# Subjective Logic and Other Challenges

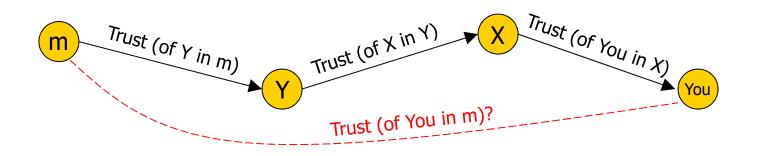
A Human-centric Sensing Perspective

## Subjective Logic

- The question it addresses is how trust (or uncertainty) "composes".
  - Your friend X said they heard their friend Y say that m might have happened.
  - What should be your estimate of the likelihood of m based on this evidence?

## Subjective Logic

- The question it addresses is how trust (or uncertainty) "composes".
  - Your friend X said they heard their friend Y say that m might have happened.
  - What should be your estimate of the likelihood of m based on this evidence?



## The Subjective Opinion

- Belief, Disbelief, and Uncertainty.
- Example: You asked John 100 questions.
  - He replied to 30 correctly
  - 20 incorrectly
  - 50 you could not tell if they were correct or not
- Your opinion of John's correctness is:
  - Belief (b) = 30/100
  - Disbelief (d) = 20/100
  - Uncertainty (u) = 50/100

- Your opinion of John is
  - (b=0.3, d=0.2, u=0.5)
- John's opinion of Sally is
  - (b=0.7, d=0.05, u=0.25)
- What should your opinion be of Sally?

- Your opinion of John is
  - (b=0.3, d=0.2, u=0.5)
- John's opinion of Sally is
  - (b=0.7, d=0.05, u=0.25)
- What should your opinion be of Sally?
  - One possibility (max uncertainty):
    - Belief (at least) = 0.3 \* 0.7
    - Disbelief (at least) = 0.3 \* 0.05
    - Uncertainty (at most) = 1 0.3 (0.7 + 0.05)

Another possibility: Divide uncertainty proportionally then continue as before

- Your opinion of John is
  - (b=0.3, d=0.2, u=0.5) → approx: (0.6, 0.4)
- John's opinion of Sally is
  - (b=0.7, d=0.05, u=0.25) → approx: (0.9333, 0.0667)
- What should your opinion be of Sally?
  - Belief = 0.6 \* 0.9333 = 0.56
  - Disbelief = 0.6 \* 0.0667 = 0.04
  - Uncertainty = 1 0.56 0.04 = 0.4

Another possibility: Divide uncertainty in half among belief and disbelief then continue as before

- Your opinion of John is
  - (b=0.3, d=0.2, u=0.5) → approx: (0.55, 0.45)
- John's opinion of Sally is
  - (b=0.7, d=0.05, u=0.25) → approx: (0.825, 0.175)
- What should your opinion be of Sally?
  - Belief = 0.55 \* 0.825 = 0.45375
  - Disbelief = 0.55 \* 0.175 = 0.09625
  - Uncertainty = 1 0.45375 0.09625 = 0.45

Other possibilities?

- Your opinion of John is
  - (b=0.3, d=0.2, u=0.5)
- John's opinion of Sally is
  - (b=0.7, d=0.05, u=0.25)

Other possibilities?

- Your opinion of John is
  - (b=0.3, d=0.2, u=0.5)
- John's opinion of Sally is
  - (b=0.7, d=0.05, u=0.25)
- Difficulties:
  - We have finite samples, not actual probabilities
  - We have uncertainty (unobserved samples)
  - We do not know the error distribution when someone is wrong: What do instances when John is wrong tell us about Sally?

Requirements of the "Discount" Operator

- Rd1: If John has pure belief in Sally, your belief in her should be equal to your belief in John.
- Example:
  - Your opinion of John is
    - (b=0.3, d=0.2, u=0.5)
  - John's opinion of Sally is
  - (b=1, d=0, u=0)
- What should your opinion be of Sally?
  - My belief in Sally should be 0.3

Requirements of the "Discount" Operator

- Rd2: If John is completely uncertain in Sally, you should be completely uncertain in her too.
- Example:
  - Your opinion of John is
    - (b=0.3, d=0.2, u=0.5)
  - John's opinion of Sally is
  - (b=0, d=0, u=1)
- What should your opinion be of Sally?
  - It should be (b=0, d=0, u=1)

Requirements of the "Discount" Operator

- Rd3: Your belief in Sally (from John's input alone) should never be higher than your belief in John
- Example:
  - Your opinion of John is
    - (b=0.3, d=0.2, u=0.5)
  - John's opinion of Sally is
  - (b<sub>John</sub>, d<sub>John</sub>, u<sub>John</sub>)
- What should your opinion be of Sally?
  - Rule says that my belief in her should not exceed 0.3

### The Fusion Operator

Given n opinions about Sally that you formed from talking to n sources, how do you fuse them into one overall opinion?

#### The Fusion Operator

- Given n opinions about Sally that you formed from talking to n sources, how do you fuse them into one overall opinion?
  - Let *your* opinion about Sally that you formed from talking to source *i* be given by (b<sub>i</sub>, d<sub>i</sub>, u<sub>i</sub>)
  - The overall opinion is the weighted average of opinions (b<sub>i</sub>, d<sub>i</sub>, u<sub>i</sub>) each weighted by your effective belief in source *i* (actual belief plus half the uncertainty)

#### Main Results

 Paper derived a discounting operator and a fusion operator that outperform others in estimating ground truth credibility of sources in the presence of reporting chains

Operator	$Md^{\star} \ d_E(O^{a_z}_{a_S _{\mathfrak{O}}}, O^{a_z}_{Exp})$	$Md^{\star} \; d_E(O^{a_z}_{a_S J}, O^{a_z}_{Exp})$	$s^{-} ( imes 10^{10})$	$s^+ (\times 10^{10})$	z	Incr. Performance <sup>†</sup>
°1	0.141	0.144	4.11	4.53	$-27.457^{\ddagger}$	$\approx +5\%$
on	0.156	0.155	4.40	3.95	$-29.586^{\ddagger}$	$\approx -5\%$
0 <sub>2</sub>	0.143	0.142	4.58	3.89	$-45.559^{\ddagger}$	pprox -8%
03	0.163	0.145	5.12	3.51	$-104.098^{\ddagger}$	$\approx -19\%$

Subjective Logic Operators in Trust Assessment: An Empirical Study