ECE 330 Exam #2, Fall 2017 Name: Solution
90 Minutes

Section (Check One) MWF 9am ______ MWF 10am ______

1. ______ / 25 2. ______ / 25

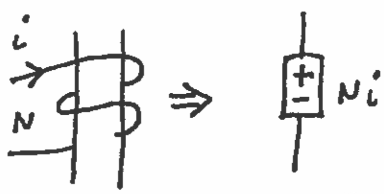
3. ______ / 25 4. ______ / 25 Total ______ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\quad \bar{V} = \bar{Z}\bar{I}$ $\quad \bar{S} = \bar{V}\bar{I}^*$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n}da$ $\quad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t}\int_S \mathbf{B} \cdot \mathbf{n}da$ $\quad MMF = Ni = \phi\Re$

$\Re = \frac{l}{\mu A}$ $\quad B = \mu H$ $\quad \phi = BA$ $\quad \lambda = N\phi$ $\quad \lambda = Li$ (if linear)



$W_m = \int_0^{\lambda} i\, d\hat{\lambda}$ $\quad W_m' = \int_0^{i} \lambda\, d\hat{i}$ $\quad W_m + W_m' = \lambda i$ $\quad f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x}$ $\quad x \to \theta$

$f^e \to T^e$

$\underset{a\to b}{EFE} = \int_a^b i\, d\lambda$ $\quad \underset{a\to b}{EFM} = -\int_a^b f^e dx$ $\quad \underset{a\to b}{EFE} + \underset{a\to b}{EFM} = W_{mb} - W_{ma}$ $\quad \lambda = \frac{\partial W_m'}{\partial i}$ $\quad i = \frac{\partial W_m}{\partial \lambda}$

For $\dot{x}_1 = f_1(x_1, x_2)$ and $\dot{x}_2 = f_2(x_1, x_2)$,

$$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} \approx \begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_{x=x^e} & \left.\frac{\partial f_1}{\partial x_2}\right|_{x=x^e} \\ \left.\frac{\partial f_2}{\partial x_1}\right|_{x=x^e} & \left.\frac{\partial f_2}{\partial x_2}\right|_{x=x^e} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

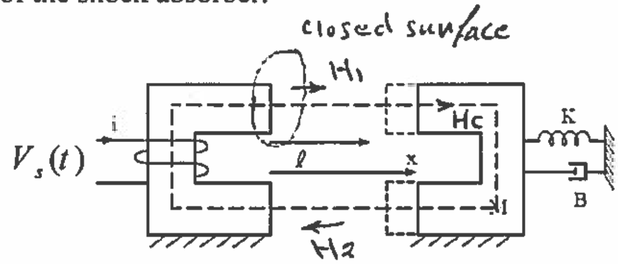
$$x(t_0 + \Delta t) \approx x(t_0) + \Delta t \cdot \left.\frac{dx}{dt}\right|_{t=t_0}$$

For $\underline{\dot{x}} = \underline{A}\underline{x}$, the eigenvalues λ of the system are given by $|\lambda \underline{I} - \underline{A}| = 0$

Problem 1 (25 points)

Consider the magnetic circuit in the Figure. The length of the magnetic path is ℓ_c and ℓ is the static equilibrium position of the moving part ($\ell > 0$). The number of the coil turns is N, and the cross section area is A. The magnetic circuit has finite μ, the mass of the moving part is M, K is the stiffness of the spring and B is the strength of the shock absorber.

(a) Find the flux linkage.
(b) Compute the force of electrical origin.
(c) Find the voltage at terminal pair.
(d) Write the mechanical equation of motion.



Method 1: Using field theory:

(a):

$$H_c \ell_c + H_1 x + H_2 x = Ni \quad \text{(Ampere's law)}$$ 2/25

$$\mu_0 H_1 A = \mu_0 H_2 A \Rightarrow H_1 = H_2 \quad \text{(Gauss's law)}$$ 2/25

Applying Gauss's law to the closed surface around the upper fixed surface:

$$\mu H_c A = \mu_0 H_1 A \Rightarrow H_c = \frac{\mu_0}{\mu} H_1$$

$$H_c \ell_c + 2 H_1 x = Ni$$

$$H_1 = \frac{Ni}{\left(\frac{\mu_0}{\mu}\ell_c + 2x\right)}$$ 2/25

The flux linkage of the coil is given by $\lambda = N\phi = N\mu_0 H_1 A$

$$= \frac{N\mu_0 Ni}{\left(\frac{\mu_0}{\mu}\ell_c + 2x\right)} A = \frac{N^2 i}{\left(\frac{\ell_c}{\mu A} + \frac{2x}{\mu_0 A}\right)}$$ 3/25

$$W'_m = \int_0^i \lambda(i, x)\, di = \frac{N^2 i^2}{2\left(\frac{\ell_c}{\mu A} + \frac{2x}{\mu_0 A}\right)}$$ 3/25

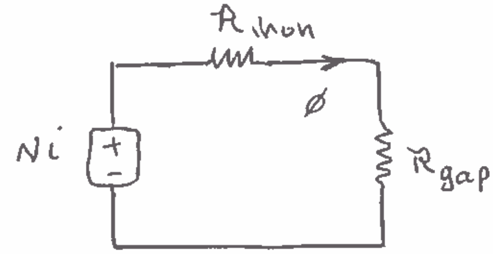
(b):

$$f^e = \frac{\partial W'_m}{\partial x} = \frac{-N^2 i^2}{2\left(\frac{\ell_c}{\mu A} + \frac{2x}{\mu_0 A}\right)^2}\left(\frac{2}{\mu_0 A}\right) = \frac{-N^2 i^2}{\mu_0 A\left(\frac{\ell_c}{\mu A} + \frac{2x}{\mu_0 A}\right)^2}$$ 3/25

Method 2: Using the concept of magnetic equivalent circuit

(a): 2/25

$$\mathcal{R}_{iron} = \frac{l_c}{\mu A}, \quad \mathcal{R}_{gap} = \frac{2x}{\mu_0 A}$$ 2/25



$$\phi = \frac{Ni}{\mathcal{R}_{iron} + \mathcal{R}_{gap}} = \frac{Ni}{\frac{l_c}{\mu A} + \frac{2x}{\mu_0 A}}$$ 2/25

$$\mathcal{R}_{iron} + \mathcal{R}_{gap} = \mathcal{R}(x), \quad \phi = \frac{Ni}{\mathcal{R}(x)}$$

$$\lambda = N\phi = \frac{N^2 i}{\mathcal{R}(x)}$$ 3/25, $$W'_m = \int_0^i \lambda(i', x)\, di' = \frac{N^2 i^2}{2\mathcal{R}(x)}$$ 3/25

(b):

$$f^e = \frac{\partial W'_m}{\partial x} = \frac{-N^2 i^2}{\mu_0 A \left(\frac{l_c}{\mu A} + \frac{2x}{\mu_0 A}\right)^2}$$ 3/25

(c): Equation on the electrical side

$$v_s = \frac{d\lambda}{dt} = \frac{N^2}{\left(R_c + \frac{2x}{\mu_0 A}\right)} \cdot \frac{di}{dt} - \frac{N^2 i}{\left(R_c + \frac{2x}{\mu_0 A}\right)^2} \cdot \frac{2}{\mu_0 A} \frac{dx}{dt}$$ 5/25

(d): Mechanical equations: $\frac{d^2(x-l)}{dt} = \frac{dx}{dt}$

5/25

$$M\frac{d^2x}{dt^2} + K(x-l) + B\frac{dx}{dt} = f^e = \frac{-N^2 i^2}{\mu_0 A\left(R_c + \frac{2x}{\mu_0 A}\right)^2}$$

Problem 2. **(25 points)**

A single-phase generator consists of a coil on the stator and a coil on the rotor with a mutual inductance variation as $0.1\cos(\theta)$ (Henries) where θ is the angle from the stator field axis to the rotor field axis. The rotor is being driven at a constant speed of 377 radians per second and the rotor coil has a constant dc current $i_r = 5$ A. The self inductances of the stator and rotor are both constants and you may assume a linear magnetic core.

(a) Compute the open-circuit stator voltage ($i_s = 0$) as a function of time (recall that $d\theta/dt=\omega$ and assume some angle $\theta= \theta o$ at time zero)

$$\lambda_s = L_s i_s + 0.1\cos\theta\, i_r$$

$$\lambda_r = 0.1\cos\theta\, i_s + L_r i_r$$

$$V_{s_{oc}} = \left.\frac{d\lambda_s}{dt}\right|_{i_s=0} = -0.1\sin\theta\, i_r \times \frac{d\theta}{dt} = -0.5\times 377\sin\theta$$

$$= -188.5\sin(377t+\theta_0)\ V$$

(b) What is the torque of electrical origin when $i_s = 10$ Amps and $\theta = 45°$?

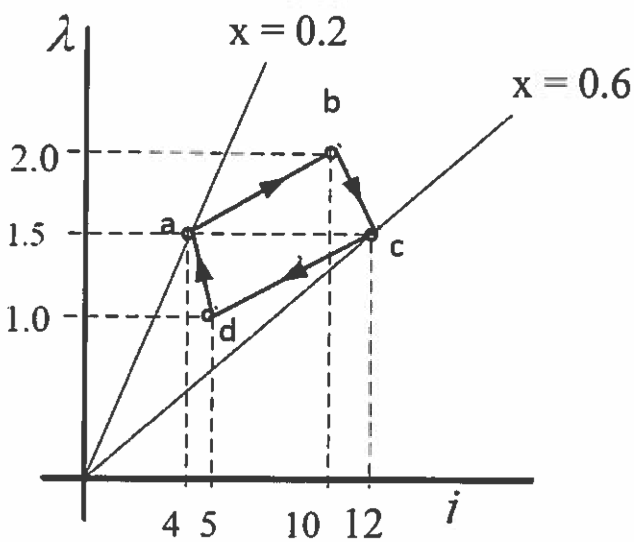
$$W_m' = \frac{1}{2}L_s i_s^2 + 0.1\cos\theta\, i_s i_r + \frac{1}{2}L_r i_r^2$$

$$T^e = -0.1\sin\theta\, i_s i_r$$

$$= -0.1\sin 45° \times 10\times 5 = -3.54\ Nm$$

Problem 3. (25 points.)

An electromechanical device is operated over the cycle abcda shown in the figure below. The system is known to be electrically linear, i.e. $\lambda = L(x)i$. The units of λ are Wb-Turns, the units of i are Amps, and the units of x are cm.



The letters EFE below stand for "Energy From the Electrical system" and EFM stands for "Energy From the Mechanical system".

(a) Calculate the energy stored in the coupling field (W_m) at points a, b, c, and d.
(b) Calculate EFE|a-b and EFM|a-b in Joules.
(c) Calculate EFE|b-c and EFM|b-c in Joules.
(d) Calculate EFE|c-d and EFM|c-d in Joules.
(e) Calculate EFE|d-a and EFM|d-a in Joules.
(f) Is the machine operating as a motor or a generator?

(Note : You must clearly show the steps for parts (a) – (e) and state the reason for your answer in part (f))

(a) $W_{ma} =$ 1.5, 0, 4, a $= 3J$ $\qquad W_{mb} =$ 2, 0, 10, b $= 10J$

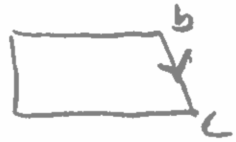
$W_{mc} =$ 1.5, 0, 12, c $= 9J$ $\qquad W_{md} =$ 1, 0, 5 $= 2.5J$

(b) – (f) next page

(b) $\underset{a-b}{EFE} =$ 

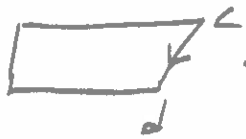
$= 3.5\,J$

$\underset{a-b}{EFm} = w_{m_b} - w_{m_a} - 3 = 7 - 3.5 = 3.5\,J$

(c) $\underset{b-c}{EFE} =$ 

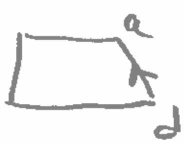
$= -5.5\,J$

$\underset{b-c}{EFm} = w_{m_c} - w_{m_b} + 5.5 = 4.5\,J$

(d) $\underset{c-d}{EFE} =$ 

$= -4.25\,J$

$\underset{c-d}{EFm} = w_{m_d} - w_{m_c} + 4.25 = -2.25\,J$

(e) $\underset{d-a}{EFE} =$ 

$= 2.25\,J$

$\underset{d-a}{EFm} = w_{m_a} - w_{m_d} - 2.25 = -1.75\,J$

(f) $\underset{a-b}{EFE} + \underset{b-c}{EFE} + \underset{c-d}{EFE} + \underset{d-a}{EFE} = -4\,J$

$\underset{cycle}{EFm} = -\underset{cycle}{EFE} = 4\,J$

Generator because

$\underset{cycle}{EFm} > 0$

Problem 4 (25 points)

A translational electromechanical system has the following equations:

$$\frac{dx_1}{dt} = x_1 - x_1 x_2$$

$$2\frac{dx_2}{dt} = x_1 x_2 - 4x_2$$

Assume the initial conditions for this system are:

$$x(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$$

(a) Find all the possible static equilibrium points.
(b) Find the eigenvalues for each of these equilibrium points.
(c) Are these points stable or unstable?
(d) Using Euler's method with time step $\Delta t = 0.1$ second, find the value of x(0.1) and x(0.2).

(a): $\left.\begin{aligned}\dot{x}_1 &= x_1 - x_1 x_2 \\ \dot{x}_2 &= 0.5 x_1 x_2 - 2x_2\end{aligned}\right\} \Rightarrow \left.\begin{aligned} 0 &= x_1 - x_1 x_2 \\ 0 &= 0.5 x_1 x_2 - 2x_2\end{aligned}\right\} \Rightarrow$ 1/25

$x_1^e = 0 \,, \quad x_2^e = 0$ 2/25

$x_1^e = 4 \,, \quad x_2^e = 1$ 2/25

2/25

(b):
(c): $A = \begin{bmatrix} 1 - x_2 & -x_1 \\ 0.5x_2 & 0.5x_1 - 2 \end{bmatrix} \Rightarrow A_1 = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$

2/25 2/25

$\begin{vmatrix} \lambda - 1 & 0 \\ 0 & \lambda + 2 \end{vmatrix} = (\lambda - 1)(\lambda + 2) = 0 \Rightarrow \lambda = 1, -2$ (unstable)

2/25 2/25 2/25

$A_2 = \begin{bmatrix} 0 & -4 \\ 0.5 & 0 \end{bmatrix} \Rightarrow \begin{vmatrix} \lambda & 4 \\ -0.5 & \lambda \end{vmatrix} = \lambda^2 + 2 = 0 \Rightarrow \lambda = \pm j\sqrt{2}$ (Marginally stable)

(d) 2/25

$x_1(0.1) = 2 + (2 - 2 \times 0.5) \times 0.1 = 2.1$ 2/25

$x_2(0.1) = 0.5 + (0.5 \times 2 \times 0.5 - 2 \times 0.5) \times 0.1 = 0.45$ 2/25

$x_1(0.2) = 2.1 + (2.1 - 2.1 \times 0.45) \times 0.1 = 2.22$

$x_2(0.2) = 0.45 + (0.5 \times 2.1 \times 0.45 - 2 \times 0.45) \times 0.1 = 0.407$ 2/25