ECE 430 Exam #1, Fall 2010 Name: SOLUTIONS
90 Minutes

Section (Check One) MWF 10am _______ MWF 2pm _______

1. ________ / 25 2. ________ / 25

3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\quad \bar{V} = \bar{Z}\bar{I}$ $\quad \bar{S} = \bar{V}\bar{I}^*$ $\quad \bar{S}_{3\phi} = \sqrt{3} V_L I_L \angle\theta$

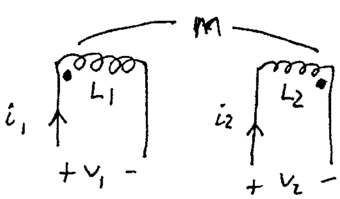
$0 < \theta < 180^\circ$ (lag)
$-180^\circ < \theta < 0$ (lead)

$I_L = \sqrt{3} I_\phi$ (delta)
$V_L = \sqrt{3} V_\phi$ (wye)

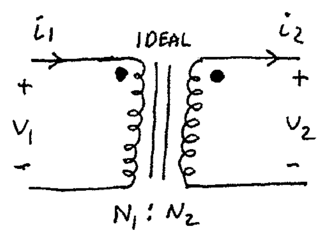
$\bar{Z}_Y = \bar{Z}_\Delta / 3$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\quad \int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\quad \Re = \frac{l}{\mu A}$ $\quad MMF = Ni = \phi\Re$

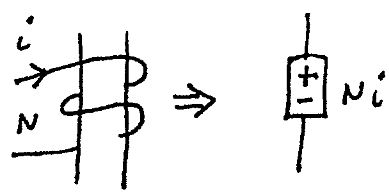
$\phi = BA$ $\quad \lambda = Li = N\phi$ $\quad k = \frac{M}{\sqrt{L_1 L_2}}$ $\quad$ 1 hp = 746 Watts



$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$



$a = \frac{N_1}{N_2}$ $\quad N_1 i_1 = N_2 i_2$ $\quad \frac{V_1}{V_2} = \frac{N_1}{N_2}$



Problem 1. **(25 points)**

A single phase source with voltage 120Volts (RMS) is serving a passive load through two wires with total impedance $(0.5 + j1.0)\Omega$. The load has a power factor of 0.85 lag and it draws a current with magnitude 8 Amps (RMS).

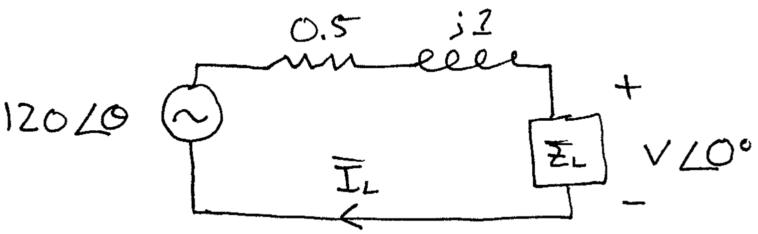
What is the magnitude of the voltage across the load?

Load: $pf = 0.85$ lag

$\rightarrow \theta_v - \theta_i = 31.8°$

$0 - \theta_i = 31.8°$

$\theta_i = -31.8°$



KVL

$$120\angle\theta = (0.5 + j1\,\Omega)(8\angle -31.8°\,A) + V\angle 0°$$

$$120\cos\theta + j120\sin\theta = 8.96\angle 31.63°\,V + V$$

Real Portion

$120\cos\theta = 7.63 + V$

$120\cos(2.24°) = 7.63 + V$

$\boxed{V = 112.28V}$

Imag. Portion

$120\sin\theta = 4.70$

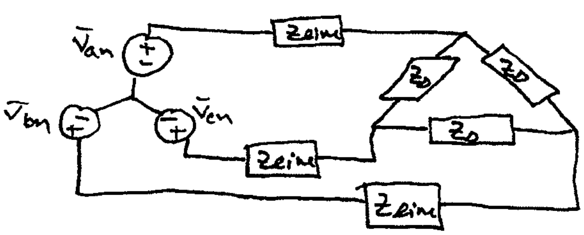
$\theta = 2.24°$

Problem 2. (25 pts)

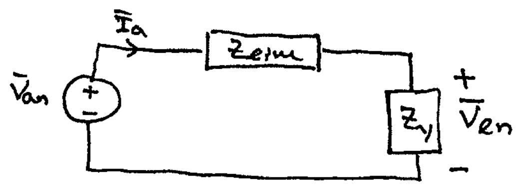
A three-phase power system consists of a wye-connected generator connected to a delta connected load through a transmission line having a per-phase impedance magnitude of 1.5 Ohms and angle 75 degrees. The delta-connected load consumes a total three-phase apparent power of 450 kVA at 0.8 power factor lag when the generator voltage is set so that the voltage at the load is 4,160 Volts (line to line).

a) Find the magnitude of the generator line current for this system.
b) Find the magnitude of the line-line voltage of the generator.
c) Find the total three-phase complex power supplied by the generator.

Total System



Per-Phase Equivalent



a) $\bar{S}_{L1\phi} = \dfrac{\bar{S}_{L3\phi}}{3} = \bar{V}_{en}\bar{I}_e^* = 150\angle\cos^{-1}(0.8)$ kVA

$$I_e^* = \frac{150\angle 36.9°}{\frac{4160}{\sqrt{3}}\angle 0°}$$

→ Load ℓ-n voltage taken as reference angle

$$\boxed{\bar{I}_e = 62.45\angle -36.9°\ A}$$

b) KVL of per-phase circuit

$\bar{V}_{an} = (\bar{Z}_{line})(\bar{I}_e) + \bar{V}_{en}$

$= (1.5\angle 75°)(62.45\angle -36.9°) + \dfrac{4160}{\sqrt{3}}\angle 0°$

$= 73.72 + j\,57.8 + 2401.8$

$\bar{V}_{an} = 2476.2\ \angle 1.33°$

↳ $\bar{V}_{\ell\ell} = \cancel{\frac{\bar{V}_{an}}{\sqrt{3}}\angle\theta+30}\ \sqrt{3}V_{an}\angle\theta + 30°$

$$\boxed{\bar{V}_{\ell\ell} = 4288.9\angle 31.33°\ V}$$

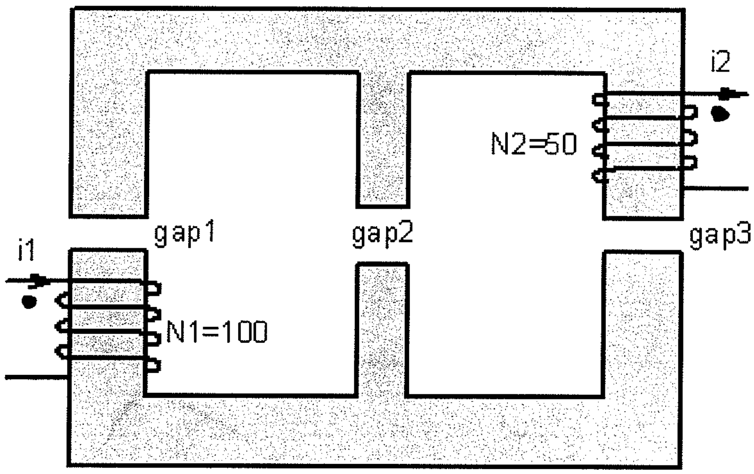
c) $\bar{S}_{T3\phi} = 3\,\bar{V}_{an}\,\bar{I}_e^*$

$= 3\,(2476.2\angle 1.33°\ V)\,(62.45\angle 36.9°\ A)$

$$\boxed{\bar{S}_{T3\phi} = 463.9\ \angle 38.23\ \text{kVA}}$$

Problem 3. (25 points)

An iron core with two coils is shown below. The permeability of the iron core is infinitely large, hence the magnetic reluctance of the metal materials can be ignored. The dimensions of the air gaps are summarized in the figure. Ignore fringing effects.



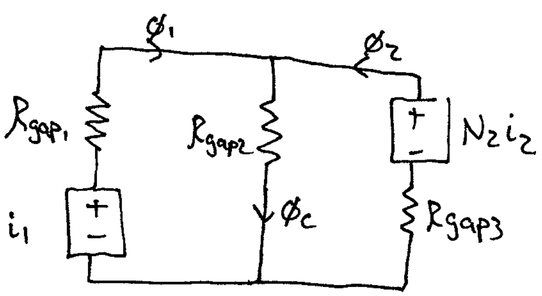
Gap 1: Distance: 5 mm; cross-sectional area: 1 cm^2
Gap 2: Distance: 6 mm; cross-sectional area: 0.5 cm^2
Gap 3: Distance: 5 mm; cross-sectional area: 1 cm^2

a) Identify the dot marking of the two coils;
b) Draw the magnetic equivalent circuit;
c) Find the flux density in air gap 1 if i_1=10 Amps (DC)and i_2=20 Amps (DC)

$$R_{gap1} = R_{gap3} = \frac{l}{\mu_0 A_c} = \frac{5\times10^{-3}\,m}{(4\pi\times10^{-7}\,\frac{H}{m})(1\times10^{-4}\,m^2)} = 3.979\times10^{7}\ \frac{At}{Wb}$$

$$R_{gap2} = \frac{l}{\mu_0 A_c} = \frac{6\times10^{-3}\,m}{(4\pi\times10^{-7}\,\frac{H}{m})(0.5\times10^{-4}\,m^2)} = 9.55\times10^{7}\ \frac{At}{Wb}$$

b)



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c) $B_{gap1} = ?$

$N_1 i_1 = 1000\ At$

$N_2 i_2 = N_1 i_1 = 1000\ At.$

KVL

$N_1 i_1 = R_{gap1}\phi_1 + R_{gap2}\phi_c$

$N_2 i_2 = R_{gap2}\phi_c + R_{gap3}\phi_2$

* $\phi_c = \phi_1 + \phi_2$ *

$N_1 i_1 = N_2 i_2 = Ni$

$N_1 i_1 = Ni = \phi_1 (R_{gap1} + R_{gap2}) + \phi_2 (R_{gap2})$

$N_2 i_2 = Ni = \phi_1 (R_{gap2}) + \phi_2 (R_{gap2} + R_{gap3})$

* Due to circuit symmetry, it can be seen that *

$\phi_1 = \phi_2 = \phi$

$\therefore\ Ni = \phi (R_{gap2} + R_{gap2} + R_{gap3})$

$$\phi = \frac{1000\ At}{(3.979\times10^7 \frac{At}{wb}) + (9.55\times10^7 \frac{At}{wb})2}$$

$\phi_1 = \phi_2 = 4.33\times10^{-6}\ wb$

$$B_{gap1} = \phi_1 \cdot A_{gap1}^{-1} = \frac{(4.33\times10^{-6} wb)}{(1\times10^{-4} m^2)} = 0.0433\ \underbrace{\frac{wb}{m^2}}_{\to T}$$

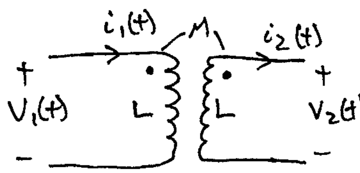
$\boxed{B_{gap1} = 0.0433 T}$

Problem 4. (25 points)

Two identical coils (each with zero resistance) are located near each other. Coil #2 is open circuited.

When a 60Hz sinusoidal voltage of 120 Volts (RMS) is applied to coil #1, the coil #1 current is 6 Amps (RMS) and the voltage measured on the open-circuited coil #2 is 70 Volts (RMS).

a) What are the self inductances of coil #1 and #2 in Henries?
b) What is the magnitude of the mutual inductance between coil #1 and coil #2 in Henries?
c) What are the current magnitudes in coil #1 and #2 if a short circuit is placed across coil #2 while the given voltage is applied across coil #1?



$v_1(t) = 120\sqrt{2}\cos(120\pi t)$
$v_2(t) = 70\sqrt{2}\cos(120\pi t)$
$|i_1(t)| = 6\sqrt{2}$

a) <u>KVL on left loop</u>

$$v_1(t) = L\frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$\int v_1(t)\,dt = \int L\,di_1 - \int M\,di_2$$

$$\frac{120\sqrt{2}\sin(120\pi t)}{120\pi} = L i_1(t) - \cancel{M i_2(t)}^{\,0}$$

$$|i_1(t)| = 6\sqrt{2} = \frac{120\sqrt{2}}{120\pi L}$$

$$L = \frac{120}{120\pi\, 6} \Rightarrow \boxed{L = 0.053\ H}$$

b) <u>KVL on right loop</u>

$$v_2(t) = L\frac{di_2}{dt} - M\frac{di_1}{dt}$$

$$\int v_2(t)\,dt = \int L\,di_2 - \int M\frac{di_1}{dt}$$

$$\frac{70\sqrt{2}\sin(120\pi t)}{120\pi} = L\cancel{i_2(t)}^{\,0} - M i_1(t)$$

$$|i_1(t)| = \frac{70\sqrt{2}}{120\pi M} \rightarrow M = \frac{70}{120\pi \cdot 6} \Rightarrow \boxed{M = 0.0309\ H}$$

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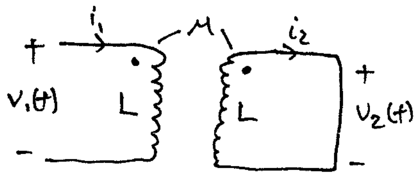
c)



$v_1(t) = 120\sqrt{2}\cos(120\pi t)$

KVL

$$\underbrace{\begin{aligned} v_1(t) &= L\frac{di_1}{dt} - M\frac{di_2}{dt} \\ 0 &= L\frac{di_2}{dt} - M\frac{di_1}{dt} \end{aligned}}$$

$$M\frac{di_1}{dt} = L\frac{di_2}{dt}$$

$$M\, i_1(t) = L\, i_2(t)$$

$$i_2(t) = \frac{M}{L} i_1(t)$$

plugging this into KVL eqn #1

$$v_1(t) = L\frac{di_1}{dt} - \frac{M\left(\frac{M}{L} i_1\right)}{dt}$$

$$v_1(t) = \left(L - \frac{M^2}{L}\right)\frac{di_1}{dt}$$

$$\int 120\sqrt{2}\cos(120\pi t)\,dt = \int\left(L - \frac{M^2}{L}\right) di_1$$

$$\frac{120\sqrt{2}\sin(120\pi t)}{120\pi\left(L - \frac{M^2}{L}\right)} = i_1(t)$$

$$|i_1(t)|_{rms} = \left(\frac{120\sqrt{2}}{120\pi\left(L - \frac{M^2}{L}\right)}\right)\cdot\frac{1}{\sqrt{2}}$$

$$= \frac{1}{\pi\left(.053\,H - \frac{(0.0309\,H)^2}{0.053\,H}\right)}$$

$$\boxed{|i_1(t)|_{rms} = 9.099A}$$

$$\boxed{|i_2(t)|_{rms} = 5.30\,A}$$