# ECE 330 POWER CIRCUITS AND ELECTROMECHANICS 

## LECTURE 1

## COURSE INTRODUCTION AND REVIEW OF PHASORS

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations.

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.

## WHY POWER CIRCUITS AND ELECTROMECHANICS?

Electromechanics combines electrical and mechanical processes and plays key roles at several levels:

- At generation level: synchronous machines convert mechanical energy to electric energy.


Source: slideshare.net

Generator


Source: water.usgs.gov

## WHY POWER CIRCUITS AND ELECTROMECHANICS?

- At transmission-distribution level: Circuit breakers trip the faults in affected areas.


Source: abb.com


Source: engineeringtutorial.com

## WHY POWER CIRCUITS AND ELECTROMECHANICS?

- At the level of loads: Electric energy is consumed by mechanical loads through motors, mostly induction (threephase and single-phase)


Source: pumpsandsystem.com


Source: electrical-know-how.com

## WHY POWER CIRCUITS AND ELECTROMECHANICS ?

- In advanced technology: renewable energy, electric cars, robots,..... etc. contain electromechanic systems.


Wind Turbine
Source: energy.gov


Electric Vehicle
Source: exchangeev.aaa.co m


Robot
Source: yaskawa.co.jp

## POWER CIRCUITS AND ELECTROMECHANICS

## Goals:

To impart basics of three-phase power circuits, transformers, electromechanical systems, and rotating machines.

## Objectives:

To build cognitive skills, such as analytical thinking and problem solving. The course integrates fundamentals of advanced math and science to enhance the ability to design a system and meet desired needs.

## COURSE OUTLINE

- Review of phasors, complex power
- Three-phase circuits, three-phase power, wye-delta conversion
- Magnetic circuits, self and mutual inductance, Maxwell's equations
- Ideal transformers, practical transformers, equivalent circuits
- Electromechanical systems, energy, co-energy, energy cycles, computation of forces and torques.
- Dynamic equations, numerical integration of electromechanical systems
- Equilibrium points, linearization, stability
- Synchronous machines
- Induction machines
- Other machines


## STRUCTURE OF POWER SYSTEMS

- Power system's main components: generation, transmission, distribution, and loads.


## POWER SYSTEM



Source: image.slidesharecdn.com

## STRUCTURE OF POWER SYSTEMS

## Generating System:

- The generating system consists of a fuel source such as coal, water, natural gas or nuclear power.
- Hydropower accounts for about $8 \%$, and nuclear power $20 \%$, renewable energy $14 \%$ of electric energy production in the US.
- The turbine acting as the prime-mover converts mechanical energy into electric energy through a threephase synchronous machine.
- The three-phase voltages of the generators range from 13.8 kV to 24 kV .


## STRUCTURE OF POWER SYSTEMS

## Transmission System:

- A three-phase transformer at each generator steps up the voltage to a high value ranging from 138 kV to 765 kV .
- The transmission lines carry power over these lines to load centers and, where appropriate, step it down to lower voltages up to 34 kV at the bulk-power substations.
- Some industrial customers are supplied from these substations. This is known as the transmission-subtransmission system.


## STRUCTURE OF POWER SYSTEMS

## Distribution System:

- Transformers step down the voltage to a range of 2.4 kV to 69 kV .
- Power is carried by main feeders to specific areas where there are lateral feeders to step it down further to customer levels, such as 208,240 , or 600 volts.
- The distribution transformers serve anywhere from 1 to 10 customers.


## STRUCTURE OF POWER SYSTEMS

The classes of entities in the electricity market are:

- Generator companies (GENCOs), also called independent power producers (IPPs).
- Transmission companies TRANSCOs,

Their primary responsibility is to transport power from generators to customers and make the transmission system available to all.

## STRUCTURE OF POWER SYSTEMS

- Distribution companies DISCOs, owning and operating the local distribution network.
- Independent system operator (ISO)

The ISO is charged with ensuring the reliability and security of the entire system.

## DYNAMICS OF POWER SYSTEMS AND COMPONENTS

- The power system is a dynamic one, it is described by a set of vector of differential equations $\dot{x}=f(x+u)$
- The time scales in the response of this equation range from milliseconds, in the case of electromagnetic transients, to a few seconds in the control of frequency, and a few hours in the case of boiler dynamics.
- Therefore, we analyze such equations for time-domain response, steady-state sinusoidal behavior, equilibrium points, stability, etc.


## DYNAMICS OF POWER SYSTEMS AND COMPONENTS



Power system dynamic structure

## DYNAMICS OF POWER SYSTEMS AND COMPONENTS

- Diagram does not show all the complex dynamic interaction between components and their controls.
- Electrical side contains mechanical dynamics(TCUL)
- Mechanical side contains components with electrical dynamics (electrical valves)
- After the model is derived we put it in the state-space form.


## REVIEW OF PHASORS

- Phasors represent quantities with magnitude and angle with respect to a reference and are commonly used in energy systems.
- Example: $v(t)=V_{m} \cos \left(\omega t+\theta_{v}\right)$

Euler's expansion: $e^{j \alpha}=\cos (\alpha)+j \sin (\alpha)$
Then, $v(t)$ can be written as
$v(t)=\operatorname{Re}\left\{V_{m} e^{j\left(\omega t+\theta_{v}\right)}\right\}=\operatorname{Re}\left\{V_{m} e^{j \omega t} e^{j \theta_{v}}\right\}=\operatorname{Re}\left\{\sqrt{2} \bar{V} e^{j \omega t}\right\}$
where $\bar{V}=\frac{V_{m}}{\sqrt{2}} e^{j \theta_{v}}=V_{m s} \angle \theta_{v}$ is the RMS phasor with cosine reference.

## REVIEW OF PHASORS

- Recall: RMS (root-mean square) where v is a function of time, $\mathrm{t}_{0}$ is the initial time, and $T$ is the period of $v$.

$$
V_{m s s}=\sqrt{\frac{1}{T} \int_{t_{0}}^{t_{o}+T} v^{2}(t) d t}=\sqrt{\frac{\frac{1}{t}^{t_{+}+T}}{\int_{t_{0}}} V_{m}^{2} \cos ^{2}\left(\omega t+\theta_{v}\right) d t}=\frac{V_{m}}{\sqrt{2}}
$$

- How do phasors apply in electric circuits?
- Example:

Find $v(t)$ for $i(t)=I_{m} \sin (\omega t) \mathrm{A}$


## REVIEW OF PHASORS

- Time-domain approach:

$$
\begin{aligned}
& v(t)=R i(t)+L \frac{d i(t)}{d t} \\
& v(t)=R I_{m} \sin (\omega t)+L \omega I_{m} \cos (\omega t)
\end{aligned}
$$



$$
v(t)=\sqrt{\left(R I_{m}\right)^{2}+\left(L \omega I_{m}\right)^{2}} \cos \left(\omega t-\tan ^{-1}\left(\frac{R I_{m}}{L \omega I_{m}}\right)\right)
$$

For $R=2 \Omega, L=1 / 377 \mathrm{H}, \omega=377 \mathrm{rad} / \mathrm{s}$, and $I_{m}=10 \mathrm{~A}$,

$$
v(t)=\sqrt{(2 \times 10)^{2}+\left(\frac{1}{377} \times 377 \times 10\right)^{2}} \cos \left(377 t-\tan ^{-1}\left(\frac{2 \times 10}{\frac{1}{377} \times 377 \times 10}\right)\right)
$$

$v(t)=22.36 \cos \left(377 t-63.4^{\circ}\right) V$

## REVIEW OF PHASORS

- Frequency-domain approach (phasors)

Time-domain $\rightarrow$ Frequency-domain

$$
\begin{aligned}
& v(t) \rightarrow \bar{V} \\
& i(t) \rightarrow \bar{I}
\end{aligned}
$$

$$
R \rightarrow R
$$

$$
L \rightarrow j \omega L=j X_{L} \quad\left(X_{L}=\omega L\right)
$$

$$
C \rightarrow \frac{1}{j \omega C}=\frac{-j}{\omega C}=j X_{C} \quad\left(X_{C}=\frac{-1}{\omega C}\right)
$$

- In frequency-domain $\bar{V}=Z \bar{I}$ (Ohm's law)where $Z$ can be a series-parallel combination of $R, X_{C}$, and/or $X_{L}$.


## REVIEW OF PHASORS

- Frequency-domain approach:

$$
\begin{aligned}
& \bar{V}=(R+j \omega L) \bar{I} \text { where } \bar{I}=\frac{10}{\sqrt{2}} \angle-90^{\circ} \mathrm{A}(\mathrm{RMS}) \\
& \bar{V}=\left(2+j 377 \frac{1}{377}\right) \frac{10}{\sqrt{2}} \angle-90^{\circ} \\
& \bar{V}=\left[\sqrt{2^{2}+1^{2}} \angle\left(\tan ^{-1}\left(\frac{1}{2}\right)\right)\right] \frac{10}{\sqrt{2}} \angle-90^{\circ} \\
& \bar{V}=\left(\frac{10 \sqrt{5}}{\sqrt{2}}\right) \angle\left(26.56^{\circ}-90^{\circ}\right) \\
& \bar{V}=15.81 \angle\left(-63.4^{\circ}\right) \mathrm{V}(\mathrm{RMS})
\end{aligned}
$$

## TWO-TERMINAL NETWORK

- A two-terminal electrical network has voltage at its terminals and current flowing in and out of its terminals.

- The instantaneous power is $p(t)=v(t) i(t)$.
- For $i(t)=I_{m} \cos \left(\omega t+\theta_{i}\right) \mathrm{A}$ and $v(t)=V_{m} \cos \left(\omega t+\theta_{v}\right) \mathrm{V}$ we get

$$
p(t)=V_{m} I_{m} \cos \left(\omega t+\theta_{v}\right) \cos \left(\omega t+\theta_{i}\right)
$$

## TWO-TERMINAL NETWORK

$\cos \alpha \cos \beta=\frac{1}{2}[\cos (\alpha-\beta)+\cos (\alpha+\beta)]$

$$
p(t)=\frac{V_{m} I_{m}}{2} \cos \left(\theta_{v}-\theta_{i}\right)+\frac{V_{m} I_{m}}{2} \cos \left(2 \omega t+\theta_{v}+\theta_{i}\right) \quad \mathrm{W}
$$

The first term is time-independent, while the second term is a sinusoid at double frequency.

## TWO-TERMINAL NETWORK

- The average power is thus

$$
\begin{gathered}
P=\frac{1}{T} \int_{0}^{T} P(t) d(t) \quad, \quad T=\frac{2 \pi}{\omega} \\
P_{i n}=\frac{V_{m} I_{m}}{2} \cos \left(\theta_{v}-\theta_{i}\right)
\end{gathered}
$$

- This is called the active or real power and its unit is watts (W).
- The power factor is the cosine of the phase angle between $v(t)$ and $\mathrm{i}(\mathrm{t})$.


## POWER FACTOR

- The power factor (P.F.) is thus P.F. $=\cos \left(\theta_{v}-\theta_{i}\right)$.
- The power factor can be:
- Lagging: $0^{\circ}<\theta_{v}-\theta_{i}<90^{\circ}$
- Leading: $-90^{\circ}<\theta_{v}-\theta_{i}<0^{\circ}$
- Unity: $\quad \theta_{v}-\theta_{i}=0$

Therefore, $0 \leq P . F . \leq 1$,
and the highest real power exists when P.F.=1.


Source: grupovision.com

## APPARENT POWER AND REACTIVE POWER

- The apparent power is $S=\frac{V_{m} I_{m}}{2}$
- The apparent power unit is volt-amps (VA).
- The reactive power is $Q_{i n}=\frac{V_{m} I_{m}}{2} \sin \left(\theta_{v}-\theta_{i}\right)$.
- The reactive power unit is volt-amps-reactive
(VARs).


## COMPLEX POWER

- The instantaneous power is

$$
p(t)=\frac{V_{m} I_{m}}{2} \cos \left(\theta_{v}-\theta_{i}\right)+\frac{V_{m} I_{m}}{2} \cos \left(2 \omega t+\theta_{v}+\theta_{i}\right) \quad \mathrm{W}
$$

- The time varying component

$$
\begin{aligned}
& \frac{V_{m} I_{m}}{2} \cos \left(2 \omega t+\theta_{v}+\theta_{i}\right)=\frac{V_{m} I_{m}}{2}\left\{\cos \left[\left(2 \omega t+2 \theta_{i}\right)+\left(\theta_{v}-\theta_{i}\right)\right]\right\} \\
& =\frac{V_{m} I_{m}}{2} \cos \left(2 \omega t+2 \theta_{i}\right) \cos \left(\theta_{v}-\theta_{i}\right)-\frac{V_{m} I_{m}}{2} \sin \left(2 \omega t+2 \theta_{i}\right) \sin \left(\theta_{v}-\theta_{i}\right)
\end{aligned}
$$

## COMPLEX POWER

- Define

$$
\begin{aligned}
& Q_{i n}=\frac{V_{m} I_{m}}{2} \sin \left(\theta_{v}-\theta_{i}\right), \quad \quad \text { (Reactive power) } \\
& p(t)=P_{i n}+P_{i n} \cos \left(2 \omega t+2 \theta_{i}\right)-Q_{i n} \sin \left(2 \omega t+\theta_{i}\right) \\
& =P_{i n}\left(1+\cos \left(2 \omega t+\theta_{i}\right)\right)-Q_{i n} \sin \left(2 \omega t+2 \theta_{i}\right)
\end{aligned}
$$

- The real power can be written as

$$
P_{i n}=\frac{V_{m} I_{m}}{2} \cos \left(\theta_{v}-\theta_{i}\right)=\mathrm{V}_{r m s} I_{r m s} \cos \left(\theta_{v}-\theta_{i}\right)
$$

## PHASOR REPRESENTATION

$$
P_{i n}=\operatorname{Re}\left\{\frac{V_{m} I_{m}}{2} e^{j \theta_{v}} e^{-j \theta_{i}}\right\}=\operatorname{Re}\left\{V_{r m s} e^{j \theta_{\theta_{1}}} I_{r m s} e^{-j \theta_{i}}\right\}
$$

- The reactive power can be written as

$$
Q_{i n}=\operatorname{Im}\left\{\frac{V_{m} I_{m}}{2} e^{j \theta_{v}} e^{-j \theta_{i}}\right\}=\operatorname{Im}\left\{V_{m s s} e^{j \theta_{v}} I_{m m s} e^{-j \theta_{i}}\right\}
$$

- The voltages and currents can be written as phasors:

$$
V_{r m s}{ }^{j \theta_{v}}=\bar{V} \text { and } I_{r m s} e^{j \theta_{i}}=\bar{I} .
$$

$$
P_{i n}=\operatorname{Re}\left(\bar{V} \bar{I}^{*}\right)=\mathrm{V}_{m s} I_{m s} \cos \left(\theta_{v}-\theta_{i}\right)
$$

$$
Q_{i n}=\operatorname{Im}\left(\bar{V} \bar{I}^{*}\right)=\mathrm{V}_{m s} I_{m s} \sin \left(\theta_{v}-\theta_{i}\right)
$$



Real Power

## Complex Power

- Define the complex power as $\bar{S}=P_{i n}+j Q_{i n}$
- Then $\bar{S}$ can be written as $\bar{S}=\bar{V} \bar{I}^{*}$
- The quantity $\bar{I}^{*}$ is the complex conjugate of $\bar{I}$.
- $\bar{S}$ can also be written as

$$
\bar{S}=S \angle\left(\theta_{v}-\theta_{i}\right)
$$

- Note that

$$
S=\frac{V_{m} I_{m}}{2}=\sqrt{P_{i n}^{2}+Q_{i n}^{2}}
$$



## ALTERNATE FORMS OF COMPLEX POWER

- If the load is $\bar{Z}=R+j X$, connected across the source $\bar{V}$ By Ohm's law: $\bar{V}=\bar{Z} \bar{I}$, but $\bar{S}=\bar{V} \bar{I}^{*}$

Then $\bar{S}$ can be written as $\bar{S}=I^{2} R+j I^{2} X$ Also, $P=I^{2} R$ and $Q=I^{2} X$, Z̄and P.F. $=\cos (\operatorname{angle}(\bar{Z}))$.

- Thus, $Q>0$ when $\bar{Z}$ is inductive, $X=\omega L$ and $Q<0$ when $\quad$ is capacitive, $X=-\frac{1}{\omega C}$
- $\bar{S}$ and $\bar{Z}$ are not phasors but complex quantities.


## EXAMPLE: LC FILTER AND R LOAD

- The circuit shown is commonly used as an LC filter to supply a load, which is resistive in this case.
- Find the current, real, reactive, and complex powers, and the P.F. for $v(t)=\sqrt{2} V_{m m s} \cos (377 t)$

$$
\begin{aligned}
& \bar{Z}=j \omega L+\left(R / / \frac{-j}{\omega C}\right) \\
& \bar{Z}=\frac{\omega L+j\left(\omega^{2} R L C-R\right)}{\omega R C-j}
\end{aligned}
$$



## EXAMPLE: LC FILTER AND R LOAD

- Let

$$
V_{r m s}=120 \mathrm{~V}, L=1 \mathrm{mH}, C=6.8 \mathrm{mF}, \text { and } \mathrm{R}=10 \Omega .
$$

$$
\begin{aligned}
& \bar{Z}=0.0197 \angle-39.41^{\circ}=0.0152-j 0.0125 \Omega \\
& \bar{I}=\frac{\bar{V}}{\bar{Z}}=\frac{120 \angle 0^{\circ}}{0.0197 \angle-39.41^{\circ}}=6091.4 \angle 39.41^{\circ} \mathrm{A}
\end{aligned}
$$

$$
i(t)=6091.4 \sqrt{2} \cos \left(377 t+39.41^{\circ}\right)
$$

$$
\bar{S}=\bar{V} \bar{I}^{*}=731 \angle-39.41^{\circ} \mathrm{kVA}
$$

$$
P_{i n}=731 \cos \left(-39.41^{\circ}\right)=564.8 \mathrm{~kW}
$$


$Q_{i n}=731 \sin \left(-39.41^{\circ}\right)=-464.1 \mathrm{kVAR}$

$$
\text { P.F. }=\cos \left(-39.41^{\circ}\right)=0.773 \text { leading }\left(\theta_{v}-\theta_{i}=-39.41^{\circ}\right)
$$

