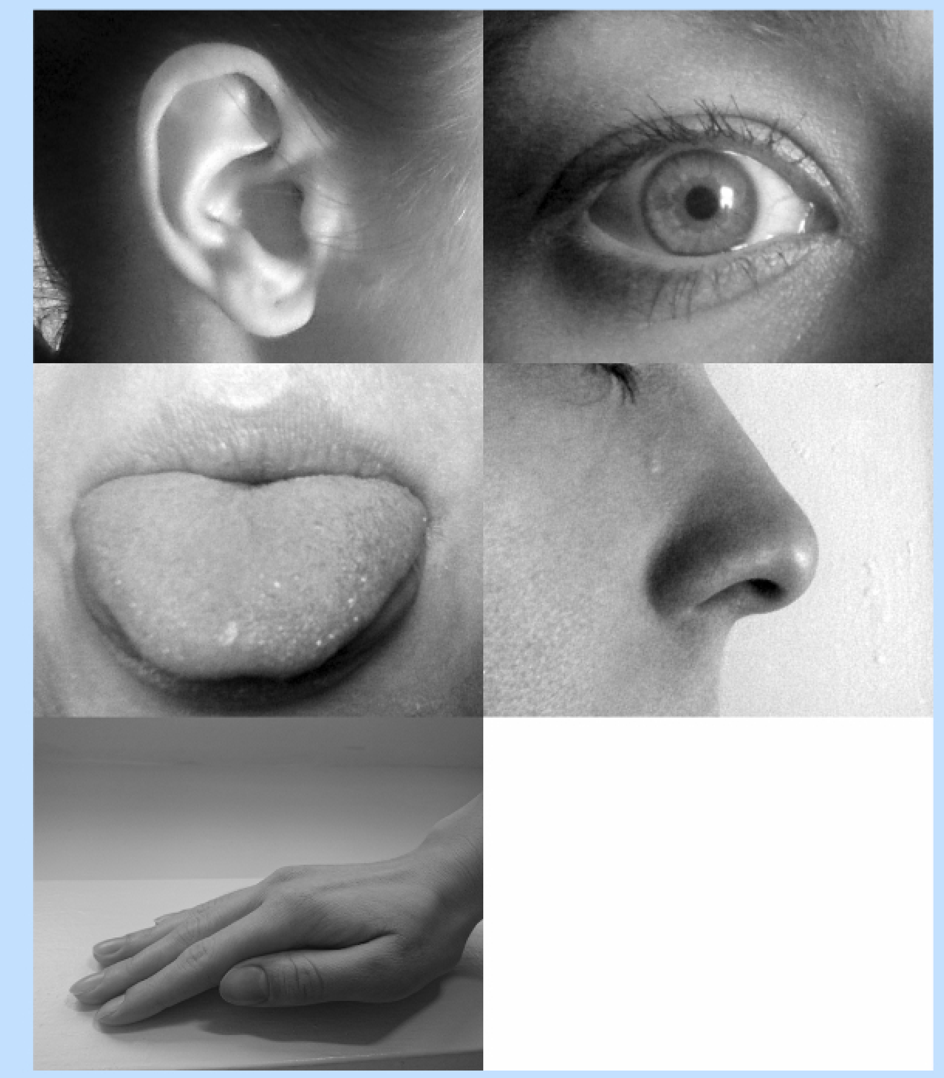
Announcements

- Work in groups - groups of 2 for MPs, groups of 2-4 for the final project.

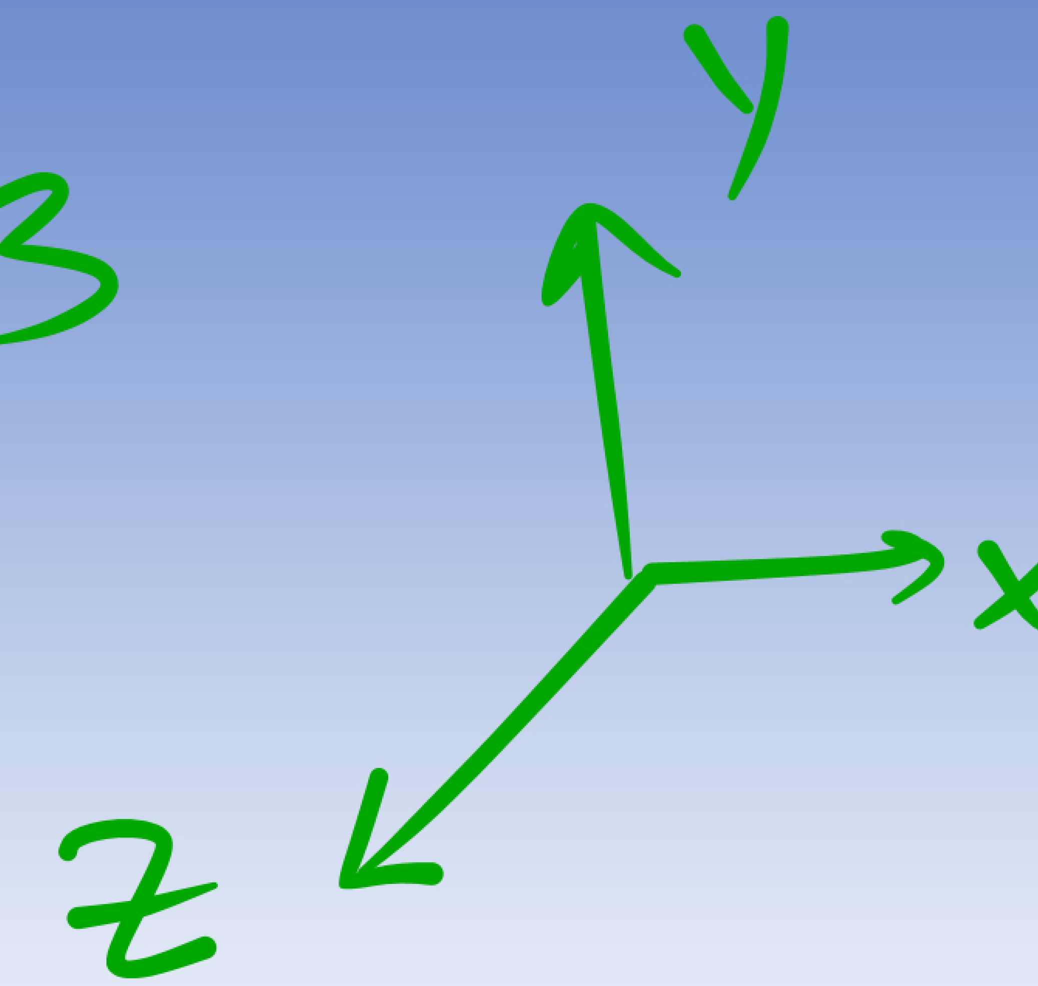
VR System: Hardware, Software and Perceptual Psychology

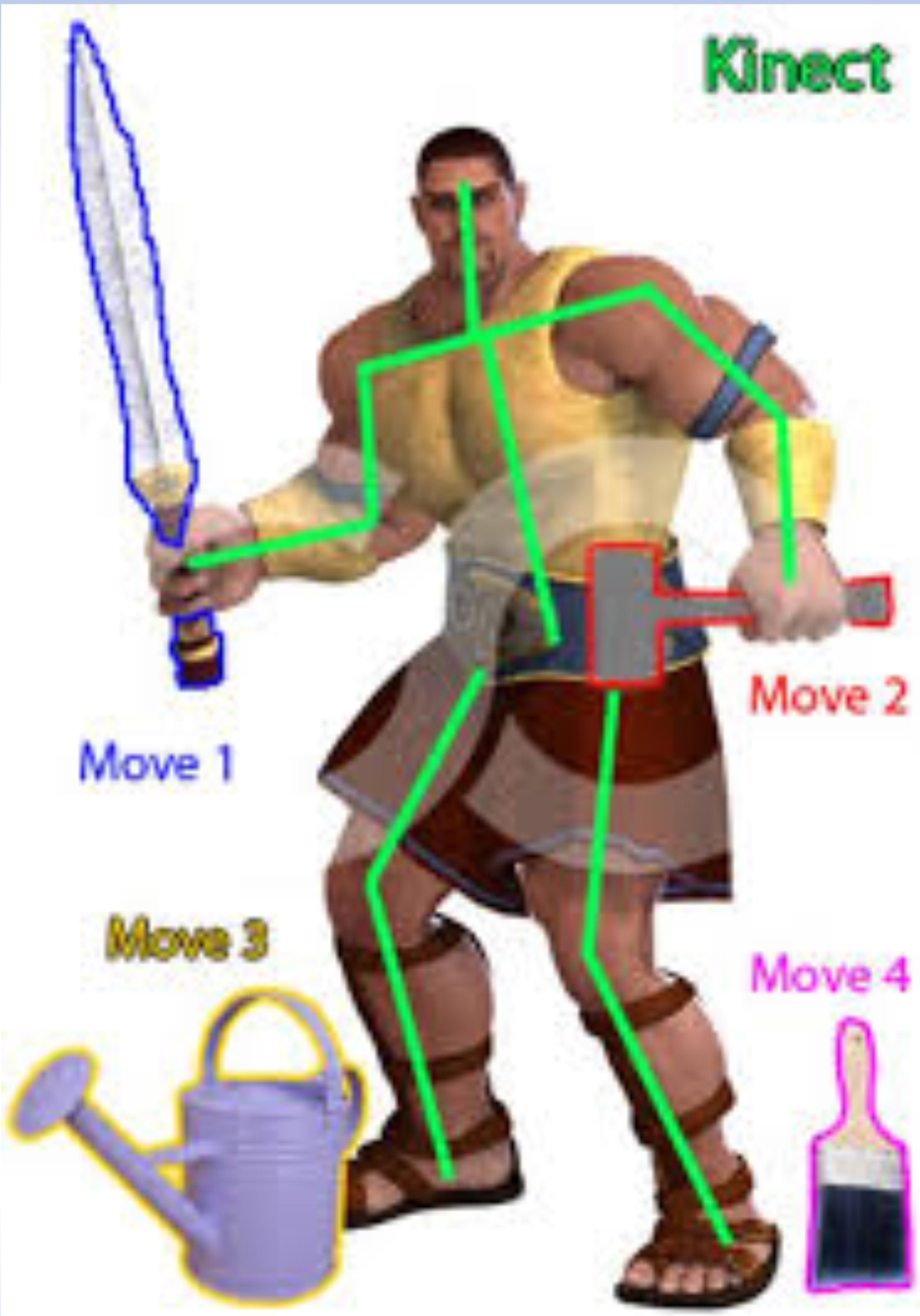
AWG

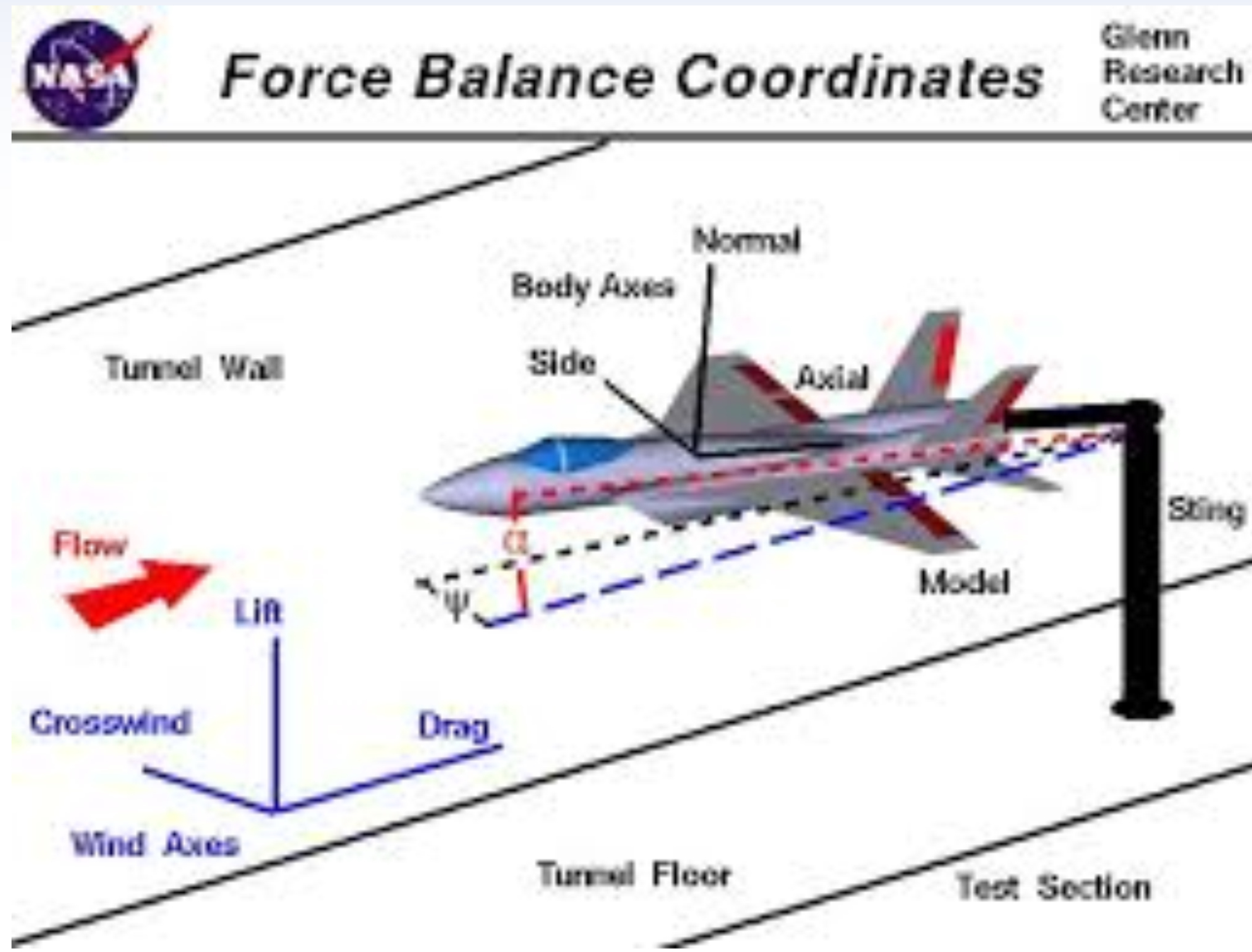


Geometric Modelling

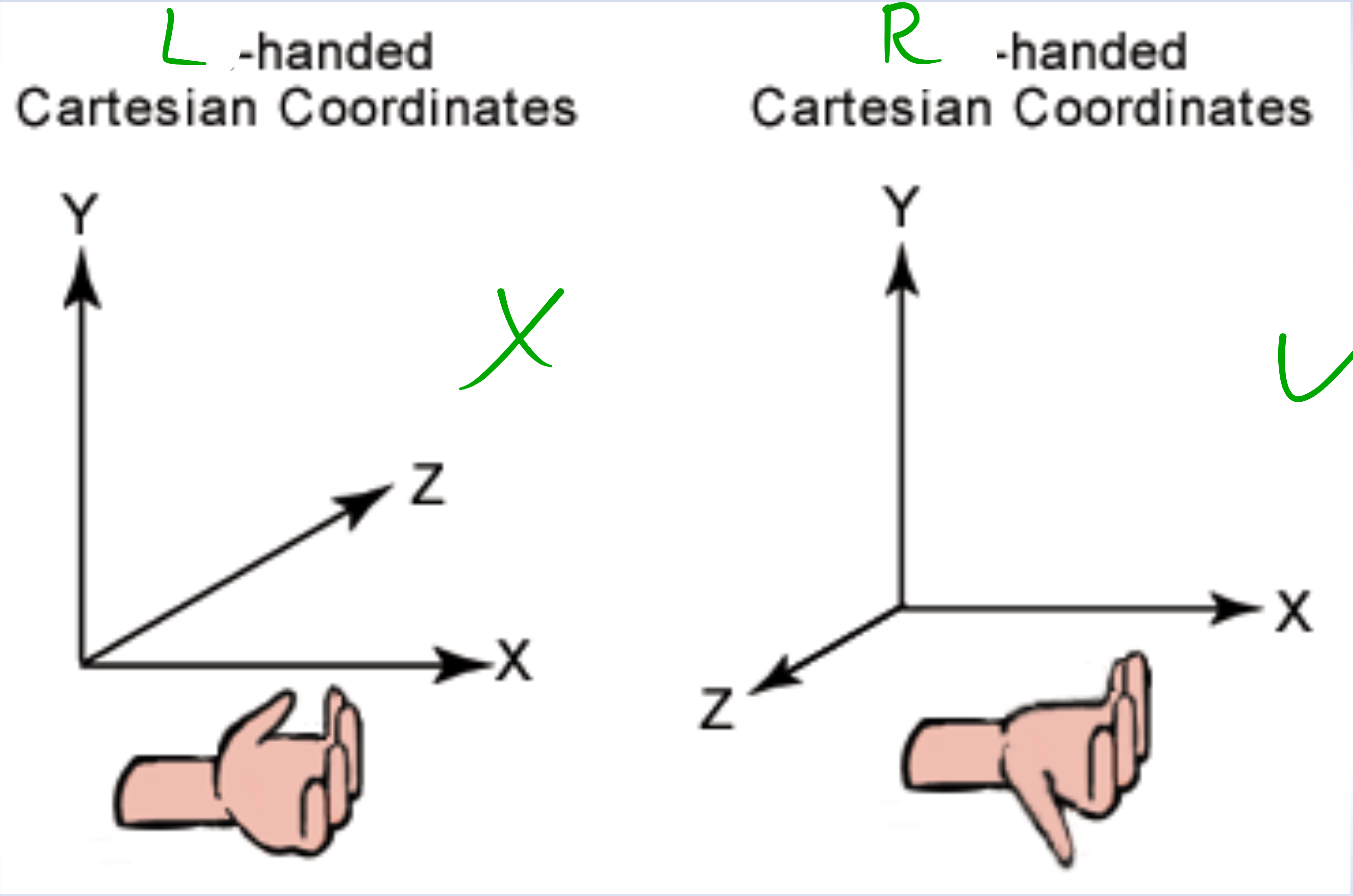
$W \subseteq \mathbb{R}^3$







Geometric Modelling



Static vs Dynamic Models

Stationary objects: described in world coordinate frame

Static vs Dynamic Models

have a space of tranformations

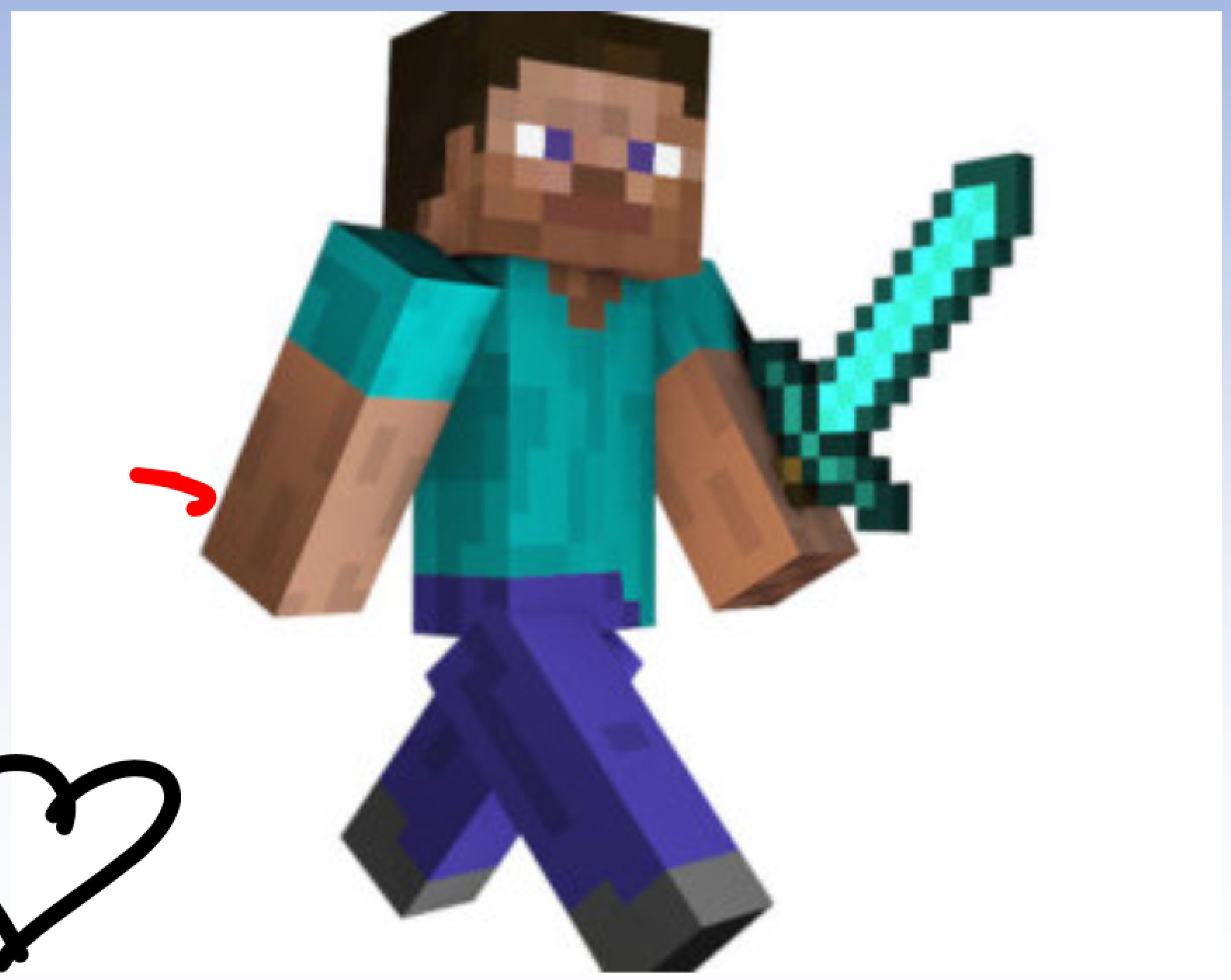
Movable objects: defined in its own coordinate frame

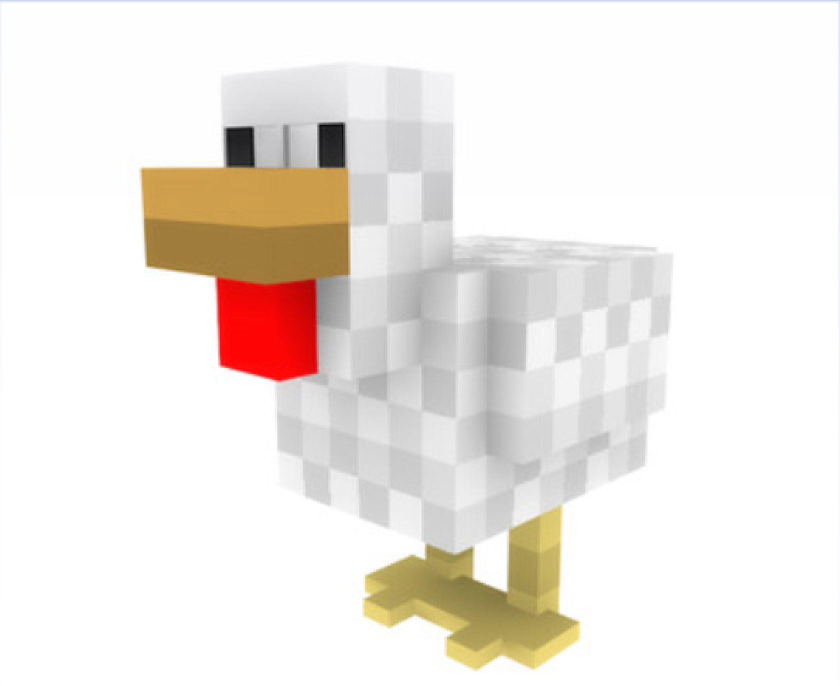
a set of rigid bodies

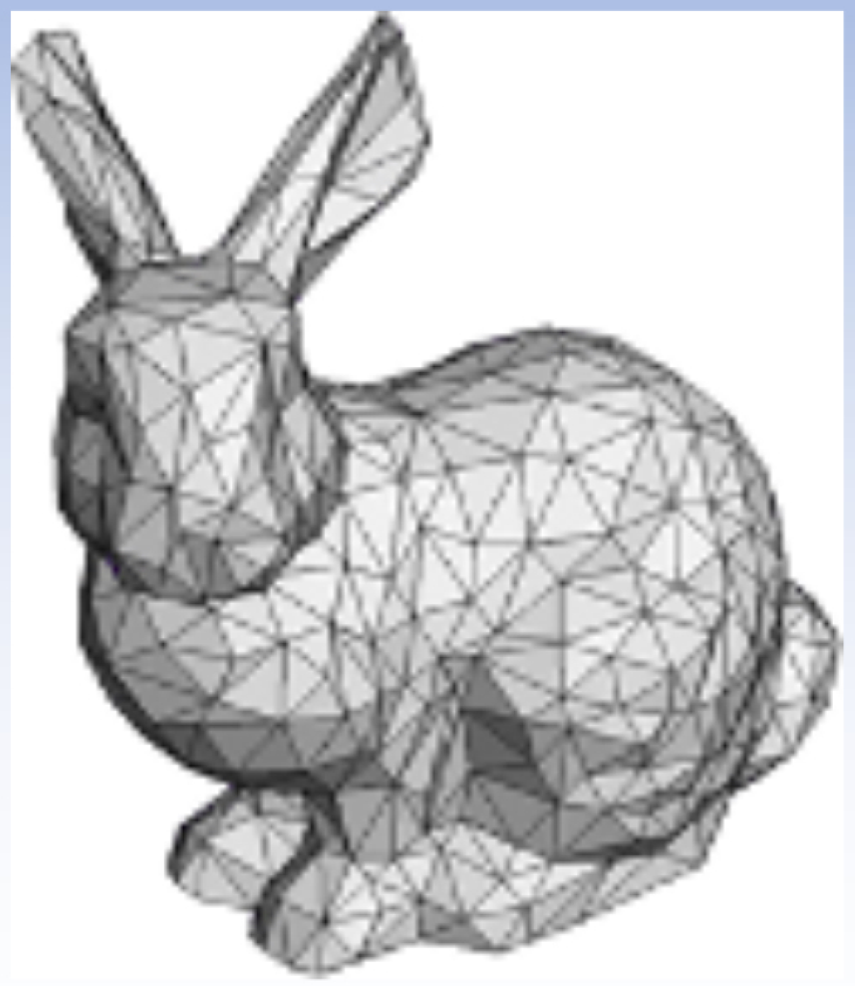
Static vs Dynamic Models

Movable objects:

3D Primitives vs 2D Boundary Representations

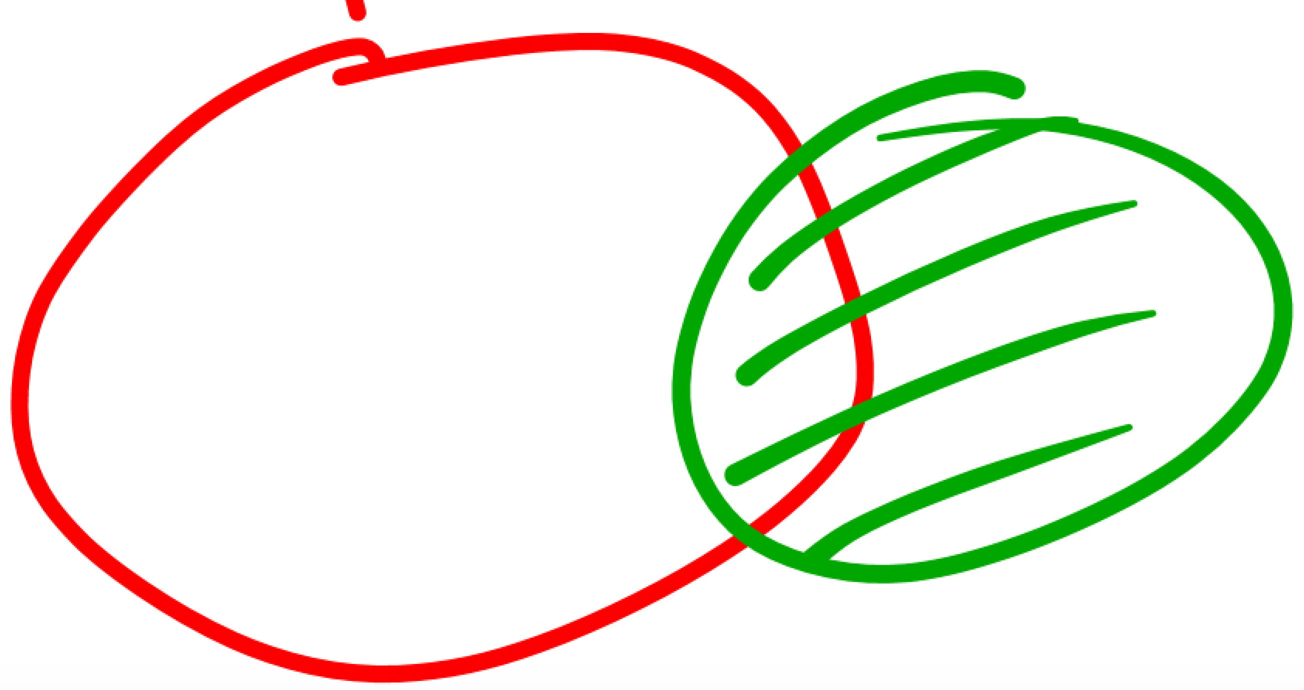


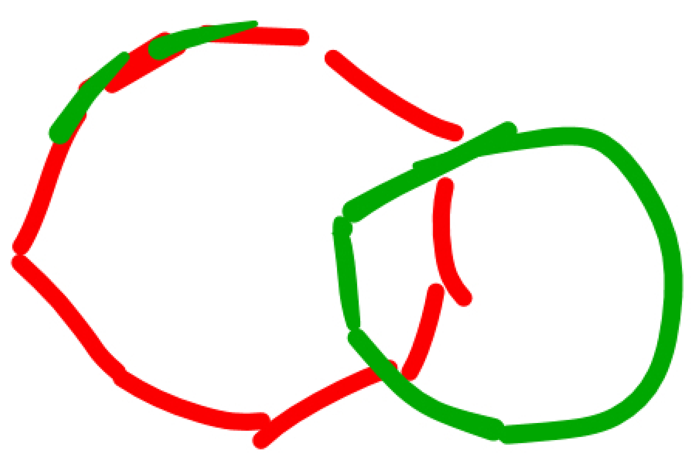




1. Solid primitive representation "3d primitives"

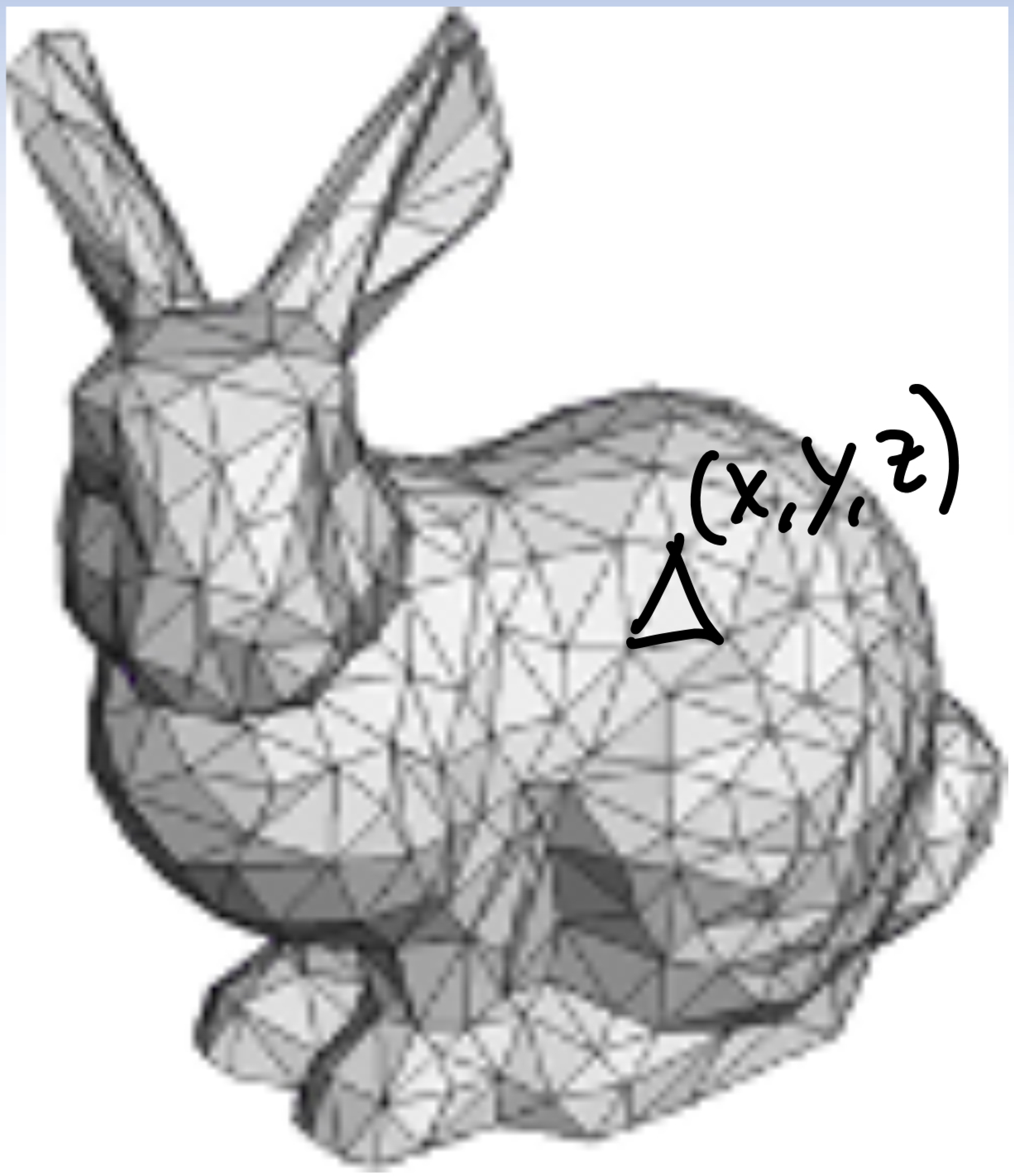
2. Boundary representation "2d primitives"

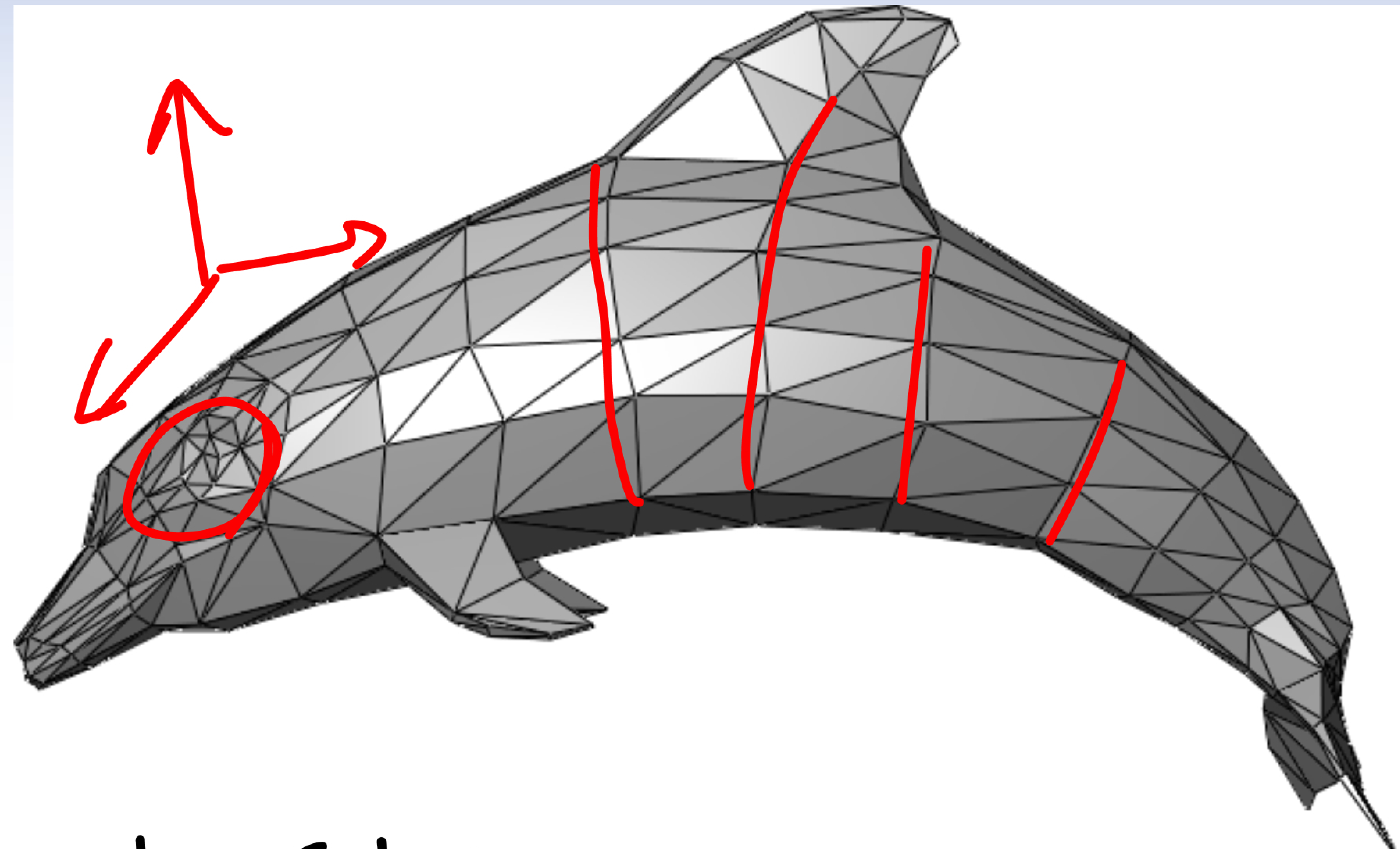




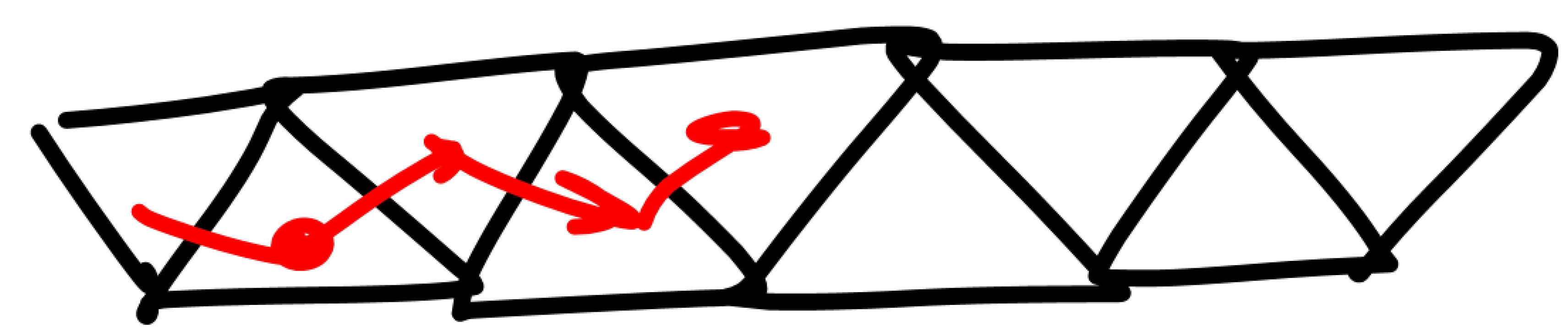
model coherency

3D Triangles Boundary Representations

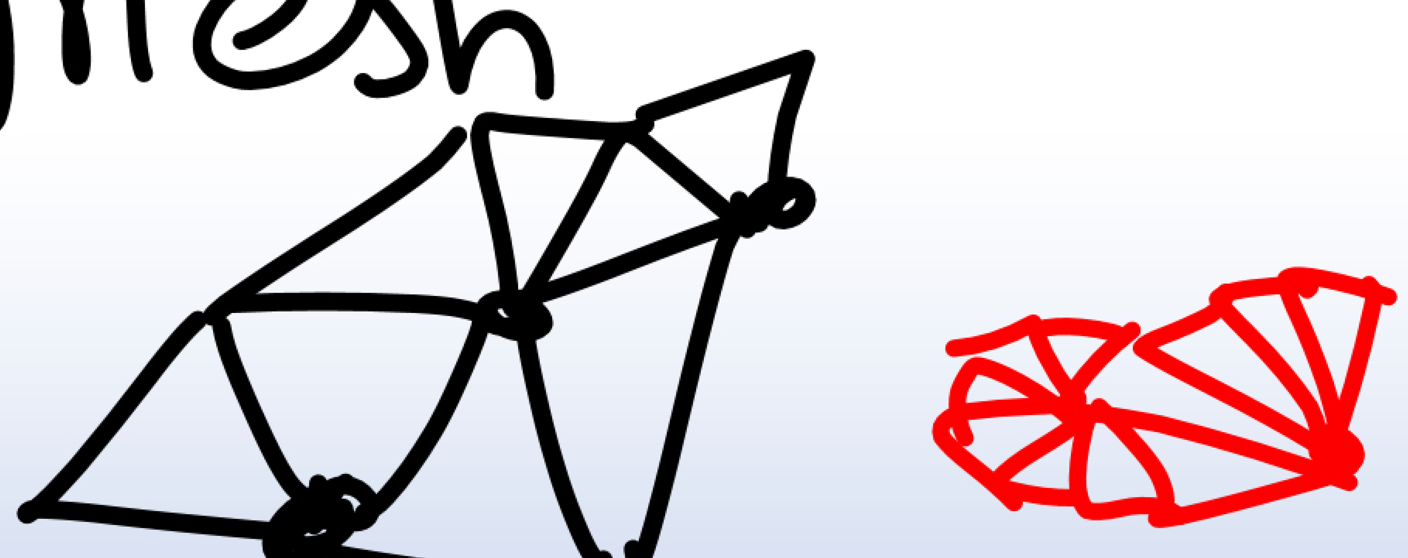




1. Strip



2. Mesh



Transforming Rigid Bodies

Why transformations?

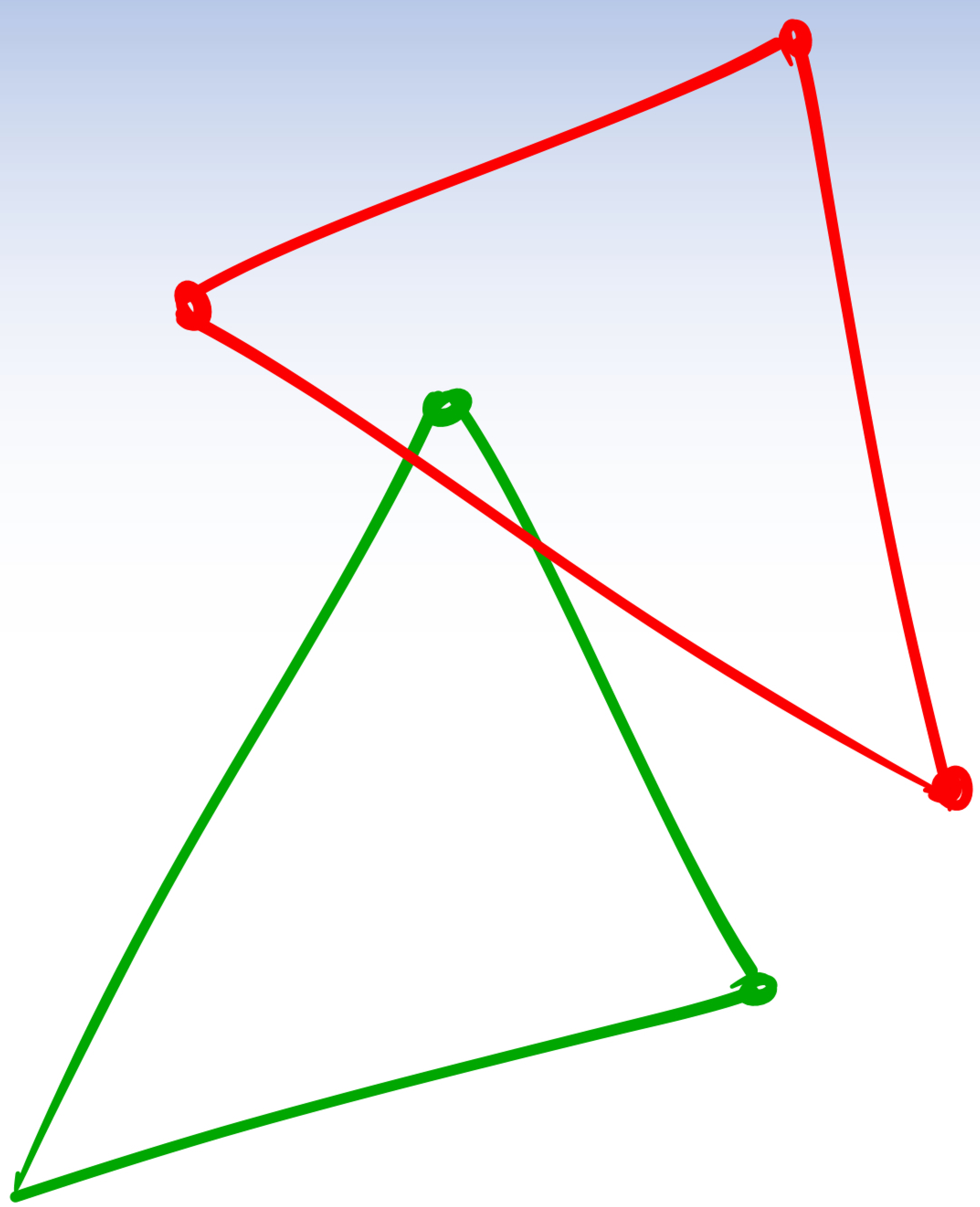
1. Movable objects

http://www.senocular.com/flash/tutorials/transformmatrix/examples/3dpicturebox.html

2. Perception of stationarity

https://www.youtube.com/watch?v=A7q3mY0iNOQ

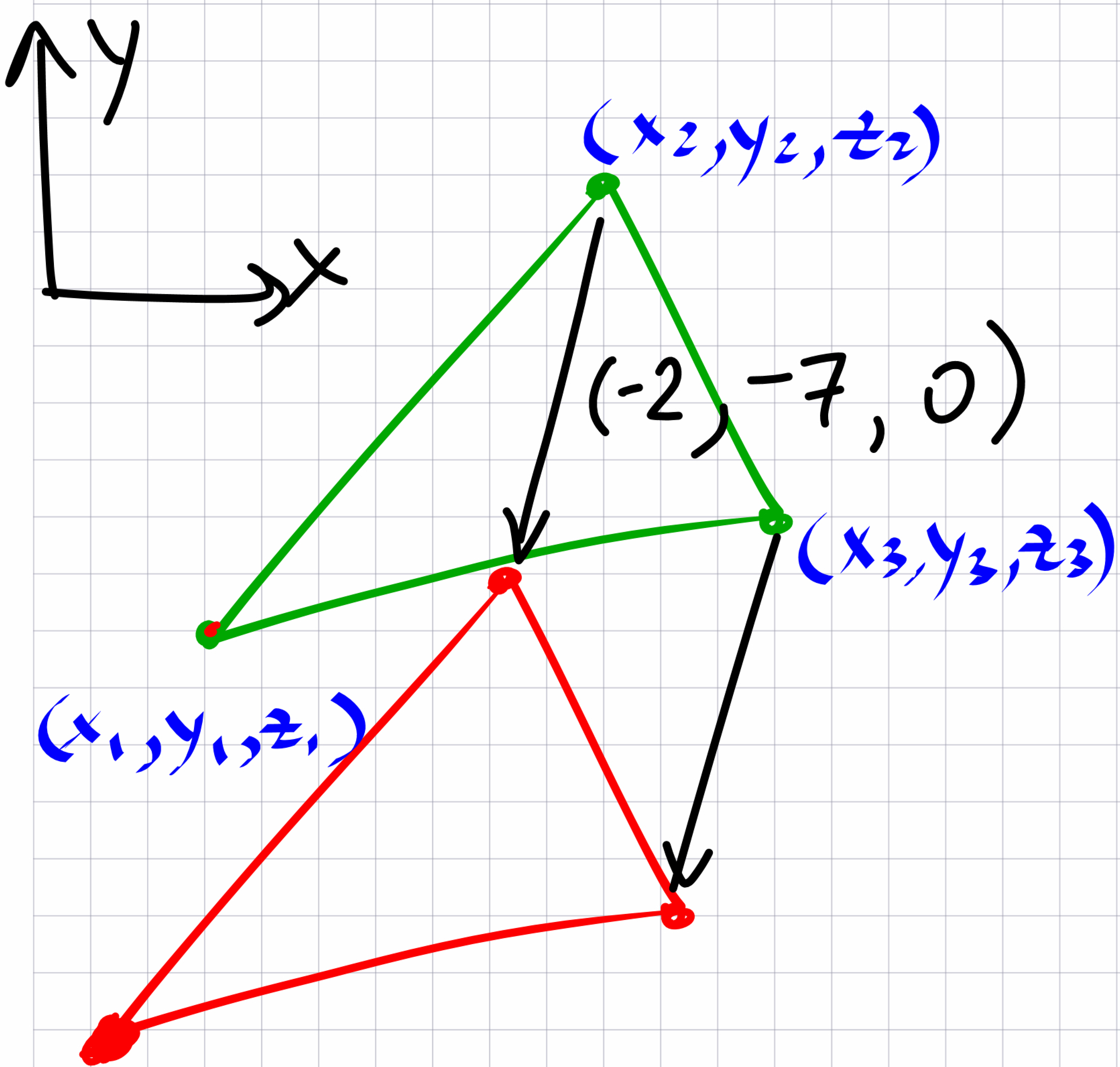
Rigid Body Transformations



DOFs

	2D	3D
1) Easy: translations	2	3
	+	+
2) More difficult: rotation	1	3 :(
	=	=
3) Most difficult rotation+translation	3	6

3D Translations



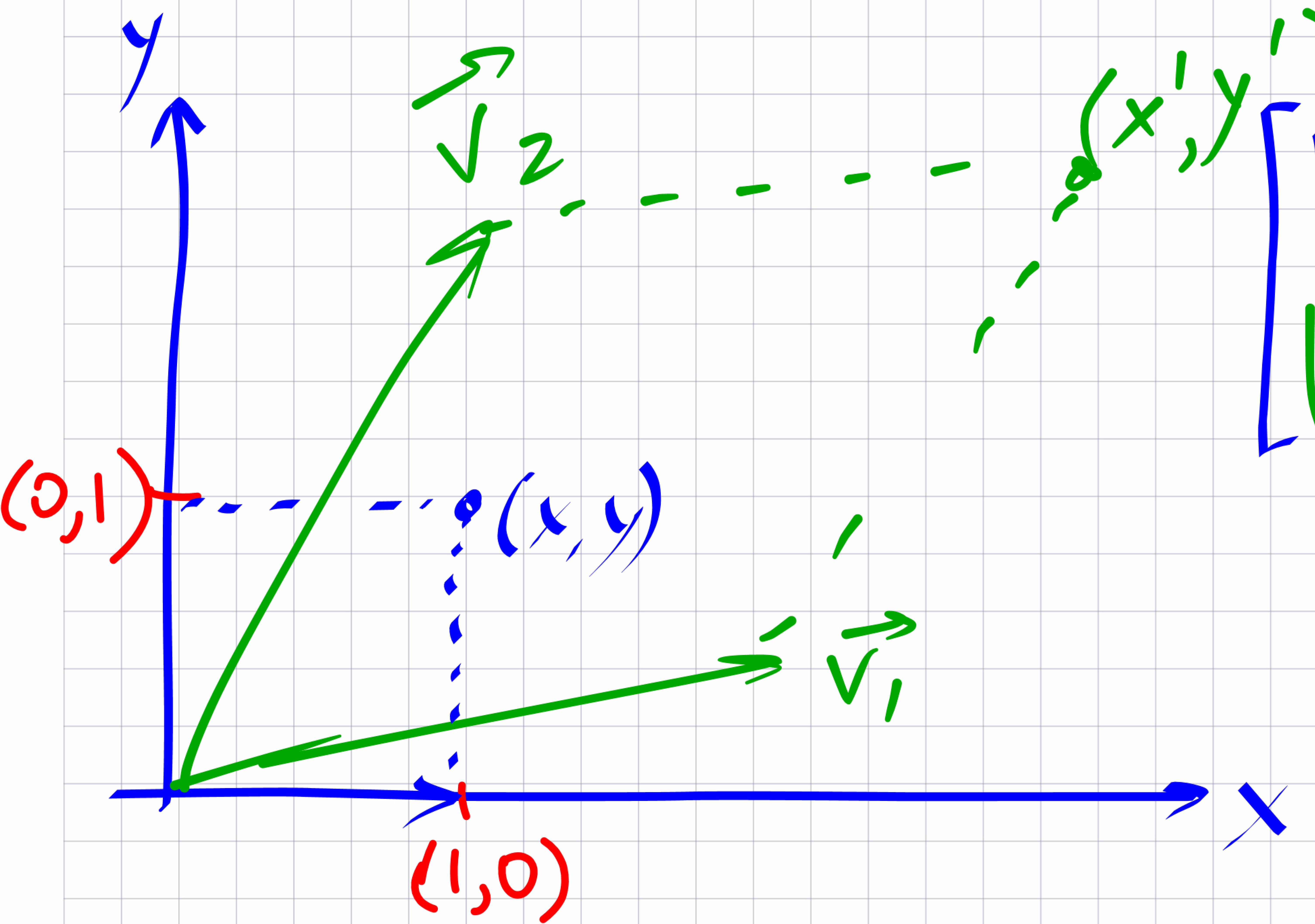
Triangle: (x_i, y_i, z_i) $i=1,2,3$

Shift: (x_t, y_t, z_t)

New Triangle:

$(x_i + x_t, y_i + y_t, z_i + z_t)$

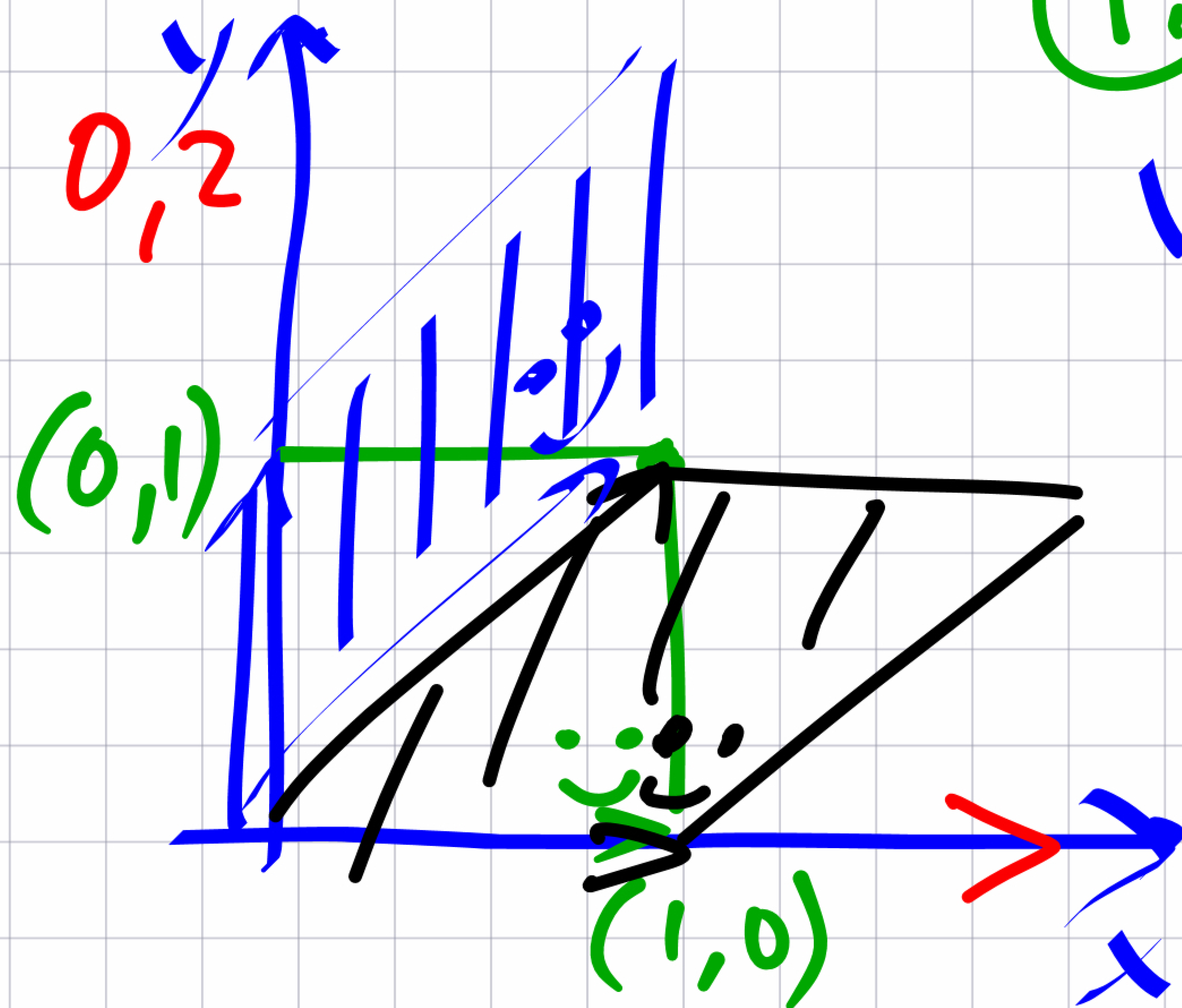
2D Linear Transformations $m_{ij} \in \mathbb{R}$



$$\begin{array}{cc} \left[\begin{array}{cc} m_{11} & m_{12} \\ m_{21} & m_{22} \end{array}\right] \\ \quad V_1 \qquad\; V_2 \end{array} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{bmatrix} m_{11} & \cancel{m_{12}} \\ m_{21} & \cancel{m_{22}} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} m_{11} \\ m_{21} \end{bmatrix} \qquad \begin{bmatrix} \cancel{m_{11}} & m_{12} \\ \cancel{m_{21}} & m_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} m_{12} \\ m_{22} \end{bmatrix}$$

2D Linear Transformations: Examples



1. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

identity matrix
no effect

2. $\begin{bmatrix} \overset{3}{\cancel{0.2}} & 0 \\ 0 & \overset{2}{\cancel{0.2}} \end{bmatrix}$

"scale"
"aspect ratio"

3. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

"reflex over y"

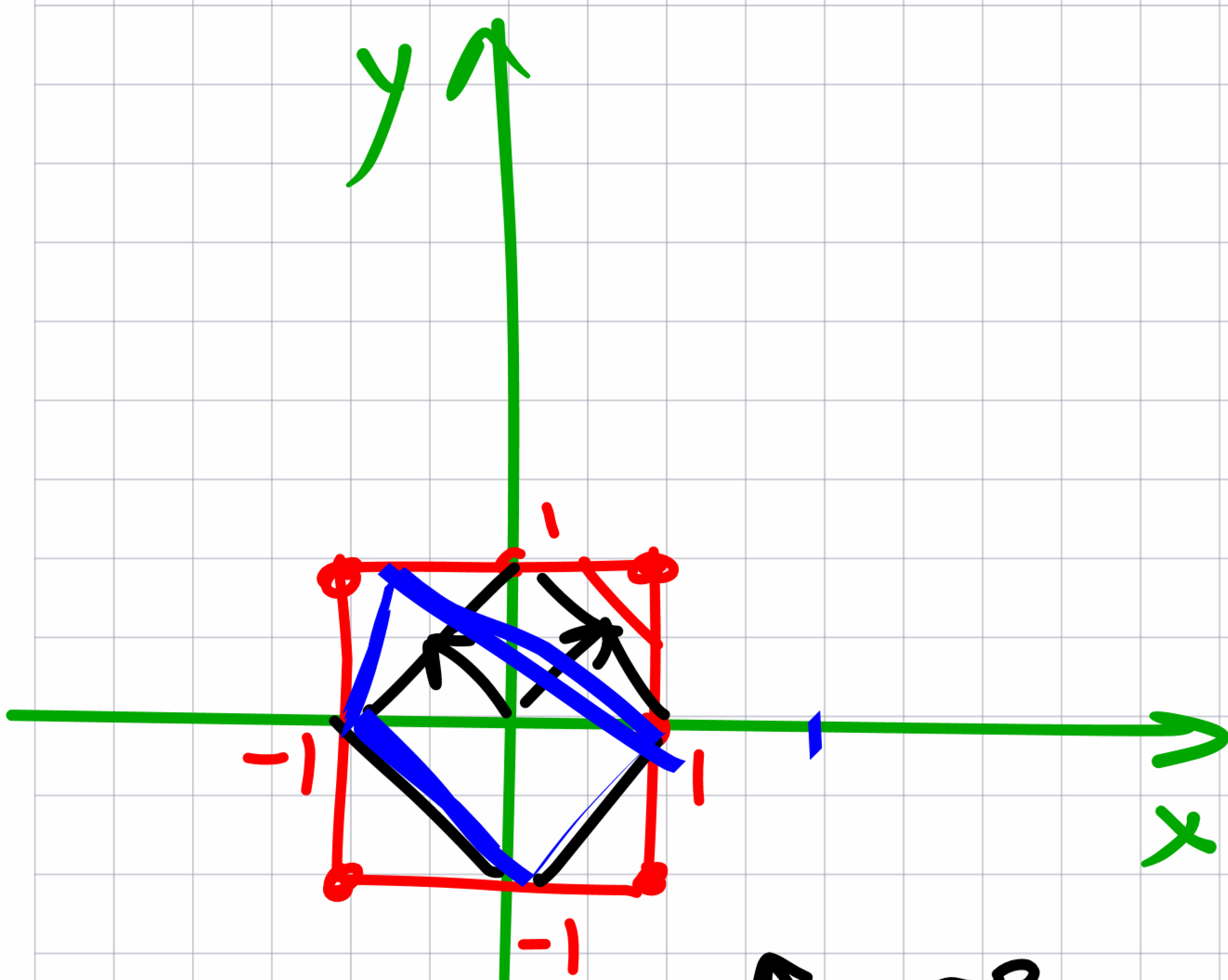
4. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

"rotation by π

5. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

x-shear y-shear

2D Linear Transformations: Examples



↑ 45°

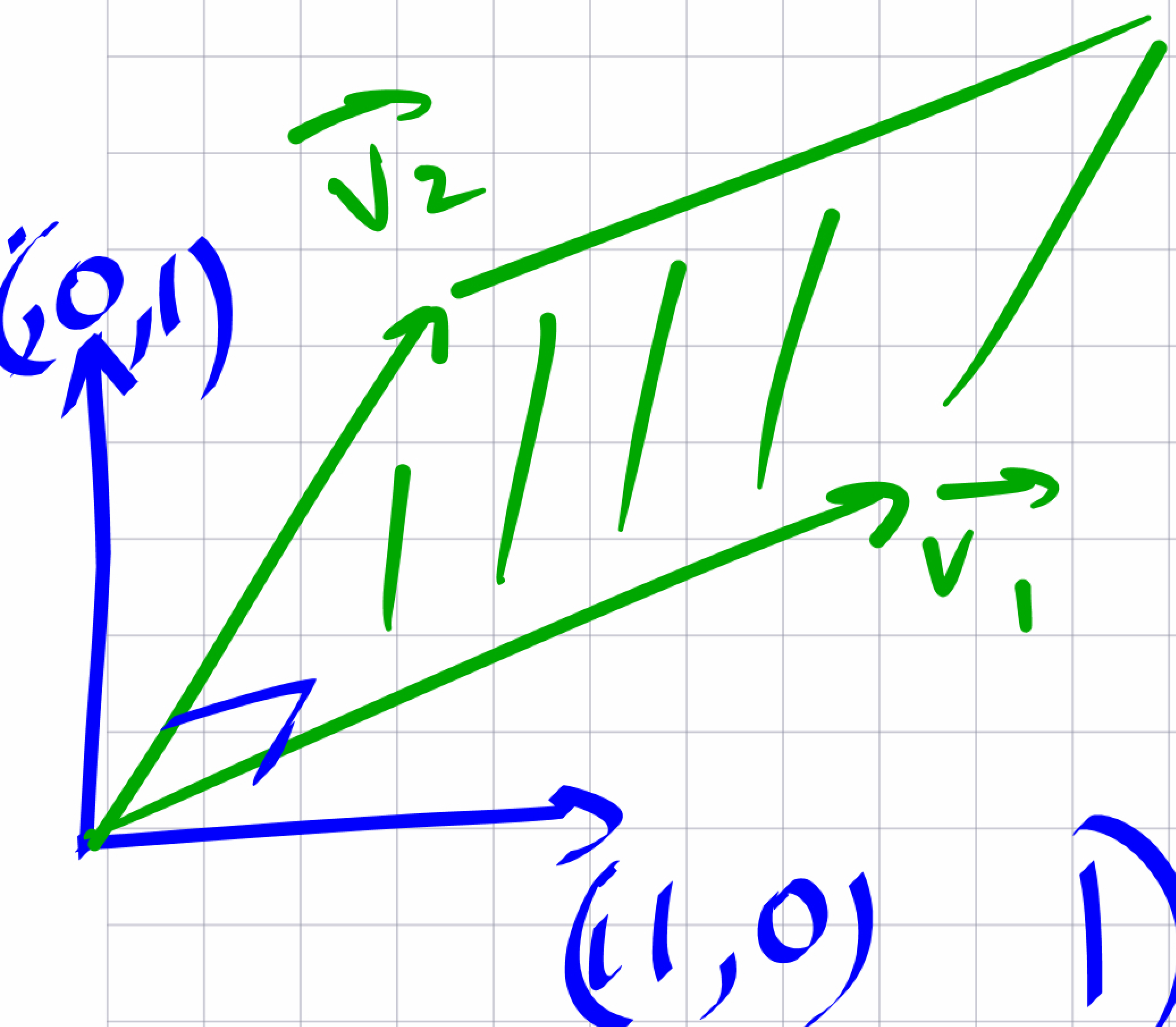
+ shrinking

1. $\begin{bmatrix} 2 & 0.5 \\ -1 & -1.5 \end{bmatrix}$

2. $\begin{bmatrix} 0.5 & -0.5 \\ 0.5 & 0.5 \end{bmatrix}$

3. Draw a polygon that IS NOT a result of linear transformation ?

2D Rotations



$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$
$$\begin{array}{cc} \underbrace{\quad}_{v_1} & \underbrace{\quad}_{v_2} \end{array}$$

4 DOFs

1) No scaling: $\|v_1\| = \|v_2\| = 1$

$$m_{11}^2 + m_{21}^2 = 1$$
$$m_{12}^2 + m_{22}^2 = 1$$

-2

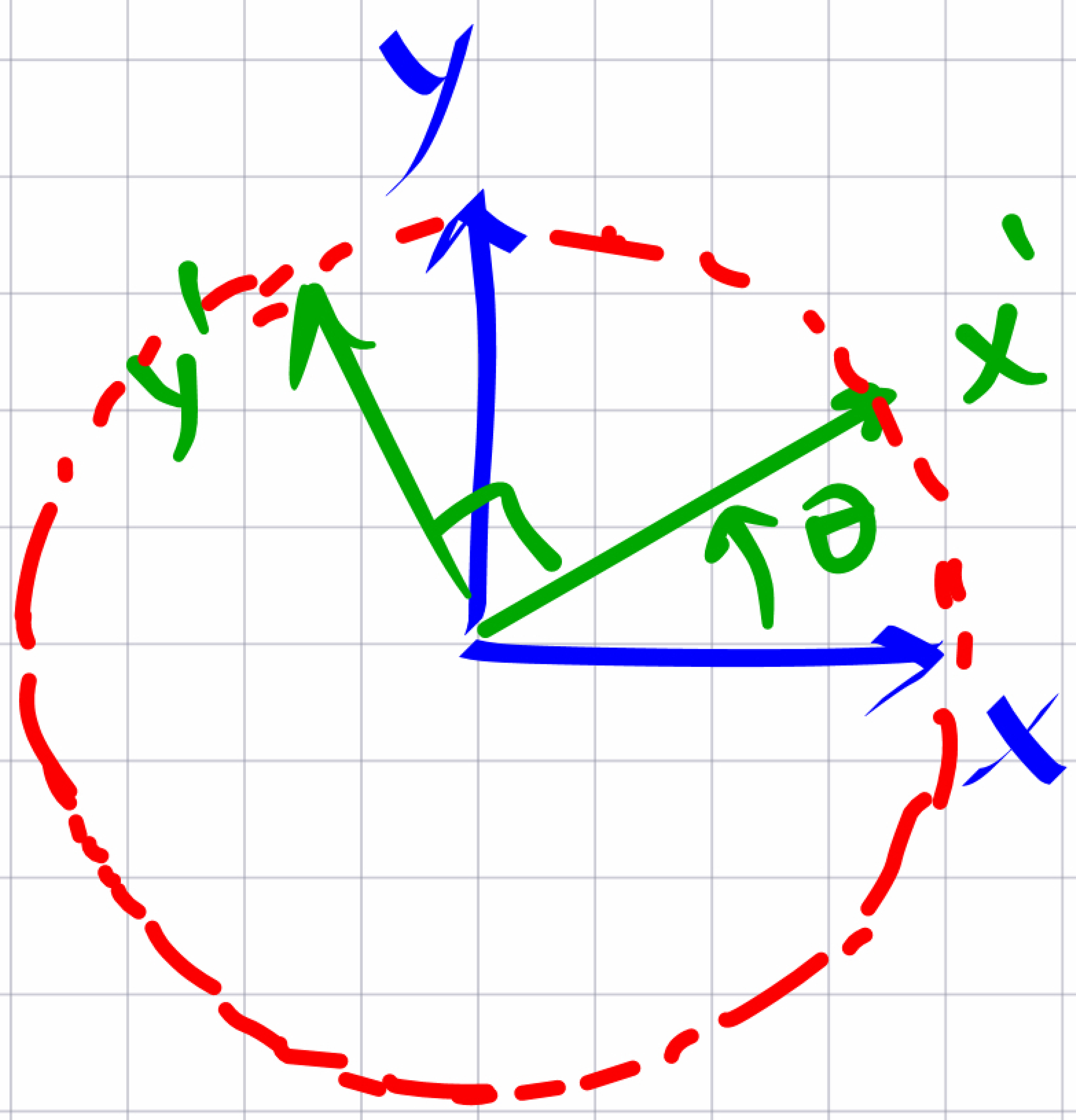
2) No shearing

$$v_1 \cdot v_2 = 0 \qquad m_{11} \cdot m_{12} + m_{21} m_{22} = 0$$

-1

3 $\det(M) = \overset{+}{\cancel{-}}1$

2D Rotations



$m_{11} = \cos\theta$

$m_{21} = \sin\theta$

then

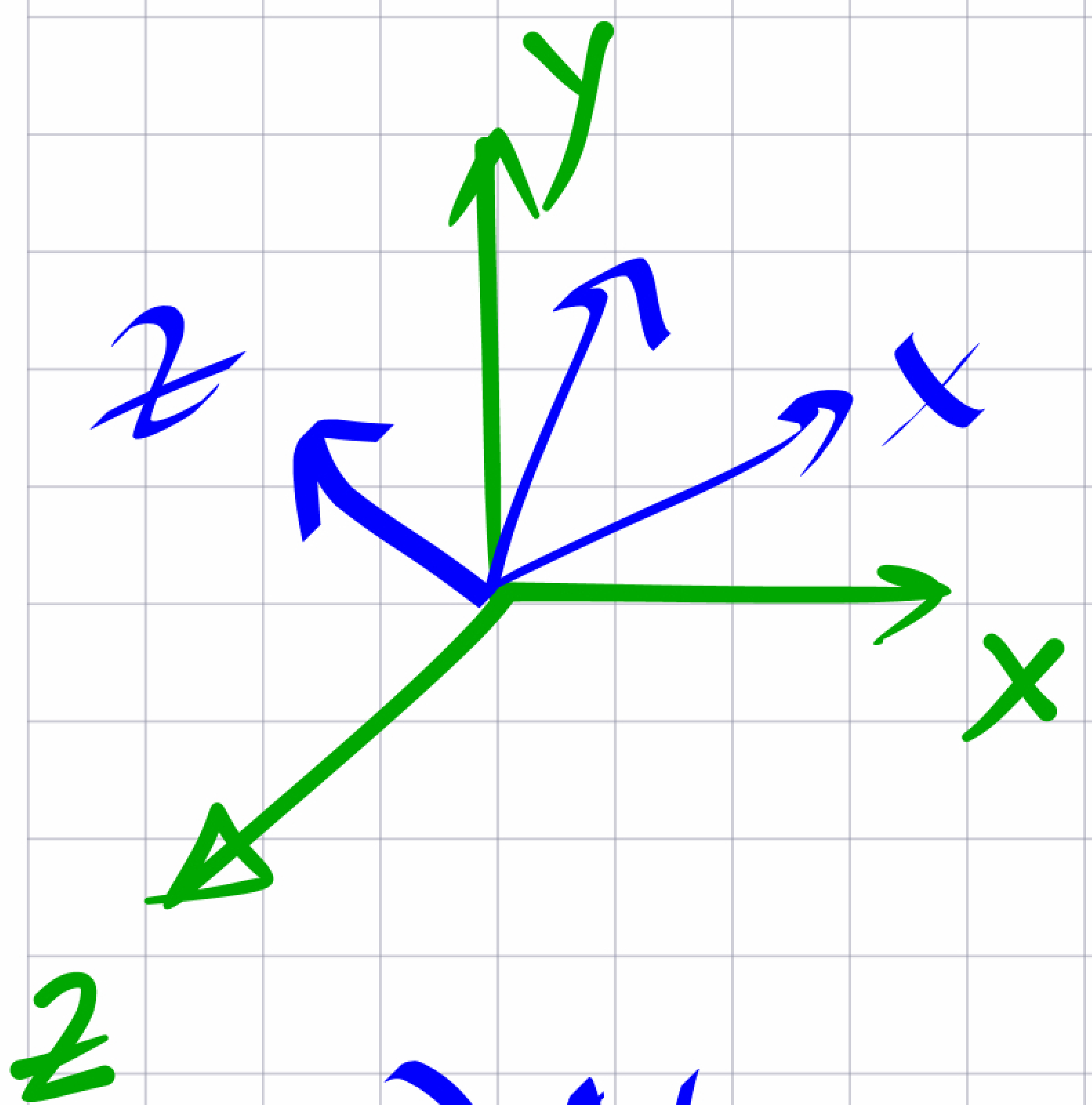
$m_{12} = -\sin\theta$

$m_{22} = \cos\theta$

θ is one parameter that represents 2D Rotations:

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

3D Rotations



$$M \Rightarrow \begin{array}{c} \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \\ \quad v_1 \qquad\; v_2 \qquad\; v_3 \end{array} \quad 9 \text{ DOFs}$$

1) No scaling

$$\|v_1\| = 1$$
$$\|v_2\| = 1$$
$$\|v_3\| = 1$$

-3

2) No shearing

$$v_1 \cdot v_2 = 0$$
$$v_2 \cdot v_3 = 0$$
$$v_1 \cdot v_3 = 0$$

-3

3) $\det(M) = \overset{+}{\cancel{-}}1$