ECE 330 Exam #1, Fall 2012 Name: *Solution*
90 Minutes

Section (Check One) MWF 10am _______ MWF 2:00pm _______

1. ________ / 25 2. ________ / 25

3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

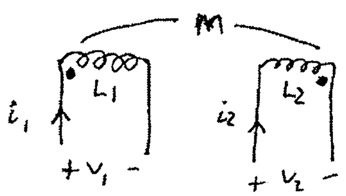
$\sin(x) = \cos(x - 90^\circ)$ $\quad \bar{V} = \bar{Z}\bar{I}$ $\quad \bar{S} = \bar{V}\bar{I}^*$ $\quad \bar{S}_{3\phi} = \sqrt{3} V_L I_L \angle\theta$

$0 < \theta < 180^\circ$ (lag) $\quad I_L = \sqrt{3} I_\phi$ (delta) $\quad \bar{Z}_Y = \bar{Z}_\Delta / 3$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

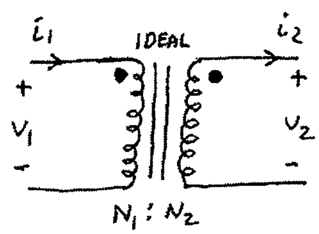
$-180^\circ < \theta < 0$ (lead) $\quad V_L = \sqrt{3} V_\phi$ (wye)

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\quad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\quad \Re = \frac{l}{\mu A}$ $\quad MMF = Ni = \phi\Re$

$\phi = BA$ $\quad \lambda = N\phi$ $\quad k = \frac{M}{\sqrt{L_1 L_2}}$ $\quad$ 1 hp = 746 Watts

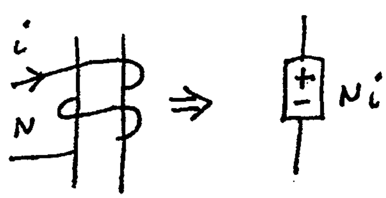


$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$



$a = \frac{N_1}{N_2}$ $\quad N_1 i_1 = N_2 i_2$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$



Problem 2. (25 pts)

A 2.4 KV (Line to Line) 60-Hz AC source supplies power to the following three WYE connected loads,

Load 1: 120 KVA at 0.8 pf lead,
Load 2: 180 KW at 0.6 pf lag,
Load 3: 30 KW at unity pf

1. Find the total complex power supplied by the source.
2. Find the magnitude of the line and phase current supplied by the source.
3. Compute the total VAR of the per phase capacitance needed to bring the PF to unity and the magnitude of the new line current after the capacitance is added.
4. Sketch the per-phase equivalent of the original circuit using the impedance of each load.

1. $\bar{S} = 120k\angle{-\cos^{-1}.8} + \frac{180k}{.6}\angle{+\cos^{-1}0.6} + 30k\angle 0$

$= 96k - j72k + 180k + j240k + 30k = \boxed{306k + j168k}$

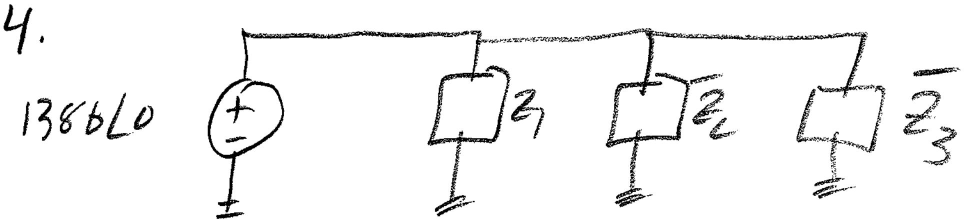
$= \boxed{349k\angle 28.8^\circ}$

2. $I_{Line} = I_\phi$, $349k = \sqrt{3} \times 2.4k \times I_L$ $I_l = I_\phi = \boxed{84A}$

3. $Q_{add\ 3\phi} = -168\,kVAR$ $Q_{per\ phase} = -56\,kVAR$

$I_{line\ new} \Rightarrow 306k = \sqrt{3} \times 2400 \times I_L$ $I_L = \boxed{73.6A}$

4.

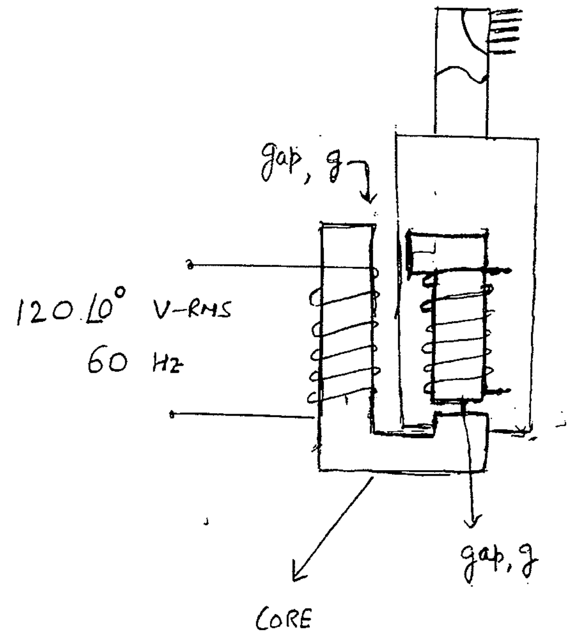


$\bar{I}_1 = \left(\frac{120k\angle{-37^\circ}}{\sqrt{3}\,2400\angle 0}\right)^* = 28.8\angle{+37^\circ}$ $\bar{Z}_1 = \frac{1386\angle 0}{28.8\angle{+37^\circ}} = \boxed{48\angle{-37^\circ}\,\Omega}$

$\bar{I}_2 = \left(\frac{\frac{180k}{.6}\angle 53^\circ}{\sqrt{3} \times 2400\angle 0}\right)^* = 72\angle{-53^\circ}$ $\bar{Z}_2 = \frac{1386\angle 0}{72\angle{-53^\circ}} = \boxed{19\angle 53^\circ\,\Omega}$

$\bar{I}_3 = \frac{30k}{\sqrt{3} \times 2400\angle 0} = 7.2\angle 0$ $\bar{Z}_3 = \frac{1386\angle 0}{7.2\angle 0} = \boxed{192.5\angle 0\,\Omega}$

Problem 3. (25 points)
Figure below shows the charging mechanism for an electric toothbrush. Coils 1 and 2 are wound on an iron core as shown. Coil 1 is powered by the $120\angle 0^{\circ}$ V-RMS, 60 Hz sinusoidal AC source. Coil 2 is magnetically coupled to Coil 1. The terminals of Coil 2 connect to a battery charging circuit when the toothbrush is placed in the charger. The current in Coil 2 then charges the toothbrush battery. The current in Coil 2 is zero when the toothbrush is not in in the charger.



1. Place the dots on the two coils.

2. Determine the cross –sectional area of the core to yield a Magnetic Flux density, B = 0.8 $\sin(120\pi t)$ T, when Coil 1 is connected to the supply and current in Coil 2 is zero. Assume Coil 1 has N_1=3000 turns.
3. Sketch the equivalent magnetic circuit of the system. Assume the reluctance of the air gap to be R_g and that of the core to be R_c.
4. Let the flux linkages in the coils be given by,

$$\lambda_1 = L_{11}i_1 + L_{12}i_2 \qquad \text{and} \qquad v_1 = \frac{d\lambda_1}{dt}, v_2 = \frac{d\lambda_2}{dt}$$
$$\lambda_2 = L_{21}i_1 + L_{22}i_2$$

where,

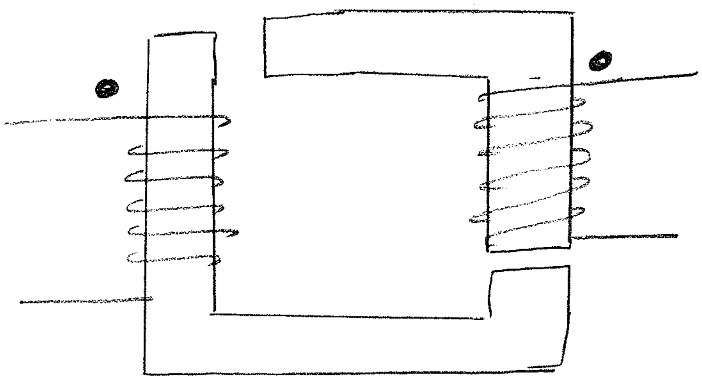
L_{11} and L_{22} are the self inductance of Coil 1 and Coil 2 respectively

$L_{21} = L_{12}$ = Mutual inductance between Coil 1 and Coil 2

i_1 and i_2 = Current in Coil 1 and Coil 2 respectively

Pick the polarity of the voltages on each Coil and the direction of the currents such that the self and mutual inductance terms are positive numbers.

①



②

$$V_1 = N_1 \frac{d\phi}{dt}$$

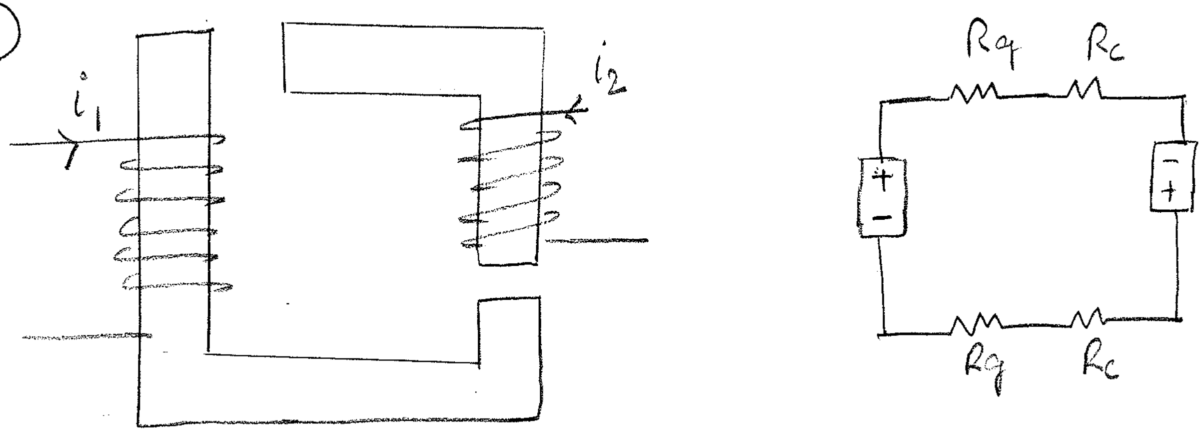
$$120\sqrt{2}\,\cos(120\pi t) = N_1 \frac{d\phi}{dt} \Rightarrow \phi = \frac{1}{N_1}\int_0^t 120\sqrt{2}\,\cos(120\pi \hat{t})\,d\hat{t}$$

$$\Rightarrow \phi = \frac{\sqrt{2}\sin 120\pi t}{3000}$$

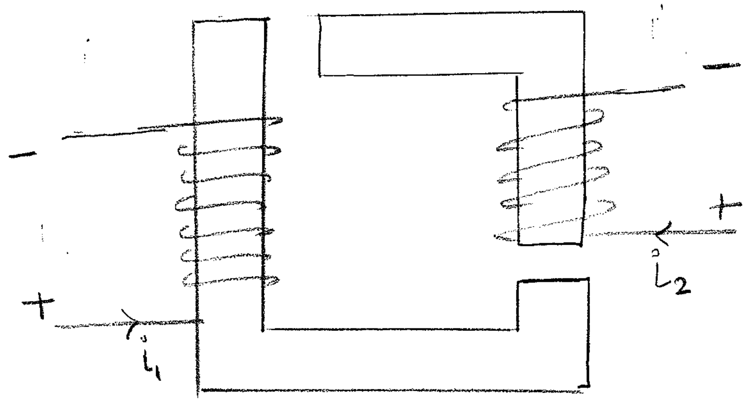
$$\Rightarrow A \times 0.8 \sin 120\pi t = \frac{\sqrt{2}\sin 120\pi t}{3000}$$

$$\Rightarrow \boxed{A = 1.8\ cm^2}$$

③



(4)



Problem 4. (25 points)
An ideal transformer is rated at 12,470V/240V, 50kVA, single phase, 60 HZ.

1. What is the current on the 12,470V side when it is delivering 35kW and 12 kVAR to your house?
2. If there is a short circuit in your house, the current on the 12,470V side is 80 Amps. How much current would this be on the 240V side?
3. Ignoring the normal house load, use this short circuit current information to estimate the impedance magnitude for the wires and transformer that supply your house (as seen on the 12,470V side).

(1) $S = 35 + j12 \text{ KVA} = 37 \angle 18.92^\circ \text{ KVA}$

$$I = \left(\frac{S}{V}\right)^* = \left(\frac{37\times10^3 \angle 18.92^\circ}{12470}\right)^* \Rightarrow \boxed{I = 2.97 \angle -18.92^\circ}$$

(2) Current on the 240 V side, $I_{240} = \frac{80 \times 12470}{240}$

$$\Rightarrow \boxed{I_{240} = 4156.67} \text{ A}$$

(3) $$Z_{240} = \frac{240}{4156.67} = 0.0577\ \Omega$$

Impedance seen from 12470 V side,

$$Z_{12470} = a^2 Z_{240}$$

$$= \left(\frac{12470}{240}\right)^2 \cdot 0.0577$$

$$\boxed{Z_{12470} = 155.74\ \Omega}$$

Problem 1. **(25 points)**

A source $v(t) = 120\sqrt{2}\cos(120\pi t)$, supplies power at 0.8 pf lagging to a load $|z| = 5\ \Omega$.

1. Find the complex impedance of the load.
2. Find the real, imaginary, and the complex power supplied by the source.
3. Find the reactive VARS to be "added" to the system so that the power factor becomes 0.9 lagging.
4. Find the new current supplied by the source after the capacitor has been added.
5. Sketch the Phasors for the voltage ~~v(t)~~ and the new current.

1. $\bar{V} = 120\angle 0 \qquad \bar{Z} = 5\angle{+\cos^{-1}.8} = \boxed{5\angle 37^\circ\ \Omega}$

2. $\bar{I} = \dfrac{120\angle 0}{5\angle 37^\circ} \qquad \bar{S} = 120\angle 0\left(\dfrac{120\angle 0}{5\angle 37}\right)^* = \dfrac{120^2}{5\angle -37^\circ} = \boxed{2880\angle 37^\circ\ \text{VA}}$

$\bar{S}_{old} = \boxed{2304 + j1728}$

3. $\bar{S}_{new} = \dfrac{2304}{.9}\angle{+\cos^{-1}.9} = 2560\angle 25.8^\circ$

$= 2304 + j1114$

$Q_{add} = 1114 - 1728$

$= \boxed{-614\ \text{vars}}$

4. $2560\angle 25.8 = 120\,\bar{I}^* \qquad |\bar{I}| = \boxed{21.3\,\text{A}} \qquad \measuredangle \bar{I} = -25.8^\circ$

5.