Name Solution
(Print Name)

Section: (circle one) 10 MWF (Sauer) 2 MWF (Kimball)

ECE330 Final Exam, Spring 2004
Tuesday, May 11, 2004, 1:30 – 4:30 PM

One sheet (2-sided) provided

Problem 1 _______

Problem 2 _______

Problem 3 _______

Problem 4 _______

Problem 5 _______

Problem 6 _______

TOTAL: _______

Receiving or giving aid in a final examination is a cause for dismissal from the University

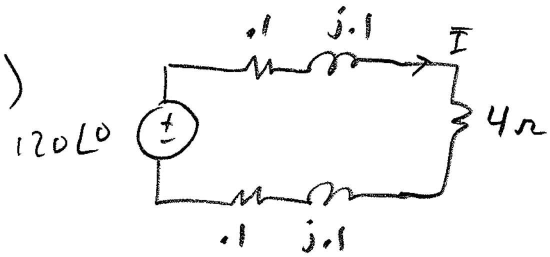
Problem 1 (25 pts.)

You have three wires of equal length sufficient to supply a load consisting of 3 resistors (12 Ohms each). The impedance of each wire is 0.1 +j0.1 Ohms. You can provide service to these resistors using three possible techniques:

a) Use two of the wires connected to a 120Volt, 60Hz, single-phase, 2-wire source and then connecting the 3 resistors in parallel at the load end
b) Use all three wires connected to a 120/208 Volt, 60Hz, three-phase, three-wire, wye-connected source and then connecting the 3 resistors in a wye configuration at the load end.
c) Use all three wires connected to a 69/120 Volt, 60Hz, three-phase, three-wire, wye-connected source and then connecting the 3 resistors in a delta configuration at the load end.

For each case, compute the magnitude of the voltage across each resistive load, the magnitude of the source line current, and the total real power lost in the lines.

a)



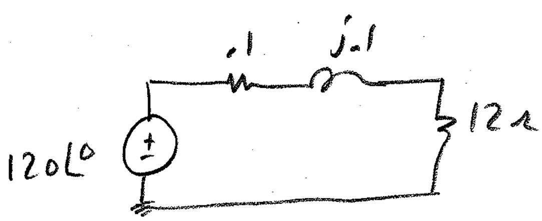
$\bar{I} = \frac{120\angle 0}{4.2 + j.2}$

$I = 28.5A$

$V = 114V$

$P_{lost} = 162.5\ W$

b)



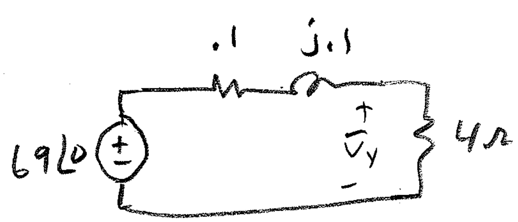
$\bar{I} = \frac{120\angle 0}{12.1 + j.1}$

$I = 9.92\ A$

$V = 119V$

$P_{lost} = 29.5\ W$

c)



$\bar{I} = \frac{69\angle 0}{4.1 + j.1}$

$I = 16.8A$

$V = 117\ V$

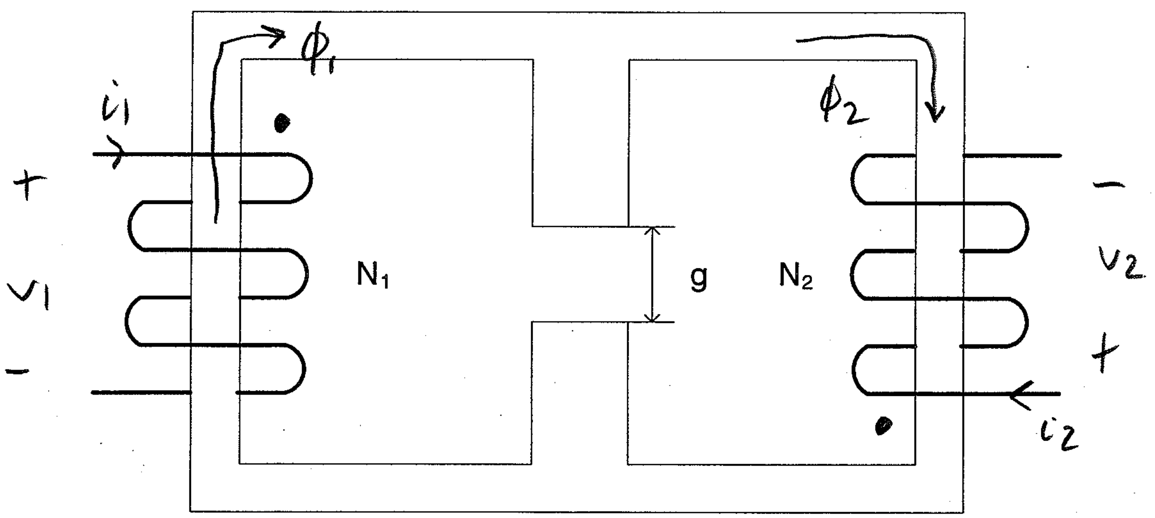
(Recall $V_\Delta = \sqrt{3}\, V_Y$)

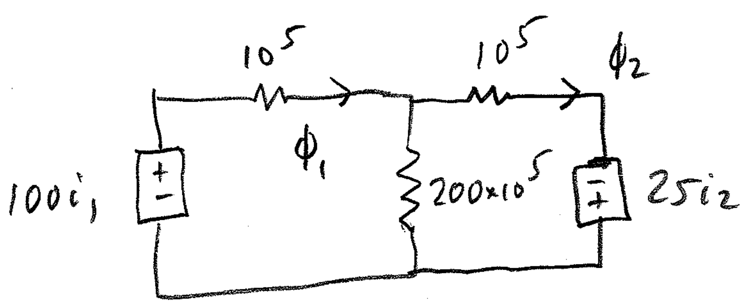
$P_{lost} = 85W$

Problem 2 (25 pts.)

In the figure below, $N_1 = 100$, $N_2 = 25$. The reluctances of the left and right branches of the core are each $1\times10^5\ H^{-1}$. The gap, g, in the center post is 1 cm, and the cross-sectional area of the center post is 4 cm^2. Neglect fringing and the effect of the steel in the center post. The resistance of the left coil (primary) is 1 ohm and the resistance of the right coil (secondary) is 0.05 ohm.

a) Label voltages v_1 and v_2 and currents i_1 and i_2 so that all self and mutual inductance terms are positive. Assign dot polarities. Assign v_1 and i_1 to the primary and v_2 and i_2 to the secondary.
b) Find L_1 (self inductance of the primary), L_2 (self inductance of the secondary), and M (mutual inductance between coils).
c) Treating this structure as a non-ideal transformer, draw the equivalent circuit with all impedances referred to the primary (assume a 60Hz supply and use the frequency domain).





$$R_{gap} = \frac{.01}{4\pi\times10^{-7}\times4\times10^{-4}} = 200\times10^5\ H^{-1}$$

$$\left.\begin{aligned} -100 i_1 + 10^5\phi_1 + 200\times10^5(\phi_1-\phi_2) &= 0 \\ -25 i_2 + 10^5\phi_2 + 200\times10^5(\phi_2-\phi_1) &= 0 \end{aligned}\right\} \begin{aligned} 201\times10^5\phi_1 - 200\times10^5\phi_2 &= 100 i_1 \\ -200\times10^5\phi_1 + 201\times10^5\phi_2 &= 25 i_2 \end{aligned}$$

$$\phi_2 = .1244\times10^{-5} i_2 + .995\phi_1$$

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$$201\times10^5\phi_1 - 199\times10^5\phi_1 = 100i_1 + 24.88i_2$$

$$2\times10^5\phi_1 = 100i_1 + 24.88i_2 \qquad \phi_1 = .0005i_1 + .000124i_2$$

$$\lambda_1 = N_1\phi_1 = .05i_1 + .0124i_2$$

$$\phi_2 = .1244\times10^{-5}i_2 + .995(.0005i_1 + .000124i_2)$$

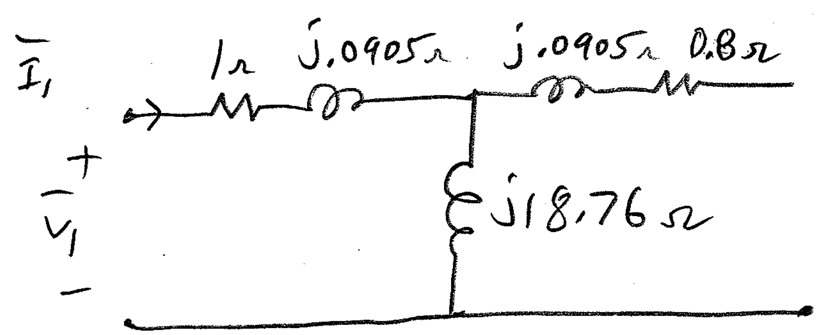
$$= .0004975i_1 + .000125i_2$$

$$\lambda_2 = N_2\phi_2 = .0124i_1 + .003125i_2$$

$$\boxed{L_1 = .05\,H \qquad M = .0124\,H \qquad L_2 = .003125\,H}$$

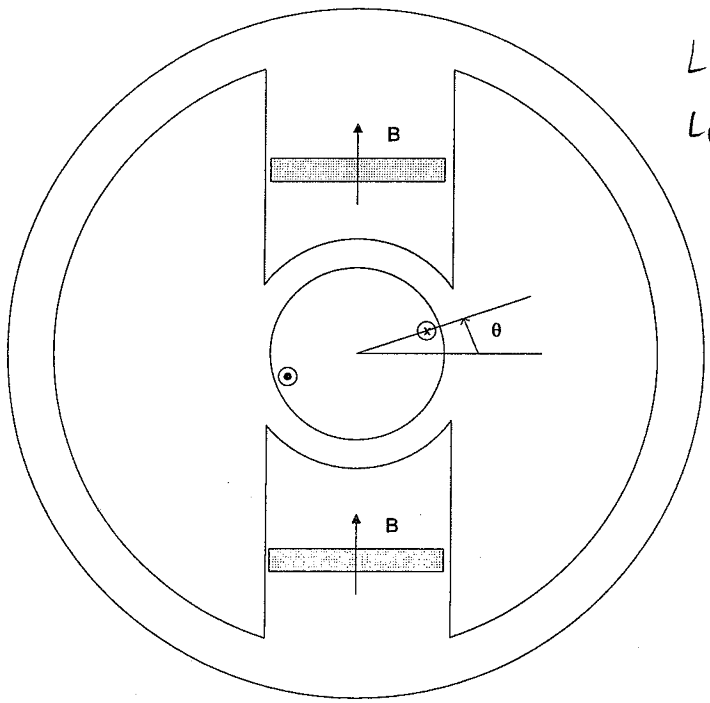
$$L_{m_1} = .0124\times\frac{100}{25} = .04976 \qquad L_{\ell_1} = L_1 - L_{m_1} = .00024\,H$$

$$.003125\left(\frac{100}{25}\right)^2 = .05 \quad \text{so} \quad L'_{\ell_2} = .00024\,H$$



Problem 3 (25 pts.)

Consider the machine below. The two gray boxes represent permanent magnets, sized so that the air gap flux density is 1.0 T (constant DC field) when the rotor current is zero. The air gap is 1 mm long, 3 cm wide (arc length), and extends 3 cm into the page. The rotor self-inductance is measured as having a maximum value of 0.3 H and a minimum value of 0.1 H. The rotor coil has 100 turns.



$L_0 + L_1 = 0.3$

$L_0 - L_1 = 0.1$

$\boxed{L_0 = .2 \quad L_1 = 0.1}$

$\boxed{K = 100 \times 1 \times 9 \times 10^{-4} = .09}$

a) Using the general form of the flux linkage given below, find the constants L_0, L_1 and K. This flux linkage λ is defined with respect to the voltage labeled as positive where the current enters the page (on the right in the figure) and $v = d\lambda/dt$.

$$\lambda = \left(L_0 + L_1 \cos(2\theta)\right)i + K\cos\theta$$

b) Find the co-energy, W_m', as a function of current i and angle θ.

c) Find the torque of electric origin, T^e.

d) Suppose the machine goes through the following four transitions along the given paths:

Transition	Initial Current	Final Current	Initial Angle	Final Angle	Path
A	3 A	3 A	0°	90°	Constant Current
B	3 A	0 A	90°	90°	Constant Position
C	0 A	0 A	90°	0°	Constant Current
D	0 A	3 A	0°	0°	Constant Position

Sketch the cycle in the λ-i plane and determine EFE_{cycle} and EFM_{cycle}.

$$\lambda = (.2 + .1\cos 2\theta)\, i + .09\cos\theta$$

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$$W_m' = \int \lambda\, di = \frac{1}{2}(.2 + .1\cos 2\theta)\, i^2 + .09\cos\theta\, i$$

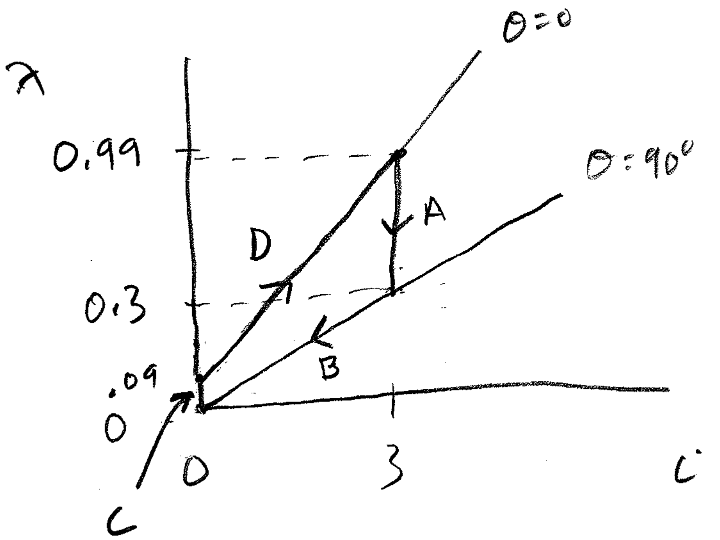
$$T^e = \frac{\partial W_m'}{\partial \theta} = -.1\sin 2\theta\, i^2 - .09\sin\theta\, i$$

$\boxed{\theta = 0 \quad \lambda = .3i + .09}$

$i = 3 \quad \lambda = 0.99$

$\boxed{\theta = 90^\circ \quad \lambda = .1i}$

$i = 3 \quad \lambda = 0.3$



$\theta = 90^\circ$
$i = 0 \quad \lambda = 0$

$\theta = 0$
$i = 3 \quad \lambda = .99$

$$\underset{cycle}{EFE} = \int_{.99}^{.3} 3\, d\lambda + \int_{.3}^{0} 10\lambda\, d\lambda + \int_{0}^{.09} 0\, d\lambda + \int_{.09}^{.99} \frac{1}{.3}(\lambda - .09)\, d\lambda$$

$$= -2.07 - 0.45 + 0 + \frac{1}{.3}\left(\frac{1}{2}.99^2 - \frac{1}{2}.09^2 - .09 \times .99 + .09 \times .09\right)$$

$$\boxed{= -1.17\ J}$$

$$\boxed{\underset{cycle}{EFM} = +1.17\ J}$$

Problem 4 (25 pts.)

A dynamic system is modeled as:

$$\dot{x}_1 = -3x_1 + 2x_2$$
$$\dot{x}_2 = x_1^2 - 2x_2 + 2$$

a) Find all equilibrium points.
b) Linearize the system at each equilibrium point.
c) Determine the eigenvalues at each equilibrium point. Determine which points are stable and which are unstable.
d) Starting near the stable equilibrium point at $\Delta x_1 = 0.01$, $\Delta x_2 = -0.01$, and using $\Delta t = 0.001$, find the values of Δx_1 and Δx_2 at the first three time steps after zero. Use the linearized state-space form and Euler's method.

a) $0 = -3x_1^e + 2x_2^e \rightarrow x_2^e = \frac{3}{2} x_1^e$

$0 = x_1^{e2} - 2x_2^e + 2$ $\quad x_1^{e2} - 3x_1^e + 2 = 0$ $\quad x_1^e = 1, 2$

$x_2^e = \frac{3}{2}, 3$

$A = \begin{bmatrix} -3 & 2 \\ 2x_1^e & -2 \end{bmatrix}$ b) $A_1 = \begin{bmatrix} -3 & 2 \\ 2 & -2 \end{bmatrix}$ $A_2 = \begin{bmatrix} -3 & 2 \\ 4 & -2 \end{bmatrix}$

c) $\begin{vmatrix} \lambda+3 & -2 \\ -2 & \lambda+2 \end{vmatrix} = \lambda^2 + 5\lambda + 2 = 0$ $\quad \lambda = -.44, -4.56$ stable

$\begin{vmatrix} \lambda+3 & -2 \\ -4 & \lambda+2 \end{vmatrix} = \lambda^2 + 5\lambda - 2 = 0$ $\quad \lambda = .37, -5.37$ unstable

d) $\dot{\Delta x}_1 = -3\Delta x_1 + 2\Delta x_2$

$\dot{\Delta x}_2 = 2\Delta x_1 - 2\Delta x_2$

$\Delta x_1(.001) = .01 + .001(-.03 - .02) = .00995$

$\Delta x_2(.001) = -.01 + .001(.02 + .02) = -.00996$

$\Delta x_1(.002) = .00995 + .001($

$\Delta x_2(.002) = -.00996 + .001($

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Problem 5 (25 pts.)

A three-phase, 60 Hz, 2 pole, wye connected synchronous generator has a rating of 600MVA and 23KV (line-line). When it delivers 400MW at unity power factor, the torque angle is 45 degrees. (neglect armature resistance)

a) What is the synchronous reactance of this machine?
b) What is the maximum reactive power this machine can deliver when it is delivering 500MW of real power at rated voltage?
c) What is the minimum reactive power this machine can deliver when it is delivering 500MW of real power at rated voltage?
d) Repeat b) and c) when it is delivering 400MW of real power at rated voltage.

a) $400\times10^6 = \dfrac{3\times\left(\frac{23k}{\sqrt3}\right)E_0}{X_S}\sin 45°$ $\dfrac{E_0}{X_S} = 14{,}201$

$0 = \dfrac{3\left(\frac{23k}{\sqrt3}\right)^2}{X_S} - \dfrac{3\,\frac{23k}{\sqrt3}E_0}{X_S}\cos 45°$ $E_0 = 18{,}782$ so $\boxed{X_S = 1.322\,\Omega}$

b) $(600\times10^6)^2 = (500\times10^6)^2 + Q_{max}^2$ $\boxed{Q_{max} = 332\text{ MVAR}}$

c) Either $Q_{min} = -332$ MVAR, or it is stability limited.

At the stability limit, $\delta = 90°$ so $Q_{MIN} = -\dfrac{3\left(\frac{23k}{\sqrt3}\right)^2}{1.322} = -400$ MVAR

Since $400 > 332$, the minimum is $\boxed{-332\text{ MVAR}}$

d) $(600\times10^6)^2 = (400\times10^6)^2 + Q_{max}^2$ $\boxed{Q_{max} = 447\text{ MVAR}}$

Since $400 < 447$, the minimum is $\boxed{-400\text{ MVAR}}$

Problem 6 (25 pts.)

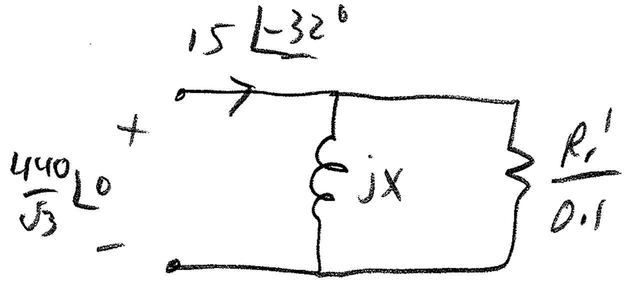
A 3-phase, 4-pole, 60 Hz, 440 V (line-line), wye-connected squirrel cage induction motor is running at 1620 RPM and draws 15 Amps at 0.85 power factor lagging. Neglect all equivalent circuit parameters except the magnetizing reactance, rotor resistance, and slip. Find the following things:

a) The frequency of the rotor currents
b) The efficiency of the machine
c) The magnetizing reactance and the rotor resistance as seen by the stator
d) The torque to the shaft
e) The load on the machine in HP

$$s = \frac{1800-1620}{1800} = 0.1 \qquad a)\ f_r = 6\,Hz$$

b) $\eta = (1-s)\times 100 = 90\%$

c), d)



$$15\angle{-32^\circ} = \frac{\frac{440}{\sqrt{3}}\angle 0}{jX} + \frac{\frac{440}{\sqrt{3}}\angle 0}{R_r'/0.1} = 12.75 - j7.9$$

$$\boxed{X = 32\,\Omega \quad R_r' = 2\,\Omega}$$

e) $$T^e = \frac{P_m}{\omega_m} = \frac{P_{AG}}{2\pi 60\frac{2}{4}} = \frac{3\left(\frac{440}{\sqrt{3}}\right)^2/20}{377/2} = \boxed{51\ Nm}$$

f) $$P_m = (1-.1)P_{AG} = .9\times 3\times\left(\frac{440}{\sqrt{3}}\right)^2/20 = 8713\ W$$

$$= \boxed{11.7\ HP}$$