University of Illinois Spring 2018

ECE 313: Hour Exam I

Wednesday, February 28, 2018
8:45 p.m. — 10:00 p.m.

Name: (in BLOCK CAPITALS) ________________________________

NetID: ________________________________

Signature: ________________________________

Section:
☐ C, 10:00 a.m. ☐ G, 11:00 a.m. ☐ D, 11:00 a.m. ☐ F, 1:00 p.m. ☐ B, 2:00 p.m.

Instructions

This exam is closed book and closed notes except that one 8.5”×11” sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of **six** problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading	
1. 12 points	____
2. 10 points	____
3. 20 points	____
4. 18 points	____
5. 18 points	____
6. 22 points	____
Total (100 points)	____

1. **[12 points]** A blood test gives readings of an indicator X according to the following likelihood matrix.

	$X=0$	$X=1$	$X=2$	$X=3$
H_1	0.0	0.35	0.6	0.05
H_0	0.4	0.3	0.2	0.1

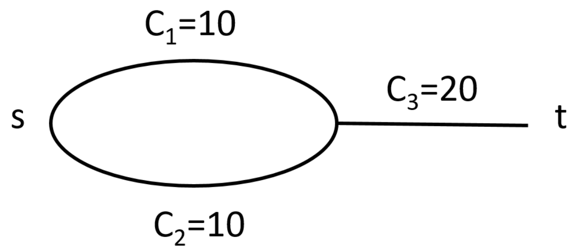
The priors are $(\pi_0, \pi_1) = (0.2, 0.8)$.

(a) What is the maximum likelihood (ML) decision rule? Compute p_{miss} for the ML decision rule.

(b) What is the maximum a posteriori (MAP) decision rule? Compute p_{e} for the MAP decision rule.

(c) What is the decision rule that minimizes p_{miss}?

2. [**10 points**] Consider the following network. Each link fails with probability p.



(a) What is the outage probability?

(b) What is the probability that the network has capacity 10?

3. **[20 points]** The three parts of this problem are unrelated.

(a) Consider a random variable Y. It is known that $E[-3Y-2] = 4$ and that $Var(-3Y-2) = 36$. Determine $E[Y]$ and $Var(Y)$.

(b) Consider rolling a fair die and flipping a fair coin. Define a random variable X which is equal to the number shown in the die if the coin shows heads and twice the number in the die if the coin shows tails. Obtain the pmf of X.

(c) Let Z be a random variable with pmf $p_Z(k) = \frac{c}{(2^k)}$ for $k \in \{1, 2, 3, 4\}$ and zero else. Determine the value of the constant c, of the mean $E[Z]$ and of $E[2^Z]$.

4. **[18 points]** Let X denote a random variable with a Binomial distribution with parameters $n \geq 1$ and $0 \leq p \leq 1$.

   (a) Assume that $p = 3\alpha$, where $\alpha \geq 0$ is unknown. Compute the maximum likelihood estimate of α, denoted by $\hat{\alpha}_{ML}$, for the case when $n = 10$, and $X = 6$.

   (b) Assume that $p = 3\alpha - 4$, where $\alpha \geq 0$ is unknown. Compute the maximum likelihood estimate of α, denoted by $\hat{\alpha}_{ML}$, for the case when $n = 10$, and $X = 0$.

5. **[18 points]** Consider flipping a biased coin, which has $P\{\text{heads}\} = p$. Define a random variable X which is equal to the number of independent successive coin flips until 5 heads show.

   (a) Determine $P\{X = 10 | \text{the 3rd head shows on the 4th coin flip}\}$.

   (b) Determine the conditional probability $P\{\text{the 3rd flip shows heads} | X = 10\}$.

6. **[22 points]** The three parts are unrelated.

(a) There are two basketball teams A and B. We estimate the probability that team A beats team B (denoted by p) by having them play against each other n times. If we want to estimate p within 0.1 and with 75% confidence (using the confidence interval based on the Chebychev bound), how many games should they play?

(b) For this part, assume that the probability that a basketball team is good is given by 0.7. We also know that the probability that a good team wins a game is 0.8, whereas if the team is not good the probability of winning is only 0.6. If a given basketball team plays three games and wins only two, what is the conditional probability that the team was good?

(c) Assume x_i for $i \in \{1, 2, \ldots, r\}$, for some integer r. Let n be another integer where $n \geq r - 1$. How many solutions are there for the equation $x_1 + x_2 + \ldots + x_r = n$ such that exactly one of the integers x_i's is zero, and all the remaining integers x_i's are positive? (Leave your answer in terms of n and r.)