ECE 313: Lecture 6

Review of probablity space, random variables, pmf, and expectation

Conditional probabilities

Independent events and independent random variables

Prob. Space $\{\Omega, \mathcal{F}, P\}$

experiment outcome ω

Ex: throw 2 dices

$\Omega = \{\omega : \omega = (i, j) ; i, j \in \{1, 2, \dots, 6\}\}$

Random Variable

$X(\omega)$: a real value func. of ω

Ex: $X(\underbrace{(i,j)}_{\omega}) = i + j$

$P(A)$ for $A \subset \Omega$

ω.

axioms of prob.

1. $0 \le P(A)$
2. $P(\Omega) = 1$
3. $A_1, A_2 \dots$ disjoint

$P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \dots$

Law of total Prob

i \ j	1	2	3	4	5	6
1	2	3	4	5		
2	3	4	5			
3	4	5				
4	5					
5						
6						

pmf (discrete)

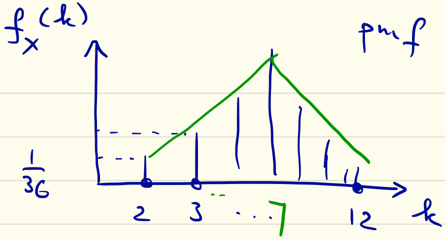
$P(\underbrace{\{X = u_i\}}_{\text{an event}}) = f_X(u_i)$

Ex:

X	f_X
2	1/36
3	2/36
4	4/36
⋮	
12	2/36

$\sum_i f_X(u_i) = 1$

$$E[X] = \sum_i u_i \underbrace{P(\{X = u_i\})}_{f_X(u_i)}$$



Ex: X = "sum of 2 dice"

$$E[X] = 2 \cdot \frac{1}{36} + 3\,\frac{2}{36} + 4\,\frac{3}{36} + \dots + 7\,\frac{6}{36} + \dots + 12 \cdot \frac{1}{36}$$

$$= 7$$ (mean / expectation)

Conditional prob.

Let A & B are events, (assume $P(B) > 0$)

$$\underbrace{P(A \mid B)}_{\text{prob of } A \text{ given } B} = \frac{P(AB)}{P(B)} \qquad AB = A \cap B$$

$$\Rightarrow \quad P(AB) = P(B)\, P(A \mid B)$$

Ex : Medical test of a disease D

actual disease state

test outcome	D	D^c
+	0.009	0.099
−	0.001	0.891

$P(D^c, +)$

$P(D, -)$

$$P(+ \mid D) = \frac{P(+, D)}{P(D)} = \frac{0.009}{0.001 + 0.009} = 0.9$$

$$P(- \mid D^c) = \frac{P(-, D^c)}{P(D^c)} = 0.9$$

$$P(D \mid +) = \frac{P(D, +)}{P(+)} = \frac{0.009}{0.009 + 0.099} \approx 0.8$$