ECE 330 Exam #1, Spring 2012 Name: Solution

90 Minutes

Section (Check One) MWF 10am _______ TR 12:30pm _______

1. ________ / 25 2. ________ / 25

3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

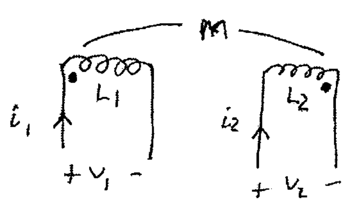
$\sin(x) = \cos(x - 90^\circ)$ $\bar{V} = \bar{Z}\bar{I}$ $\bar{S} = \bar{V}\bar{I}^*$ $\bar{S}_{3\phi} = \sqrt{3}V_L I_L \angle\theta$

$0 < \theta < 180^\circ$ (lag) $I_L = \sqrt{3}I_\phi$ (delta) $\bar{Z}_Y = \bar{Z}_\Delta / 3$ $\mu_0 = 4\pi \cdot 10^{-7}$ H/m

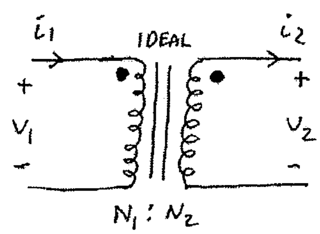
$-180^\circ < \theta < 0$ (lead) $V_L = \sqrt{3}V_\phi$ (wye)

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t}\int_S \mathbf{B} \cdot \mathbf{n} da$ $\Re = \frac{l}{\mu A}$ $MMF = Ni = \phi\Re$

$\phi = BA$ $\lambda = N\phi$ $k = \frac{M}{\sqrt{L_1 L_2}}$ 1 hp = 746 Watts

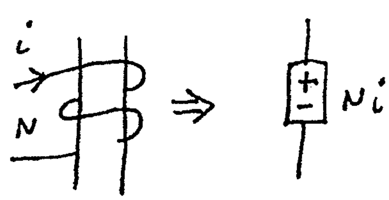


$v_1 = L_1 \frac{di_1}{dt} - M\frac{di_2}{dt}$



$a = \frac{N_1}{N_2}$ $N_1 i_1 = N_2 i_2$

$\frac{v_1}{v_2} = \frac{N_1}{N_2}$

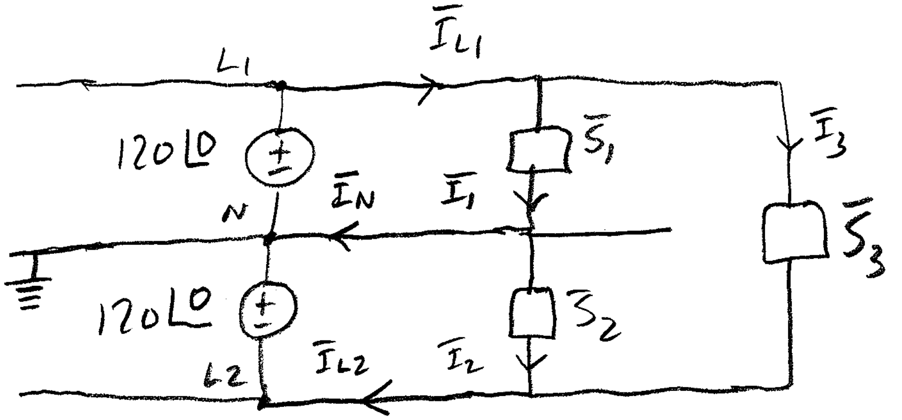


Problem 1. (25 points)

The electrical service to your house is called 120/240 Volts, single phase, 3 wire. It consists of 3 wires named L_1, L_2, and N coming into your house from the transformer in the back. The voltage drop from L_1 to N is 120 volts angle zero. The voltage drop from L_2 to N is 120 volts angle 180 degrees. The neutral wire (N) is at ground potential. Suppose you hook up loads to these circuits as follows:

- A fan between L_1 and N that draws 5 Amps at a power factor of 0.8 lag.
- A toaster between L_2 and N that draws 11 Amps at unity power factor.
- An air conditioner between L_1 and L_2 that draws 24 Amps at a power factor of 0.85 lag.

a) What is the total real power consumed by your house when the things above are turned on?
b) How much current is in each of the three wires (magnitude only) when all the things above are turned on?



$\bar{S}_1 = 120 \times 5 \angle +37^\circ$

$\bar{S}_2 = 120 \times 11 \angle 0^\circ$

$\bar{S}_3 = 240 \times 24 \angle +32^\circ$

a) $\bar{S}_{TOT} = 479 + j361 + 1320 + j0 + 4885 + j3052$

$= \boxed{6684} + j3413 = 7505 \angle 27^\circ$

b) $\bar{I}_1 = 5 \angle -37^\circ \quad \bar{I}_2 = 11 \angle 0^\circ \quad \bar{I}_3 = 24 \angle -32^\circ$

$\bar{I}_{L_1} = \bar{I}_1 + \bar{I}_3 = 4 - j3 + 20.4 - j12.7 = 24.4 - j15.7 = 29 \angle -32.8^\circ$

$\bar{I}_{L2} = \bar{I}_2 + \bar{I}_3 = 11 + j0 + 20.4 - j12.7 = 31.4 - j12.7 = 33.87 \angle -22^\circ$

$\bar{I}_N = \bar{I}_1 - \bar{I}_2 = 4 - j3 - 11 + j0 = -7 - j3 = 7.6 \angle 23.2^\circ + 180^\circ$

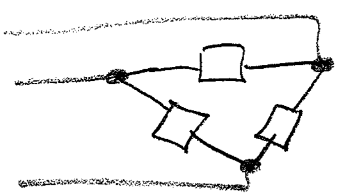
$= 7.6 \angle +203.2$

Problem 2. (25 pts)

A 345 KV(line-to-line) three-phase line supplies 750 MVA (3-phase) at 0.8 PF lagging to a three-phase load which is delta connected.

a) Find the complex impedance per phase of the load
b) Find the magnitudes of the line and phase currents
c) Find the MVAR (3-phase) rating of a capacitor bank needed to improve the power factor to be 0.95 lagging.
d) What will the line current be after the capacitors are installed.

a) $\bar{S}_{1\phi} = \frac{750\times10^6}{3}\angle +\cos^{-1}0.8 = (345\times10^3\angle\theta)\left(\frac{345\times10^3\angle\theta}{\bar{Z}_\Delta}\right)^*$



$$= \frac{(345\times10^3)^2}{\bar{Z}_\Delta^*} \qquad \bar{Z}_\Delta = \frac{(345\times10^3)^2}{250\times10^6\angle-37^\circ}$$

$$\boxed{\bar{Z}_\Delta = 476\angle 37^\circ\ \Omega}$$

b) $750\times10^6 = \sqrt{3}\times345\times10^3\times I_L$ $\quad\boxed{I_L = 1255\text{A}}$

$$\boxed{I_\phi = \frac{1255}{\sqrt{3}} = 725\text{A}}$$

c) $\bar{S}_{new} = \frac{600\times10^6}{0.95}\angle +\cos^{-1}.95 = 600\times10^6 + j197\times10^6$

$\bar{S}_{old} = 750\times10^6\angle +\cos^{-1}0.8 = 600\times10^6 + j450\times10^6$

$Q_{add} = 450\times10^6 - 197\times10^6 = 253\times10^6$

of capacitive VARS

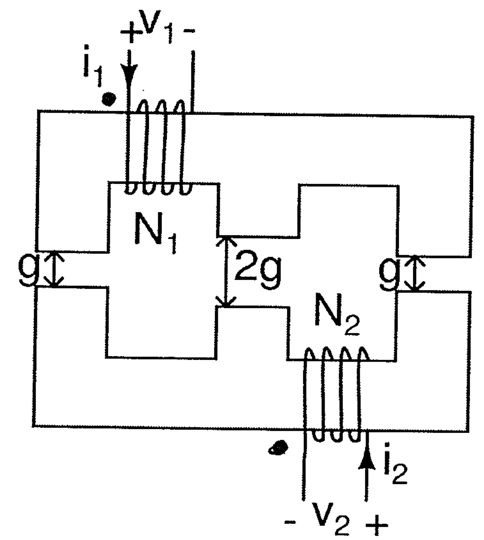
d) $I_{L_{new}} = \frac{600\times10^6}{.95\times\sqrt{3}\times345{,}000} = \boxed{1057\text{A}}$

Problem 3. (25 points)

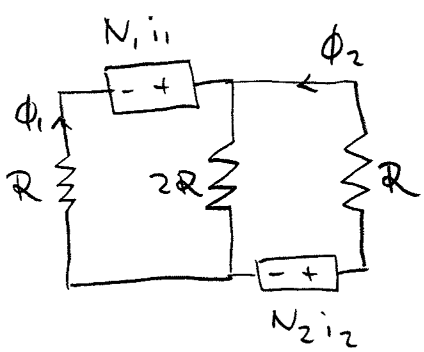
A magnetic core with infinite permeability ($\mu=\infty$) is shown below.

$\mathcal{R} = \dfrac{g}{\mu_0 A}$

$k = \dfrac{M}{\sqrt{L_1 L_2}}$



b)



For this problem, you can ignore any resistance in the coil, and any fringing effects in the gaps.

The cross sectional area of the core is 1 cm^2, $N_1 = 10$, $N_2 = 40$, $g = 1$ mm.

a) Identify the dot markings of the two windings.

b) Draw the magnetic equivalent circuit.

c) Find the magnitudes of the self-inductances, the mutual inductance, and the coupling coefficient k. **Note, this part of the problem requires quite a bit of math. If you are short on time, you may want to consider doing the next problem and coming back to this part if time permits.**

c) (A) $N_1 i_1 = \Phi_1 \mathcal{R} + 2\mathcal{R}(\Phi_1 + \Phi_2)$

(B) $N_2 i_2 = \Phi_2 \mathcal{R} + 2\mathcal{R}(\Phi_1 + \Phi_2)$

(A) − (B) $\Rightarrow N_1 i_1 - N_2 i_2 = (\Phi_1 - \Phi_2)\mathcal{R}$

$\Phi_1 = \dfrac{N_1 i_1 - N_2 i_2}{\mathcal{R}} + \Phi_2$ ← plug into (B)

$\Rightarrow N_2 i_2 = \Phi_2 \mathcal{R} + 2\mathcal{R}\left(\dfrac{N_1 i_1 - N_2 i_2}{\mathcal{R}} + \Phi_2\right) + 2\mathcal{R}\Phi_2$

$N_2 i_2 = \Phi_2 \mathcal{R} + 2N_1 i_1 - 2N_2 i_2 + 4\mathcal{R}\Phi_2$

$\Phi_2 = \dfrac{3}{5}\dfrac{N_2 i_2}{\mathcal{R}} - \dfrac{2}{5}\dfrac{N_1 i_1}{\mathcal{R}}$

$\Phi_1 = \dfrac{N_1 i_1}{\mathcal{R}} - \dfrac{N_2 i_2}{\mathcal{R}} + \dfrac{3}{5}\dfrac{N_2 i_2}{\mathcal{R}} - \dfrac{2}{5}\dfrac{N_1 i_1}{\mathcal{R}}$

$\Phi_1 = \dfrac{3}{5}\dfrac{N_1 i_1}{\mathcal{R}} - \dfrac{2}{5}\dfrac{N_2 i_2}{\mathcal{R}}$

$\lambda_1 = N_1 \Phi_1 = \underbrace{\dfrac{3}{5}\dfrac{N_1^2}{\mathcal{R}}}_{L_1} i_1 - \underbrace{\dfrac{2}{5}\dfrac{N_1 N_2}{\mathcal{R}}}_{M} i_2$

$\lambda_2 = N_2 \Phi_2$

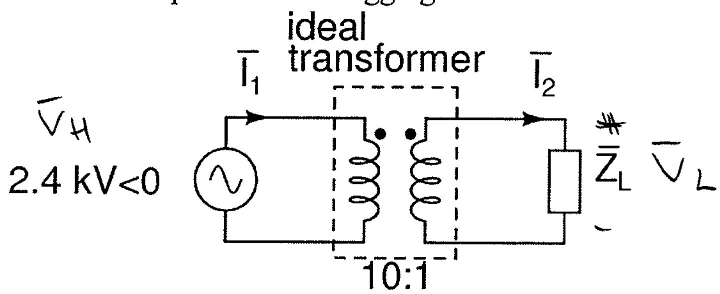
$\lambda_2 = -\underbrace{\dfrac{2}{5}\dfrac{N_2 N_1}{\mathcal{R}}}_{M} i_1 + \underbrace{\dfrac{3}{5}\dfrac{N_2^2}{\mathcal{R}}}_{L_2} i_2$

$L_1 = 7.53\,\mu H$ $M = 20.1\,\mu H$

$L_2 = 120\,\mu H$ $k = 0.67$

Problem 4. (25 points)

a) An ideal single-phase 60 Hz transformer as shown below is used to supply power to a complex load drawing 1 KW at 0.8 power factor lagging.



1) Find the load impedance as seen from the high voltage side.

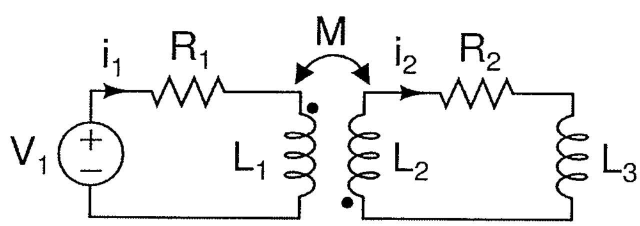
$\bar{S} = \frac{1000}{0.8} \angle \arccos 0.8 = 1250 \angle 37°$ $|V_H| = 2400$

$\vec{Z}_H = |V_H|^2 / S^* = 4608 \angle 37° \ \Omega$

2) Find the magnitudes of currents I_1 and I_2.

$I_1 = \frac{|V_H|}{|Z_H|} = \frac{2400}{4608} = 0.52 \text{ A}$ $I_2 = \frac{N_1}{N_2} I_1 = 10 \cdot I_1 = 5.2 \text{ A}$

b) The coupled coils L_1 and L_2 are connected in the circuit below (note that L_3 is a regular inductor, and is not coupled).



1) Write the two loop equations. You do not need to solve them.

$$V_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$0 = i_2 R_2 + L_3 \frac{di_2}{dt} + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$