ECE 430 Exam #2, Spring 2011 Name: SOLUTION
90 Minutes

Section (Check One) MWF 10am ______ MF 2pm ______

1. ________ / 25 2. ________ / 25

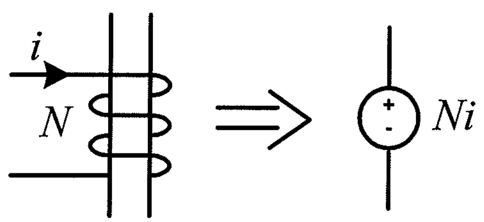
3. ________ / 25 4. ________ / 25 Total ________ / 100

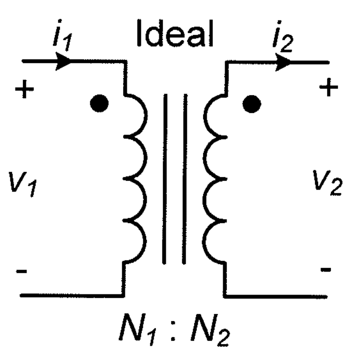
Useful Information

$\sin(x) = \cos(x - 90°)$ $\bar{V} = \bar{Z} \cdot \bar{I}$ $\bar{S} = \bar{V} \cdot \bar{I}^*$ $\bar{S}_{3\phi} = \sqrt{3} V_L I_L \angle\theta$ 1 hp = 746 W

$\oint_c \vec{H} \cdot d\vec{l} = \int_s \vec{J} \cdot \vec{n}\, ds$ $\oint_c \vec{E} \cdot d\vec{l} = -\int_s \frac{d}{dt}\vec{B} \cdot \vec{n}\, ds$ $\mathcal{R} = \frac{l}{\mu A}$ $mmf = Ni = \mathcal{R}\phi$

$\phi = BA$ $\lambda = N\phi$ $L = \frac{\partial \lambda}{\partial i}$ $M_{12} = \frac{\partial \lambda_1}{\partial i_2}$ $k = \frac{M}{\sqrt{L_1 L_2}}$ $\mu_0 = 4\pi \cdot 10^{-7} H/m$





$W_m = \int_0^\lambda i\, d\hat{\lambda}$ $W_m' = \int_0^i \lambda\, d\hat{\imath}$ $W_m + W_m' = \lambda i$ $f^e = \frac{\partial W_m'}{\partial x} = \frac{\partial W_m}{\partial x}$ $i = \frac{\partial W_m}{\partial \lambda}$

$\underset{a\to b}{EFE} = \int_a^b i\, d\lambda$ $\underset{a\to b}{EFM} = -\int_a^b f^e dx$ $\underset{a\to b}{EFE} + \underset{a\to b}{EFM} = W_{m,b} - W_{m,a}$ $\lambda = \frac{\partial W_m'}{\partial i}$

Rotational systems: $x \to \theta$ and $f^e \to T^e$

Problem 1 (25 points)

The electromagnet in Figure 1 consists of a core and plunger. The plunger M experiences gravity g, i.e. it falls in the direction indicated by x if the electromagnet is turned off. The core is fixed.

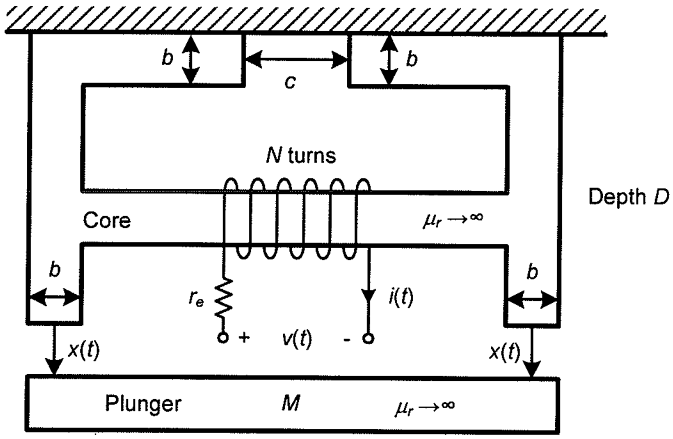


Figure 1. Electromagnet.

a) Calculate the reluctance of the magnetic circuit in terms of the given variables. You can assume that core and plunger have infinite permeability

b) Write the differential equation of the electrical circuit based on the given variables

c) Calculate the force on the plunger in terms of i and x

d) Write the mechanical differential equation

e) What is the steady-state position of the plunger if you assume a constant voltage excitation $v(t) = V_0$?

a) $\mathcal{R}_1 = \frac{c}{\mu_o b D}$, $\mathcal{R}_2 = \frac{2x}{\mu_o b D}$, $\mathcal{R}_{total} = \mathcal{R}_1 // \mathcal{R}_2 = \frac{\left(\frac{c}{\mu_o b D}\right)\left(\frac{2x}{\mu_o b D}\right)}{\frac{c}{\mu_o b D} + \frac{2x}{\mu_o b D}} = \boxed{\frac{1}{\mu_o b D}\left(\frac{2xc}{c+2x}\right)}$

b) $v(t) = r_e i(t) + \frac{d\lambda(t)}{dt}$

$v(t) = r_e i(t) + \frac{\partial \lambda}{\partial i}\frac{di}{dt} + \frac{\partial \lambda}{\partial x}\frac{dx}{dt}$

$\lambda = N\phi = N\left(\frac{Ni}{\mathcal{R}_{total}}\right) = N^2 i \mu_o b D\left(\frac{1}{c} + \frac{1}{2x}\right)$

$\frac{\partial \lambda}{\partial i} = N^2 \mu_o b D\left(\frac{1}{c} + \frac{1}{2x}\right)$, $\frac{\partial \lambda}{\partial x} = \frac{-N^2 i \mu_o b D}{2x^2}$

$$\boxed{v(t) = r_e i(t) + N^2 \mu_o b D\left(\frac{1}{c} + \frac{1}{2x}\right)\frac{di}{dt} + \left(-\frac{N^2 i \mu_o b D}{2x^2}\right)\frac{dx}{dt}}$$

c) $W_m' = \int_0^i \lambda \, di = \frac{N^2 i^2 \mu_o b D}{2}\left(\frac{1}{c} + \frac{1}{2x}\right)$, $f^e = \frac{\partial W_m'}{\partial x} = \boxed{\frac{-N^2 i^2 \mu_o b D}{4x^2}}$

d) $Ma = \Sigma F$

$$M\ddot{x} = f^e + Mg$$

$$\ddot{x} = g + \frac{f^e}{M}$$

$$\boxed{\ddot{x} = g - \frac{N^2 i^2 \mu_o b D}{M 4 x^2}}$$

e) Choosing states:

$$\underline{X} = \begin{bmatrix} i \\ x \\ \dot{x} \end{bmatrix} \qquad \dot{\underline{X}} = \begin{bmatrix} \dot{i} \\ \dot{x} \\ \ddot{x} \end{bmatrix}$$

thus, at steady state $\frac{di}{dt} = 0$

$$\frac{dx}{dt} = 0$$

$$\frac{d^2x}{dt^2} = 0$$

$$V(t) = V_o$$

$$\therefore \; i^e = \frac{V_o}{r_e}$$

$$(x^e)^2 = \frac{N^2 i^{e2} \mu_o b D}{4Mg}$$

$$x^e = +\sqrt{\frac{N^2 i^{e2} \mu_o b D}{4Mg}}$$

$$\boxed{x^e = \frac{N V_o}{2 r_e}\sqrt{\frac{\mu_o b D}{Mg}}}$$

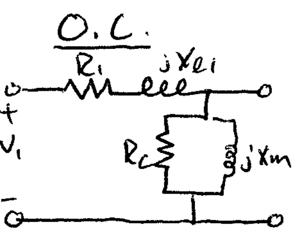
Problem 2 (25 points)

A single-phase 1 kVA, 240V/120V, transformer is tested and the following measurements are recorded. Both tests and measurements are performed on the high voltage side, and all voltages and currents are sinusoidal.

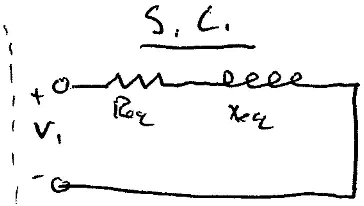
Open Circuit Test	P_{OC}= 25 W	V_{OC}= 240 V	I_{OC}= 0.5 A
Short Circuit Test	P_{SC}= 80 W	V_{SC}= 24 V	I_{SC}= 4.17 A

a) Find the equivalent circuit parameters and the equivalent circuit model referred to the high voltage side. Draw and label the approximate equivalent circuit model, including referred voltages and currents (10 points)

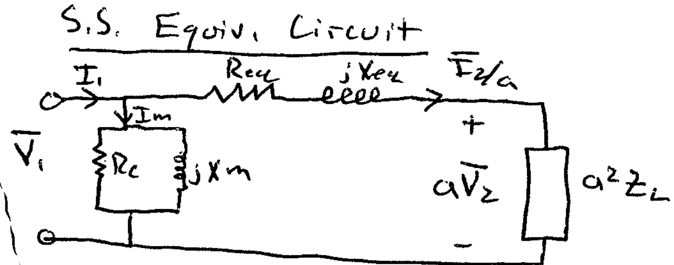
O.C.



S.C.



S.S. Equiv. Circuit



$$Z_{SC} = \frac{24V}{4.17A} = 5.755\Omega$$

$$Z_{oc} = \frac{240V}{0.5A} = 480\Omega$$

$$a = \frac{240}{120} = 2$$

$$R_{eq} = \frac{P_{SC}}{I_{SC}^2}$$

$$P_{oc} = \frac{V_{oc}^2}{R_c}$$

$$Q = \sqrt{(240V \cdot 0.5A)^2 - (25W)^2}$$

$$\boxed{R_{eq} = 4.6\Omega}$$

$$Q = 117.37 \text{ VAR}$$

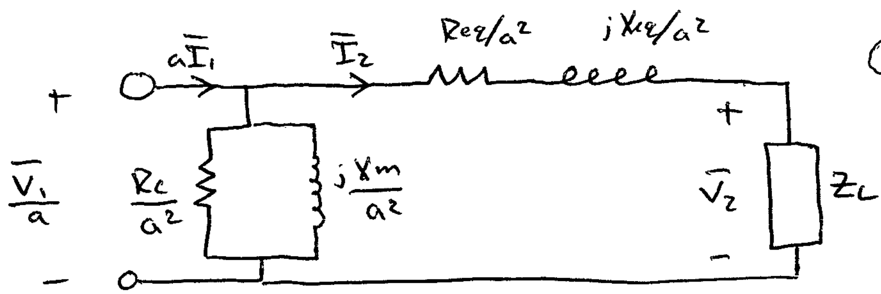
$$\boxed{R_c = 2304 W}$$

$$X_m = \frac{V_{oc}^2}{Q} = \boxed{490.8\Omega = X_m}$$

$$X_{eq} = \sqrt{(5.755\Omega)^2 - (4.6\Omega)^2}$$

$$\boxed{X_{eq} = 3.46\Omega}$$

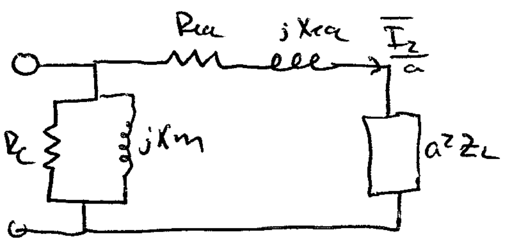
b) Find the equivalent circuit model referred to the low voltage side. Draw and label the approximate equivalent circuit model, including referred voltages and currents (7.5 points)



where: $a = 2$

c) A resistive load is connected to the low voltage side, and the load voltage magnitude is 120 V. The load operates at the transformer rated power. Use the equivalent circuit from part b to find the transformer efficiency and input power [Hint: consider copper and core losses] (7.5 points)

$S_{rated} = 1\text{ kVA}$



$$I_{2rated} = \frac{1\text{kVA}}{120\text{V}} = \frac{25}{3}\text{ A}$$

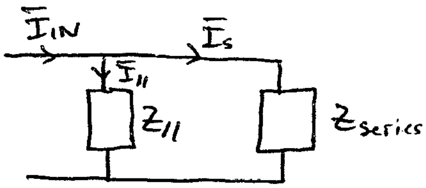
$$\frac{I_{2rated}}{a} = \frac{25}{6}\text{ A}$$

$$P_{cu} = R_{eq}\left(\frac{25}{6}\text{A}\right)^2$$
$$= (4.6\,\Omega)\left(\frac{625}{36}\text{A}^2\right)$$
$$\underline{P_{cu} = 79.86\text{ W}}$$

$$a^2 Z_L = (2)^2\left(\frac{120\text{V}}{\left(\frac{25}{3}\text{A}\right)}\right) = 57.6\,\Omega$$

$$R_{eq} + jX_{eq} + a^2 Z_L = Z_{series} = 62.2 + j3.46\,\Omega$$

$$R_c \,//\, jX_m = Z_{//} = \frac{(2304)(j490.8)}{(2304 + j490.8)} = 100.01 + j469.5\,\Omega$$



$$I_{//} = \frac{Z_{series}}{Z_{//}} I_s = 0.541\angle -74.8°$$

$$|V_{//}| = |I_{//}|\cdot|Z_{//}| = 259.7\text{V}$$

$$P_{core} = \frac{259.7\text{V}^2}{2304\,\Omega} = 29.27\text{W}$$

$$\therefore P_{in} = P_{out} + P_{cu} + P_{core}$$
$$= 1\text{kW} + 79.86\text{W} + 29.27\text{W}$$
$$\boxed{P_{in} = 1109.13\text{ W}}$$

$$\Rightarrow \quad \eta = \frac{1000\text{W}}{1109.13\text{W}} \times 100$$

$$\boxed{\eta = 90.16\%}$$

Problem 3 (25 points)

A nonlinear system is given as:

$$\dot{x}_1 = x_1 + x_2 - x_3$$
$$\dot{x}_2 = 2x_1 - x_2 - 5x_3$$
$$\dot{x}_3 = -2x_2 - x_3 + 3u$$

a) Write the system state-space model as $\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}u$ where $\mathbf{x}$ is the state vector, while $\mathbf{A}$ is a matrix and $\mathbf{B}$ is a vector

$$\dot{\underline{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -1 & -5 \\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} u$$

b) Find all equilibrium points of the system at $\boxed{u=1}$

$$\dot{\underline{x}} = 0 \Rightarrow \begin{aligned} 0 &= x_1^e + x_2^e - x_3^e \\ 0 &= 2x_1^e - x_2^e - 5x_3^e \\ 0 &= -2x_2^e - x_3^e - 3(1) \end{aligned}$$

$$\boxed{\begin{aligned} x_1^e &= -6 \\ x_2^e &= 3 \\ x_3^e &= -3 \end{aligned}}$$

c) The input u is set to be $u=x_1^2$. The initial conditions are $x_1(0)=1$, $x_2(0)=0$, and $x_3(0)=0$. For a time step $\Delta t=0.1$ second, find $x_1(0.2)$

$$\underline{x}_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{x}_{0.1} = \underline{x}_0 + \Delta t\, \dot{\underline{x}}_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 0.1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1.1 \\ 0.2 \\ 0.3 \end{bmatrix}$$

$$\underline{x}_{0.2} = \begin{bmatrix} 1.1 \\ 0.2 \\ 0.3 \end{bmatrix} + 0.1 \begin{bmatrix} 1 \\ 0.5 \\ 2.93 \end{bmatrix} = \begin{bmatrix} 1.2 \\ 0.25 \\ 0.593 \end{bmatrix}$$

$$\boxed{x_1(0.2) = 1.2}$$

Problem 4 (25 points)

Given is the energy conversion cycle in Figure 2. The underlying electromechanical system cycles from a – b – c – d – a and can be described by

$$\lambda(i,x) = \frac{5i}{x+1}\,.$$

The dashed lines in Figure 2 indicate the λ-i relationship at constant values for x (note that points c, d, and e lie on the $x = x_c$ axis).

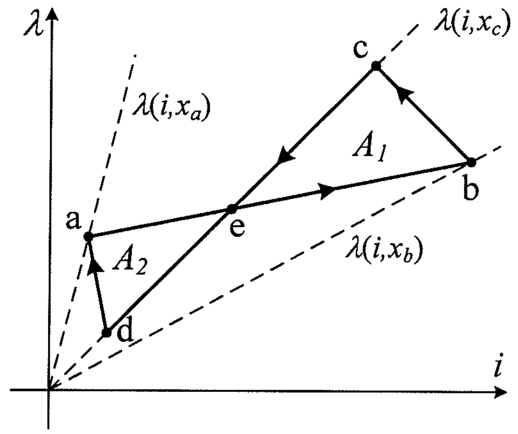


Figure 2. Energy conversion cycle.

a) Calculate values for the variables in the table below:

	a	B	c	d
i	i_a	1	0.8	i_d
λ	1	λ_b	λ_c	0.78
x	0	1.5	0.6	0.6
Energy	$W_{m,a}$	$W_{m,b}$	$W_{m,c}$	$W_{m,d}$
Force	f_a^e	f_b^e	f_c^e	f_d^e

b) For the specific values of λ and i, is this a motor or a generator?

c) What is the general condition for this system to be a generator or a motor, expressed in terms of areas A_1 (triangle *cbe*) and A_2 (triangle *ade*)? Show all your work for full credit. Hint: solve graphically

a) $i = \frac{\lambda}{5}(x+1)$

$\boxed{i_a = 0.2A}$

$\boxed{i_d = 0.25A}$

$\boxed{\lambda_b = 2\,wb}$

$\boxed{\lambda_c = 2.5\,wb}$

$W_m = \int_0^{\lambda} i\, d\hat{\lambda} = \frac{\lambda^2}{10}(x+1)$

$\boxed{W_m = [0.1 \quad 1.0 \quad 2.0 \quad 0.097]\ J}$

$f^e = \frac{-\partial W_m}{\partial x} = \frac{-\lambda^2}{10}$

Additional page for work space

$\boxed{f^e = [-0.1 \quad -0.4 \quad -0.625 \quad -0.061]\ N}$

b) $EFE_{ab} = \int_a^b i\,d\lambda = i_a(\lambda_b - \lambda_a) + \frac{1}{2}(\lambda_b - \lambda_a)(i_b - i_a)$

$= (.2)(1) + \frac{1}{2}(1)(.8)$

$= 0.6\ J$

$EFE_{bc} = \int_b^c i\,d\lambda = i_c(\lambda_c - \lambda_b) + \frac{1}{2}(-\lambda_b + \lambda_c)(i_b - i_c)$

$= 0.8(0.5) + \frac{1}{2}(0.5)(.2)$

$= 0.45\ J$

$EFE_{cd} = \int_c^d i\,d\lambda = -\left[i_d(\lambda_c - \lambda_d) + \frac{1}{2}(i_c - i_d)(\lambda_c - \lambda_d)\right]$

$= -\left[.25(1.72) + \frac{1}{2}(.55)(1.72)\right]$

$= -0.903\ J$

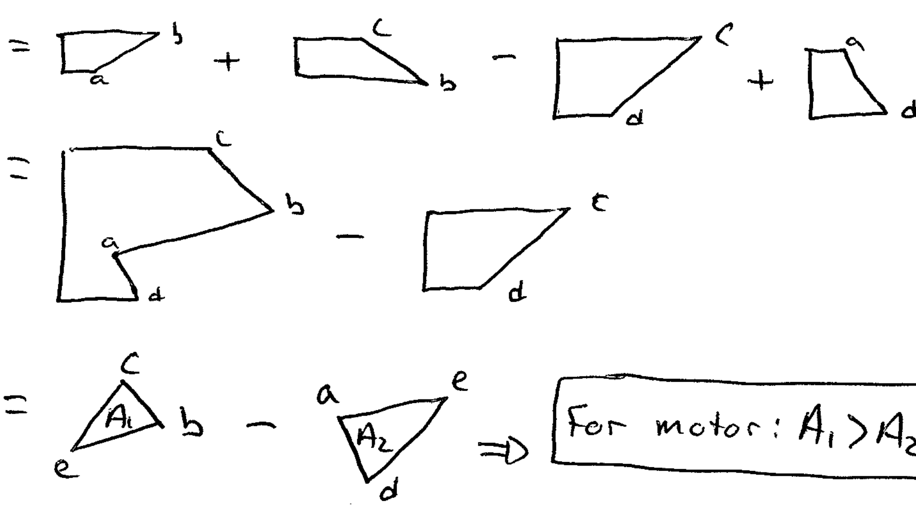
$EFE_{da} = \int_d^a i\,d\lambda = i_a(\lambda_a - \lambda_d) + \frac{1}{2}(\lambda_a - \lambda_a)(i_d - i_a)$

$= 0.2(0.22) + \frac{1}{2}(0.22)(0.05A)$

$= 0.0495\ J$

$EFE_{cycle} = \Sigma EFE = \boxed{\underbrace{0.20\ J}_{+EFE = \text{MOTOR}}}$

c) $EFE_{cycle} = \Sigma EFE =$



$\Rightarrow$ $\boxed{\text{For motor: } A_1 > A_2}$