CS447: Natural Language Processing
http://courses.engr.illinois.edu/cs447

Lecture 4: Smoothing

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Last lecture's key concepts

Basic probability review:

joint probability, conditional probability

Probability models

Independence assumptions

Parameter estimation: relative frequency estimation (aka maximum likelihood estimation)

Language models

N-gram language models:

unigram, bigram, trigram…

N-gram language models

A language model is a distribution $P(\mathrm{W})$
over the (infinite) set of strings in a language L

To define a distribution over this infinite set,
we have to make independence assumptions.

N-gram language models assume that each word $\mathrm{w_i}$
depends only on the $\mathrm{n-1}$ preceding words:

$$P_{\text{n-gram}}(w_1 \ldots w_T) := \prod_{i=1..T} P(\mathrm{w_i} \mid \mathrm{w_{i-1}}, \ldots, \mathrm{w_{i-(n-1)}})$$

$$P_{\text{unigram}}(w_1 \ldots w_T) := \prod_{i=1..T} P(\mathrm{w_i})$$

$$P_{\text{bigram}}(w_1 \ldots w_T) := \prod_{i=1..T} P(\mathrm{w_i} \mid \mathrm{w_{i-1}})$$

$$P_{\text{trigram}}(w_1 \ldots w_T) := \prod_{i=1..T} P(\mathrm{w_i} \mid \mathrm{w_{i-1}}, \mathrm{w_{i-2}})$$

Quick note re. notation

Consider the sentence $W = $ "John loves Mary"

For a trigram model we could write:

$$P(w_3 = Mary \mid w_1\, w_2 = \text{“}John\ loves\text{”})$$

This notation implies that we treat the preceding bigram $w_1 w_2$ as *one* single conditioning variable $P(X \mid Y)$

Instead, we typically write:

$$P(w_3 = Mary \mid w_2 = loves,\, w_1 = John)$$

Although this is less readable (*John loves* → *loves, John*), this notation gives us more flexibility, since it implies that we treat the preceding bigram $w_1 w_2$ as *two* conditioning variables $P(X \mid Y, Z)$

Parameter estimation (training)

Parameters: the actual probabilities (numbers)

$P(w_i = \text{'the'} \mid w_{i-1} = \text{'on'}) = 0.0123$

We need (a large amount of) text as training data to estimate the parameters of a language model.

The most basic estimation technique:
relative frequency estimation (= counts)

$P(w_i = \text{'the'} \mid w_{i-1} = \text{'on'}) = C(\text{'on the'}) / C(\text{'on'})$

This assigns *all* probability mass to events in the training corpus.

Also called Maximum Likelihood Estimation (MLE)

Testing: unseen events will occur

Recall the Shakespeare example:

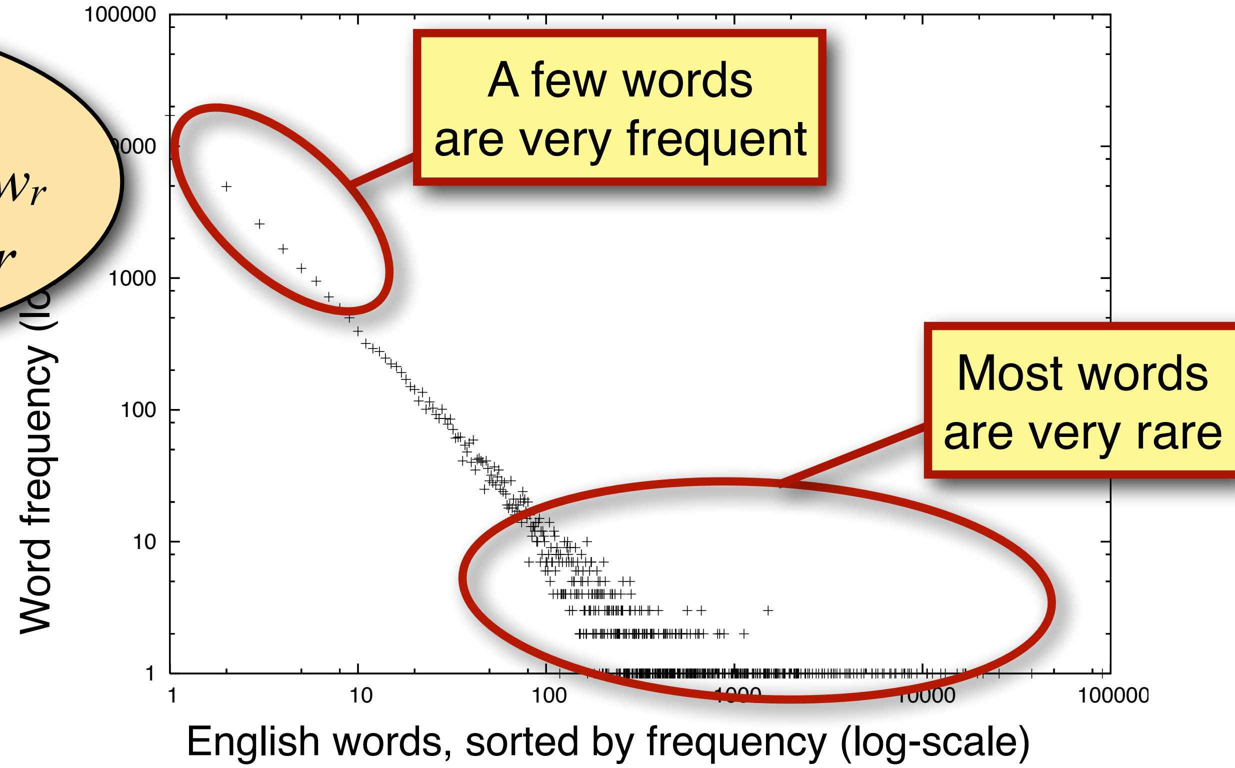
Only 30,000 word types occurred.
Any word that does not occur in the training data has zero probability!

Only 0.04% of all possible bigrams occurred.
Any bigram that does not occur in the training data has zero probability!

Zipf’s law: the long tail

How many words occur once, twice, 100 times, 1000 times?

the *r*-th most common word w_r has $P(w_r) \propto 1/r$



$w_1 = \textit{the}$, $w_2 = \textit{to}$, …., $w_{5346} = \textit{computer}$, ...

In natural language:

- A small number of events (e.g. words) occur with high frequency
- A large number of events occur with very low frequency

So….

… we can't actually evaluate our MLE models on unseen test data (or system output)…

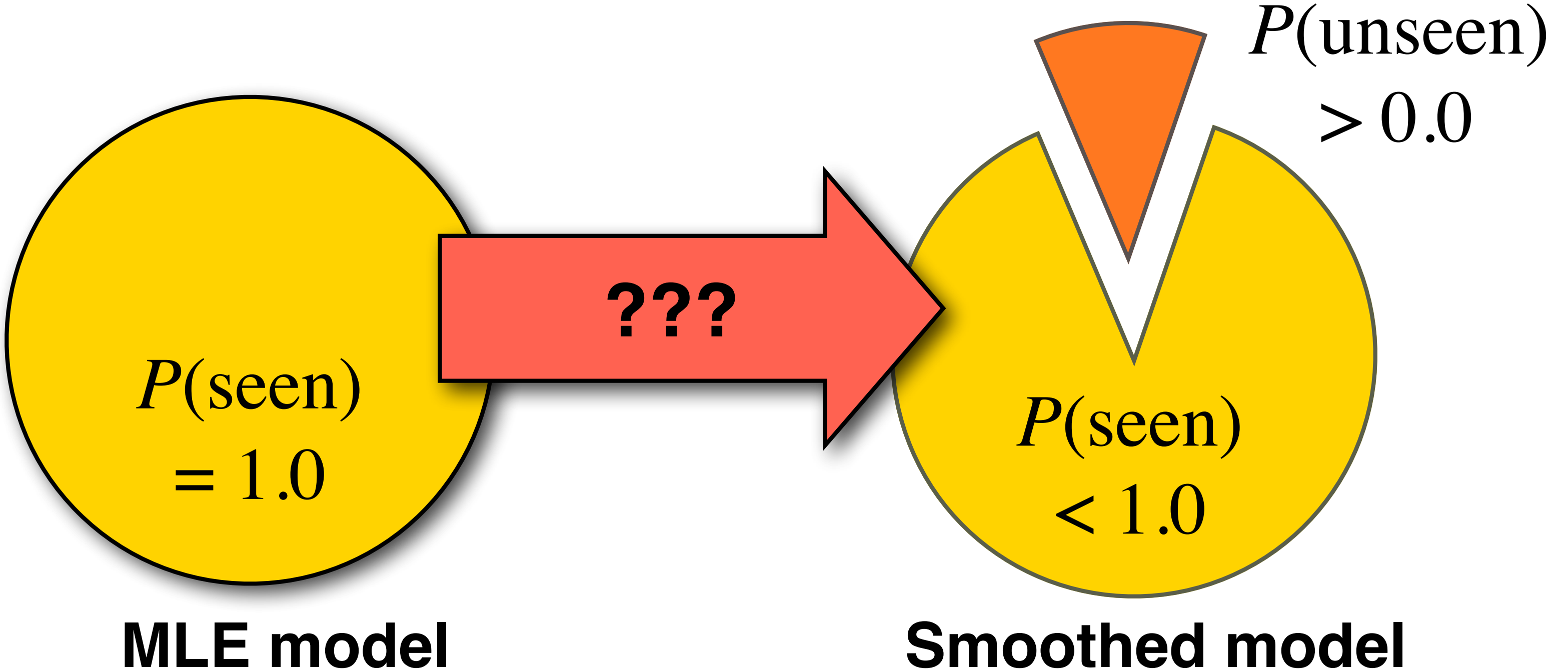
… because both are likely to contain words/n-grams that these models assign zero probability to.

We need language models that assign some probability mass to unseen words and n-grams.

Today's lecture

How can we design language models*
that can deal with previously unseen events?

*actually, probabilistic models in general



Dealing with unseen events

Relative frequency estimation assigns all probability mass to events in the training corpus

But we need to reserve *some* probability mass to events that don't occur in the training data

- Unseen events = new words, new bigrams

Important questions:

- What possible events are there?
- How much probability mass should they get?

What unseen events may occur?

Simple distributions:

$$P(X = x)$$

(e.g. unigram models)

Possibility:

The outcome x has not occurred during training (i.e. is unknown):

- We need to reserve mass in $P(X)$ for x

Questions:

- What outcomes x are possible?
- How much mass should they get?

What unseen events may occur?

Simple *conditional* distributions:

$$P(X = x \mid Y = y)$$

(e.g. bigram models)

Case 1: The outcome x has been seen,
but not in the context of $Y = y$:

- We need to reserve mass in $P(X \mid Y{=}y)$ for $X = x$

Case 2: The conditioning variable y has not been seen:

- We have no $P(X \mid Y = y)$ distribution.
- We need to drop the conditioning variable $Y = y$ and use $P(X)$ instead.

What unseen events may occur?

Complex conditional distributions
(with multiple conditioning variables)

$$P(X = x \mid Y = y, Z = z)$$

(e.g. trigram models)

Case 1: The outcome $X = x$ was seen, but not in the context of $(Y{=}y, Z{=}z)$:

- We need to reserve mass in $P(X \mid Y = y, Z = z)$

Case 2: The joint conditioning event $(Y{=}y, Z{=}z)$ hasn't been seen:

- We have no $P(X \mid Y{=}y, Z{=}z)$ distribution.
- But we can drop z and use $P(X \mid Y{=}y)$ instead.

Examples

Training data: The wolf is an endangered species

Test data: The wallaby is endangered

Unigram	Bigram	Trigram
$P(\text{the})$	$P(\text{the} \mid \text{<s>})$	$P(\text{the} \mid \text{<s>})$
$\times\, P(\text{wallaby})$	$\times\, P(\text{wallaby} \mid \text{the})$	$\times\, P(\text{wallaby} \mid \text{the, <s>})$
$\times\, P(\text{is})$	$\times\, P(\text{is} \mid \text{wallaby})$	$\times\, P(\text{is} \mid \text{wallaby, the})$
$\times\, P(\text{endangered})$	$\times\, P(\text{endangered} \mid \text{is})$	$\times\, P(\text{endangered} \mid \text{is, wallaby})$

- **Case 1:** $P(\text{wallaby})$, $P(\text{wallaby} \mid \text{the})$, $P(\text{wallaby} \mid \text{the, <s>})$:
  What is the probability of an unknown word (in any context)?
- **Case 2:** $P(\text{endangered} \mid \text{is})$
  What is the probability of a known word in a known context, if that word hasn’t been seen in that context?
- **Case 3:** $P(\text{is} \mid \text{wallaby})$ $P(\text{is} \mid \text{wallaby, the})$ $P(\text{endangered} \mid \text{is, wallaby})$:
  What is the probability of a known word in an unseen context?

Smoothing: Reserving mass in $P(X)$ for unseen events

Dealing with unknown words: The simple solution

Training:

- Assume a fixed vocabulary
  (e.g. all words that occur at least twice (or n times) in the corpus)
- Replace all other words by a token <UNK>
- Estimate the model on this corpus.

Testing:

- Replace all unknown words by <UNK>
- Run the model.

This requires a large training corpus to work well.

Dealing with unknown events

Use a different estimation technique:

- Add-1(Laplace) Smoothing
- Good-Turing Discounting

Idea: *Replace MLE estimate* $P(w) = \frac{C(w)}{N}$

Combine a complex model with a simpler model:

- Linear Interpolation
- Modified Kneser-Ney smoothing

Idea: use bigram probabilities of w_i $P(w_i|w_{i-1})$
to calculate trigram probabilities of w_i $P(w_i|w_{i-n}...w_{i-1})$

Add-1 (Laplace) smoothing

Assume every (seen or unseen) event occurred once more than it did in the training data.

Example: unigram probabilities

Estimated from a corpus with N tokens and a vocabulary (number of word types) of size V.

$$\text{MLE} \quad P(w_i) = \frac{C(w_i)}{\sum_j C(w_j)} = \frac{C(w_i)}{N}$$

$$\text{Add One} \quad P(w_i) = \frac{C(w_i)+\mathbf{1}}{\sum_j (C(w_j)+\mathbf{1})} = \frac{C(w_i)+\mathbf{1}}{N+\mathbf{V}}$$

Bigram counts

Original:

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Smoothed:

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Bigram probabilities

Original:

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Smoothed:

	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

Problem:

Add-one moves too much probability mass from seen to unseen events!

Reconstituting the counts

We can "reconstitute" pseudo-counts c^* for our training set of size N from our estimate:

Unigrams:

$$c_i^* = P(w_i) \cdot N$$

$P(w_i)$: probability that the next word is w_i.
N: number of word tokens we generate

$$= \frac{C(w_i)+1}{N+V} \cdot N$$

Plug in the model definition of $P(w_i)$
V: size of vocabulary

$$= (C(w_i)+1) \cdot \frac{N}{N+V}$$

Rearrange
(to see dependence on N and V)

Bigrams:

$$c^*(w_i|w_{i-1}) = P(w_i|w_{i-1}) \cdot C(w_{i-1})$$

$P(w_{i-1}w_i)$: probability of bigram "$w_{i-1}w_i$".
$C(w_{i-1})$: frequency of w_{i-1} (in training data)

$$= \frac{C(w_{i-1}w_i)+1}{C(w_{i-1})+V} \cdot C(w_{i-1})$$

Plug in the model definition of $P(w_i \mid w_{i-1})$

Reconstituted Bigram counts

Original:

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Reconstituted:

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Summary: Add-One smoothing

Advantage:

Very simple to implement

Disadvantage:

Takes away too much probability mass from seen events.

Assigns too much total probability mass to unseen events.

The Shakespeare example
(V = 30,000 word types; '*the*' occurs 25,545 times)
Bigram probabilities for 'the …':

$$P(w_i|w_{i-1} = the) = \frac{C(the\ w_i)+\mathbf{1}}{25,545+\mathbf{30,000}}$$

Add-K smoothing

Variant of Add-One smoothing:
For any $k > 0$ (typically, $k < 1$)

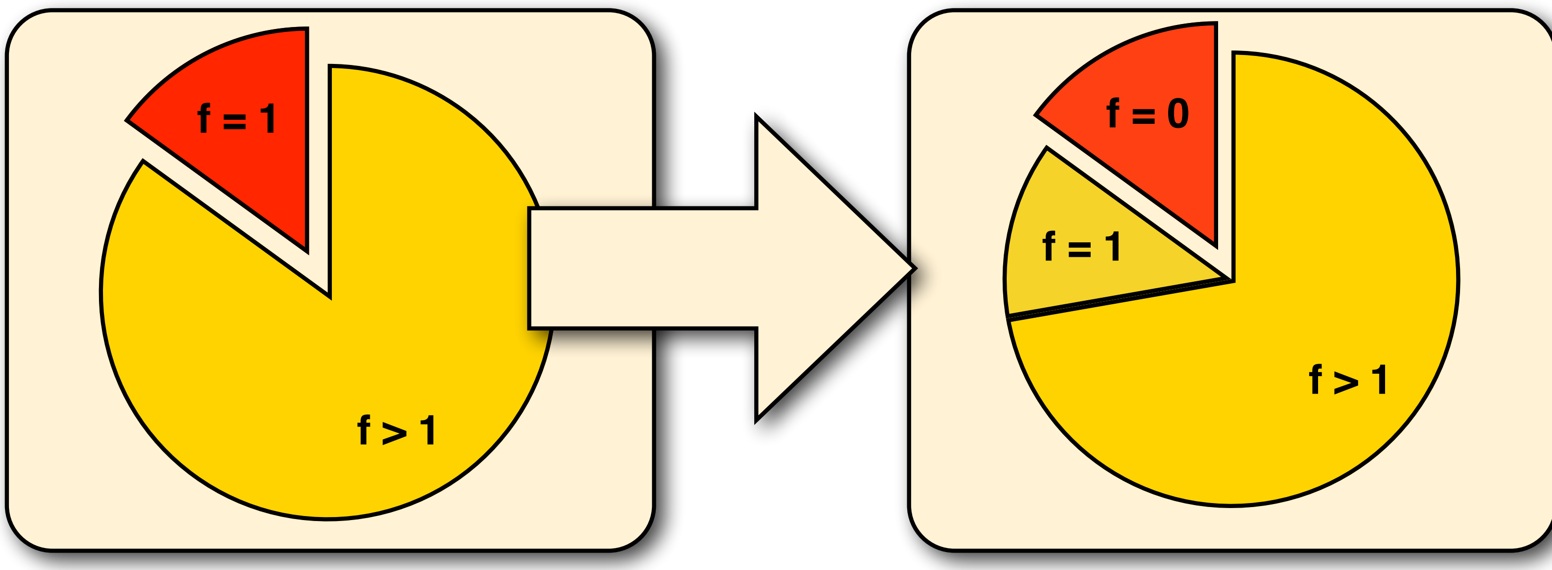
$$\text{Add K} \quad P(w_i) = \frac{C(w_i) + k}{N + kV}$$

This is still too simplistic to work well.

Good-Turing smoothing

Basic idea: Use **total frequency of events that occur only once** to estimate **how much mass to shift to unseen events**

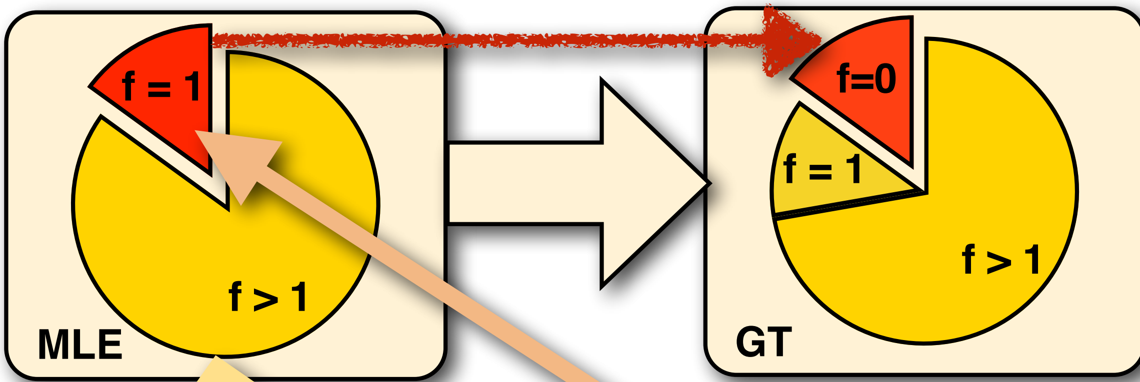
- "occur only once" (in training data): frequency f = 1
- "unseen" (in training data): frequency f = 0 (didn't occur)



Relative Frequency Estimate

Good Turing Estimate

Good-Turing smoothing



	$P(seen)$	+	$P(unseen)$	=	1
MLE	$\frac{N}{N}$	+	0	=	1
Good Turing	$\frac{2 \cdot N_2 + \ldots + m \cdot N_m}{\sum_{i=1}^{m} i \cdot N_i}$	+	$\frac{1 \cdot N_1}{\sum_{i=1}^{m} i \cdot N_i}$	=	$\frac{\sum_{i=1}^{m} i \cdot N_i}{\sum_{i=1}^{m} i \cdot N_i}$

N_c: number of *event types* that occur c times *(can be counted)*

N_1: number of *event types that occur once*

$N = 1N_1 + \ldots + mN_m$: total number of observed event tokens

Good-Turing Smoothing

General principle:
Reassign the probability mass of all events that occur k times in the training data to all events that occur k–1 times.

N_k events occur k times, with a total frequency of $k \cdot N_k$

The probability mass of all words that appear k–1 times becomes:

$$\sum_{w:C(w)=k-1} P_{GT}(w) = \sum_{w':C(w')=k} P_{MLE}(w') = \sum_{w':C(w')=k} \frac{k}{N}$$

$$= \frac{k \cdot N_k}{N}$$

There are N_{k-1} words w that occur k–1 times in the training data.
Good-Turing replaces the original count c_{k-1} of w with a new count c^*_{k-1}:

$$c^*_{k-1} = \frac{k \cdot N_k}{N_{k-1}}$$

Good-Turing smoothing

The Maximum Likelihood estimate of the probability of a word w that occurs k–1 times $P_{MLE}(w) = C(w)/N$

$$P_{MLE}(w) = \frac{c_{k-1}}{N} = \frac{k-1}{N}$$

The Good-Turing estimate of the probability of a word w that occurs k–1 times: $P_{\mathrm{GT}}(w) = c*_{k-1} / N$:

$$P_{GT}(w) = \frac{c^*_{k-1}}{N} = \frac{\left(\frac{k \cdot N_k}{N_{k-1}}\right)}{N} = \frac{k \cdot N_k}{N \cdot N_{k-1}}$$

Problems with Good-Turing

Problem 1:
What happens to the most frequent event?

Problem 2:
We don't observe events for every k.

Variant: Simple Good-Turing

Replace N_n with a fitted function $f(n)$:

$$f(n) = a + b \log(n)$$

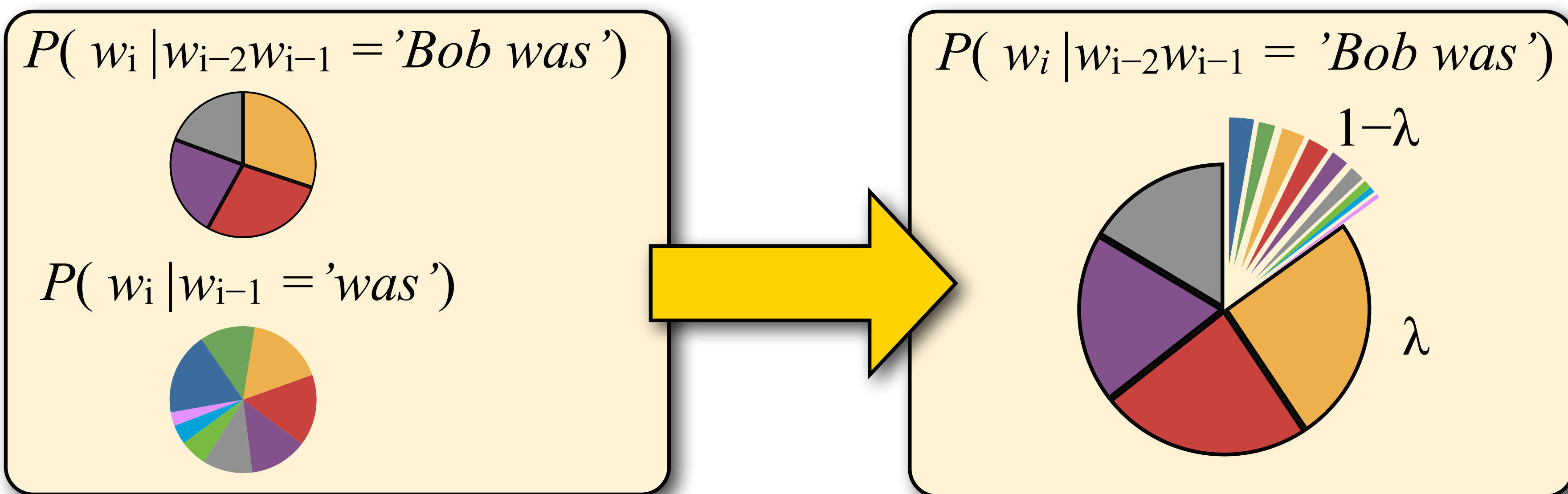
Requires parameter tuning (on held-out data):
Set a,b so that $f(n) \cong N_n$ for known values.
Use c_n^* only for small n

Smoothing: Reserving mass in $P(X \mid Y)$ for unseen events

Linear Interpolation (1)

We don't see "*Bob was reading*", but we see "__ *was reading*".
We estimate $P(reading \mid \text{'Bob was'}) = 0$ but $P(reading \mid \text{'was'}) > 0$

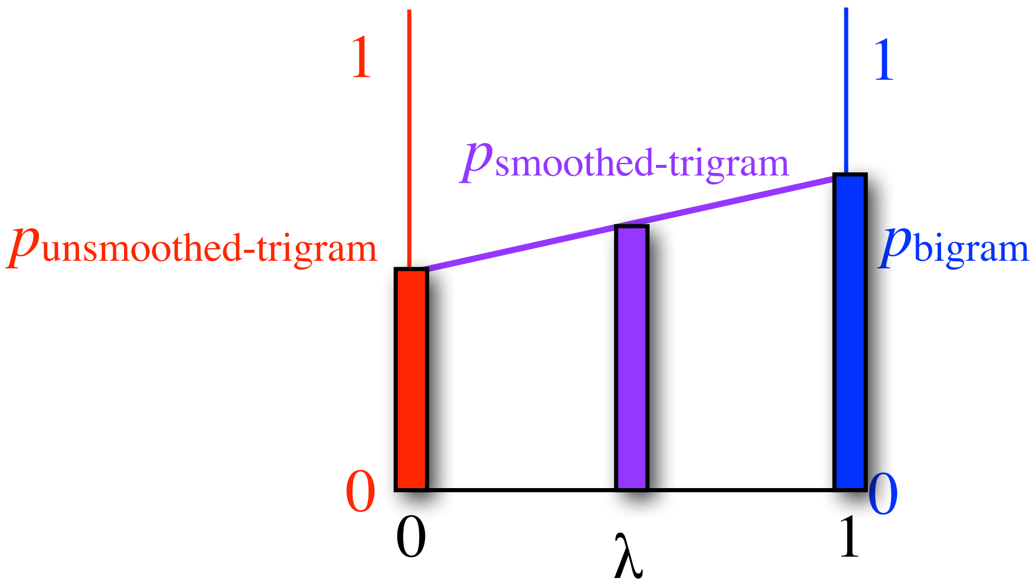
Use (n –1)-gram probabilities to smooth n-gram probabilities:

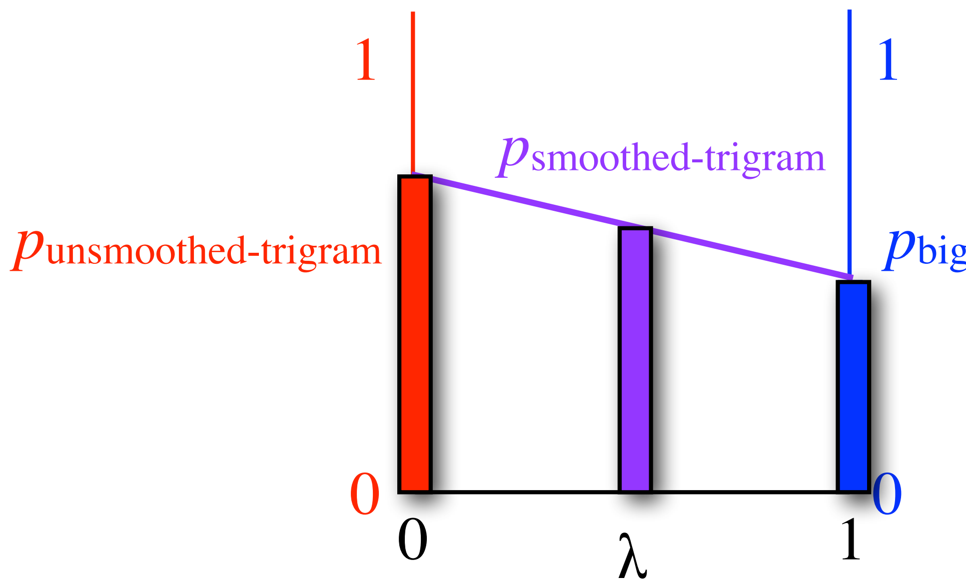


$$\underbrace{\tilde{P}_{LI}(w_i|w_{i-n}w_{i-n+1}\ldots w_{i-2}w_{i-1})}_{\text{smoothed n-gram}} = \lambda\,\underbrace{\hat{P}(w_i|w_{i-n}w_{i-n+1}\ldots w_{i-2}w_{i-1})}_{\text{unsmoothed n-gram}} + (1-\lambda)\,\underbrace{\tilde{P}_{LI}(w_i|w_{i-n+1}\ldots w_{i-2}w_{i-1})}_{\text{smoothed (n-1)-gram}}$$

What happens to $P(\mathrm{w} \mid \ldots)$?

The smoothed probability $P_{\text{smoothed-trigram}}(w_i \mid w_{i-2}\, w_{i-1})$ is a linear combination of $P_{\text{unsmoothed-trigram}}(w_i \mid w_{i-2}\, w_{i-1})$ and $P_{\text{bigram}}(w_i \mid w_{i-1})$:





Linear Interpolation (2)

We've never seen "*Bob was reading*",
but we might have seen "__ *was reading*",
and we've certainly seen "__ *reading*" (or <UNK>)

$$\tilde{P}(w_i|w_{i-1},w_{i-2}) = \lambda_3 \cdot \hat{P}(w_i|w_{i-1},w_{i-2})$$
$$+ \lambda_2 \cdot \hat{P}(w_i|w_{i-1})$$
$$+ \lambda_1 \cdot \hat{P}(w_i)$$
$$\text{for } \lambda_1 + \lambda_2 + \lambda_3 = 1$$

$P_{\text{smoothed}}(w_i = \textit{reading} \mid w_{i-1} = \textit{was}, w_{i-2} = \textit{Bob}) =$
$\lambda_3 \, P_{\text{unsmoothed-trigram}}(w_i = \textit{reading} \mid w_{i-1} = \textit{was}, w_{i-2} = \textit{Bob})$
$+ \lambda_2 \, P_{\text{unsmoothed-bigram}}(w_i = \textit{reading} \mid w_{i-1} = \textit{was})$
$+ \lambda_1 \, P_{\text{unsmoothed-unigram}}(w_i = \textit{reading})$

Interpolation: Setting the λs

Method A: Held-out estimation

Divide data into training and held-out data.

Estimate models on training data.

Use held-out data (and some optimization technique) to find the λ that gives best model performance.

Often: λ is a learned function of the frequencies of $w_{i-n} \ldots w_{i-1}$

Method B:

λ is some (deterministic) function of the frequencies of $w_{i-n} \ldots w_{i-1}$

Absolute discounting

Subtract a constant factor $D < 1$ from each nonzero *n*-gram count, and interpolate with $P_{AD}(w_i \mid w_{i-1})$:

non-zero if trigram $w_{i-2}w_{i-1}w_i$ is seen

$$P_{AD}(w_i|w_{i-1},w_{i-2}) = \frac{\max(C(w_{i-2}w_{i-1}w_i) - D, 0)}{C(w_{i-2}w_{i-1})} + (1-\lambda)P_{AD}(w_i|w_{i-1})$$

If S seen word types occur after $w_{i-2}\, w_{i-1}$ in the training data, this reserves the probability mass $P(U) = (S \times D)/C(w_{i-2}w_{i-1})$ to be computed according to $P(w_i \mid w_{i-1})$. Set:

$$(1-\lambda) = P(U) = \frac{S \cdot D}{C(w_{i-2}w_{i-1})}$$

N.B.: with N_1, N_2 the number of *n*-grams that occur once or twice, $D = N_1/(N_1+2N_2)$ works well in practice

Kneser-Ney smoothing

Observation: *"San Francisco"* is frequent,
but *"Francisco"* only occurs after "*San*".

Solution: the unigram probability $P(w)$ should not depend on the frequency of w, but on the number of contexts in which w appears

$N_{+1}(\bullet w)$: number of contexts in which w appears
= number of word types w' which precede w
$N_{+1}(\bullet\bullet) = \sum_{w'} N_{+1}(\bullet w')$

Kneser-Ney smoothing: Use absolute discounting,
but use $P(w) = N_{+1}(\bullet w)/N_{+1}(\bullet\bullet)$

Modified Kneser-Ney smoothing: Use different *D for bigrams and trigrams (Chen & Goodman '98)*

To recap….

Today's key concepts

Dealing with unknown words
Dealing with unseen events
Good-Turing smoothing
Linear Interpolation
Absolute Discounting
Kneser-Ney smoothing

Today's reading:
Jurafsky and Martin, Chapter 4, sections 1-4