

# CS 440/ECE448 Lecture 9: Game Theory

Slides by Svetlana Lazebnik, 9/2016

Modified by Mark Hasegawa-Johnson, 2/2019

<b>Prisoner A \ Prisoner B</b>	<b>Prisoner B stays silent (<i>cooperates</i>)</b>	<b>Prisoner B betrays (<i>defects</i>)</b>
<b>Prisoner A stays silent (<i>cooperates</i>)</b>	Each serves 1 year	Prisoner A: 3 years Prisoner B: goes free
<b>Prisoner A betrays (<i>defects</i>)</b>	Prisoner A: goes free Prisoner B: 3 years	Each serves 2 years

[https://en.wikipedia.org/wiki/Prisoner's\\_dilemma](https://en.wikipedia.org/wiki/Prisoner's_dilemma)

# Game theory

- **Game theory** deals with systems of **interacting agents** where the **outcome** for an agent depends on the **actions of all the other agents**
  - Applied in sociology, politics, economics, biology, and, of course, AI
- **Agent design**: determining the **best strategy for a rational agent** in a given game
- **Mechanism design**: how to **set the rules of the game** to ensure a desirable outcome

Modelling behaviour

## Game theory in practice

Computing: Software that models human behaviour can make forecasts, outfox rivals and transform negotiations

Sep 3rd 2011 | from the print edition



Christian Montenegro

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# FACEBOOK DOESN'T MAKE AS MUCH MONEY AS IT COULD—ON PURPOSE



YOU CAN THINK of John Hegeman as Facebook's chief economist. He spends his days thinking about the economics of Facebook advertising.

That's an enormous thing. Facebook pulled in \$4.04 billion in the second quarter of this year. And the overall economy of Facebook advertising, as Hegeman describes it, is far larger. Advertising, you see, is very much a part of everything else on the world's largest social network. Hegeman doesn't just think about ads. He thinks about how ads fit with the rest of Facebook.

When he joined Facebook in 2007, after getting a master's in economics at Stanford University, Hegeman helped build the online auction that drives the company's advertising system. Auctions are the standard way that online services accept ads from advertisers and place them on web pages and inside smartphone apps. That's what Google uses with [AdWords](#), the system that serves up all those ads when you look for stuff on the company's Internet search engine. Advertisers bid (in dollars) for placement on the results page when you key in a particular word or group of words. But in building Facebook's advertising system, Hegeman and team took online auctions in a new direction.

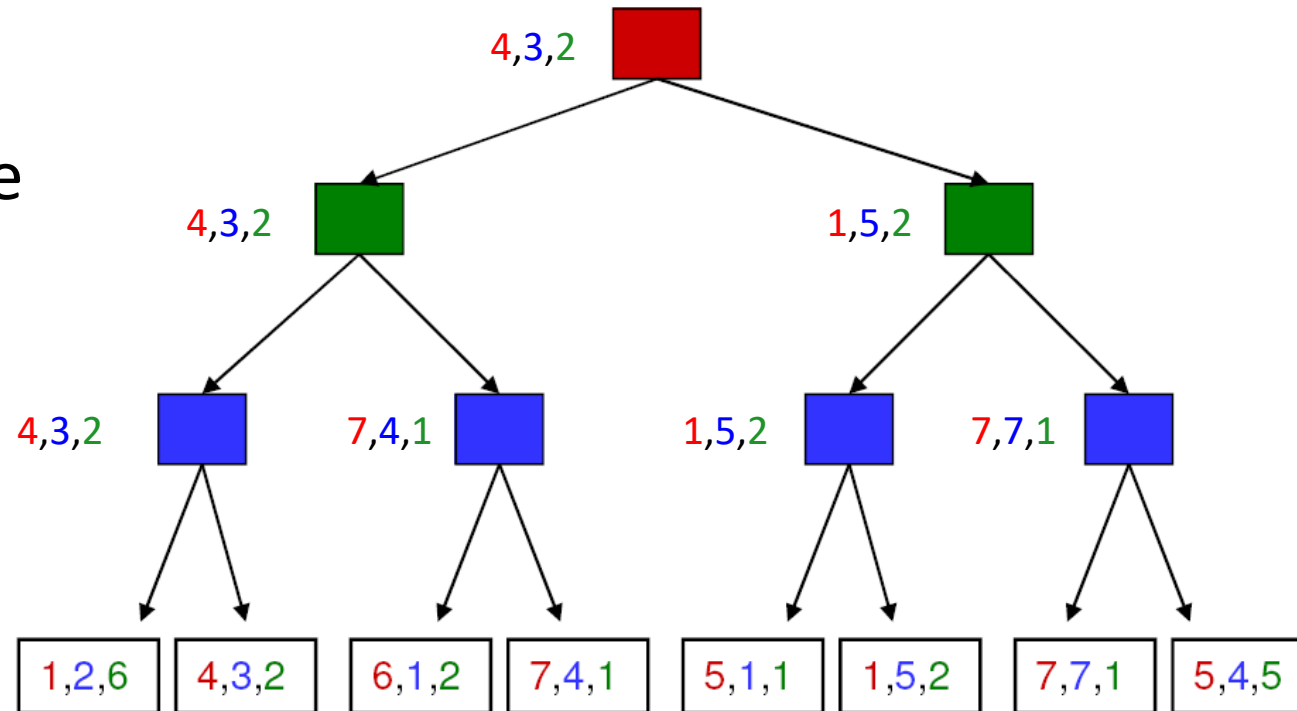
# Outline of today's lecture

- Nash equilibrium, Dominant strategy, and Pareto optimality
- Stag Hunt: Coordination Games
- Chicken: Anti-Coordination Games, Mixed Strategies
- The Ultimatum Game: Continuous and Repeated Games
- Mechanism Design: Inverse Game Theory

# Nash Equilibria, Dominant Strategies, and Pareto Optimal Solutions

# Recall: Multi-player, non-zero-sum game

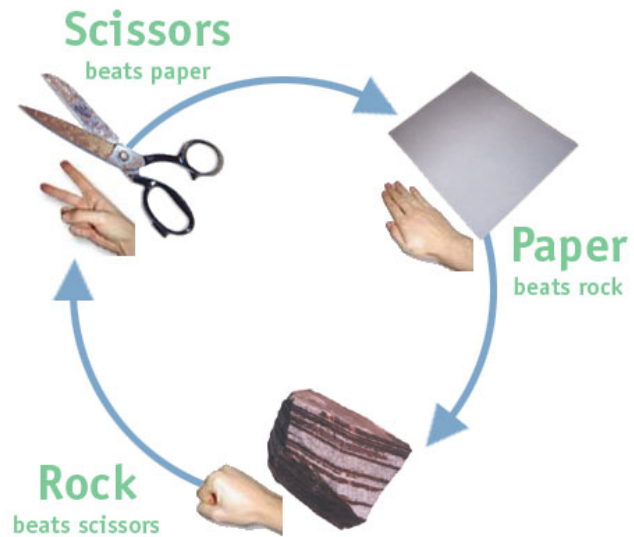
- Players act in sequence.
- Each player makes the move that is best for them, when it's their turn to move.








# Simultaneous single-move games

- Players must choose their actions at the same time, without knowing what the others will do
  - Form of partial observability



**Normal form** representation:

Player 1

			
	0,0	1,-1	-1,1
	-1,1	0,0	1,-1
	1,-1	-1,1	0,0

**Payoff matrix**

(Player 1's utility is listed first)

Is this a zero-sum game?

# Prisoner's dilemma

- Two criminals (Alice and Bob) have been arrested and the police visit them separately
- If **one player testifies** against the other and **the other refuses**, the **one who testified goes free** and the **one who refused gets a 10-year sentence**
- If **both players testify** against each other, they **each get a 5-year sentence**
- If **both refuse** to testify, they **each get a 1-year sentence**

	Alice: Testify	Alice: Refuse
Bob: Testify		
Bob: Refuse		

# Prisoner's dilemma

- Alice's reasoning:
  - Suppose Bob testifies. Then I get 5 years if I testify and 10 years if I refuse. So I should testify.
  - Suppose Bob refuses. Then I go free if I testify, and get 1 year if I refuse. So I should testify.
- **Nash equilibrium:** A pair of strategies such that **no player can get a bigger payoff by switching strategies, provided the other player sticks with the same strategy**
  - (Testify, Testify) is a Nash equilibrium

	Alice: Testify	Alice: Refuse	
Bob: Testify	-5,-5	-10,0	-5 > -10
Bob: Refuse	0,-10	-1,-1	0 > -1
	-5 > -10	0 > -1	

# Prisoner's dilemma

- **Dominant strategy:** A strategy whose outcome is better for the player *regardless of the strategy chosen by the other player*.
  - **TESTIFY!**
- **Pareto optimal outcome:** It is impossible to make one of the players better off without making another one worse off.
  - (Testify, Refuse)
  - (Refuse, Refuse)
  - (Refuse, Testify)
- Other games can be constructed in which there is no dominant strategy – we'll see some later

	Alice: Testify	Alice: Refuse
Bob: Testify	-5,-5	-10,0
Bob: Refuse	0,-10	-1,-1

# Prisoner's dilemma in real life

- Price war
- Arms race
- Steroid use
- [Diner's dilemma](#)
- Collective action in politics

	Defect	Cooperate
Defect	Lose – lose	Lose big – win big
Cooperate	Win big – lose big	Win – win

# Is there any way to get a better answer?

- Superrationality
  - Assume that the answer to a symmetric problem will be the same for both players
  - Maximize the payoff to each player while considering only identical strategies
  - Not a conventional model in game theory
  - ... same thing as the Categorical Imperative?
- Repeated games
  - If the number of rounds is fixed and known in advance, the equilibrium strategy is still to defect
  - If the number of rounds is unknown, cooperation may become an equilibrium strategy

# The Stag Hunt: Coordination Games

# Stag hunt



	Hunter 1: Stag	Hunter 1: Hare
Hunter 2: Stag	2,2	1,0
Hunter 2: Hare	0,1	1,1



- Both hunters cooperate in hunting for the stag  
→ each gets to take home half a stag  
(which is worth more than a hare)
- Both hunters defect, and hunt for rabbit instead  
→ each gets to take home a rabbit
- One cooperates, one defects → the defector gets a bunny,  
the cooperator gets nothing at all



# Stag hunt



	Hunter 1: Stag	Hunter 1: Hare
Hunter 2: Stag	2,2	1,0
Hunter 2: Hare	0,1	1,1



- What is the Pareto Optimal solution?
- Is there a Nash Equilibrium?
- Is there a Dominant Strategy for either player?
- Model for cooperative activity under conditions of incomplete information (the issue: trust)

# Prisoner's dilemma vs. stag hunt

## Prisoner's dilemma

	Cooperate	Defect
Cooperate	Win – win	Win big – lose big
Defect	Lose big – win big	Lose – lose

Players **improve** their winnings by **defecting unilaterally**

## Stag hunt

	Cooperate	Defect
Cooperate	Win big – win big	Win – lose
Defect	Lose – win	Win – win

Players **reduce** their winnings by **defecting unilaterally**

# Chicken: Anti-Coordination Games, Mixed Strategies

# Game of Chicken



- Two players each bet \$1000 that the other player will chicken out
- Outcomes:
  - If one player chickens out, the other wins \$1000
  - If both players chicken out, neither wins anything
  - If neither player chickens out, they both lose \$10,000 (the cost of the car)

# Prisoner's dilemma vs. Chicken

## Prisoner's dilemma

	Cooperate	Defect
Cooperate	Win – win	Win big – Lose big
Defect	Lose big – Win big	Lose – Lose

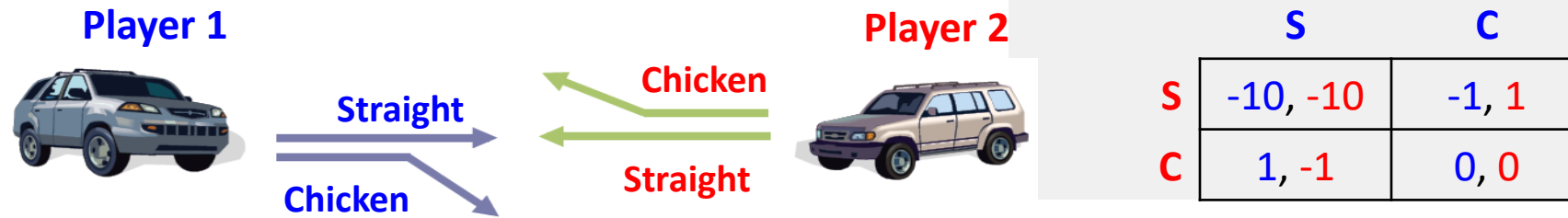
Players can't improve  
their winnings by  
*unilaterally* cooperating

## Chicken

	Chicken	Straight
Chicken	Nil – Nil	Win – Lose
Straight	Lose – Win	Lose big – Lose big

The best strategy is always  
the **opposite of what the  
other player does**

# Game of Chicken



- Is there a dominant strategy for either player?
- Is there a Nash equilibrium?  
(straight, chicken) or (chicken, straight)
- *Anti-coordination* game: it is mutually beneficial for the two players to choose different strategies
  - Model of escalated conflict in humans and animals (hawk-dove game)
- How are the players to decide what to do?
  - Pre-commitment or threats
  - Different roles: the “hawk” is the territory owner and the “dove” is the intruder, or vice versa

# Mixed strategy equilibria



- **Mixed strategy:** a player chooses between the moves according to a probability distribution
- Suppose each player chooses S with probability  $1/10$ . Is that a Nash equilibrium?
- Consider payoffs to P1 while keeping P2's strategy fixed
  - The payoff of P1 choosing S is  $(1/10)(-10) + (9/10)1 = -1/10$
  - The payoff of P1 choosing C is  $(1/10)(-1) + (9/10)0 = -1/10$
  - Can P1 change their strategy to get a better payoff?
  - Same reasoning applies to P2

# Finding mixed strategy equilibria

	P1: Choose S with prob. $p$	P1: Choose C with prob. $1-p$
P2: Choose S with prob. $q$	-10, -10	-1, 1
P2: Choose C with prob. $1-q$	1, -1	0, 0

- Expected payoffs for P1 given P2's strategy:
  - P1 chooses S:  $q(-10) + (1-q)1 = -11q + 1$
  - P1 chooses C:  $q(-1) + (1-q)0 = -q$
- In order for P2's strategy to be part of a Nash equilibrium, P1 has to be indifferent between its two actions:
  - $-11q + 1 = -q$  or  $q = 1/10$
  - Similarly,  $p = 1/10$



# Existence of Nash equilibria

- Any game with a finite set of actions has at least one Nash equilibrium (which may be a mixed-strategy equilibrium)
- If a player has a dominant strategy, there exists a Nash equilibrium in which the player plays that strategy and the other player plays the *best response* to that strategy
- If both players have *strictly dominant* strategies, there exists a Nash equilibrium in which they play those strategies

# Computing Nash equilibria

- For a two-player zero-sum game, simple linear programming problem
- For non-zero-sum games, the algorithm has worst-case running time that is exponential in the number of actions
- For more than two players, and for sequential games, things get pretty hairy

# Nash equilibria and rational decisions

- If a game has a *unique* Nash equilibrium, it will be adopted if each player
  - is rational and the payoff matrix is accurate
  - doesn't make mistakes in execution
  - is capable of computing the Nash equilibrium
  - believes that a deviation in strategy on their part will not cause the other players to deviate
  - there is *common knowledge* that all players meet these conditions

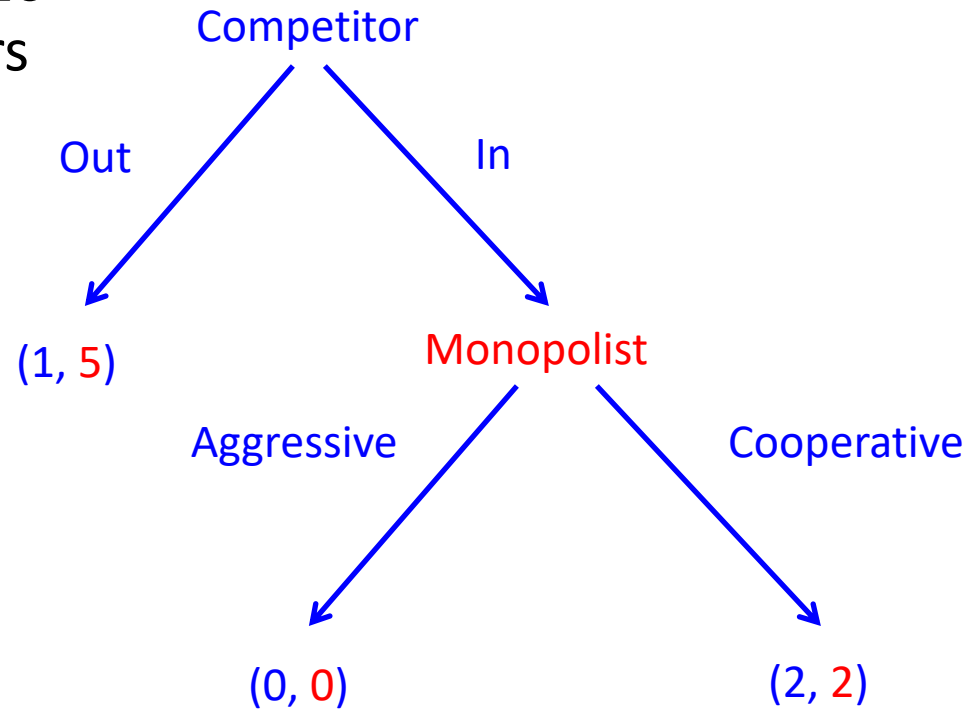
# The Ultimatum Game: Continuous and Repeated Games

# Continuous actions: Ultimatum game

- Alice and Bob are given a sum of money  $S$  to divide
  - Alice picks  $A$ , the amount she wants to keep for herself
  - Bob picks  $B$ , the smallest amount of money he is willing to accept
  - If  $S - A \geq B$ , Alice gets  $A$  and Bob gets  $S - A$
  - If  $S - A < B$ , both players get nothing
- What is the Nash equilibrium?
  - Alice offers Bob the smallest amount of money he will accept:  
 $S - A = B$
  - Alice and Bob both want to keep the full amount:  $A = S, B = S$   
(both players get nothing)
- How would humans behave in this game?
  - If Bob perceives Alice's offer as unfair, Bob will be likely to refuse
  - Is this rational?
    - Maybe Bob gets some positive utility for "punishing" Alice?

# Sequential/repeated games and threats: Chain store paradox

- A monopolist has branches in 20 towns and faces 20 competitors successively
  - Threat: respond to “in” with “aggressive”



# Mechanism Design: Inverse Game Theory

# Mechanism design (inverse game theory)

- Assuming that agents pick rational strategies, how should we design the game to achieve a socially desirable outcome?
- We have multiple agents and a **center** that collects their choices and determines the outcome



# Auctions

- Goals
  - Maximize revenue to the seller
  - Efficiency: make sure the buyer who values the goods the most gets them
  - Minimize transaction costs for buyer and sellers

# Ascending-bid auction

- What's the optimal strategy for a buyer?
  - Bid until the current bid value exceeds your *private value*
- Usually revenue-maximizing and efficient, unless the reserve price is set too low or too high
- Disadvantages
  - Collusion
  - Lack of competition
  - Has high communication costs

# Sealed-bid auction

- Each buyer makes a single bid and communicates it to the auctioneer, but not to the other bidders
  - Simpler communication
  - More complicated decision-making: the strategy of a buyer depends on what they believe about the other buyers
  - Not necessarily efficient
- **Sealed-bid second-price auction:** the winner pays the price of the second-highest bid
  - Let  $V$  be your private value and  $B$  be the highest bid by any other buyer
  - If  $V > B$ , your optimal strategy is to bid above  $B$  – in particular, bid  $V$
  - If  $V < B$ , your optimal strategy is to bid below  $B$  – in particular, bid  $V$
  - Therefore, your dominant strategy is to bid  $V$
  - This is a **truth revealing** mechanism

# Dollar auction

A malevolent twist on the second-price auction:

- Highest bidder gets to buy the object, and pays whatever they bid
- Second-highest bidder is required to pay whatever they bid, but gets nothing at all in return
- Dramatization: <https://www.youtube.com/watch?v=pA-SNscNADk>

# Dollar auction

- A dollar bill is auctioned off to the highest bidder, but the second-highest bidder has to pay the amount of his last bid
  - Player 1 bids 1 cent
  - Player 2 bids 2 cents
  - ...
  - Player 2 bids 98 cents
  - Player 1 bids 99 cents
    - If Player 2 passes, he loses 98 cents, if he bids \$1, he might still come out even
  - So Player 2 bids \$1
    - Now, if Player 1 passes, he loses 99 cents, if he bids \$1.01, he only loses 1 cent
  - ...
- What went wrong?
  - When figuring out the expected utility of a bid, a rational player should take into account the future course of the game
- What if Player 1 starts by bidding 99 cents?

# Regulatory mechanism design: Tragedy of the commons

- States want to set their policies for controlling emissions
  - Each state can reduce their emissions at a cost of  $-10$  or continue to pollute at a cost of  $-5$
  - If a state decides to pollute,  $-1$  is added to the utility of every other state
- What is the dominant strategy for each state?
  - Continue to pollute
  - Each state incurs cost of  $-5-49 = -54$
  - If they all decided to deal with emissions, they would incur a cost of only  $-10$  each
- Mechanism for fixing the problem:
  - Tax each state by the total amount by which they reduce the global utility (**externality cost**)
  - This way, continuing to pollute would now cost  $-54$

# Review: Game theory

- Normal form representation of a game
- Dominant strategies
- Nash equilibria
- Pareto optimal outcomes
- Pure strategies and mixed strategies
- Examples of games
- Mechanism design
  - Auctions: ascending bid, sealed bid, sealed bid second-price, “dollar auction”