ECE 330 Exam #1, Spring 2014
90 Minutes

Name: Solution

Section (Check One) MWF 10am ________ MWF 2pm ________

1. ________ / 25 2. ________ / 25

3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

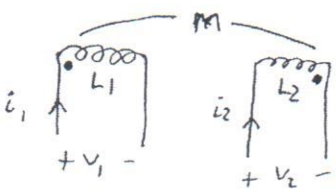
$\sin(x) = \cos(x - 90^\circ)$ $\quad \bar{V} = \bar{Z}\bar{I}$ $\quad \bar{S} = \bar{V}\bar{I}^*$ $\quad \bar{S}_{3\phi} = \sqrt{3}V_L I_L \angle\theta$

$0 < \theta < 180^\circ$ (lag) $\quad I_L = \sqrt{3}I_\phi$ (delta) $\quad \bar{Z}_Y = \bar{Z}_\Delta / 3$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

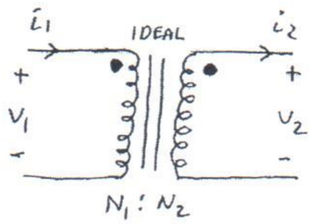
$-180^\circ < \theta < 0$ (lead) $\quad V_L = \sqrt{3}V_\phi$ (wye)

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n}da$ $\quad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t}\int_S \mathbf{B} \cdot \mathbf{n}da$ $\quad \Re = \frac{l}{\mu A}$ $\quad MMF = Ni = \phi\Re$

$\phi = BA$ $\quad \lambda = N\phi$ $\quad v = d\lambda/dt$ $\quad k = \frac{M}{\sqrt{L_1 L_2}}$ $\quad$ 1 hp = 746 Watts

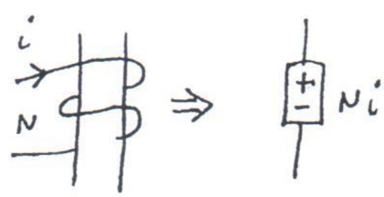


$v_1 = L_1 \frac{di_1}{dt} - M\frac{di_2}{dt}$



$a = \frac{N_1}{N_2}$ $\quad N_1 i_1 = N_2 i_2$

$\frac{v_1}{v_2} = \frac{N_1}{N_2}$



Problem 1. (25 points)

Three single-phase loads are connected in parallel across a 60Hz source of 120 Volts (RMS).

Load #1: 3 KVA at 0.8 power factor lagging
Load #2: 20 Amps (RMS) and 2 KW real power (inductive load) $pf = \frac{2000}{20 \times 120}$
Load #3: $(10 + j10)\ \Omega$

a) Find the total complex power consumed by these three loads.
b) Find the equivalent impedance as viewed from the source.
c) Find the value of capacitive VARS that should be added in parallel to these three loads to make the overall power factor 0.95 lag.

a) $\bar{S}_1 = 3000 \angle 37^\circ = 2400 + j1800$

$\bar{S}_2 = 20 \times 120 \angle 33.6^\circ = 2000 + j1328$

$\bar{S}_3 = 120 \left(\frac{120}{10\sqrt{2} \angle 45} \right)^* = 1018 \angle 45^\circ = 720 + j720$

$\boxed{\bar{S}_{tot} = 5120 + j3848} = 120 I \angle \theta$

$I \angle \theta = 53 \angle -36.9^\circ$

b) $\bar{Z} = \frac{120 \angle 0}{53 \angle -36.9^\circ} = 2.26 \angle 36.9^\circ\ \Omega$

c) $\bar{S}_{new} = \frac{5120}{0.95} \angle 18.2^\circ = 5120 + j1683$

$3848 + Q_{add} = 1683$ $\boxed{Q_{add} = -2165}$

$pf = \frac{3100}{\sqrt{3} \times 208 \times 10} = 0.86$

Problem 2. (25 points)

A balanced, 60 Hz, 120/208 Volt, 3 phase, 4 wire source is supplying impedance loads in parallel. One is 4-wire, Wye connected and draws 10 Amps line current and 3,100 Watts (3-phase) of real power (inductive). The other is 3-wire Delta connected and draws 8 Amps of line current and zero VARs (3-phase) of reactive power. — $pf = 1.0$

a) What is the source total line current magnitude?
b) What is the total real power (3-phase) delivered by the source?
c) Repeat a) and b) if the second load is rewired (same impedances) to be 4-wire, Wye connected.

a $\bar{S}_1 = \sqrt{3} \times 208 \times 10 \angle 30.6^\circ = 3100 + j1834$

$\bar{S}_2 = \sqrt{3} \times 208 \times 8 \angle 0 = 2882 + j0$

$\boxed{S_{TOT} = 5982 + j1834} = 6257 \angle$

$= \sqrt{3} \times 208 \times I \angle$

$\boxed{I = 17.4\ A}$

b) $P_{TOT} = 5982\ W$

c) $\frac{208^2}{R_{load}} = \frac{2882}{3}$ $R_{load} = 45\ \Omega$

$\bar{S}_{2Y} = 3 \times \frac{120^2}{45} = 960\ W$

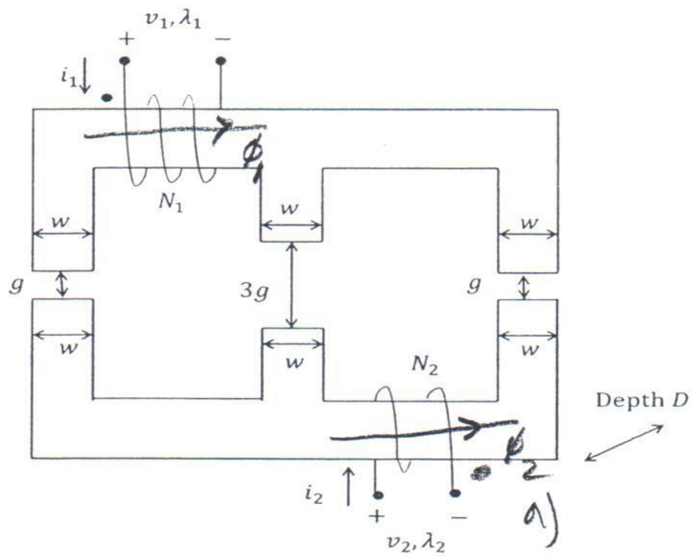
$\boxed{\bar{S}_{TOT} = 4060 + j1834}$

$4155 = \sqrt{3} \times 208 \times I$

$\boxed{P_{TOT} = 4060\ W}$

$\boxed{I = 12.4\ A}$

Problem 3. (25 points)
An electric device has both upper and lower pieces composed of infinitely permeable magnetic material. It has depth D, and all other lengths are shown in the figure: $D = 2cm$, $w = 3cm$, and $g = 3mm$. The number of turns are $N_1 = 100$, $N_2 = 50$. Assume no fringing or flux leakage.



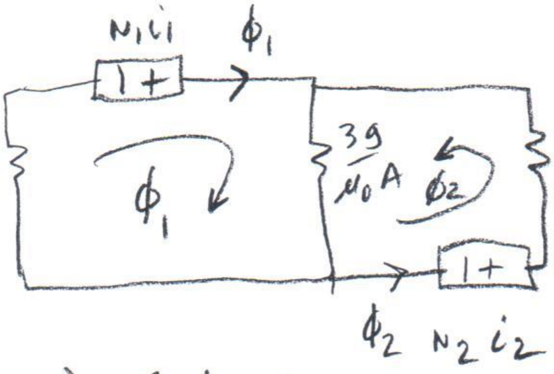
$\lambda_1 = N_1 \phi_1$

$\lambda_2 = N_2 \phi_2$

a) Given the dot marking for coil 1, finish the dot marking for coil 2 in the above figure;
b) Draw the magnetic equivalent circuit, and find the specific value of each reluctance in the magnetic circuit. Write two "loop" equations that you could use to find the *flux* through the two coils in terms of i_1 and i_2 – do not simplify or solve ;
c) Ignore coil 2, and find the inductance L_1 of coil 1;
d) Ignore coil 1, and find the inductance L_2 of coil 2;
e) Assume the self-inductance values of coils 1 and 2 are given by L_1 and L_2 as in Parts c) and d), and their coupling coefficient is $k = 0.75$. Write v_1 and v_2 in terms of i_1 and i_2.

b)

$R_g = \frac{g}{\mu_0 A}$



$R_g = \frac{3\times10^{-3}}{4\pi\times10^{-7}\times6\times10^{-4}}$

$R_g = 3.98\times10^{6}$

$3R_g = 11.94\times10^{6}$

$-N_2 i_2 + \frac{g}{\mu_0 A}\phi_2 + \frac{3g}{\mu_0 A}(\phi_2+\phi_1) = 0$

$-N_1 i_1 + \frac{3g}{\mu_0 A}(\phi_1+\phi_2) + \frac{g}{\mu_0 A}\phi_1 = 0$

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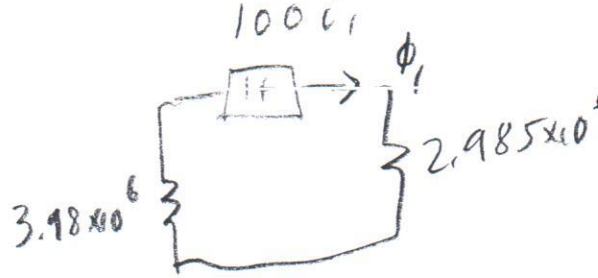
c) $i_2 = 0$



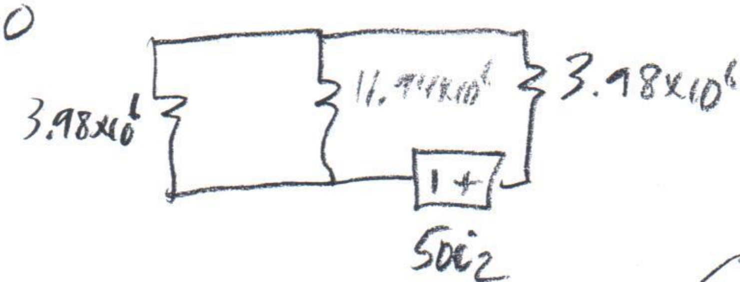
$$\phi_1 = \frac{100}{6.965\times10^{6}} i_1$$

$$= 14.94\times10^{-6} i_1$$

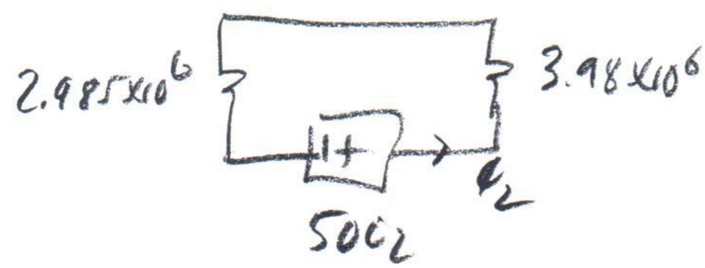
$$\lambda_1 = N_1\phi_1 = 14.94\times10^{-4} i_1$$

$$L_1 = 14.94\times10^{-4}\ H$$

d) $i_1 = 0$



$$v_2 = -5.49\times10^{-4}\frac{di_1}{dt} + 3.589\times10^{-4}\frac{di_2}{dt}$$



$$50 i_2 = 6.965\times10^{6}\phi_2$$

$$\phi_2 = 7.178\times10^{-6} i_2$$

$$\lambda_2 = 50\phi_2 = 358.9\times10^{-6} i_2$$

$$L_2 = 3.589\times10^{-4}\ H$$

$$k = \frac{M}{\sqrt{L_1 L_2}} = 0.75 = \frac{M}{7.32\times10^{-4}}$$

$$M = 5.49\times10^{-4}$$

$$v_1 = 14.94\times10^{-4}\frac{di_1}{dt} - 5.49\times10^{-4}\frac{di_2}{dt}$$

Problem 4. (25 points)

A voltage of 17,633cos(377t) Volts (12,470 V RMS) is applied to a single-phase transformer coil primary (source side). The primary coil has 6,500 turns.

a) What is the magnitude of flux density in the transformer primary coil iron core that has a cross-section are of 0.008 square meters?

b) If this is an ideal transformer, with 125 turns on the secondary (load side), how much current (RMS) will a 7 Ohm load draw on the secondary side?

c) For part b), what will this impedance look like (how many Ohms) from the high voltage side?

a) $v = 17{,}633 \cos(377t) = N\frac{d\phi}{dt} = 6500\frac{d\phi}{dt}$

$$\phi = 0.0072 \sin(377t) = BA = B \times .008$$

$$B = 0.9 \sin(377t)\,T$$

$$\boxed{|B| = 0.9\ T}$$

b) $v_{prim} = 12470\,V_{RMS}$ $\quad v_{sec} = 12470\,\frac{125}{6500} = 240\,V_{RMS}$

$I_{sec} = \frac{240}{7} = 34.3$ Amps RMS $\qquad I_{pri} = 0.66$ A

c) $Z_{High} = \frac{12470\,V}{0.66} = 18{,}893\,\Omega \;\left(\text{or } 7\left(\frac{6500}{125}\right)^2\right)$