ECE 330 Exam #1, Fall 2017   Name: ______________________________
90 Minutes

Section (Check One)   MWF 9 am _______   MWF 10 am _______

1. ________ / 25   2. ________ / 25

3. ________ / 25   4. _________ / 25   Total   ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\overline{V} = \overline{ZI}$ $\overline{S} = \overline{VI}^* = P + jQ$ $\overline{S}_{3\phi} = \sqrt{3} V_L I_L \angle\theta$

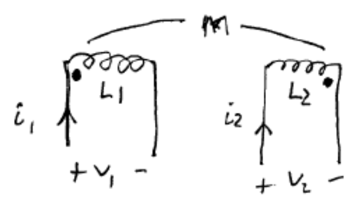
$0 < \theta < 180^\circ$ (lag) $I_L = \sqrt{3} I_\phi$ (delta) $\overline{Z}_Y = \overline{Z}_\Delta / 3$ $\mu_0 = 4\pi \cdot 10^{-7}$ H/m

$-180^\circ < \theta < 0$ (lead) $V_L = \sqrt{3} V_\phi$ (wye)

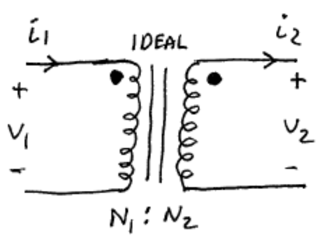
ABC sequence has A at zero, B at minus 120 degrees, and C at plus 120 degrees

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\mathfrak{R} = \frac{l}{\mu A}$ $MMF = Ni = \phi \mathfrak{R}$

$\varphi = B A$ $\lambda = N\varphi = Li$ (if linear) $v = d\lambda/dt$ $k = \frac{M}{\sqrt{L_1 L_2}}$ 1 hp = 746 Watts

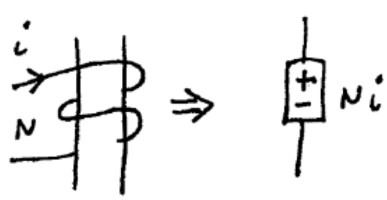


$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$



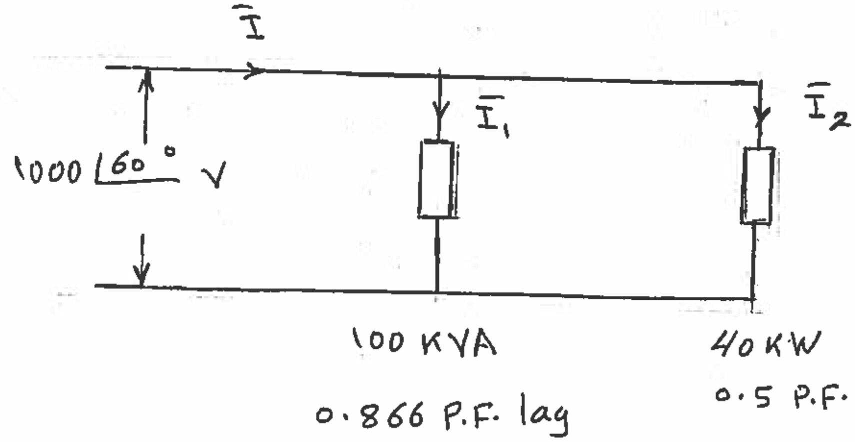
$a = \frac{N_1}{N_2}$ $N_1 i_1 = N_2 i_2$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$



Problem 1. (25 points)

Two loads are connected in parallel across a source whose voltage is $1000\angle 60^\circ$ V. Load 1 is 100 kVA at 0.866 PF lagging and load 2 is 40 kW at 0.5 PF lagging. Find the total complex power, the total current $\bar{I}$ supplied by the source, and the currents in each of the loads. What is the kVAR of the capacitor needed to make the overall PF 0.95 lag?. The capacitor is connected across the parallel combination.



load 1: $\bar{S}_1 = \bar{V}\bar{I}_1^* = 100K\angle \cos^{-1} 0.866 = 1000\angle 60^\circ \, I_1^*$

$\Rightarrow \bar{I}_1 = 100\angle 30^\circ$ A $\qquad \frac{4}{25}$

Load 2: $P_2 = V_2 I_2 \cos\theta \Rightarrow 40K = 1000 I_2 \, 0.5$

$\Rightarrow |I_2| = 80$, P.F. angle $= 60^\circ - \cos^{-1} 0.5 = 0 \Rightarrow \bar{I}_2 = 80\angle 0^\circ$ A $\qquad \frac{4}{25}$

$\bar{I} = \bar{I}_1 + \bar{I}_2 = 100\angle 30^\circ + 80\angle 0^\circ = 173.9\angle 16.7^\circ$ A $\qquad \frac{4}{25}$

$\bar{S} = \bar{V}\bar{I}^* = 1000\angle 60^\circ \times 173.9\angle -16.7^\circ = 126.6 + j\,119.3$

$= 173.9\angle 43.3^\circ$ KVA $\qquad \frac{3}{25}$

New P.F. is 0.95 $\Rightarrow \cos\theta_{new} = 0.95$, $\theta_{new} = 18.194^\circ$ $\qquad \frac{3}{25}$

$Q_{new} = P\tan\theta_{new} \Rightarrow Q_{new} = 126.6 \tan 18.194^\circ \Rightarrow Q_{new} = 41.61$ KVAR

3/25

$Q_{new} = Q_{old} + Q_{cap} \Rightarrow Q_{cap} = 41.61 - 119.3 = -77.69$ KVAR

4/25

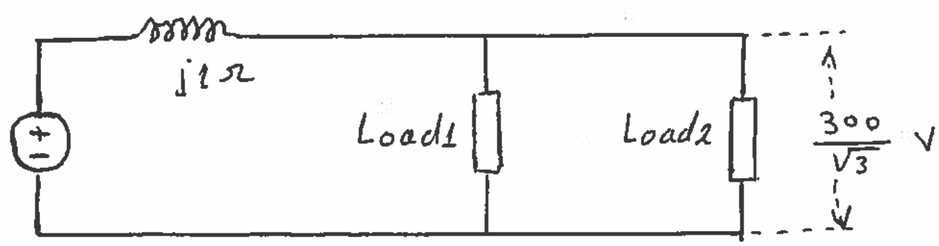
Problem 2. (25 points)

A three-phase power system consists of a wye-connected source, 60Hz, connected to two loads in parallel through a transmission line having an impedance of $Z_{line} = 0 + j1\ \Omega$.
The first load is wye connected and draws 10 Amps line current at 0.85 P.F. lagging. The second load is delta connected and consumes $1581.138\angle 18.434^{o}\ VA$. Phase to neutral voltage at the load is $\frac{300}{\sqrt{3}}\ V$.

a) Draw the per-phase equivalent circuit with all given voltages, currents, powers, and impedances labeled. [5 points]
b) Find the voltage at the source.[10 points]
c) Find the complex power delivered by the source.[4 points]
d) Find the per-phase impedance of the delta connected load.[6 points]

5/25

a)



$1581.138\angle 18.434^{0}$ VA

b) $I_1 = 10\angle -\cos^{-1} 0.85 = 10\angle -31.788^{0}\ A$ 2/25
$8.5 - j5.2$

$$\bar{S}_{2ph} = \frac{S_{2\,3\phi}}{3} = \frac{1581.138\angle 18.434^{0}}{3} = 500 + j166.667\ VA$$

$$\bar{I}_{2ph} = \left(\frac{S_{2ph}}{V_{ph}}\right)^{*} = \left(\frac{500 + j166.667}{300/\sqrt{3}}\right)^{*} = 3.04\angle -18.4^{0}\ A$$ 3/25

or, $\bar{I}_{2ph} = \left(\frac{S_{3\phi}}{3V_{ph}}\right)^{*} = \frac{1500 + j500}{3 \times \frac{300}{\sqrt{3}}} = 3.04\angle -18.4^{0}\ A$ 3/25
$2.884 - j0.959$

$$\bar{I}_{source} = I_1 + I_2 = 10\angle -31.788^{0} + 3.04\angle -18.4^{0}\ A = 12.96\angle -28.57^{0}\ A$$
$11.38 - j6.19$ 3/25

$$\bar{V}_{source} = \frac{300}{\sqrt{3}} + 12.96\angle -28.57^{0} \times j1 = 179.76\angle 3.63^{0}\ V$$ 3/25
$179.429 + j11.38$

c) $\bar{S}_{source} = 3\bar{V}\bar{I}^{*} = 3 \times 179.76\angle 3.63^{0} \times 12.96\angle 28.57^{0} = 6989\angle 32.2\ VA$

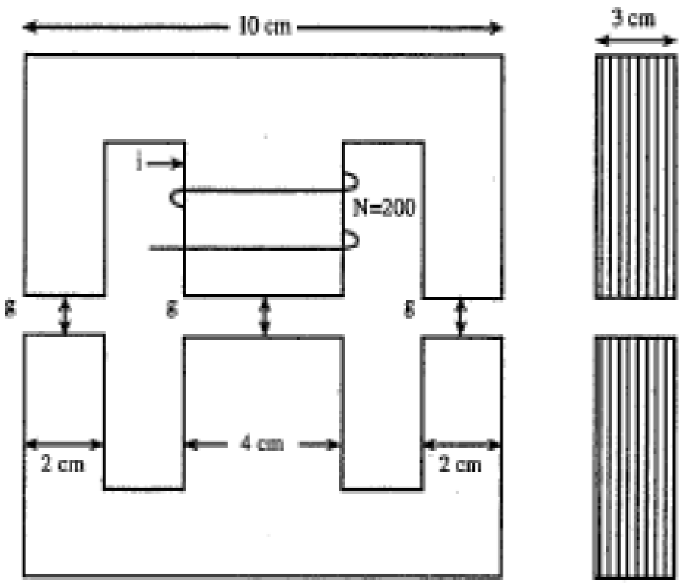
3/25

d) $$\bar{Z}_{2,Y} = \frac{\bar{V}_2}{\bar{I}_2} = \frac{\frac{300}{\sqrt{3}}}{3.04\angle -18.4^{0}} = 56.98\angle 18.4^{0}\ \Omega$$ 3/25

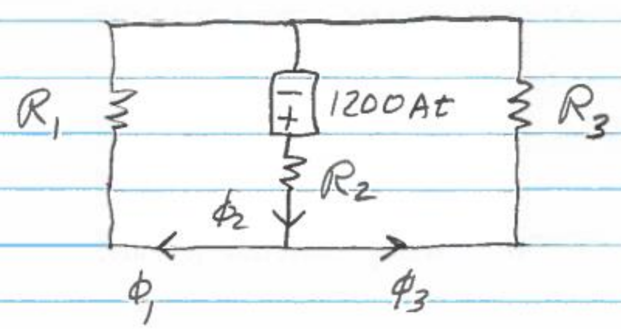
$$\bar{Z}_{2,\Delta} = 3\bar{Z}_{2,Y} = 171\angle 18.4^{0}\ \Omega$$ 3/25

Problem 3. (25 points)

For the figure below, for a current i = 6 A (assume DC) and gap g = 0.1 cm, find the magnetic flux density (in Tesla) in the center leg and each of the outer legs and the inductance of the coil. You may neglect fringing and assume the permeability of the iron is infinite. Note that the number of turns is given as 200. The side view of the device is also given on the right.



#3

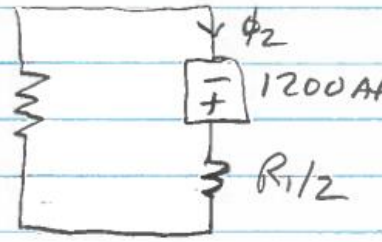


$$B = \frac{\phi}{A}$$

$$\mathcal{R}_1 = \frac{g}{\mu_0 A_1} = \frac{0.1\times10^{-2}}{4\pi\times10^{-7}\times6\times10^{-4}} = 1326292\,\frac{At}{Wb} = \mathcal{R}_3$$

$$\mathcal{R}_2 = \frac{g}{\mu_0 A_2} = \frac{0.1\times10^{-2}}{4\pi\times10^{-7}\times12\times10^{-4}} = 663146\,\frac{At}{Wb} = \mathcal{R}_1/2$$

$R_1 // R_3 = \frac{R_1}{2}$



$-1200At + R_1 \phi_2 = 0$

$\phi_2 = 0.000905$ wb

$\phi_1 = \phi_3 = \frac{\phi_2}{2} = 0.0004524$ wb

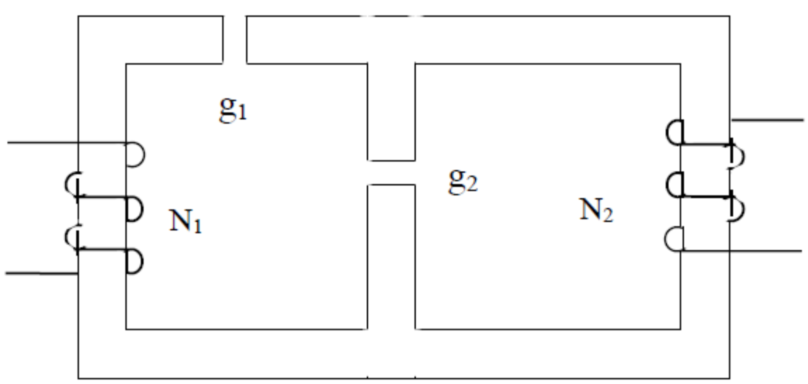
$B_1 = B_3 = 0.75\,T \qquad B_2 = 0.75\,T$

Assume v, λ polarity + on top

$\lambda = N\phi_2 = 0.181$ wbtn $= L \times 6$ Amps $\qquad L = 0.03\ H$

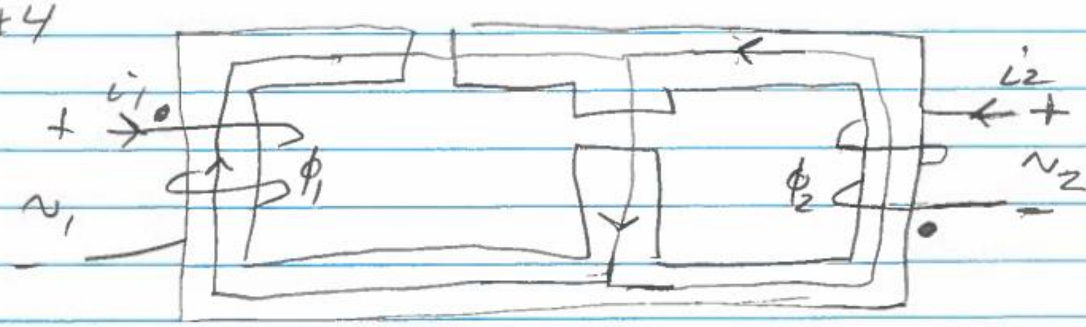
Problem 4. (25 points)

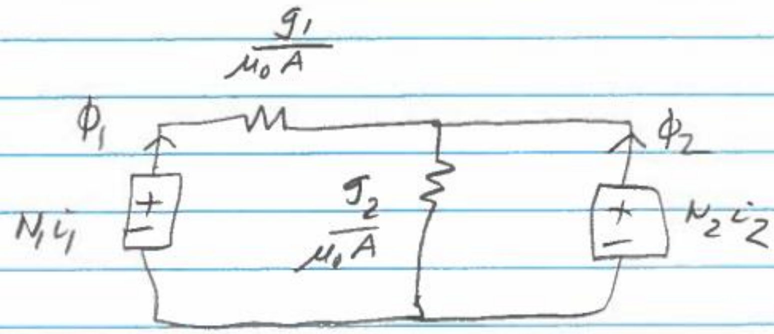
A multi-coil magnetic circuit is illustrated in the diagram below. The cross-sectional area of the magnetic core is A everywhere. The permeability of the magnetic core is infinite. There are two air gaps with lengths g_1 and g_2 as shown and permeability μ_o. You may neglect fringing. Label the voltages and currents - you may define the two voltages and two currents any way that you like.



a) (5 pts) Put the polarity "dots" on the proper side of each coil
b) (10 pts) Find expressions for L1, L2, and M in terms of the parameters
c) (5 pts) Find an expression for the coefficient of coupling in terms of only g1 and g2
d) (5 pts) What happens to the coefficient of coupling as g1 gets smaller and g2 gets bigger?

#4





$v_1 = N_1 \frac{d\phi_1}{dt}$ $\qquad$ $v_2 = N_2 \frac{d\phi_2}{dt}$

$$-N_1 v_1 + \frac{g_1}{\mu_0 A}\phi_1 + \frac{g_2}{\mu_0 A}(\phi_1 + \phi_2) = 0$$

$$-N_2 i_2 - \frac{g_1}{\mu_0 A}\phi_1 + N_1 i_1 = 0 \qquad \phi_1 = \frac{\mu_0 A N_1}{g_1} i_1 - \frac{\mu_0 A}{g_1} N_2 i_2$$

$$\lambda_1 = \underbrace{\frac{\mu_0 A N_1^2}{g_1}}_{= L_1} i_1 - \underbrace{\frac{\mu_0 A N_1 N_2}{g_1}}_{= M} i_2$$

$$-N_1 i_1 + \left(\frac{g_1}{\mu_0 A} + \frac{g_2}{\mu_0 A}\right)\left(\frac{\mu_0 A N_1 i_1}{g_1} - \frac{\mu_0 A N_2 i_2}{g_1}\right) + \frac{g_2 \phi_2}{\mu_0 A} = 0$$

$$\phi_2 = \cancel{\frac{\mu_0 A N_1}{g_2} i_1} - \left(\cancel{\frac{g_1}{g_2}} + 1\right)\left(\frac{\mu_0 A N_1}{g_1} i_1\right) + \left(\frac{g_1}{g_2} + 1\right)\left(\frac{\mu_0 A N_2}{g_1} i_2\right)$$

$$\lambda_2 = -\underbrace{\frac{\mu_0 A N_1 N_2}{g_1}}_{= M} i_1 + \underbrace{\left(\frac{g_1}{g_2} + 1\right)\left(\frac{\mu_0 A N_2^2}{g_1}\right)}_{= L_2} i_2$$

c) $$k = \frac{\frac{\mu_0 A N_1 N_2}{g_1}}{\sqrt{\frac{\mu_0 A N_1^2}{g_1} \frac{\mu_0 A N_2^2}{g_1} \left(\frac{g_1}{g_2} + 1\right)}} = \frac{1}{\sqrt{\left(\frac{g_1}{g_2} + 1\right)}}$$

d) If g_1 gets smaller and g_2 gets bigger

k approaches 1