Problem 1 (25 pts.)

The following three-phase balanced loads are connected in parallel across a three-phase wye-connected, 50 Hz source of 480 V (line to line)

Load #1   94 kVA (3-phase) at 0.8 PF lag (Wye connected)
Load #2   67 kW (3-phase) at 0.7 PF lag (Wye connected)
Load #3   75 Amps phase current, with 0.9 power factor lag (Delta connected)

Find the following things:

a) The total complex power (3-phase) consumed by the three loads
b) The total source line current (magnitude).
c) The capacitive reactive power needed per phase (for a delta connection) so that the overall power factor is 0.95 lag.
d) The new source line current with the power factor correction of c) installed.

a) $\bar{S} = 94k\underline{/37°} + \dfrac{67k}{0.7}\underline{/46°} + 3 \times 480 \times 75\underline{/26°}$

$= \left[(75 + j56) + (66 + j69) + (97 + j47) \right]k = 238 + j172k$

$= \boxed{294k \underline{/36°}}$

b) $294k = \sqrt{3} \times 480 \times I_L$ $\boxed{I_L = 354A}$

c) $\bar{S} = \dfrac{238}{0.95}\underline{/18°} = 279k + j77k$

$Q_{Req} = \dfrac{172k - 77k}{3} = \boxed{32\,kVAR \quad per\ phase}$

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d) $\dfrac{238k}{.95} = 251k = \sqrt{3} \times 480 \times I_L$ $\boxed{I_L = 302A}$

Problem 2 (25 pts.)

μ large   1 mm   This structure is 100 mm long, into the paper.

$+\, i_1$

v_1

$\mu = \mu_0$

$N_1 = 100$

$+\, v_2\, -$

i_2

μ large

$120°$

θ

0

$N_2 = 50$

40 mm
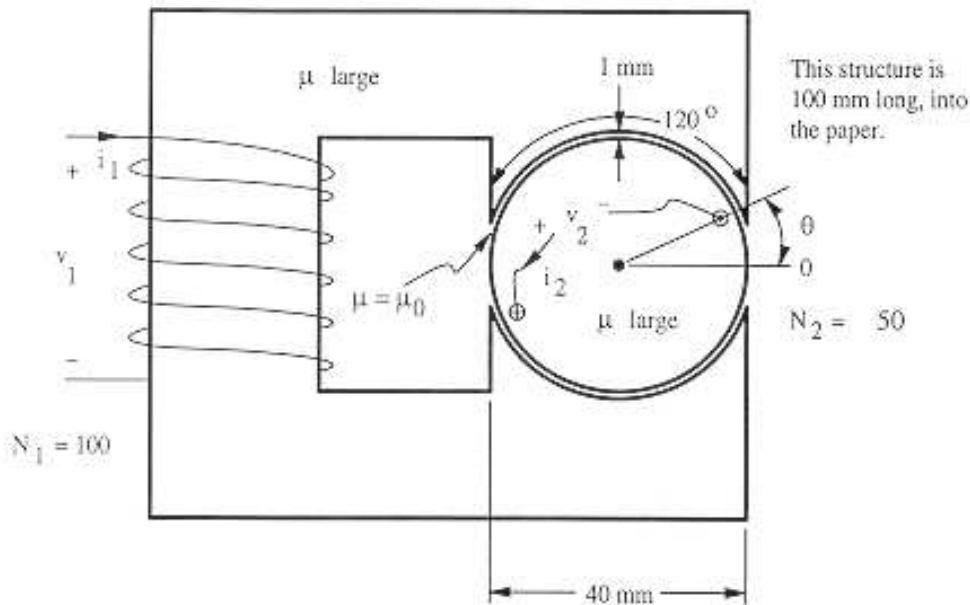
a) For the structure above, find $\lambda_1(i_1,i_2,\theta)$ and $\lambda_2(i_1,i_2,\theta)$. The winding resistances are small. (Hint: consider the reluctance of the structure. You may use sinusoidal approximations for position effects as appropriate.)

b) Let $i_1 = 6$ A. If $d\theta/dt = 120\pi$ rad/s, what is the measured voltage $v_2(t)$ when $i_2 = 0$?

$$\mathcal{R} = \frac{l}{\mu_0 A} \rightarrow \frac{\frac{2\pi}{3/2}\, 0.002}{\mu_0 \left(\frac{2\pi}{3} RL\right)} \qquad R = 20\,mm \quad L = 100\,mm$$

$$= 3.80 \times 10^5 \, H^{-1} \qquad L_1 = \frac{N_1^2}{\mathcal{R}} = 2.63 \times 10^{-2}$$

$$\lambda_1 = L_1 i_1 + M i_2 \qquad \lambda_2 = L_2 i_2 + M i_1$$

$$L_1 = \frac{N_1^2}{\mathcal{R}} = 2.63 \times 10^{-2} \qquad M = \frac{N_1 N_2}{\mathcal{R}} \cos\theta \qquad L_2 = \frac{N_2^2}{\mathcal{R}} = 6.58 \times 10^{-3}$$

$$= 1.32 \times 10^{-2}$$

$$\lambda_1 = 2.63 \times 10^{-2} i_1 + 1.32 \times 10^{-2} \cos\theta \, i_2 \qquad \lambda_2 = 6.58 \times 10^{-3} i_2 + 1.32 \times 10^{-2} \cos\theta \, i_1$$

$$i_1 = 6, \; i_2 = 0, \; \frac{d\theta}{dt} = 120\pi \qquad v_2 = \frac{d\lambda_2}{dt} = -7.90 \times 10^{-2} \sin\theta \frac{d\theta}{dt}$$

$$= 29.8 \sin(120\pi t + \theta_0)$$

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Problem 3 (25 pts.)

An electromechanical system is described by the following flux-linkage vs current characteristic:

$$\lambda = \frac{.04}{x - .01}\, i$$

It is operated on the closed cycle a - b - c - d - e as indicated below, with x constant during a - b and also c - d. The current is constant during b - c and also d - e.

	a	b	c	d	e
i (Amps)	0	i_b	i_b	0	0
λ (Wb turns)	0	8	λ_c	0	0
x (meters)	.03	.03	.02	.02	.03

$$W_m' = \frac{.02\, i^2}{x - .01}$$

$$f' = \frac{-.02\, i^2}{(x - .01)^2}$$

Find the following things:

a) i_b and λ_c.
b) Energy transferred from the electrical system into the coupling field during a - b.
c) Energy transferred from the mechanical system into the coupling field during b - c.
d) Energy transferred from the electrical system into the coupling field during c - d.
e) Energy transferred from the mechanical system into the coupling field during d - e.

a) $\quad 8 = \frac{.011}{.02} i_b \qquad \boxed{i_b = 4A} \qquad \lambda_c = \frac{.04}{.01} 4 = \boxed{16 \text{ wb Tn}}$

b) $\quad \underset{a-b}{EFE} = \int_0^8 \frac{\lambda}{2} d\lambda = \left.\frac{\lambda^2}{4}\right|_0^8 = 16 - 0 = \boxed{16 \text{ J}}$

c) $\quad \underset{b-c}{EFM} = \int_{.03}^{.02} + \frac{.02 \times 16}{(x - .01)^2} dx = - \left.\frac{.32}{(x - .01)}\right|_{.03}^{.02} = -32 + 16 = \boxed{-16 \text{ J}}$

d) $\quad \underset{c-d}{EFE} = \int_{16}^0 \frac{\lambda}{4} d\lambda = \left.\frac{\lambda^2}{8}\right|_{16}^0 = \boxed{-32 \text{ J}}$

e) $\quad \underset{d-e}{EFM} = \boxed{0}$

$\boxed{\text{check}}$

$$\underset{b-c}{EFE} = \int_8^{16} 4\, d\lambda = 32 \text{ J}$$

$\underset{a-b}{EFM} = 0$

$\underset{c-d}{EFM} = 0$

$\underset{d-e}{EFE} = 0$

$\underset{cycle}{EFE} = 16 \text{ J}$

$\boxed{\underset{cycle}{EFM} = -16 \text{ J}}$

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Problem 4 (25 pts.)

μ large

This structure is 20 mm long, into the paper.

+ i

v, λ

5 mm

15 mm

$\mu = \mu_0$

x

0

The isolated mass has high permeability and a mass of ten grams.

$N_1 = 100$

20 mm
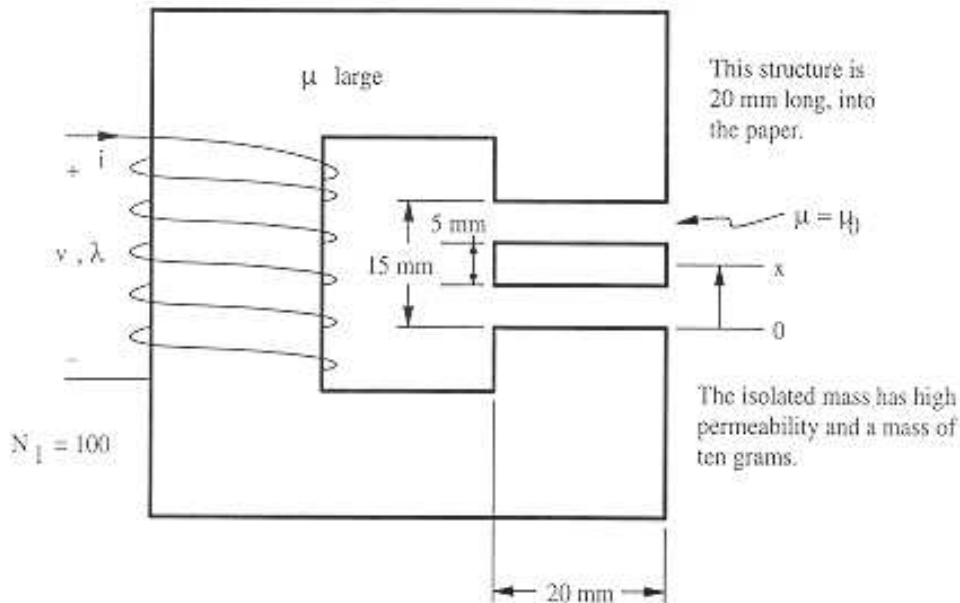
The physical arrangement shown is proposed as a way to levitate an object. The following analysis tries to determine whether this will work. Use the acceleration due to gravity (pointed down) as 10 m/s².

a) Find $\lambda(i,x)$ for this structure.

b) Find either the stored energy $W_m(\lambda,x)$ or co-energy $W_m'(i,x)$ for this structure.

c) It is proposed that if the position of the ten gram object is sensed, and a current $i(x) = 20000\,x^2$ (with x in meters and i in amps) is imposed, then the object can be levitated (held stable and stationary in open space). What is $f^e(x)$ for this choice of current?

d) Write equations for $d\lambda/dt$, $dx/dt = V$ (velocity) and dV/dt for this system, in state-space form, given that $i = i(x) = 20000\,x^2$. Use λ, x, and V as state variables.

e) Linearize your result about the point $x = 5$ mm. Do you think it will work?

a) $\mathcal{R} = \dfrac{\ell}{\mu A} = \dfrac{.01 \cap}{\mu_0 (400 \times 10^{-6})} = 19.9 \times 10^{6}$ $\quad \dfrac{N^2}{\mathcal{R}} = 0.503\ mH = L \quad \lambda = 5.03 \times 10^{-4}\,i$

b) $w_m' = \frac{1}{2} L i^2 = 2.51 \times 10^{-4}\,i^2$

c) $w_m' = 10^5 x^4 \quad f^e = \dfrac{\partial w_m'}{\partial x} = 4 \times 10^5 x^3$

d) $\dfrac{d\lambda}{dt} = 10\,\dfrac{dx}{dt} = 10\,V \quad \dfrac{dV}{dt} = 4 \times 10^5 x^3 - 0.1$

$\dfrac{dx}{dt} = V$

At 5 mm:

.05 + ... -0.05

v FALLING

$x = .005 + \Delta x$

$\dfrac{d\Delta\lambda}{dt} = 10\,\dfrac{dx}{dt}, \dfrac{d\Delta x}{dt}$

$\dfrac{d\Delta x}{dt} = V$

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$(.005 + \Delta x)^3$

$= (.005 + \Delta x)(.005 + \Delta x)$

$= 0.05 + 30\Delta x$

$\dfrac{d\Delta V}{dt} = 30\Delta x - 0.05$

Extra work space (please show which problem is involved).

$$(.005 + \Delta x)^2 (.005 + \Delta x)$$

$$= (.005^2 + .010x + \Delta x^2)(.005 + \Delta x)$$

$$= .005^3 + 50 \times 10^{-6} \Delta x + .005 \Delta x^2$$
$$+ .005^2 \Delta x + .01 \Delta x^2 + \Delta x^3$$

$$4 \times 10^5 \ (.005^3 + 75 \times 10^{-6} \Delta x)$$

$$0.05 + 30 \Delta x$$

$$\begin{bmatrix} \dot{\Delta x} \\ \dot{\Delta v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 30 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta v \end{bmatrix} \begin{bmatrix} 0 \\ -.06 \end{bmatrix}$$

$$(sI - A)^{-1}$$

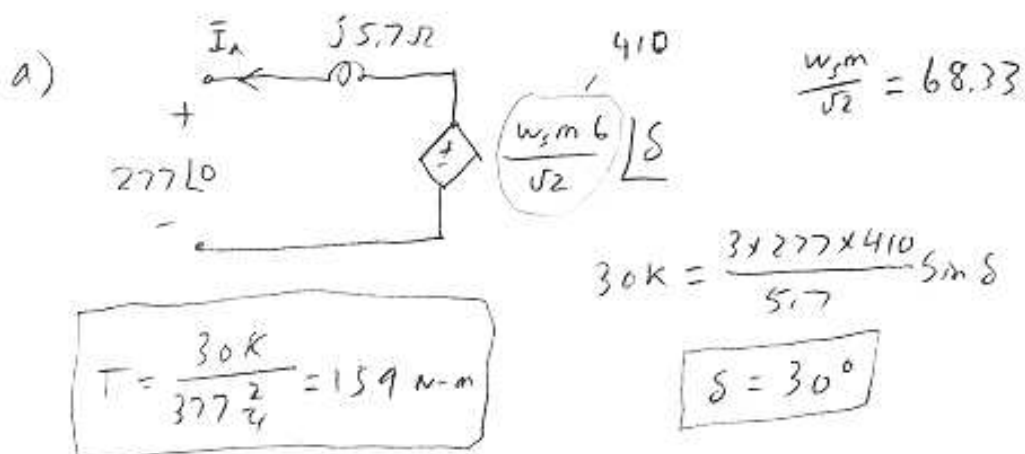$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 30 & 0 \end{bmatrix}$$

$$det \begin{bmatrix} s & -1 \\ -30 & s \end{bmatrix}$$

$$s^2 + 30 = 0$$
$$s = \pm \sqrt{30} \quad \underline{NO}$$

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Problem 5 (25 pts.)

A three-phase, 60 Hz, 4 pole, wye connected synchronous generator is delivering 30 kW (3-phase) into a system at the rated voltage of 277 V (line to neutral). The field current I_f = 6 ADC produces an internal excitation voltage magnitude E_{ar} of 410 V (line to neutral). The synchronous reactance X_S = 5.7 Ohms. Compute the following things (neglect stator (armature) resistance and saturation of the iron):

a) The torque angle δ and the torque in NM.
b) The generator line current magnitude
c) The power factor of the power being delivered by the generator
d) Is the generator over-excited or under-excited? (explain your answer)

a)

$$\frac{w_s m}{\sqrt{2}} = 68.33$$

$$30K = \frac{3 \times 277 \times 410}{5.7} \sin \delta$$

$$T = \frac{30K}{377\frac{2}{4}} = 159 \text{ N-m}$$

$$\boxed{\delta = 30°}$$

b) $$\bar{J}_A = \frac{410\lfloor 30 - 277\lfloor 0}{j\,5.7} = 72\lfloor -60° - 48.6\lfloor -90°$$

$$= 36 - j62 + 0 + j48.6 = 36 - j13.4 = \boxed{38\lfloor -20°}$$

c) $$pf = \cos(0 + 20°) = \boxed{0.94 \text{ lag}}$$

d) $$\boxed{\text{Overexcited because } Q_{out} > 0}$$

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Problem 6 (25 pts.)

An industrial plant is supplied by the electric utility with 3 φ, 4160 V (line-line), 60 Hz. The total plant load is 5000 kW (3-phase). Each major process in the plant now runs with an induction motor that has a power factor of 0.85 lagging. It has been suggested that one of these major processes can be converted over to a synchronous motor, which would also improve the overall power factor. The proposed machine has a series reactance per phase $X_s = 6\,\Omega$. It will serve a process that requires 2000 kW (3-phase) after installation.

a) Find the current delivered to the plant by the utility before the replacement of the induction motor by the synchronous motor.

b) The motor is installed, and set up to draw 2000 kW (out of the same 5000 kW total) and S = 2750 kVA, with leading power factor. What is the current delivered to the plant now? What is the plant's overall power factor?

c) With the motor in place, drawing P = 2000 kW and S = 2750 kVA, find the torque angle and the internal voltage magnitude E_{ar} in the machine.

a) $P = 5000\,kW$, $S = 5882\,kVA = \sqrt{3}\,V_\ell I_\ell$ $I_\ell = 816.4$ $Q = 3099\,kVAR$

b) $P = 5000\,kW$

$Q = \underset{I.m}{\cancel{3000}}\ Q = 1854\,kVAR$ $Q_{syn-m} \sim 1887$

$Q_{TOTAL} = 28.45\,kVAR\ LEAD$

$S_{TOTAL} = 5000\,kVA$ $I_\ell = 694\,A$

OVERALL $pf = 1$

c) $P = \dfrac{3\,|V_{as}||E_{ar}|\sin\delta}{6} = \dfrac{|V_{\ell\ell}||E_{ar}|\sin\delta}{6} = 2000\,kW$

$E_{ar}\sin\delta = 1665$

$Q = \dfrac{3\,V_{as}^2}{6} - \dfrac{3\,V_{as}\,E_{ar}}{6}\cos\delta = -1887\,kVAR$

$9.97\times10^5 = \dfrac{3\,V_{as}\,E_{ar}}{6}\cos\delta$

$830.4 = E_{ar}\cos\delta$ $\tan\delta = \dfrac{1665}{830.4} = 2$
$1665 = E_{ar}\sin\delta$ $\delta = 63.5°$

$9.968\times10^5 = \dfrac{3\,V_{as}\,E_{ar}}{6}\cos\delta$ $\dfrac{1665}{3973}$

$830 = E_{ar}\cos\delta$
or $3973 = E_{ar}\cos\delta$ $\tan\delta = 0.419$

8    $E_{ar} = 4308$ ✓ $\delta = 22.73°$ ✓

Problem 7 (25 pts.)

A 3-phase, 6-pole, 60 Hz, 440 V (line-line) wye-connected squirrel cage induction motor is running at 1120 RPM. The shunt magnetizing reactance (on the stator side) is 20 Ohms per phase, the stator a rotor series leakage reactances are each 1.5 Ohms per phase (on the stator side), and the rotor resistance referred to the stator is 0.5 Ohms per phase. Neglect stator resistance, iron core losses and rotational losses. Find the following things:

a) The frequency of the rotor currents in Hz at this level of load
b) The magnitude of the stator line current at this level of load
c) The torque to the shaft of the machine in N-m at this level of load
d) What is the starting current if this machine was started with full voltage suddenly applied while it is at standstill?

a) $f_r = s f_s = \left(\dfrac{1200 - 1120}{1200} \right) 60 = \boxed{4 \, H_3}$ $\quad s = .0666$

b)
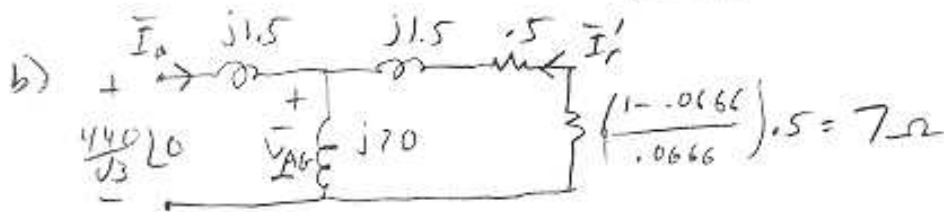
$\bar{Z}_{TOT} = j1.5 + \dfrac{j70\,(7.5 + j1.5)}{7.5 + j21.5} = j1.5 + 5.8 + j3.35 = 7.56 \,\underline{|40°}$

$|\bar{I}_a| = \dfrac{254}{7.56} = \boxed{33.6\,A} \qquad \theta = -40°$

c) $-254\underline{|0} + j1.5 \times 33.6 \,\underline{|-40°} + \bar{V}_{AG} = 0 \qquad \bar{V}_{AG} = 254\underline{|0} - 50.4\underline{|50°}$

$\bar{V}_{AG} = 222 - j39 = 225 \,\underline{|-10°} \qquad \bar{I}_r' = \dfrac{225\,\underline{|-10°}}{7.5 + j1.5} = 29.4 \,\underline{|\quad}$

$T = \dfrac{3 \times 29.4^2 \left(\frac{.5}{.0666} \right)}{1200 \times \frac{2\pi}{60}} = \boxed{155 \, N\text{-}m}$

d) $s = 1 \qquad \bar{Z} = j1.5 + \dfrac{(.5 + j1.5)\,j20}{.5 + j21.5} = .43 + j2.9 = 2.93\,\underline{|\quad}$

$|\bar{I}_a| = \dfrac{254}{2.93} = \boxed{87\,A}$

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Problem 8 (25 pts.)

An induction machine has nameplate ratings of 3φ, 60 Hz, 230 V (line to line), 24 A, 1740 RPM, 10 HP (horsepower). This motor has been tested, and is known to have $R_s = 0.1\ \Omega$ and $X_{ls} = 0.6\ \Omega$. The motor operates at rated voltage. When it is unloaded, it draws 10 A and total power of 32 W while running at a speed just slightly under 1800 RPM. When it is running at rated load and speed, it draws 24 A and 7890 W. Recall that 1 HP = 746 W. You may ignore magnetic losses and rotational losses.

a) Find X_m (hint: consider the unloaded information).

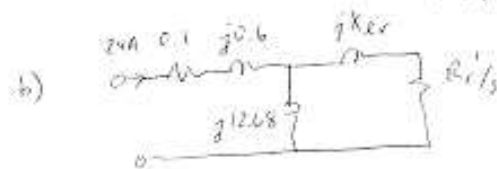b) Find R_r' and X_{lr}'. What is the efficiency of this motor at rated load and speed?
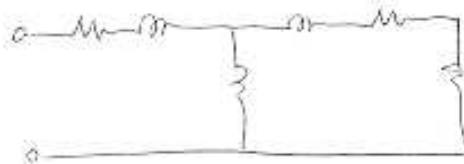
a)

$$|Z| = 13.3\Omega = \sqrt{0.1^2 + (0.6 + X_m)^2}$$

$$176.3 = (0.6 + X_m)^2$$

$$13.28 = 0.6 + X_m$$

$$X_m = 12.68\ \Omega$$

b)

$$S = \sqrt{3}\ V_\ell I_\ell$$
$$= 9561 \qquad P = 7890$$
$$Q = 5400$$
$$\angle I = -34.4°$$

$$24\ \underline{/-34.4°}$$
$$-10\ \underline{/-90}$$
$$= 19.81 - j\ 23.56$$

$$\eta = \frac{7460}{7890} = 94.6\%$$