ECE 313: Lecture 19
Continuous-type random variables
Cumulative distribution function (CDF)
Probability density functions (pdf)

X is a random verible : $X = X(\omega)$ real

$\omega \to \mathbb{R}$

CDF

$$F_X(c) = P\{X \leq c\}$$

Discrete-type

$X \in$ finite or countably infinite

$X \in \{k\}_{k=1,2,\ldots}$

$$F_X(c) = \sum_{k \leq c} p_X(k)$$

(pmf) prob mass func

Continuous-type

$X \in$ uncountable

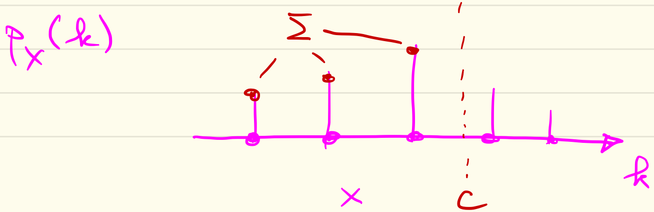
eg. any real val.

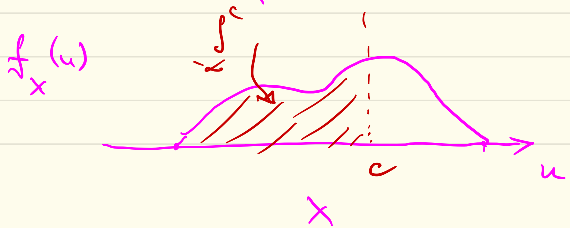
$X \in \mathbb{R}$

$$f_X = F_X'$$

$$F_X(c) = \int_{u \leq c} f_X(u)\, du = \int_{-\infty}^{c} f_X(u)\, du$$

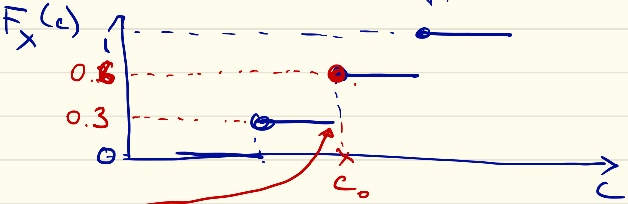
(pdf) prob distr fu



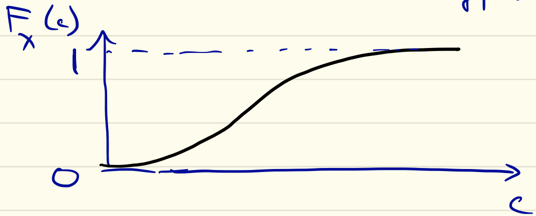


How to interpret a CDF?

Discrete-type



Continuous-type



$F_X(c)$ has a jump at $c = c_0$

$$F_X(c_0) = 0.6$$

$$F_X(c_0-) = \lim_{\substack{c \to c_0 \\ c < c_0}} F_X(c) = 0.3$$

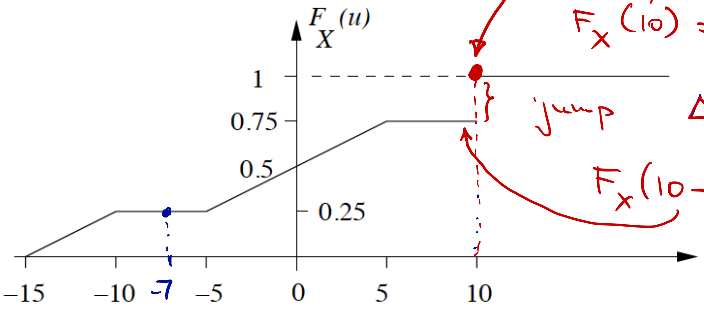
$$\Delta F_X(c_0) \overset{\text{def}}{=} \underbrace{F_X(c_0)} - \underbrace{F_X(c_0-)}$$

$$= P\{X \leq c_0\} - P\{X < c\}$$

$$= P\{X = c_0\}$$

$F_X(c) = P\{X \le c\}$

3.1. **[Using a CDF I]**
Let X be a random variable with the CDF shown.



$F_X(10) = 1$

jump

$\Delta F_X(10) = F_X(10) - F_X(10-)$

$F_X(10-) = 0.75$

Compute the following probabilities:

(a) $P\{X \le 10\}$ $= F_X(10) = 1$

(b) $P\{X \ge -7\}$ $= 1 - P\{X < -7\}$

$= 1 - F_X((-7)-) = 1 - 0.25$

$= 0.75$

(c) $P\{|X| < 10\}$

$X \le c$ $\quad$ $X > c$

$X < c$ $\quad$ $X \ge c$

$= P\{-10 < X < 10\}$

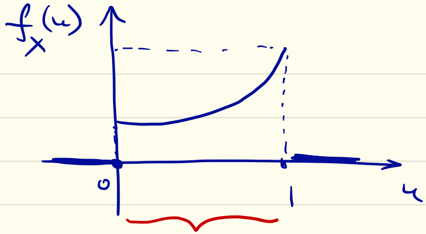
$= P\{X < 10\} - P\{X \le -10\}$

$= F_X(10-) - F_X(-10) = 0.75 - 0.25 = 0.5$

3.5. **[Continuous-type random variables II]**
The pdf of a random variable X is given by:

$$f_X(u) = \begin{cases} a + bu^2, & 0 \le u \le 1 \\ 0, & \text{else} \end{cases}$$

If E[X]=5/8, find a and b.



support of f_X
(i.e. u such that $f_X(u) \neq 0$)

1, $$1 = \int_{-\infty}^{+\infty} f_X(u)\, du = \int_0^1 (a + bu^2)\, du$$

$$= \left(au + \frac{1}{3} bu^3\right)\Big|_0^1$$

$$= a + \frac{1}{3} b$$

$X = u$

2, $$\frac{5}{8} = E[X] \stackrel{def}{=} \int_{-\infty}^{\infty} u \cdot f_X(u)\, du = \int_0^1 u(a + bu^2)\, du$$

$$= \left(\frac{1}{2} au^2 + \frac{1}{4} bu^4\right)\Big|_0^1$$

$$= \frac{1}{2} a + \frac{1}{4} b$$