ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

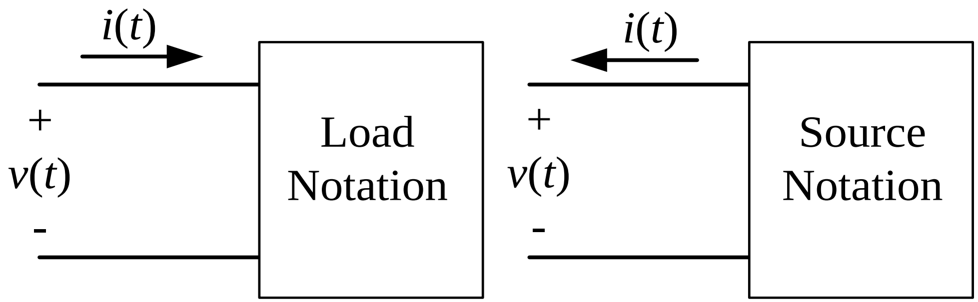
LECTURE 2
ACTIVE, REACTIVE, AND COMPLEX POWER

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.

TWO-TERMINAL NETWORK

- A two-terminal electrical network has voltage at its terminals and current flowing in and out of its terminals.



- The instantaneous power is $p(t) = v(t)\,i(t)$.
- For $i(t) = I_m \cos(\omega t + \theta_i)$ A and $v(t) = V_m \cos(\omega t + \theta_v)$ V we get

$$p(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

TWO-TERMINAL NETWORK

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$$

$$p(t) = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2\omega t + \theta_v + \theta_i) \quad \text{W}$$

The first term is time-independent, while the second term is a sinusoid at double frequency.

TWO-TERMINAL NETWORK

- The average power is thus

$$P = \frac{1}{T}\int_{0}^{T} P(t)\,d(t) \quad , \qquad T = \frac{2\pi}{\omega}$$

$$P_{in} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i).$$

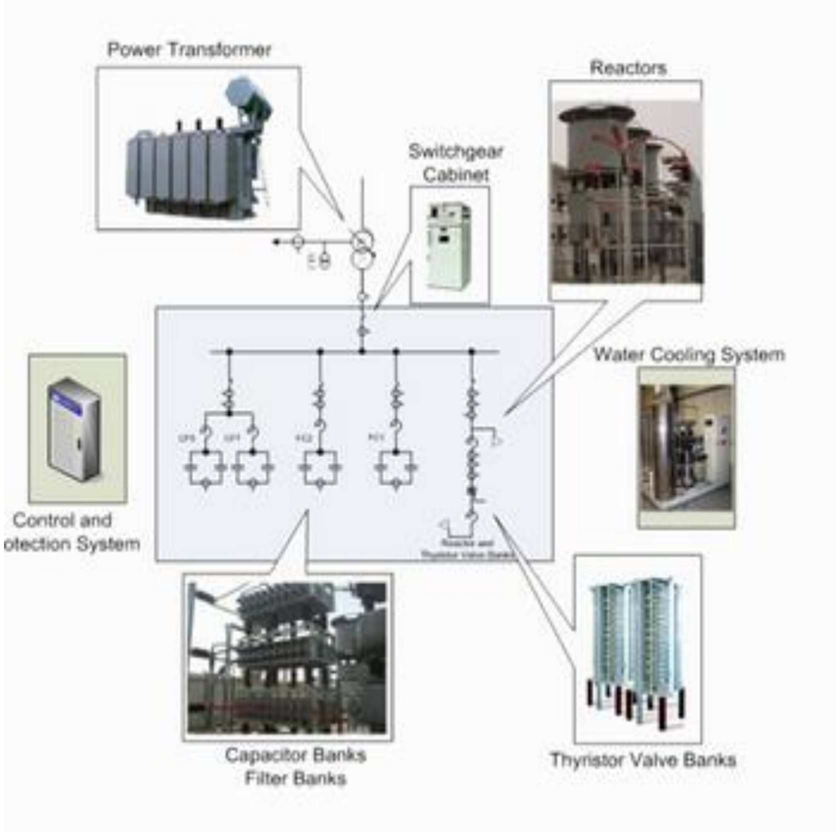
- This is called the active or real power and its unit is watts (W).
- The power factor is the cosine of the phase angle between $v(t)$ and i(t).

POWER FACTOR

- The power factor (*P.F.*) is thus $P.F. = \cos(\theta_v - \theta_i)$.
- The power factor can be:
  - Lagging: $0^o < \theta_v - \theta_i < 90^o$
  - Leading: $-90^o < \theta_v - \theta_i < 0^o$
  - Unity: $\theta_v - \theta_i = 0$

  Therefore, $0 \le P.F. \le 1$,

  and the highest real power exists when *P.F.*=1.



Source: grupovision.com

APPARENT POWER AND REACTIVE POWER

- The apparent power is $S = \frac{V_m I_m}{2}$
- The apparent power unit is volt-amps (VA).
- The reactive power is $Q_{in} = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$.
- The reactive power unit is volt-amps-reactive

  (VARs).

COMPLEX POWER

- The instantaneous power is

$$p(t) = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2\omega t + \theta_v + \theta_i) \quad \text{W}$$

- The time varying component

$$\frac{V_m I_m}{2}\cos(2\omega t + \theta_v + \theta_i) = \frac{V_m I_m}{2}\left\{\cos\left[(2\omega t + 2\theta_i) + (\theta_v - \theta_i)\right]\right\}$$

$$= \frac{V_m I_m}{2}\cos(2\omega t + 2\theta_i)\cos(\theta_v - \theta_i) - \frac{V_m I_m}{2}\sin(2\omega t + 2\theta_i)\sin(\theta_v - \theta_i)$$

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COMPLEX POWER

- Define

$$Q_{in} = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i), \qquad \text{(Reactive power)}$$

$$p(t) = P_{in} + P_{in}\cos(2\omega t + 2\theta_i) - Q_{in}\sin(2\omega t + \theta_i)$$
$$= P_{in}(1 + \cos(2\omega t + \theta_i)) - Q_{in}\sin(2\omega t + 2\theta_i)$$

- The real power can be written as

$$P_{in} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \mathrm{V}_{rms} I_{rms}\cos(\theta_v - \theta_i)$$

PHASOR REPRESENTATION

$$P_{in} = \mathrm{Re}\{\frac{V_m I_m}{2} e^{j\theta_v} e^{-j\theta_i}\} = \mathrm{Re}\{V_{rms} e^{j\theta_v} I_{rms} e^{-j\theta_i}\}$$

- The reactive power can be written as

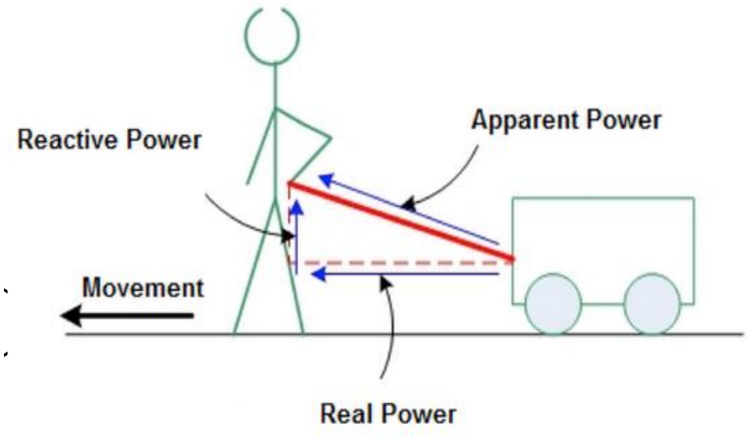
$$Q_{in} = \mathrm{Im}\{\frac{V_m I_m}{2} e^{j\theta_v} e^{-j\theta_i}\} = \mathrm{Im}\{V_{rms} e^{j\theta_v} I_{rms} e^{-j\theta_i}\}$$

- The voltages and currents can be written as phasors:

$V_{rms} e^{j\theta_v} = \bar{V}$ and $I_{rms} e^{j\theta_i} = \bar{I}$.

$$P_{in} = \mathrm{Re}(\bar{V}\ \bar{I}^*) = \mathrm{V}_{rms}\ I_{rms} \cos(\theta_v - \theta_i$$

$$Q_{in} = \mathrm{Im}(\bar{V}\ \bar{I}^*) = \mathrm{V}_{rms}\ I_{rms} \sin(\theta_v - \theta_i$$



Source: Tonex.com

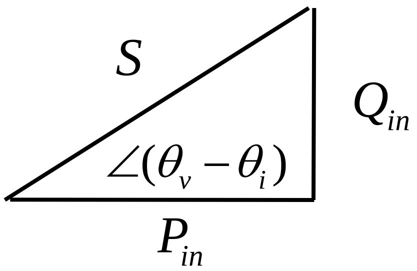
Complex Power

- Define the complex power as $\bar{S} = P_{in} + jQ_{in}$
- Then $\bar{S}$ can be written as $\bar{S} = \bar{V}\,\bar{I}^*$
- The quantity $\bar{I}^*$ is the complex conjugate of $\bar{I}$.
- $\bar{S}$ can also be written as

$$\bar{S} = S\angle(\theta_v - \theta_i)$$

- Note that

$$S = \frac{V_m I_m}{2} = \sqrt{P_{in}^2 + Q_{in}^2}$$



ALTERNATE FORMS OF COMPLEX POWER

- If the load is $\bar{Z} = R + jX$, connected across the source $\bar{V}$

By Ohm's law: $\bar{V} = \bar{Z}\ \bar{I}$, but $\bar{S} = \bar{V}\ \bar{I}^*$

Then $\bar{S}$ can be written as $\bar{S} = I^2 R + jI^2 X$ Also,

$P = I^2 R$ and $Q = I^2 X$, $\bar{Z}$and $P.F. = \cos(angle(\bar{Z}))$.

- Thus, $Q > 0$ when $\bar{Z}$ is inductive, $X = \omega L$

and $Q < 0$ when is capacitive, $X = -\dfrac{1}{\omega C}$

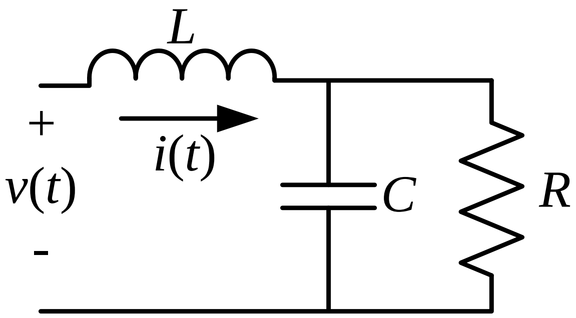
- $\bar{S}$ and $\bar{Z}$ are not phasors but complex quantities.

EXAMPLE: LC FILTER AND R LOAD

- The circuit shown is commonly used as an LC filter to supply a load, which is resistive in this case.
- Find the current, real, reactive, and complex powers, and the P.F. for $v(t) = \sqrt{2}V_{rms}\cos(377t)$

$$\bar{Z} = j\omega L + \left(R // \frac{-j}{\omega C} \right)$$

$$\bar{Z} = \frac{\omega L + j(\omega^2 RLC - R)}{\omega RC - j}$$



EXAMPLE: LC FILTER AND R LOAD

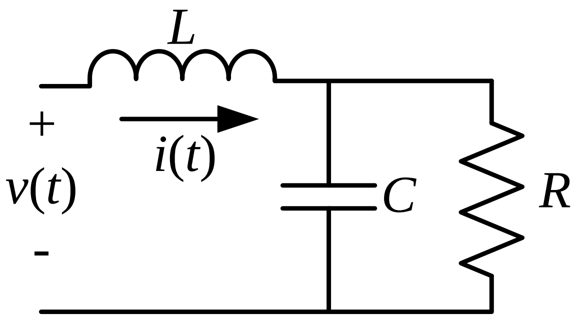
- Let

$$V_{rms} = 120\text{V}, L = 1\text{mH}, C = 6.8\text{mF}, \text{ and R} = 10\Omega.$$

$$\bar{Z} = 0.0197\angle -39.41^o = 0.0152 - j0.0125\Omega$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{120\angle 0^o}{0.0197\angle -39.41^o} = 6091.4\angle 39.41^o\,\text{A}$$

$$i(t) = 6091.4\sqrt{2}\cos(377t + 39.41^o)$$



$$\bar{S} = \bar{V}\ \bar{I}^* = 731\angle -39.41^o\,\text{kVA}$$

$$P_{in} = 731\cos(-39.41^o) = 564.8\text{kW}$$

$$Q_{in} = 731\sin(-39.41^o) = -464.1\text{kVAR}$$

$$\text{P.F.} = \cos(-39.41^o) = 0.773 \text{ leading } (\theta_v - \theta_i = -39.41^o)$$

READING MATERIAL

- Reading material: Chapter 2 sections 2.1 – 2.3.

- Recommended reading for next time: section 2.4.