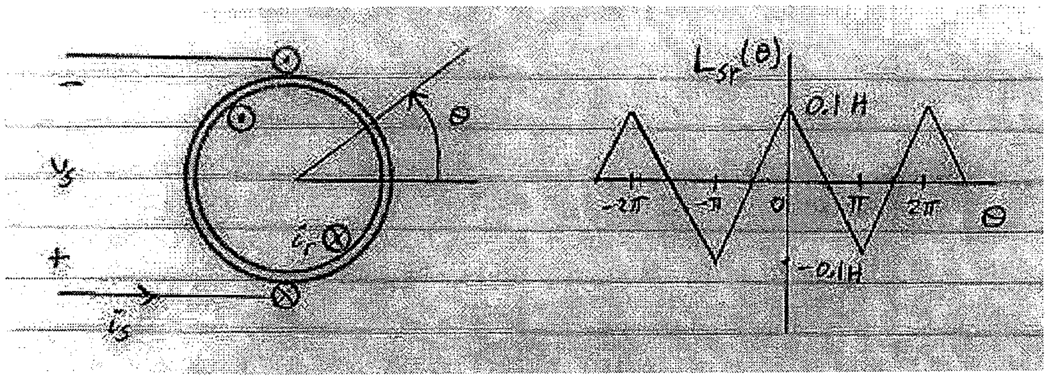
1. (25 points)

A single-phase generator consists of a coil on the stator and a coil on the rotor with a mutual inductance variation with θ as shown in the figure below. The rotor is being driven at a constant speed of 377 radians per second and the rotor coil has a constant dc current $i_r = 5$ A. The self inductances are constants and you may assume a linear magnetic core.



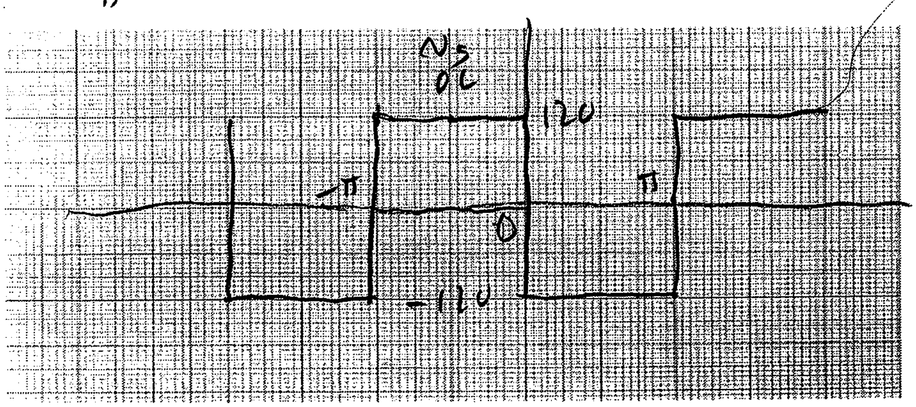
(a) Plot the open circuit voltage ($i_S = 0$) as a function of θ (label all points)

$$\lambda_s = L_s i_s + \left(0.1 - \frac{0.2}{\pi}\theta\right) i_r$$

$$v_s = \frac{d\lambda_s}{dt}$$

$$\lambda_r = \left(0.1 - \frac{0.2}{\pi}\theta\right) i_s + L_r i_r$$

$$v_{s\,OC}\ (0-\pi) = -\frac{0.2}{\pi}\omega \times 5 = -\frac{0.2}{\pi} 2\pi 60 \times 5 = -120\text{V}$$



(b) What is the torque of electrical origin when $i_S = 10$ Amps and θ = 45°?

$$W_m' = \tfrac{1}{2} L_s i_s^2 + \left(0.1 - \frac{0.2}{\pi}\theta\right) i_s i_r + \tfrac{1}{2} L_r i_r^2 \quad 0-\pi$$

$$T^e = -\frac{0.2}{\pi} i_s i_r \quad 0-\pi$$

$$T^{el} = -\frac{0.2}{\pi} \times 10 \times 5 = -\frac{10}{\pi}\ \text{NM}$$

$i_s = 10 \quad i_r = 5 \quad \theta = 45°$

i_s $\quad$ i_r

2. (25 points)

The machine of problem 1 is being operated such that the currents i_s and i_r can be assumed to be constants at $I_s = 10$Amps, and $I_r = 5$ Amps respectively while the shaft is rotated from θ equals zero to θ equals $\pi/2$.

For this change from "point a" to point b", find:

a) The energy transferred from the electrical system into the coupling field as the system moved from point a to point b with constant currents.

b) The energy transferred from the mechanical system into the coupling field as the system moved from point a to point b with constant current.

$$\lambda_s = L_s i_s + L_{sr}(\theta) i_r$$
$$\lambda_r = L_{sr}(\theta) i_s + L_r i_r$$

a) $$\underset{a-b}{EFE} = \int_{L_s i_s + 0.1\times 5}^{L_s i_s + 0} 10\, d\lambda_s + \int_{L_r i_r + 0.1\times 10}^{L_r i_r + 0} 5\, d\lambda_r = -5-5 = \boxed{-10\text{ J}}$$

b) linear, so $W_{mb} = W'_{mb} = \frac{1}{2} L_s i_s^2 + \cancel{\left(0.1 - \frac{0.2}{\pi}\theta\right)} i_s i_r + \frac{1}{2} L_r i_r^2$

$$W_{ma} = W'_{ma} = \frac{1}{2} L_s i_s^2 + (0.1 - 0) i_s i_r + \frac{1}{2} L_r i_r^2$$

$$W_{mb} - W_{ma} = -0.1\times 5\times 10 = \underline{-5\text{ J}}$$

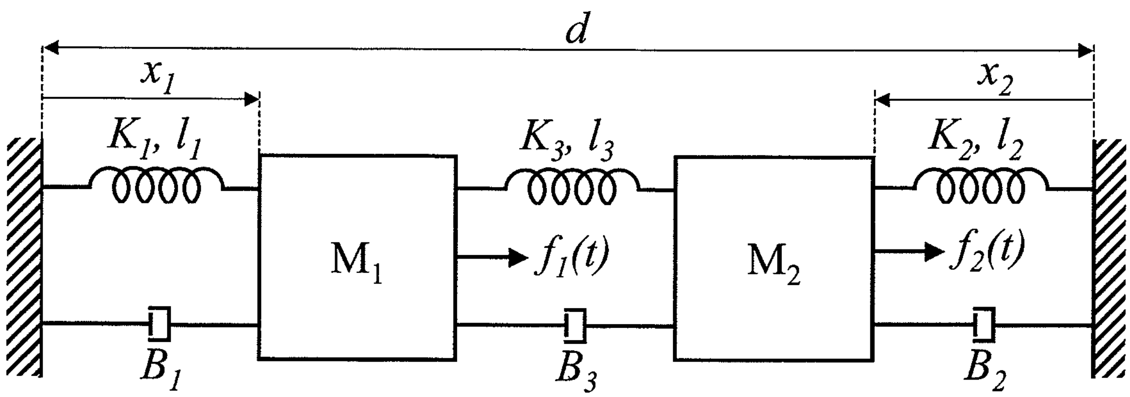
So $$\underset{a-b}{EFM} = -5-(-10) = \boxed{5\text{ J}}$$

$$T^e = -\frac{0.2}{\pi} i_s i_r = -\frac{10}{\pi}$$

Check $$W'_m = \frac{1}{2} L_s i_s^2 + \left(0.1 - \frac{0.2}{\pi}\theta\right) i_s i_r + \frac{1}{2} L_r i_r^2$$

$$EFM = -(-)\int^{\pi/2} \frac{10}{\pi}\, d\theta = \frac{10}{\pi}\cdot\frac{\pi}{2} = 5\text{ J}$$

3. (25 points)



For the system shown above:

- when $x_1=l_1$, the force due to the linear spring with constant k_1 is 0,
- when $x_2=l_2$, the force due to the linear spring with constant k_2 is 0, and
- when d-x_1- x_2= l_3, the force due to the linear spring with constant k_3 is 0.

a) Write the system state-space equations taking x_1, x_2 and their time derivatives as state variables.

b) Assume $f_1(t)=0$ for t<0, and $f_1(t)=F_1$ for t ≥0; and $f_2(t)=0$ for t<0, and $f_2(t)=F_2$ for $t \geq 0$, where F_1 and F_2 are constant. Find the equilibrim point for $t>0$.

~~c) After the system reaches the equilibrium in part b),~~ the spring with constant K_2 breaks, resulting in $K_2=0$. ~~Find the new equilibrim point.~~

if

a)

$$\frac{dx_1}{dt} = v_1 \qquad \frac{dx_2}{dt} = v_2$$

$$m_1 \frac{dv_1}{dt} = f_1 - k_1(x_1 - l_1) - B_1 v_1 + k_3(d - x_1 - x_2 - l_3) + B(-v_1 - v_2)$$

$$m_2 \frac{dv_2}{dt} = -f_2 - k_2(x_2 - l_2) + k_3(d - x_1 - x_2 - l_3) + B(-v_1 - v_2)$$

b)

$$0 = F_1 - K_1(x_1 - l_1) + K_3(d - x_1 - x_2 - l_3)$$

$$0 = -F_2 + K_3(d - x_1 - x_2 - l_3)$$

$$0 = f_1 - K_1(x_1 - l_1) + F_2$$

Blank page for work

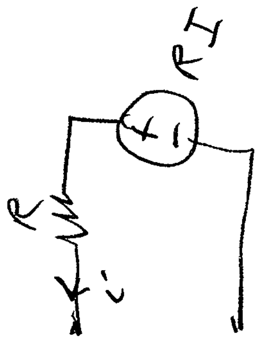
$$\frac{F_2}{K_3} = d - x_1 - x_2 - l_3$$

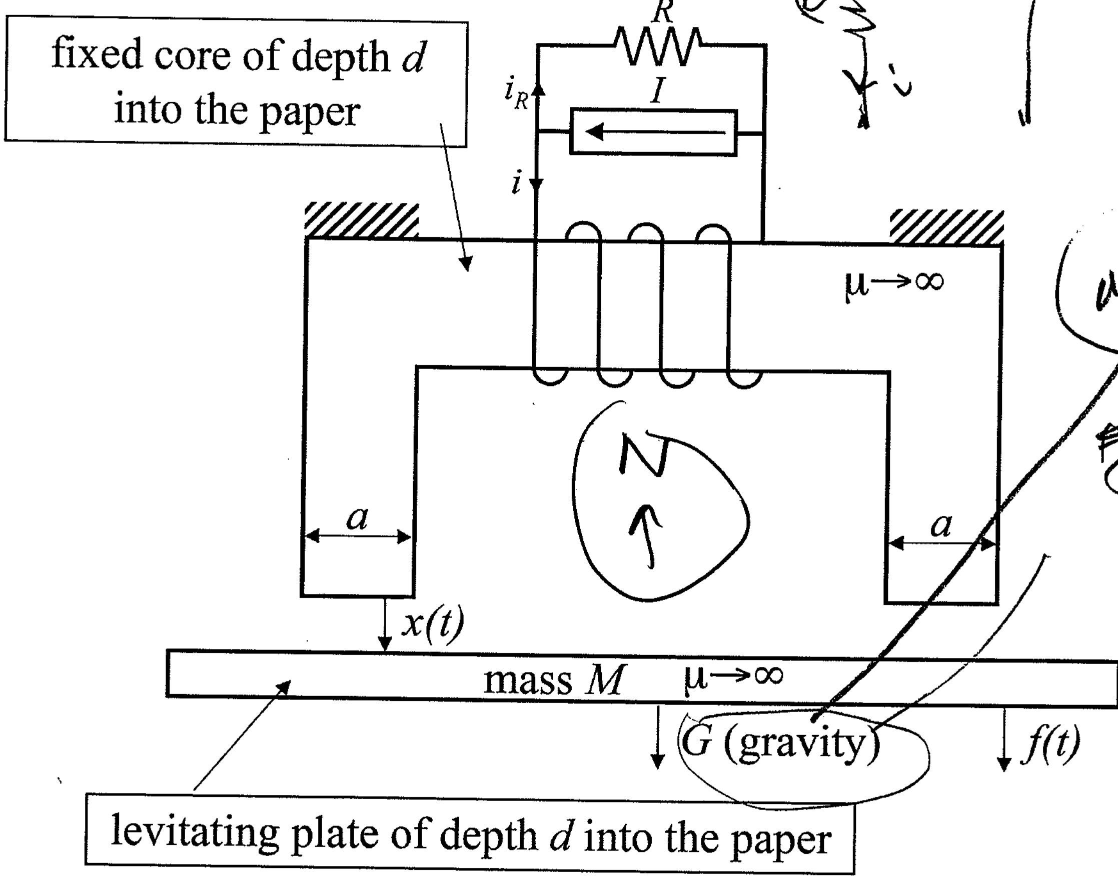
$$x_1 + x_2 = d - l_3 - \frac{F_2}{K_3}$$

$$x_2 = d - l_3 - \frac{F_2}{K_3} - x_1$$

$$x_2 = d - l_3 - \frac{F_2}{K_3} - l_1 - \frac{F_1 + F_2}{K_1}$$

4. (25 points)



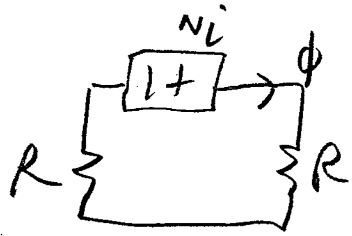


For the system show above:

(you may neglect fringing)

a) Find the force of electrical origin $f^e(i,x)$ on the plate in the x-direction.
b) Write the system state-space equations taking i, x and the time derivative of x as state variables.
c) Find the equilibrim point of the system when $f(t)=0$.
d) Show graphically whether the equilibrium point is stable or not.

$$\mathcal{R} = \frac{x}{\mu_0 a d}$$



$$\phi = \frac{Ni \, \mu_0 a d}{2x}$$

$$\lambda = \frac{\mu_0 a d N^2}{2x} i$$

$$W_m' = \frac{\mu_0 a d N^2}{4x} i^2$$

$$f^e = -\frac{\mu_0 a d N^2 i^2}{4x^2}$$

$$RI = iR + \frac{\mu_0 a d N^2}{2x}\frac{di}{dt} - \frac{\mu_0 a d N^2 i v}{2x^2}$$

Equilibrium point

$v = 0$

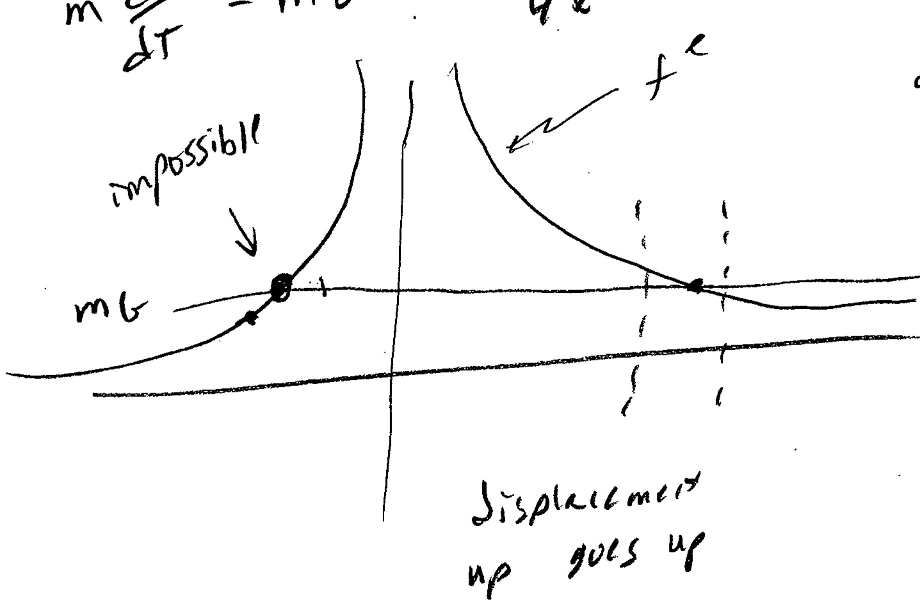
$i = I$

$$mg = \frac{\mu_0 a d N^2 i^2}{4x^2}$$

$$\boxed{x = \pm\sqrt{\frac{\mu_0 a d N^2 I^2}{4mg}} \quad x < 0 \text{ impossible}}$$

$$\frac{dx}{dt} = v$$

$$m\frac{dv}{dt} = mg - \frac{\mu_0 a d N^2 i^2}{4x^2}$$



displacement to down goes down

displacement up goes up