ECE430
Exam #2
Spring 2005

Name Solution PWS
(Print Name)

Section: **(Circle One)** **10 MWF (Sauer)** **12:30 TuTh (Liu)**

Problem 1 ______ **Problem 2** ______ **Problem 3** ______ **Problem 4** ______

TOTAL: ________

USEFUL INFORMATION

$$\oint_c \underline{H}\cdot \underline{dl} = \int_s \underline{J}\cdot\underline{n}\, da \qquad \oint_c \underline{E}\cdot\underline{dl} = -\frac{d}{dt}\int_s \underline{B}\cdot\underline{n}\, da \qquad \oint_s \underline{B}\cdot\underline{n}\, da = 0$$

$$mmf = Ni = \phi R \qquad R = \frac{l}{\mu A} \qquad \phi = BA \qquad B = \mu H \qquad \lambda = N\phi$$

$$W_m = \int i \Big|_{x=const} d\lambda \qquad W_m' = \int \lambda \Big|_{x=const} di \qquad W_m + W_m' = \lambda i$$

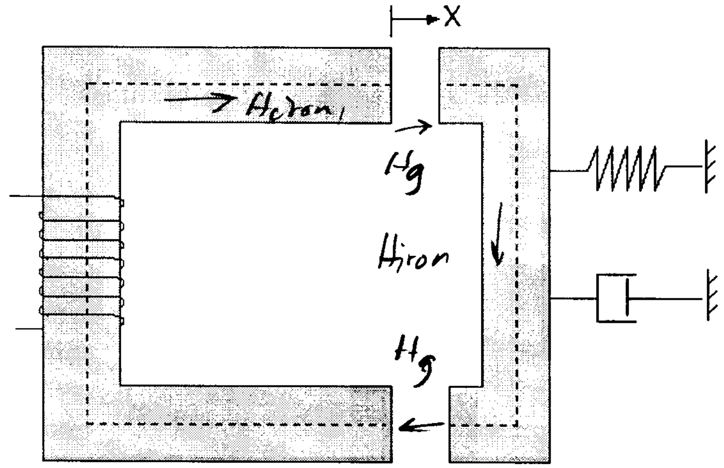
$$f^e = -\frac{\partial W_m}{\partial x} \qquad f^e = \frac{\partial W_m'}{\partial x} \qquad \text{For rotation, } x \to \theta,\ f \to T$$

$$\underset{a-b}{EFE} = \int_{\lambda_a}^{\lambda_b} i \Big|_{path} d\lambda \qquad \underset{a-b}{EFM} = -\int_{x_a}^{x_b} f^e \Big|_{path} dx$$

$$W_{m_b} - W_{m_a} = \underset{a-b}{EFE} + \underset{a-b}{EFM} \qquad x(t_0+\Delta t) \approx x(t_0) + \frac{dx}{dt}\Big|_{t_0} \Delta t$$

Problem 1 (25 pts.)

A coupled electro-mechanical system is diagrammed below. The current through the coil is designated as i. The cross-section of the magnetic core is A. The total length of the dotted line is l_c. The number of turns in the coil is N. The permeability of the magnetic core is μ_c.



a) Find the equation that relates the current and the flux linkage (you define the current direction and flux linkage polarity).
b) Find the analytical expression of the force of electric origin.
c) Write the second-order dynamic equation that governs the displacement of the movable piece. Suppose that the spring force constant is K (with a zero-force distance of C) and the damping factor is B. The mass of the movable piece is M.
d) Write the state space equations that correspond to the mechanical dynamic equation found in c) above.

a) Define top of coil as + and assume i into + sign.
By conservation of flux (and neglecting fringing and leakage)
There is only one H_{iron} and only one H_g

$$H_{iron} l_c + 2 H_g X = Ni \qquad \mu_0 H_g A = \mu_c H_{iron} A \quad / \; H_g = \frac{\mu_c}{\mu_0} H_{iron}$$

$$H_{iron}\left(l_c + 2X\frac{\mu_c}{\mu_0}\right) = Ni \qquad H_{iron} = \frac{\mu_0 N i}{\mu_0 l_c + 2\mu_c X}$$

$$\lambda = N(\mu_c H_{iron} A) = \frac{\mu_c \mu_0 N^2 A}{\mu_0 l_c + 2\mu_c X}\, i$$

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b) $$W_m' = \frac{1}{2}\frac{\mu_c \mu_0 N^2 A}{(\mu_0 l_c + 2\mu_c x)} i^2$$

$$f^e = -\frac{\mu_c^2 \mu_0 N^2 A i^2}{(\mu_0 l_c + 2\mu_c x)^2}$$

c) $$m\frac{d^2x}{dt^2} = -\frac{\mu_c^2 \mu_0 N^2 A i^2}{(\mu_0 l_c + 2\mu_c x)^2} - k(x-c) - B\frac{dx}{dt}$$

d) $$x_1 \triangleq x \qquad x_2 \triangleq \frac{dx}{dt}$$

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = \frac{1}{m}\left[-\frac{\mu_c^2 \mu_0 N^2 A i^2}{(\mu_0 l_c + 2\mu_c x_1)^2} - k(x_1 - c) - B x_2\right]$$

Problem 2 (25 pts) (April 5)

A single-phase rotating machine has one coil on the stator with current i_s and one coil on the rotor with current i_r. The inductances for this machine are (assume linear magnetic core):

$L_{ss} = L_s$ $\qquad$ $L_{sr} = M\cos(\theta)$

$L_{rs} = M\cos(\theta)$ $\qquad$ $L_{rr} = L_r$

The machine is being operated such that the currents i_s and i_r can be assumed to be constants at I_s and I_r respectively while the shaft is rotated from θ equals zero to θ equals $\pi/2$.

For this change from "point a" to point b", find:

a) The energy transferred from the mechanical system into the coupling field as the system moved from point a to point b with constant currents.

b) The energy transferred from the electrical system into the coupling field as the system moved from point a to point b with constant current.

a) $\lambda_s = L_s i_s + M\cos\theta\, i_r$ $\qquad$ $W_m' = \frac{1}{2} L_s i_s^2 + M\cos\theta\, i_s i_r + \frac{1}{2} L_r i_r^2$

$\lambda_r = M\cos\theta\, i_s + L_r i_r$

$T^e = -M\sin\theta\, i_s i_r$

$$EFM_{a-b} = -\int_0^{\pi/2} (-M\sin\theta) I_s I_r \, d\theta = -M\cos\theta\, I_s I_r \Big|_0^{\pi/2} = M I_s I_r$$

b) $$EFE_{a-b} = \int_{L_s I_s + M I_r}^{L_s I_s} I_s \, d\lambda_s + \int_{M I_s + L_r I_r}^{L_r I_r} I_r \, d\lambda_r = -M I_s I_r - M I_s I_r = -2 M I_s I_r$$

OR

$W_{mb} = W_{mb}' = \frac{1}{2} L_s I_s^2 + 0 + \frac{1}{2} L_r I_r^2$ $\qquad$ $W_{ma} = W_{ma}' = \frac{1}{2} L_s I_s^2 + M I_s I_r + \frac{1}{2} L_r I_r$

$W_{mb} - W_{ma} = -M I_s I_r = EFE_{a-b} + EFM_{a-b} = -M I_s I_r$ ✓ Checks

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Problem 2 (25 pts) (April 7,8)

An electromechanical device has the following flux-linkage vs current relationship with constants a through f and translational displacement x:

$$\lambda_1 = a\, i_1 + (b/x)\, i_2 + (c/x)\, i_3$$

$$\lambda_2 = (b/x)\, i_1 + d\, i_2 + (e/x)\, i_3$$

$$\lambda_3 = (c/x)\, i_1 + (e/x)\, i_2 + f\, i_3$$

The machine is being operated such that the currents i_1, i_2, and i_3 can be assumed to be constants while the movable mechanical member position is changed from x equals 0.10 to 0.2.

For this change in operating conditions, find:

a) The energy transferred from the mechanical system into the coupling field.

b) The energy transferred from the electrical system into the coupling field.

a) $W_m' = \frac{1}{2} a i_1^2 + \frac{b i_1 i_2}{x} + \frac{1}{2} d i_2^2 + \frac{c i_1 i_3}{x} + \frac{e i_2 i_3}{x} + \frac{1}{2} f i_3^2$

$f^e = -\frac{b i_1 i_2}{x^2} - \frac{c i_1 i_3}{x^2} - \frac{e i_2 i_3}{x^2}$

$EFM = -\int_{.1}^{.2} -\frac{1}{x^2}\left[b i_1 i_2 + c i_1 i_3 + e i_2 i_3\right] dx = -\frac{1}{x}\left[b i_1 i_2 + c i_1 i_3 + e i_2 i_3\right]\Big|_{.1}^{.2}$

$$\boxed{EFM = 5\left[b i_1 i_2 + c i_1 i_3 + e i_2 i_3\right]}$$

b) $W_{m,.2} = W'_{m,.2} = \frac{1}{2} a i_1^2 + 5 b i_1 i_2 + \frac{1}{2} d i_2^2 + 5 c i_1 i_3 + 5 e i_2 i_3 + \frac{1}{2} f i_3^2$

$W_{m,.1} = W'_{m,.1} = \frac{1}{2} a i_1^2 + 10 b i_1 i_2 + \frac{1}{2} d i_2^2 + 10 c i_1 i_3 + 10 e i_2 i_3 + \frac{1}{2} f i_3^2$

$W_{m,2} - W_{m,1} = -5\left[b i_1 i_2 + c i_1 i_3 + e i_2 i_3\right] = EFE + 5\left[b i_1 i_2 + c i_1 i_3 + e i_2 i_3\right]$

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$EFE = -10\left[b i_1 i_2 + c i_1 i_3 + e i_2 i_3\right]$ OR Also use $\int i\, d\lambda$

See next sheet

$$EFE_{a-b} = \int_{\lambda_{1,1}}^{\lambda_{1,2}} i_1 \, d\lambda_1 + \int_{\lambda_{2,1}}^{\lambda_{2,2}} i_2 \, d\lambda_2 + \int_{\lambda_{3,1}}^{\lambda_{3,2}} i_3 \, d\lambda_3$$

$$= \left[i_1 (b i_2 + c i_3) + i_2 (b i_1 + e i_3) + i_3 (c i_1 + e i_2) \right] (5 - 10)$$

$$\boxed{EFE = -10 \left(b i_1 i_2 + c i_1 i_3 + e i_2 i_3 \right)}$$

✓ checks

Problem 3 (25 pts.)

An electromechanical system is described by the following flux-linkage vs current characteristic:

$$\lambda = \frac{.04}{x - .01}\, i$$

It is operated on the closed cycle a - b - c - d - e as indicated below, with x constant during a - b and also c - d. The current is constant during b - c and also d - e.

	a	b	c	d	e
i (Amps)	0	i_b	i_b	0	0
λ (Wb turns)	0	8	λ_c	0	0
x (meters)	.03	.03	.02	.02	.03

Find the following things:

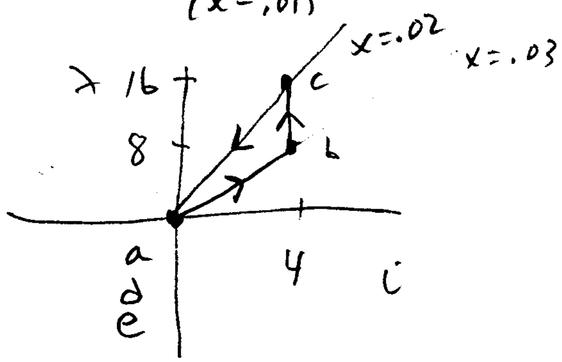
a) i_b and λ_c.
b) The energy stored in the coupling field at points b and c.
c) The force of electric origin at points b and c.
d) Sketch this cycle in the λ vs i plane (label points a, b, c, d, and e)
e) Sketch this cycle in the force vs x plane (label points a, b, c, d, and e)
f) Is this is a motor or a generator?

a) $8 = \frac{.04}{.02} i_b$ $\quad i_b = 4$ Amps $\quad \lambda_c = \frac{.04}{.01} i_b = 16$ WbTN

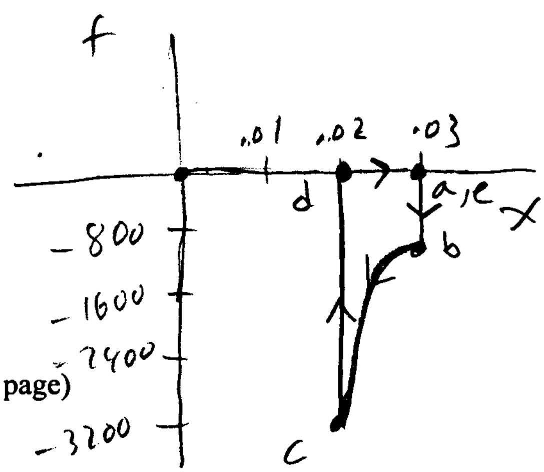
b) $w_m = w_m' = \frac{.02 i^2}{x - .01}$ $\quad w_{mb} = \frac{.02 \times 16}{.02} = 16\ J$ $\quad w_{mc} = \frac{.02 \times 16}{.01} = 32\ J$

c) $f^e = -\frac{.02 i^2}{(x - .01)^2}$ $\quad f_b^e = -\frac{.02 \times 16}{.02 \times .02} = -800\ N$ $\quad f_c^e = -\frac{.02 \times 16}{.01 \times .01} = -3200\ N$

d)



e)



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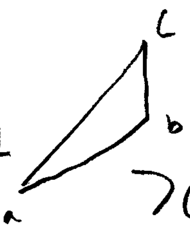
f) EFE cycle = +



> 0 so motor

Problem 4 (25 pts.)

A system can be described by the following equations

$x_1 = x$

$x_2 = \frac{dx}{dt}$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_1 x_2 + 16 - x_1^2$$

a) Write the equivalent second order differential equation that matches the given state space equations.
b) Find all equilibrium points of this system.
c) If $x_1(t=0) = 0.5, x_2(t=0) = 10$, find the values of x_1 and x_2 at t=0.001 and 0.002 s.

a) $$\frac{d^2x}{dt^2} = x\frac{dx}{dt} + 16 - x^2$$

b) $x_2^e = 0$

$0 = 16 - x_1^{e^2}$ $\quad x_1^e = \pm 4$

$x_1^e = -4$, $x_2^e = 0$

$\boxed{x_1^e = +4, \; x_2^e = 0}$

c) $x_1(.001) \approx 0.5 + (10) \times .001 = 0.51$

$x_2(.001) = 10 + (.5 \times 10 + 16 - .5^2) \times .001 = 10.0208$

$x_1(.002) = 0.51 + (10.0208) \times .001 = 0.52$

$x_2(.002) = 10.02 + (.51 \times 10.02 + 16 - .51^2) \times .001 = 10.04$