ECE 330 Exam #2, Fall 2012 Name: Solution
90 Minutes

Section (Check One) MWF 10am _______ MWF 2pm _______

1. ________ / 25 2. ________ / 25

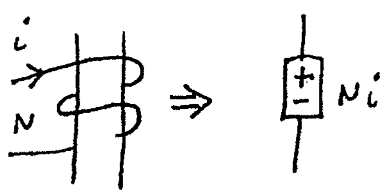
3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\qquad \overline{V} = \overline{ZI}$ $\qquad \overline{S} = \overline{VI}^*$ $\qquad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\qquad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\qquad \Re = \frac{l}{\mu A}$ $\qquad MMF = Ni = \phi\Re$

$\Re = \frac{l}{\mu A}$ $\qquad B = \mu H$ $\qquad \phi = BA$ $\qquad \lambda = N\phi$ $\qquad \lambda = Li$ (if linear)



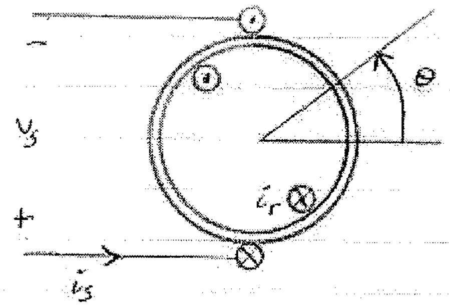
$W_m = \int_0^\lambda i d\hat{\lambda}$ $\qquad W_m' = \int_0^i \lambda d\hat{i}$ $\qquad W_m + W_m' = \lambda i$ $\qquad f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x}$ $\qquad x \to \theta$

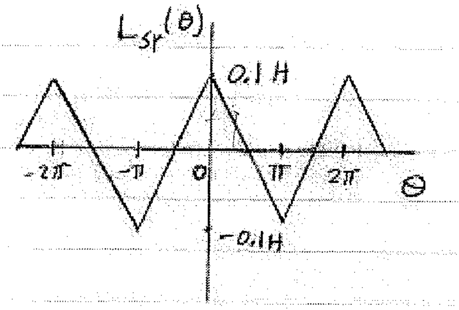
$f^e \to T^e$

$\underset{a\to b}{EFE} = \int_a^b i d\lambda$ $\qquad \underset{a\to b}{EFM} = -\int_a^b f^e dx$ $\qquad \underset{a\to b}{EFE} + \underset{a\to b}{EFM} = W_{mb} - W_{ma}$ $\qquad \lambda = \frac{\partial W_m'}{\partial i}$ $\qquad i = \frac{\partial W_m}{\partial \lambda}$

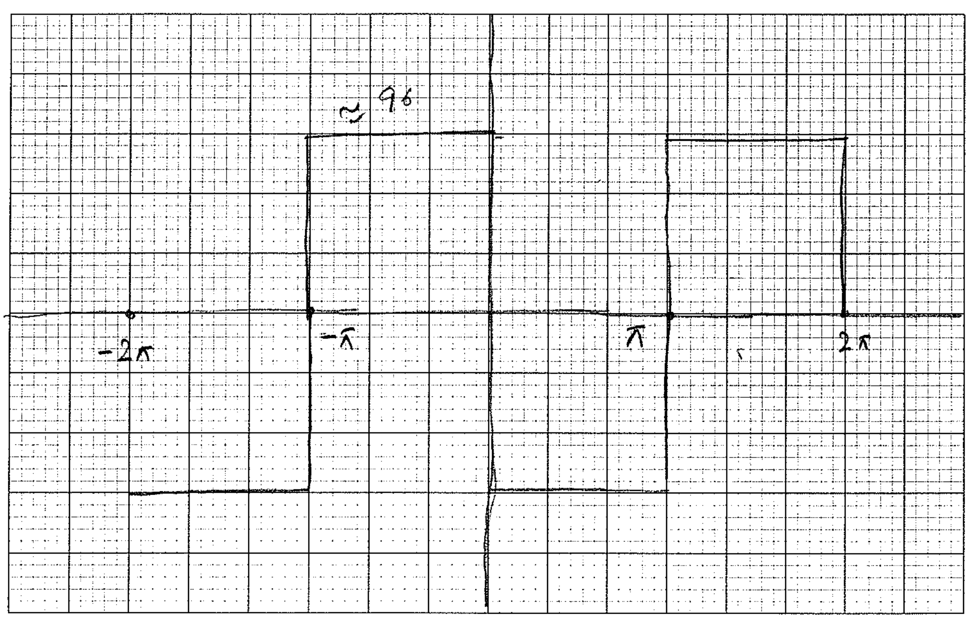
Problem 1. (25 points)

A single-phase generator consists of a coil on the stator and a coil on the rotor with a mutual inductance variation with θ as shown in the figure below. The rotor is being driven at a constant speed of 377 radians per second and the rotor coil has a constant dc current $i_r = 4$ A.





(a) Plot the open circuit voltage (the case with $i_S = 0$) as a function of θ (label all points and do not use sinusoidal approximations)



$V = \frac{d\lambda}{dt}$ 3

$= \frac{d(Li)}{dt}$

$= \frac{d(L_{sr}(\theta) \times 4)}{dt}$

Note: $L_{sr}(\theta) = 0.1 - \frac{0.2\theta}{\pi}$

$0 < \theta < \pi$ 3

$L_{sr}(\theta) = 0.1 + \frac{0.2\theta}{\pi}$

$\Rightarrow V = -\frac{0.2\omega \times 4}{\pi} \quad 0 < \theta < \pi$

2 $= \frac{0.2\omega \times 4}{\pi} \quad -\pi < \theta < 0$

(b) What is the torque of electrical origin when $i_S = 8$ A and $\theta = 30°$?

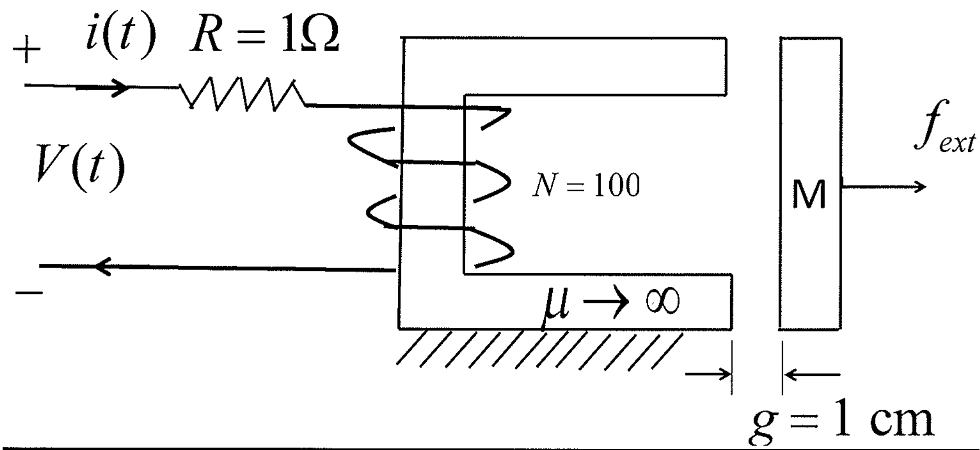
(5) $W_m' = \frac{1}{2} L_{ss} i_s^2 + L_{sr}(\theta) i_s i_r + \frac{1}{2} L_{rr} i_r^2$

$\Rightarrow T^e = \frac{\partial W_m'}{\partial \theta} = i_s i_r \cdot \left(\frac{0.2}{\pi}\right) \Rightarrow T^e = -8 \times 4 \times \frac{0.2}{\pi}$

(2) $T^e = -\frac{6.4}{\pi} \Rightarrow$ $\boxed{T^e = 2.038 \text{ N-m}}$

Problem 2. (25 points.)

Consider the electromechanical system shown in the figure below. Let the air-gap length between the fixed and the moving parts be g = 1 cm. The cross sectional area of the fixed and the moving part is $A = 100\ \text{cm}^2$.

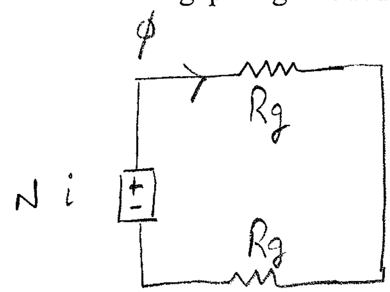


a) Compute the external force f_{ext} required to maintain the air-gap length between the fixed and moving part at g = 1cm when a voltage of V(t) = 12 VDC is applied.

b) If the external force is removed and the mass M is allowed to move, compute the energy from the mechanical system (EFM) when mass M moves and closes the air gap.

c) When the air-gap length is reduced to nearly zero, what happens to the flux in the core?

a)



$$Ni = 2R_g\phi$$

$$\Rightarrow \phi = \frac{Ni}{2R_g} = \frac{\mu_0 A N i}{2g}$$

$$\Rightarrow \lambda(i,g) = N\phi = \frac{\mu_0 A N^2 i}{2g}$$

$$\Rightarrow W_m'(i,g) = \int_0^{i} \frac{\mu_0 A N^2 i}{2g}\, di$$

$$= \frac{\mu_0 A N^2 i^2}{4g}$$

(Blank page follows this page)

$$\Rightarrow f^e(i,g) = -\frac{\mu_0 A N^2 i^2}{4g^2}$$

for $i = 12\text{A}$, $g = 1\text{cm}$

$$f^e(i,g) = -45.23\ \text{N}$$

$$\Rightarrow \boxed{f_{ext} = 45.23\ \text{N}}$$

b) $$EFM|_{a\to b} = -\int_{g_a}^{g_b} f_e \, dg$$

$$= -\int_{+0.01}^{0.00001} -\frac{\mu_0 A N^2 i^2}{4g^2} \, dg$$

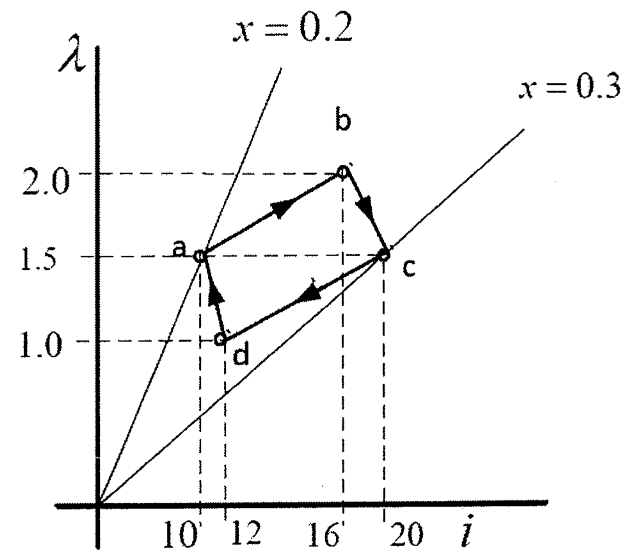
$$= \frac{\mu_0 A N^2 i^2}{4} \cdot \left. \frac{-1}{g} \right|_{0.01}^{0.00001}$$

$$= -451.9 \text{ J}$$

c) As the gap is reduced nearly to zero, the assumption of $\mu \to \infty$ (iron) no longer holds.

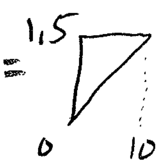
Problem 3. (25 points.)

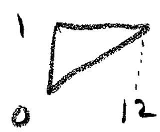
An electromechanical device is operated over the cycle abcda shown in the figure below. The system is known to be electrically linear, i.e., $\lambda = L(x)i$.



a) Calculate the energy stored in the coupling field (W_m) at points 'a' and 'd'.
b) Calculate EFE|cycle in Joules.
c) Calculate EFM|cycle in Joules.
d) Is the machine operating as a motor or a generator?

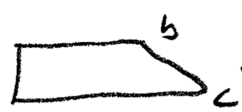
(Note : You must clearly show the steps for parts a) - c) and state the reason for your answer in part d))

a) $W_{m_a} =$ 

$= 7.5\ J$ $W_{m_d} =$ 

$= 6\ J$

b) $EFE_{cycle} =$ 

$-$ 

$-$ 

$+$ 

$= 6.5 - 9 - 8 + 5.5 = -5\ J$

c) $EFM_{cycle} = -EFE_{cycle} = 5\ J$

d) $EFM_{cycle} > 0$ so generator

(Blank page follows this page)

Problem 4. (25 points.)

An electro-mechanical device has the following nonlinear dynamic model:

$$0.04\frac{d^2\delta}{dt^2} = 2-3\sin\delta$$

a) Write this model in standard state-space form.
b) Find all of the equilibrium points in the interval with δ between -180 degrees and + 180 degrees.
c) Speculate on which of these equilibrium points might be stable (indicate why)
d) Demonstrate that you might be right by using Euler's method on the nonlinear model with time step $\Delta t = 0.1\text{s}$ to compute the values of the two state variables at $t = 0.1\text{s}$ and $t = 0.2\text{s}$ if the initial conditions are $\delta(0)$ = the equilibrium angle you think is stable plus 1 degree and $d\delta/dt$ (at $t = 0$) = 0

a) $x_1 = \delta$ $\quad \dot{x}_1 = x_2$

$x_2 = \frac{d\delta}{dt}$ $\quad \dot{x}_2 = 25\times 2 - 25\times 3 \sin x_1$

b) $0 = x_{2e}$ $\quad x_{2e} = 0$

$0 = 50 - 75\sin x_{1e}$ $\quad x_{1e} = 41.8^\circ$ or 138.2°

c) 41.81 is stable because if it moves a little to the right, the $\sin x_1$ gets bigger which will make $\dot{x}_2$ negative which will make x_2 negative, pulling it back. Reverse if it is 138.2°

d) $x_1(0.1) = (41.81^\circ + 1^\circ)\frac{\pi}{180} + 0.1(0) = 42.81^\circ \frac{\pi}{180}$ radians

$x_2(0.1) = 0 + (50 - 75\sin 42.81^\circ)\times 0.1 = -0.968\times .1 = -.097 \frac{\text{rad}}{\text{sec}}$

$x_1(0.2) = 42.81^\circ\frac{\pi}{180} + 0.1(-0.097) = 42.25^\circ\frac{\pi}{180}$ radians

$x_2(0.2) = -0.097 + 0.1(-.970) = -0.194$ rad/sec

yes, moving back