Problem 1

A 3-phase synchronous generator supplies two loads connected in parallel through a lossless transmission line. The line-to-line voltage V_L across the load is 4.16 kV. The synchronous generator stator resistance is $R_s = 0\ \Omega$ and stator reactance is $X_s = 1\ \Omega$.

- Load 1 is wye-connected with a per phase impedance of $60\angle 25°\ \Omega$.
- Load 2 is delta-connected with a per phase impedance of $90\angle 10°\ \Omega$.

Find the per phase internal voltage E_{ar} and the torque angle $\angle\delta$ for the synchronous generator.

Solution Steps

(a) Draw the per phase equivalent circuit including the equivalent circuit for the synchronous generator and the per phase equivalent load. The circuit is shown in the figure below. Note, that $\overline{Z}_{eq,Y} = \overline{Z}_{1,Y} || \overline{Z}_{2,Y}$ where $\overline{Z}_{2,Y} = \frac{1}{3}\overline{Z}_{2,\Delta}$.

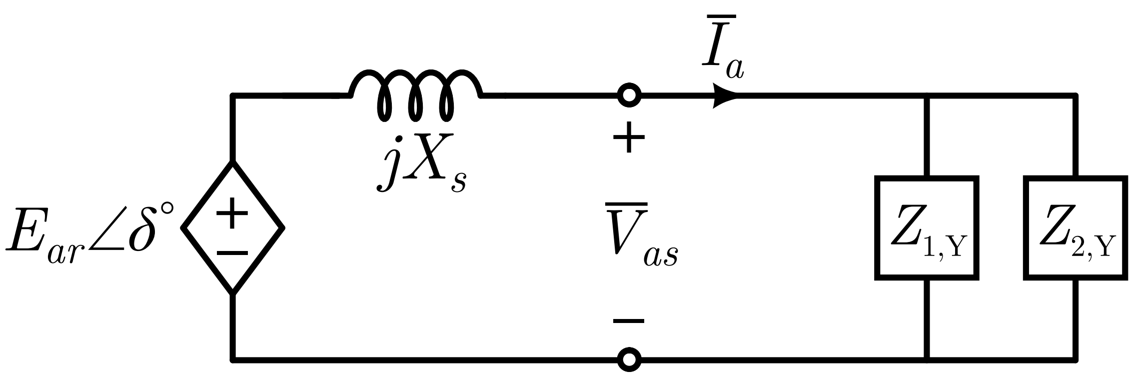

Figure 1: Power system per-phase equivalent circuit.

(b) Now compute the current in the transmission line, $\overline{I}_a = \frac{\overline{V}_{as}}{\overline{Z}_{eq,Y}}$ where $V_{as} = \frac{V_L}{\sqrt{3}}$.

(c) Since the transmission line has no impedance, the per phase voltage at the synchronous generator terminals is equal to the per phase voltage at the load. Now, using $\overline{I}_a$ and $\overline{V}_{as}$, compute the synchronous generator internal voltage and angle given by $E_{ar}\angle\delta = \overline{I}_a \cdot jX_s + \overline{V}_{as}$.

Problem 2

A three-phase, four-pole, 533 V line-to-line, 60 Hz, wye-connected induction motor is operating at rated voltage and at a speed of 1510 RPM. The per-phase equivalent circuit is shown in the figure below.

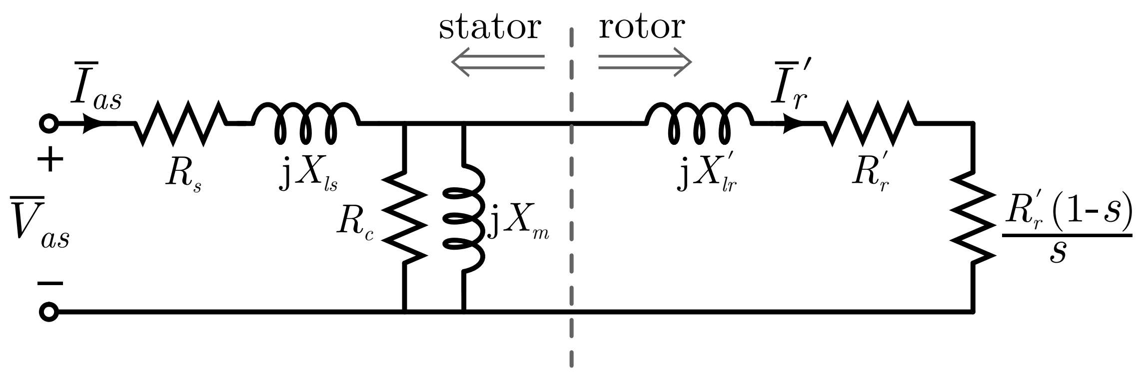

Figure 2: Full induction motor per-phase equivalent circuit with rotor referred to the stator side winding.

Use the following values for the stator leakage reactance $X_{ls} = 0.5\ \Omega$, the magnetizing reactance $X_m = 50\ \Omega$, the rotor leakage reactance (referred to the stator side) $X_{lr}' = 0.6\ \Omega$, and the rotor winding resistance (referred to the stator side) $R_r' = 1.0\ \Omega$. Assume stator winding and core losses are negligible (i.e., $R_s = 0$ and $R_c = \infty$).

Note: All values of power below are three-phase.

(a) Find the slip s of the motor.

(b) Find the power across the air gap P_{ag} of the motor.

(c) Find the mechanical power developed by the motor shaft P_m.

(d) Find the power delivered to the motor shaft P_{shaft} assuming rotational losses of $P_{rot} =$ 500 W.

(e) Find the overall efficiency of motor.

Solution Steps

(a) The value of slip s relates the synchronous speed ω_s and the mechanical speed ω_m of the motor. Identify the mechanical speed in the problem statement (1510 RPM) and convert to units of radians per second making sure to include the pole count of the motor. Then use the frequency relationship of $\frac{p}{2}\omega_m = (1-s)\omega_s$ to solve for s. Values of s range from zero to one with typical values being very close to zero.

(b) The quantity P_{ag} delineates the real power transferred across the air gap from the stator to the rotor. Thus, determine the total real power delivered to the rotor side of the gap to determine P_{ag}.

First, use fundamental circuit analysis to find $\overline{I}_r'$. Then find the total (three-phase) power through the two resistors R_r' and $\frac{R_r'(1-s)}{s}$ which are also sometimes expressed as one resistor $\frac{R_r'}{s}$. The result is P_{ag}.

(c) The mechanical power delivered to the motor P_m is the power across the air gap delivered to the rotor P_{ag} minus the power loss through the rotor windings ($P_{r,loss} = 3|\overline{I}_r'|^2 R_r'$). This is often simplified as $P_m = P_{ag}(1-s)$.

(d) The power delivered to the motor shaft P_{shaft} is the mechanical power delivered to the motor P_m minus the frictional losses of the rotating shaft P_{rot}.

(e) Efficiency η is the ratio of mechanical output power from and electrical input power to the motor—i.e., $\eta = \frac{P_{shaft}}{P_{in}}$. In this case, since the core loss and stator winding losses are zero, the power delivered across the air gap and the input power to the stator are equivalent (i.e., $P_{in} = P_{ag}$). Answer $\eta = 82.74\%$.

Problem 3

A translational electromechanical dynamical system is shown in Figure 3.

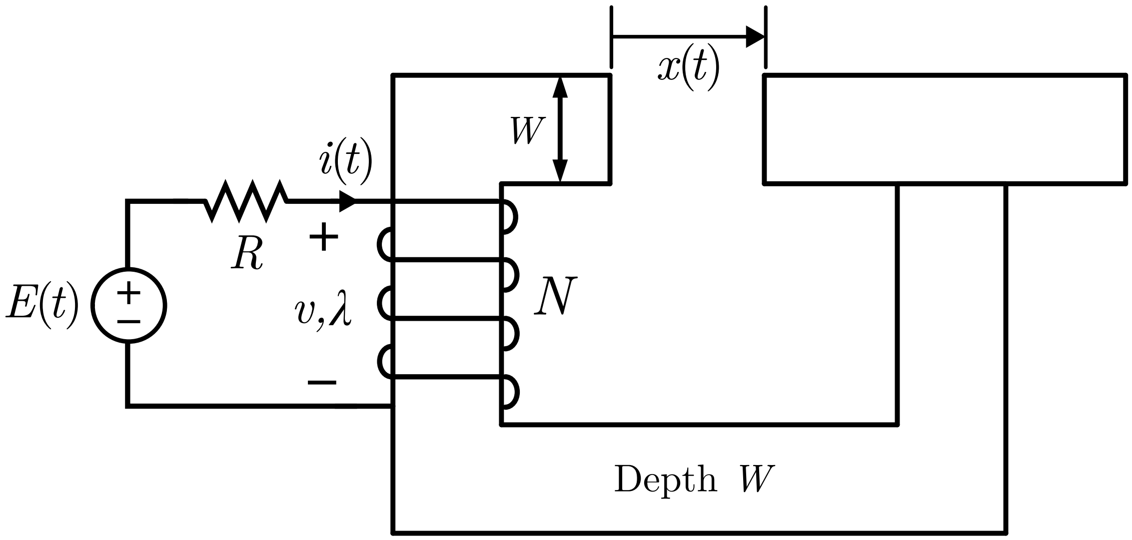

Figure 3: Electromechanical system.

It consists of a fixed and a movable piece. Both pieces are made from infinitely permeable material and have depth W into the paper. The air gap has permeability μ_0 and length $x(t)$. A voltage source E in series with a resistor R induces a current $i(t)$ that drives the terminals of an N-turn coil wound on the fixed piece producing a flux linkage $\lambda(t)$ in the coil.

(a) Compute the flux linkage (λ) in the coil.

(b) Derive the force of electric origin $f^e(i, x)$ acting on the movable piece.

(c) Write the mechanical and electrical equation for the system as a function of the inputs i, x and E and the parameters N, W and R.

(d) Convert the dynamical description of the differential equations derived in part (c) into state space form. Use $X = [x(t), \dot{x}(t), i(t)]$ as your state vector.

(e) Given the initial state space vector, $X(0) = [0, 0, 1]$, and time step of 0.1 second, find using Euler's method, $X(t)$, at time $t = 0.1$ second and $t = 0.2$ second. Take $N = 100$, $R = 5\Omega$, $W = 1cm$.

Solution Steps

(a) To compute the flux linkage, compute the reluctance $\mathcal{R}$. Use that to compute the magnetic flux ϕ and then flux linkage $\lambda = N \cdot \phi$.

(b) To compute the force of electric origin, first compute the coenergy $W'_m(i, x) = \int_0^i \lambda(\hat{i}, x) d\hat{i}$. Then differentiate coenergy with respect to x to find $f^e(i, x)$.

(c) The mechanical and the electrical equations are given by:

$$m\ddot{x}(t) = f^e(i(t), x(t))$$
$$E = i(t)R + \frac{d\lambda}{dt}$$

Note that, λ is a function of both $i(t)$ and $x(t)$, so use partial derivatives to compute $\frac{d\lambda}{dt}$

(d) The key idea is to bring the second order mechanical differential equation down to two first order differential equations and combine it with the first order electrical differential equation to express the system in the state space form. Use $x_1 = x(t)$, $x_2 = \dot{x}(t)$ and $x_3 = i(t)$ define the state vector.

(e) For the state space form, $\dot{X} = F(X)$, *Euler*'s method gives the state vector, X, at time step, k, as:
$X(t_k) \approx X(t_{k-1}) + (t_k - t_{k-1}) \cdot F\big(X(t_{k-1})\big)$