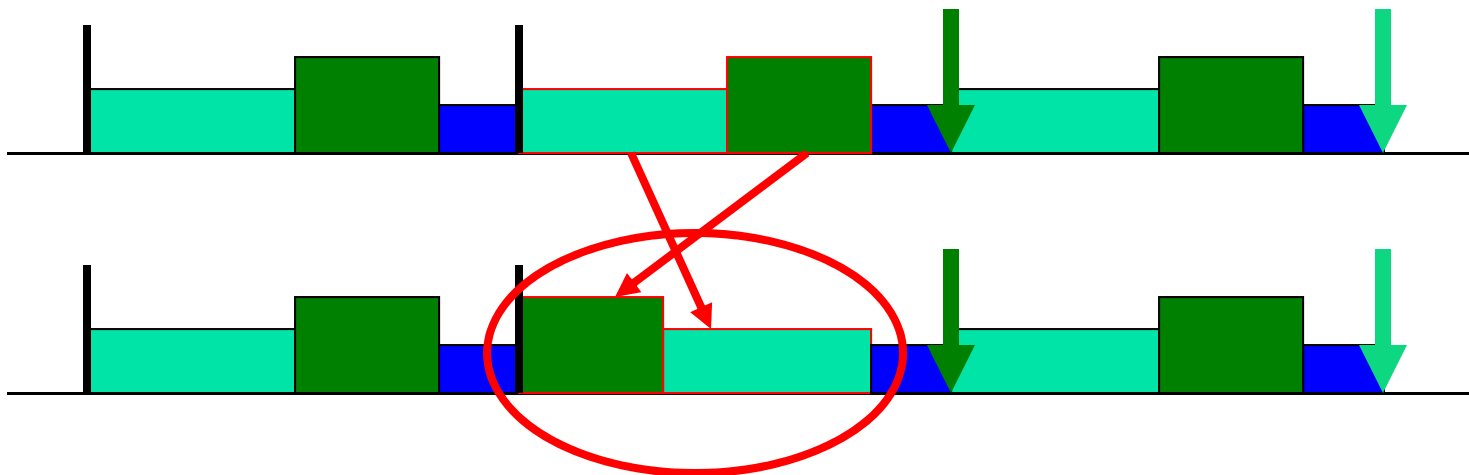
Utilization Bound of EDF

- Pick any two execution chunks that are not in EDF order and swap them



Utilization Bound of EDF

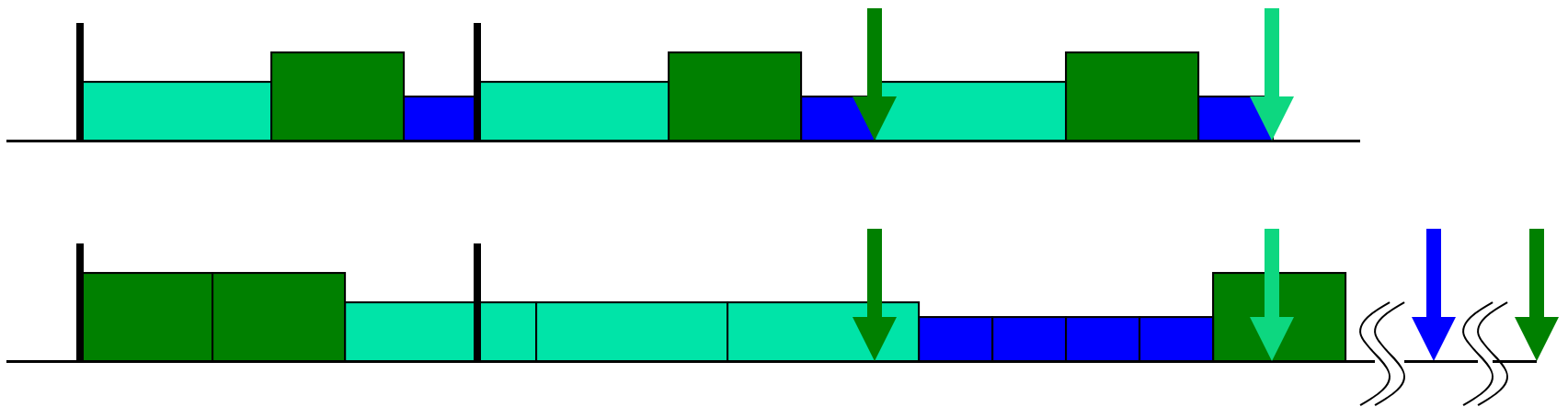
- Pick any two execution chunks that are not in EDF order and swap them



- Still meets deadlines!

Utilization Bound of EDF

- Pick any two execution chunks that are not in EDF order and swap them



- Still meets deadlines!
- Repeat swap until all in EDF order
→ EDF meets deadlines



Rate Monotonic Scheduling

- Rate monotonic scheduling is the optimal fixed-priority scheduling policy for periodic tasks (with period = deadline).

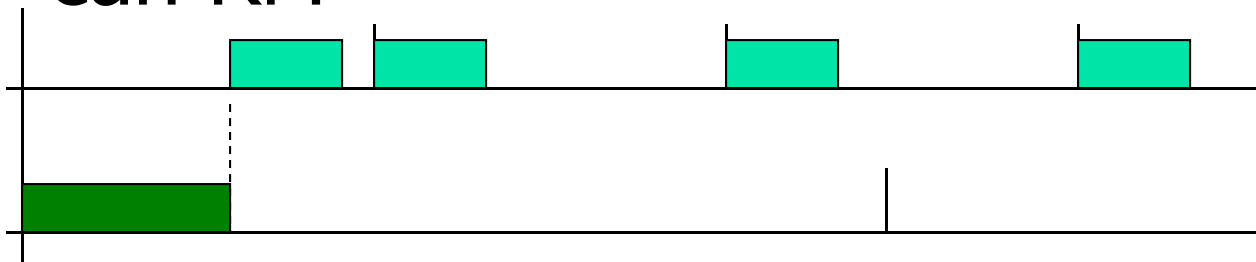


The Worst-Case Scenario

- Consider the worst case where all tasks arrive at the same time.
- If any fixed priority scheduling policy can meet deadline, rate monotonic can!

Optimality of Rate Monotonic

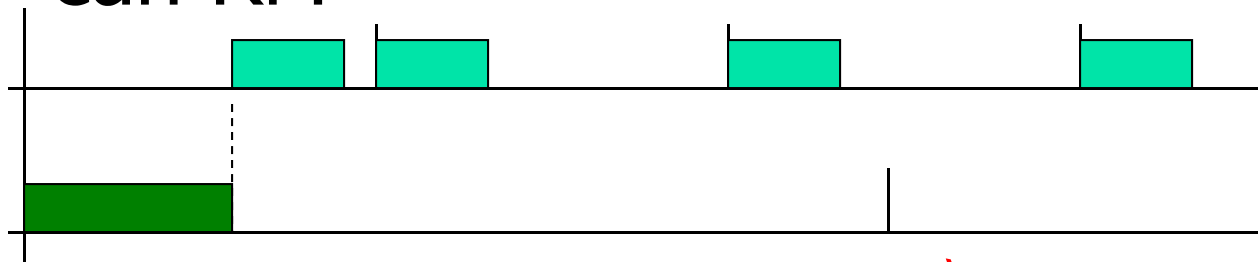
- If any other policy can meet deadlines so can RM



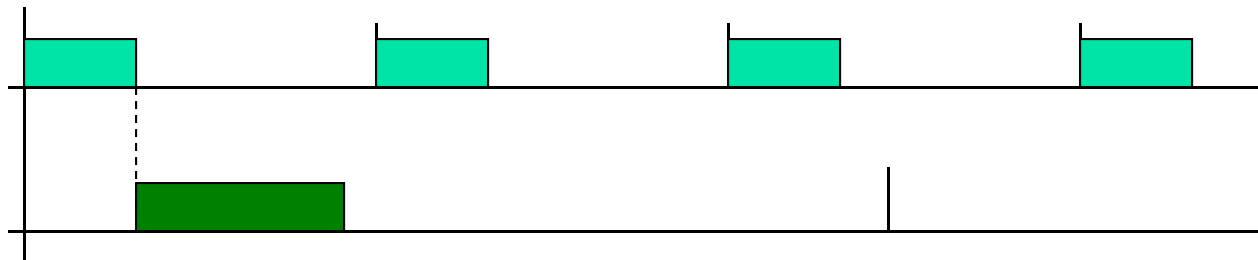
Policy X meets deadlines?

Optimality of Rate Monotonic

- If any other policy can meet deadlines so can RM



Policy X meets deadlines? *YES*
→ RM meets deadlines





Utilization Bounds

- Intuitively:
 - The lower the processor utilization, U , the easier it is to meet deadlines.
 - The higher the processor utilization, U , the more difficult it is to meet deadlines.
- Question: is there a threshold U_{bound} such that
 - When $U < U_{bound}$ deadlines are met
 - When $U > U_{bound}$ deadlines are missed

Example (Rate-Monotonic Scheduling)

Task 1

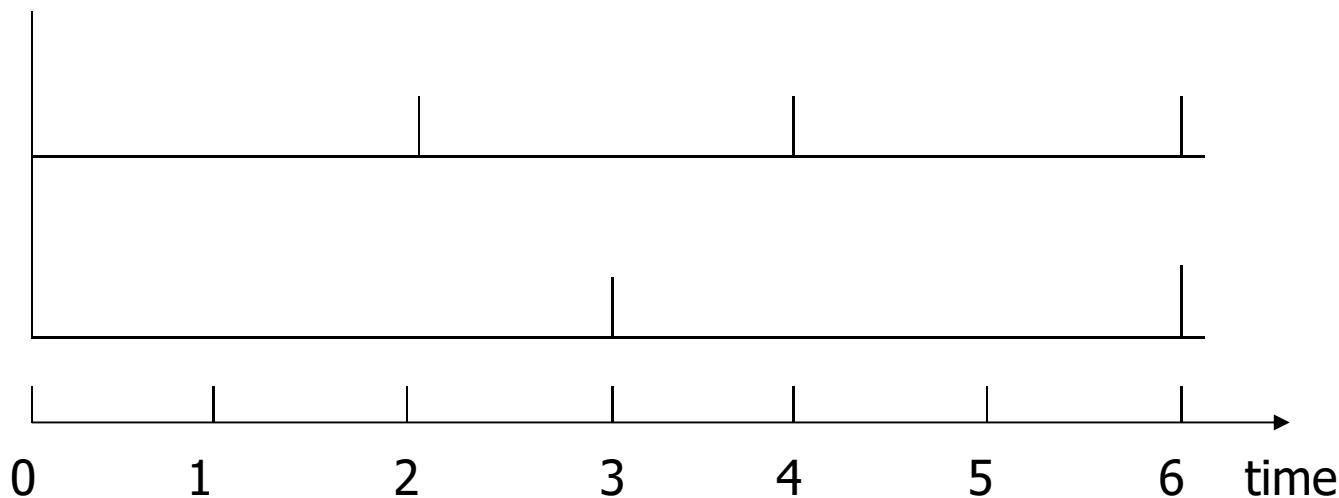
$$P_1=2$$

$$C_1=1$$

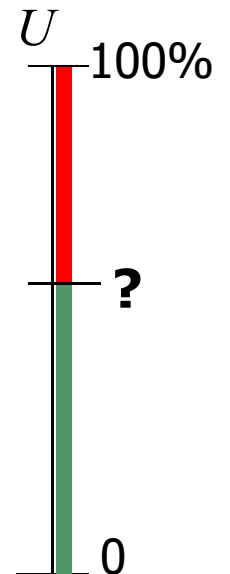
Task 2

$$P_2=3$$

$$C_2=1.01$$



$$U = \frac{C_1}{P_1} + \frac{C_2}{P_2} = \frac{1}{2} + \frac{1.01}{3} \approx 83.3\%$$



- Question: is there a threshold U_{bound} such that
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Example (Rate-Monotonic Scheduling)

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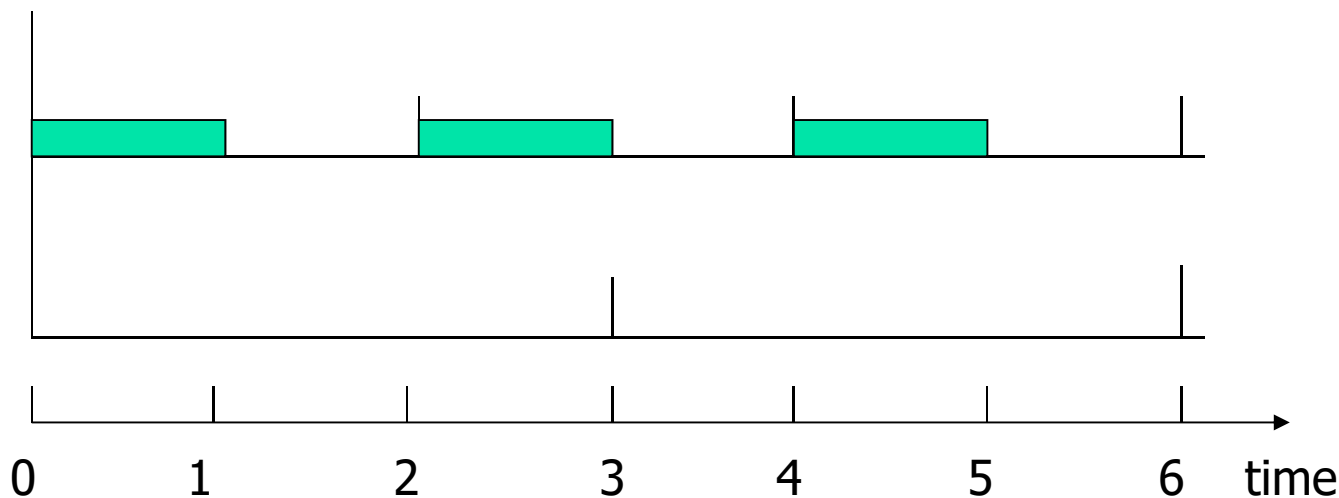
$$P_1=2$$

$$C_1=1$$

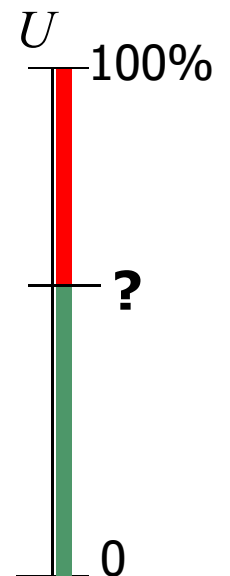
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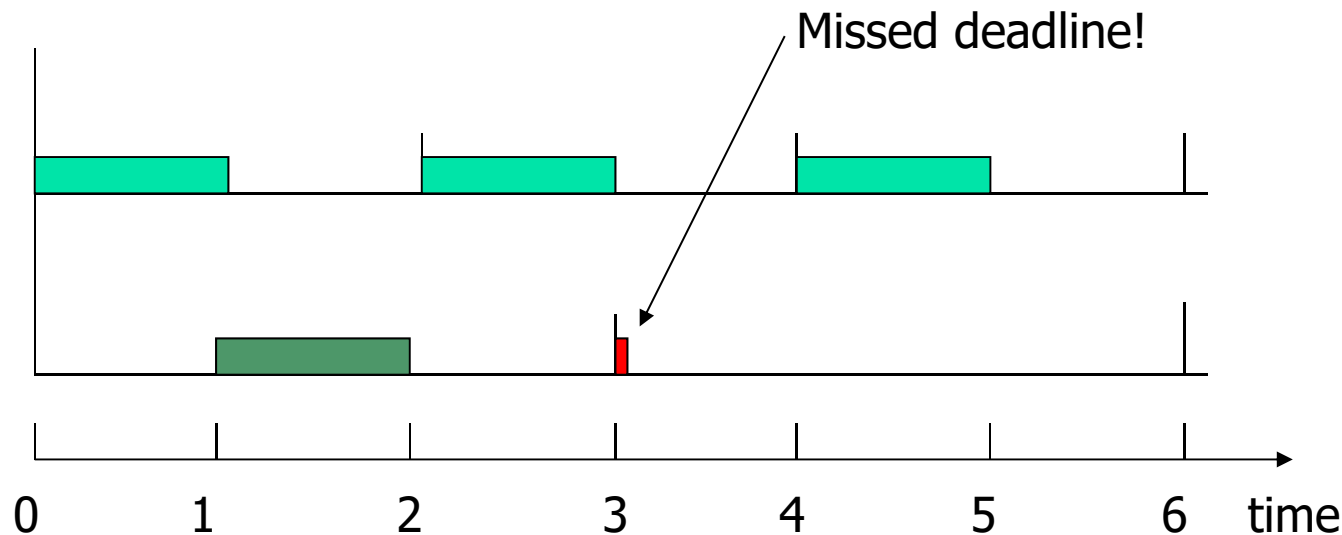
$$P_1=2$$

$$C_1=1$$

Task 2

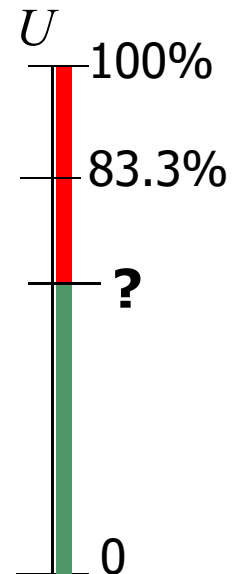
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$$C_2=1.01$$



$$U = \frac{C_1}{P_1} + \frac{C_2}{P_2} = \frac{1}{2} + \frac{1.01}{3} \approx 83.3\%$$

Unschedulable



- Question: is there a threshold U_{bound} such that
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Another Example (Rate-Monotonic Scheduling)

Task 1

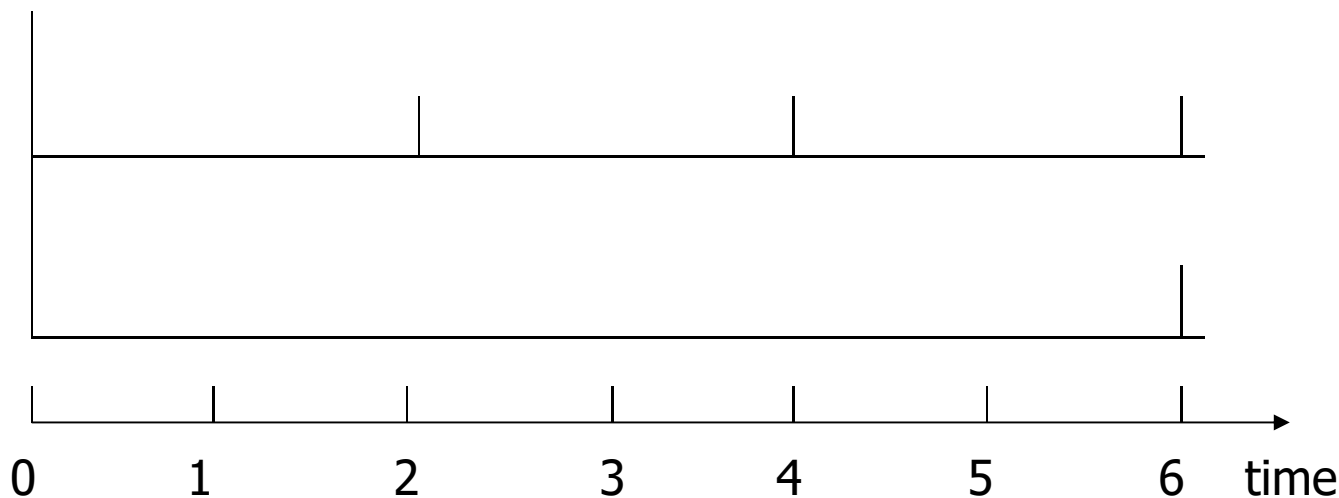
$$P_1=2$$

$$C_1=1$$

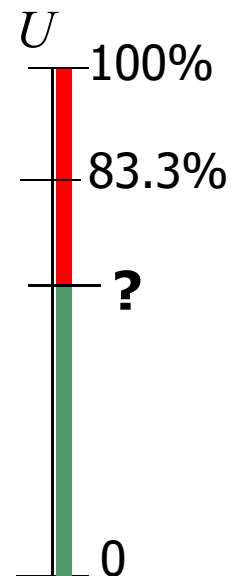
Task 2

$$P_2=6$$

$$C_2=2.4$$



$$U = \frac{C_1}{P_1} + \frac{C_2}{P_2} = \frac{1}{2} + \frac{2.4}{6} = 90\%$$



- Question: is there a threshold U_{bound} such that
 - When $U < U_{bound}$ deadlines are met
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Another Example (Rate-Monotonic Scheduling)

Task 1

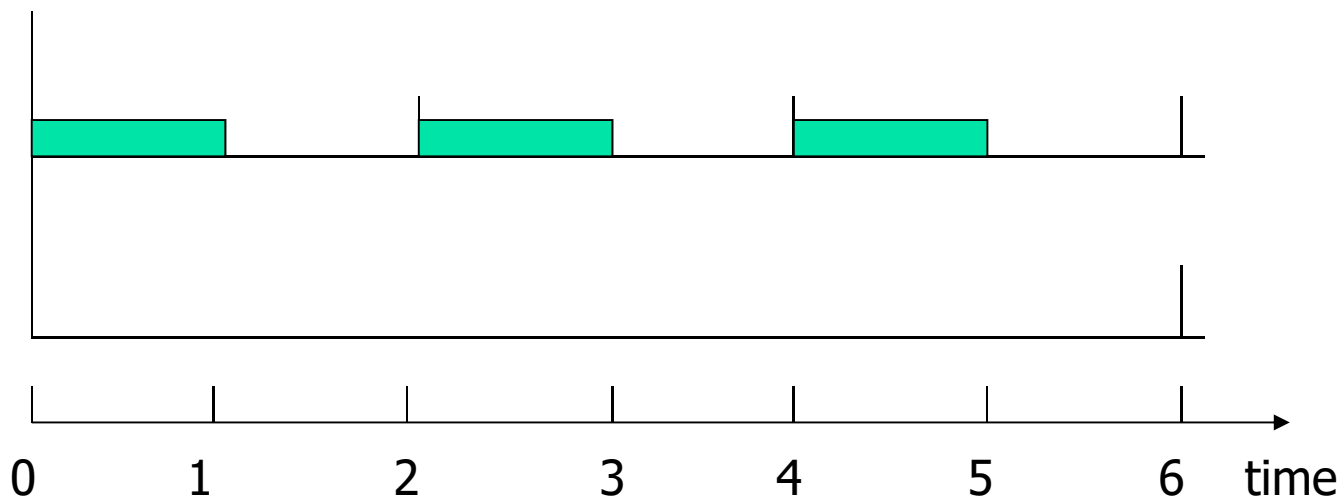
$$P_1=2$$

$$C_1=1$$

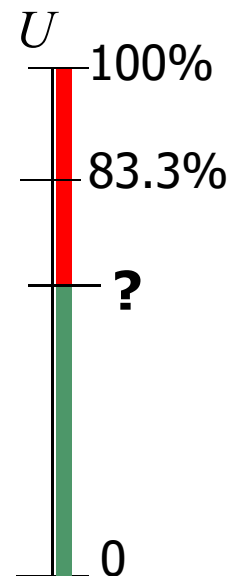
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Another Example (Rate-Monotonic Scheduling)

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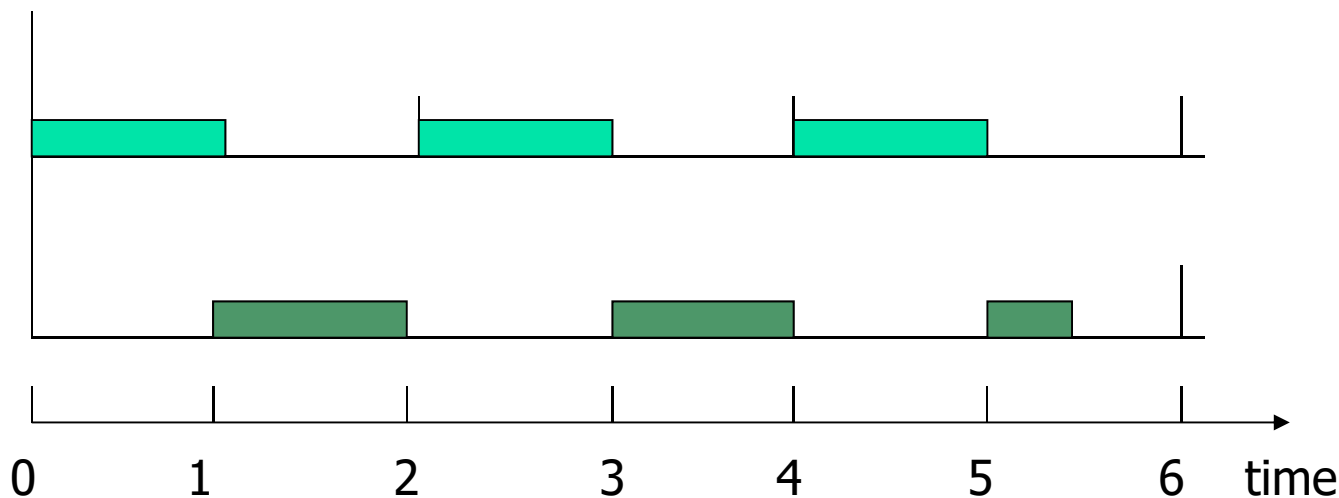
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Task 2

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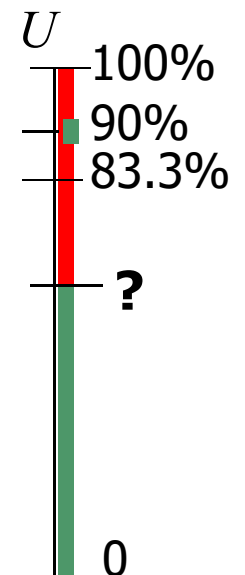
$$C_2=2.4$$



$$U = \frac{C_1}{P_1} + \frac{C_2}{P_2} = \frac{1}{2} + \frac{2.4}{6} = 90\%$$

Schedulable!

- Question: is there a threshold U_{bound} such that
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Another Example (Rate-Monotonic Scheduling)

Task 1

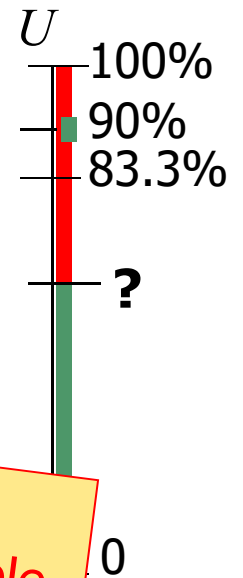
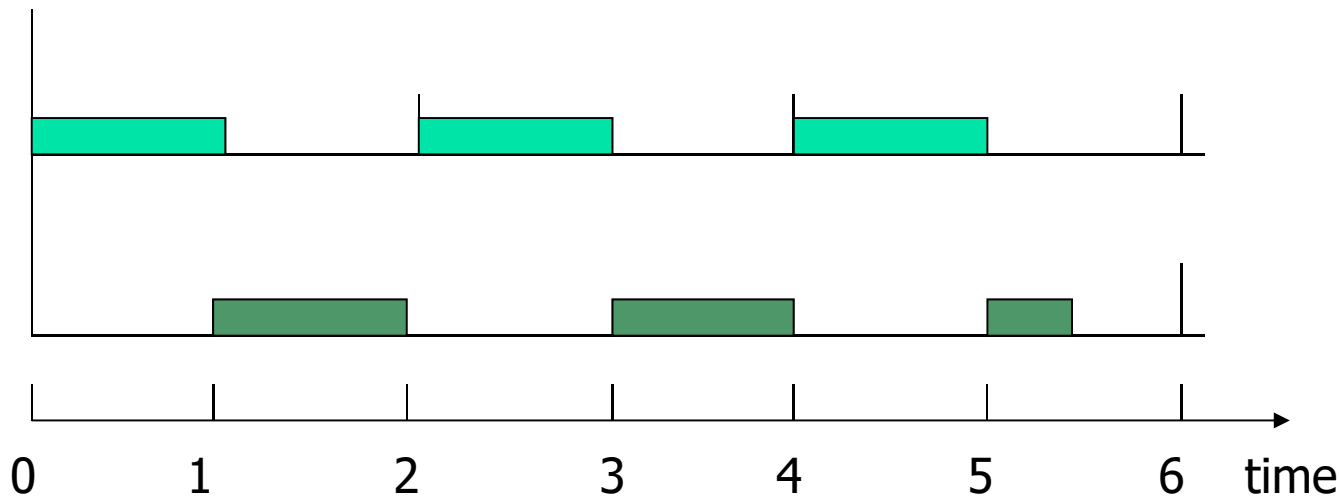
$$P_1=2$$

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Task 2

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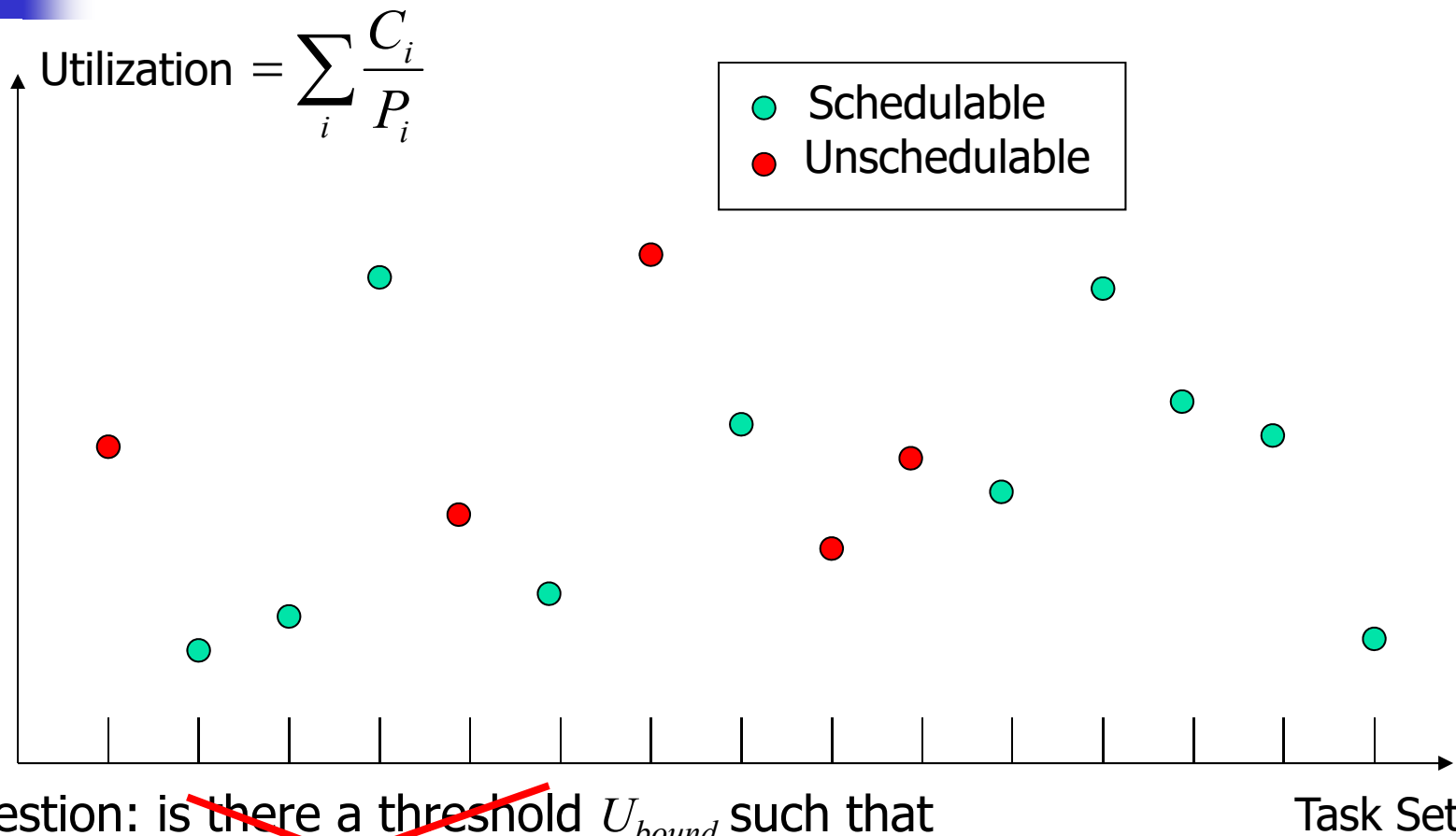
$$C_2=2.4$$



$$C_1 + C_2 = 1 + 2.4 = 90\%$$
 Schedulability depends on task set!
 No clean utilization threshold between schedulable and unschedulable task sets!

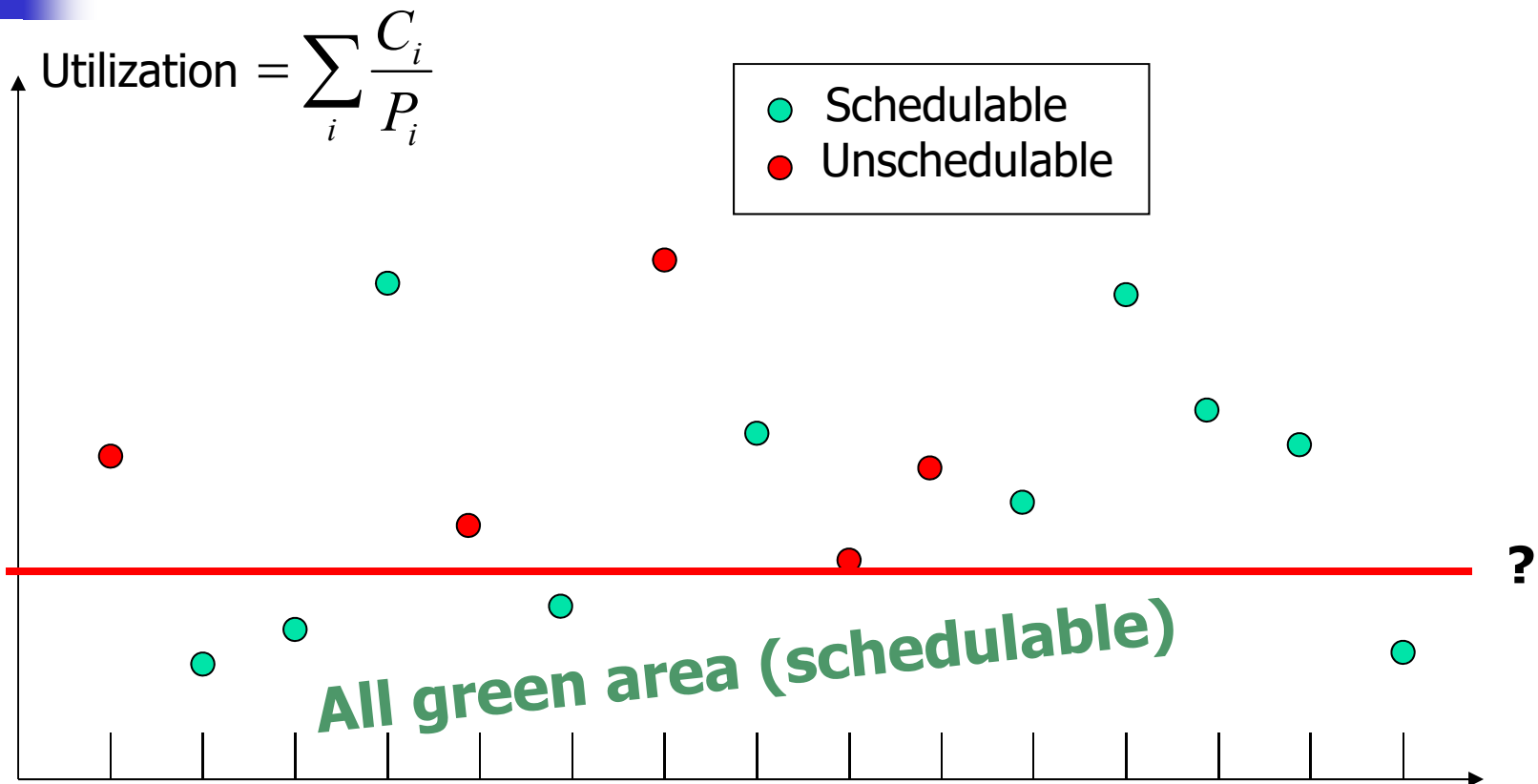
- Question: is there a threshold?
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A Conceptual View of Schedulability



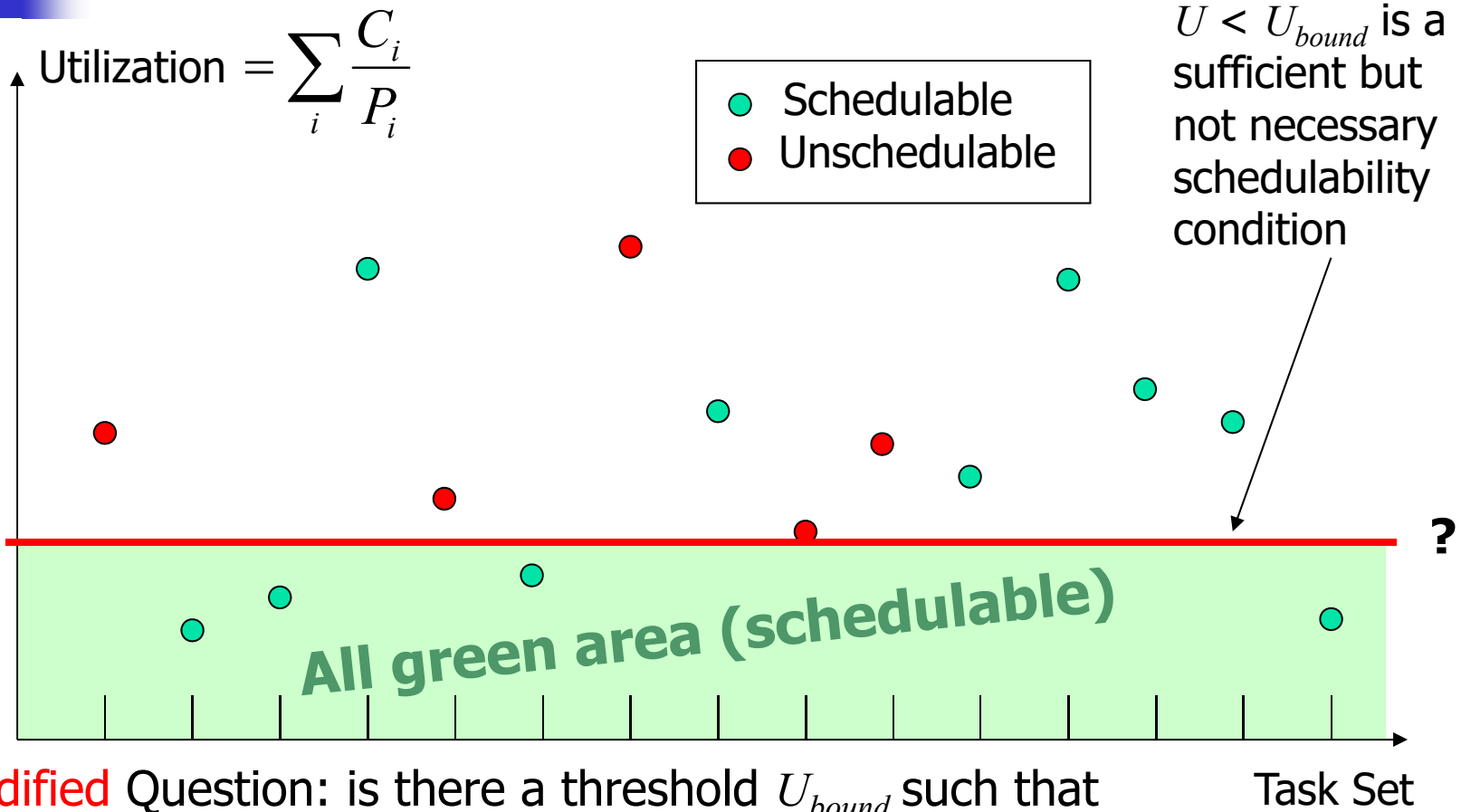
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A Conceptual View of Schedulability



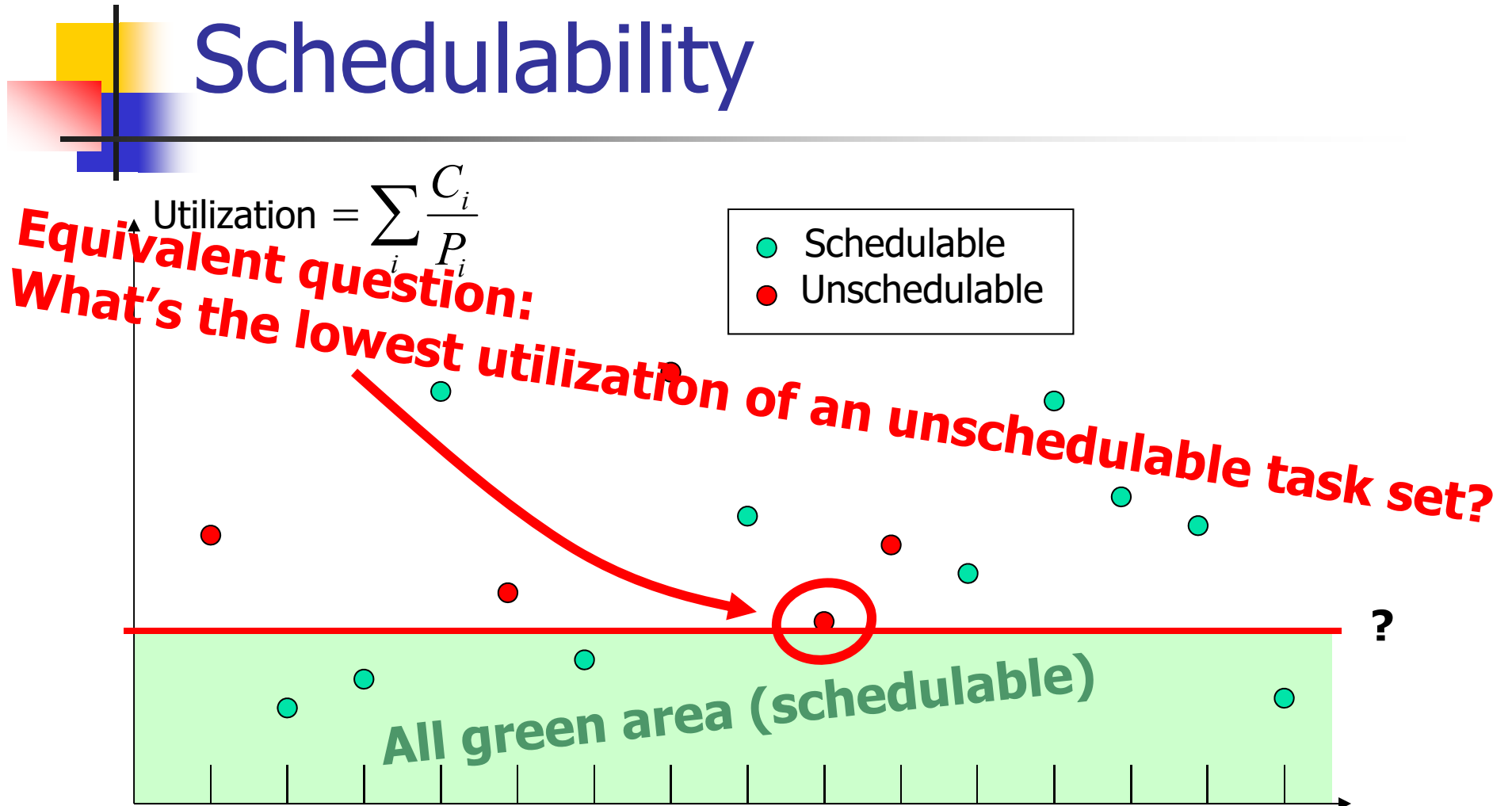
- **Modified** Question: is there a threshold U_{bound} such that
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A Conceptual View of Schedulability



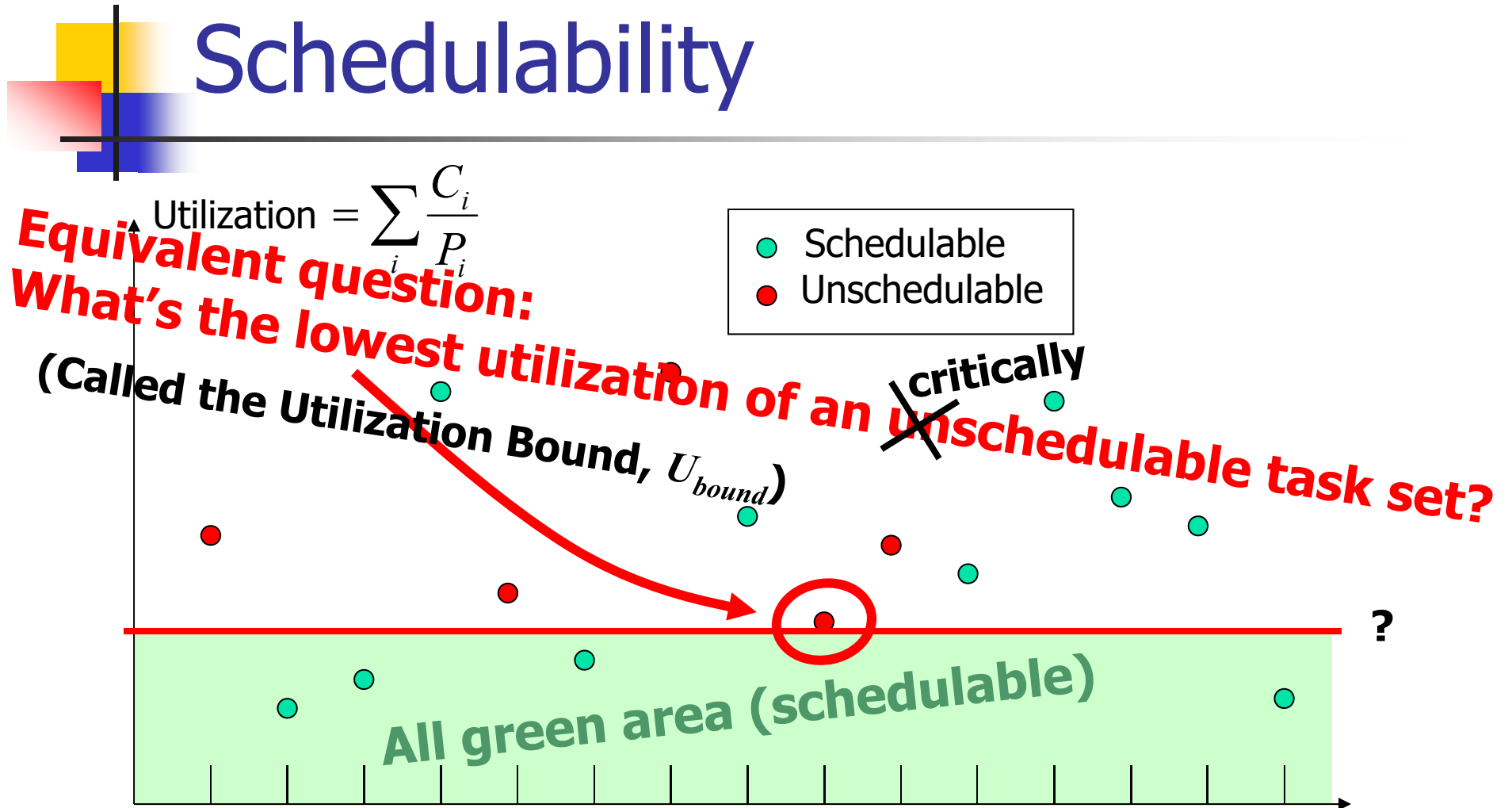
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A Conceptual View of Schedulability



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A Conceptual View of Schedulability



- **Modified** Question: is there a threshold U_{bound} such that
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The Schedulability Condition

For n independent periodic tasks with periods equal to deadlines, the utilization bound is:

$$U = n \left(2^{1/n} - 1 \right)$$

$$n \rightarrow \infty \quad U \rightarrow \ln 2$$



Done Today

