# ECE 330 POWER CIRCUITS AND ELECTROMECHANICS

#### LECTURE 1

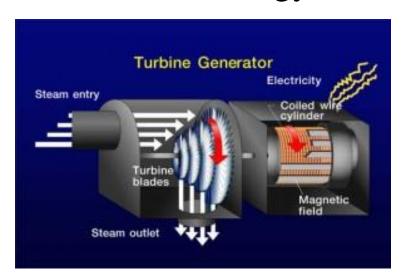
## COURSE INTRODUCTION AND REVIEW OF PHASORS

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations.

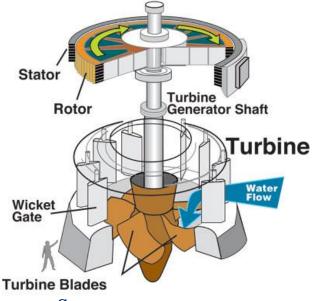
Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.

Electromechanics combines electrical and mechanical processes and plays key roles at several levels:

 At generation level: synchronous machines convert mechanical energy to electric energy.



Source: slideshare.net



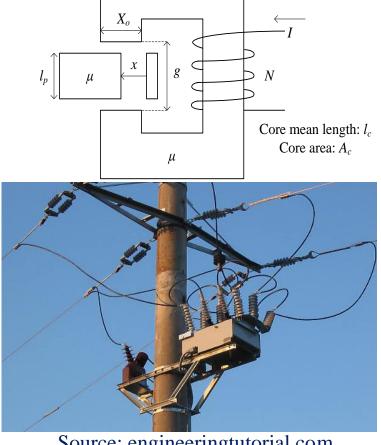
Source: water.usgs.gov

At transmission-distribution level: Circuit breakers trip the

faults in affected areas.

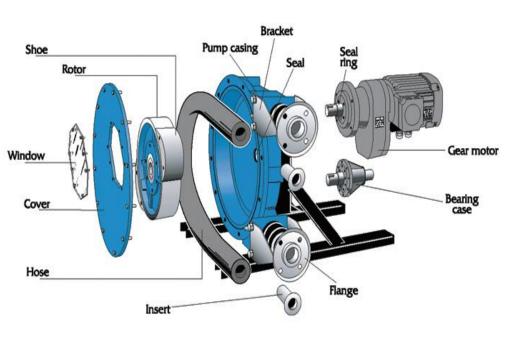


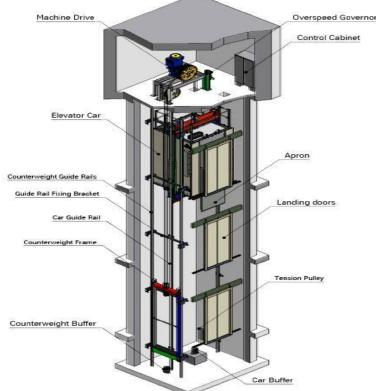
Source: abb.com



Source: engineeringtutorial.com

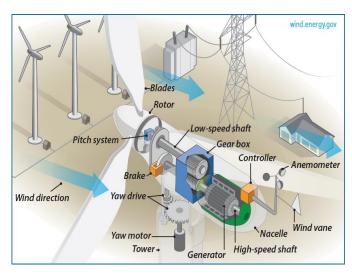
At the level of loads: Electric energy is consumed by mechanical loads through motors, mostly induction (three-phase and single-phase)

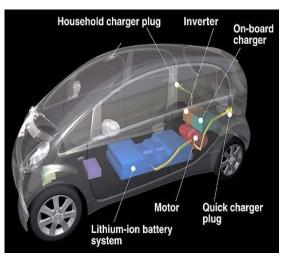


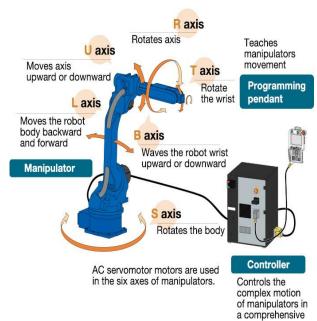


Source: electrical-know-how.com

 In advanced technology: renewable energy, electric cars, robots,..... etc. contain electromechanic systems.







Wind Turbine

Source: energy.gov

Electric Vehicle

Source: exchangeev.aaa.co m

Robot

Source: yaskawa.co.jp

#### Goals:

To impart basics of three-phase power circuits, transformers, electromechanical systems, and rotating machines.

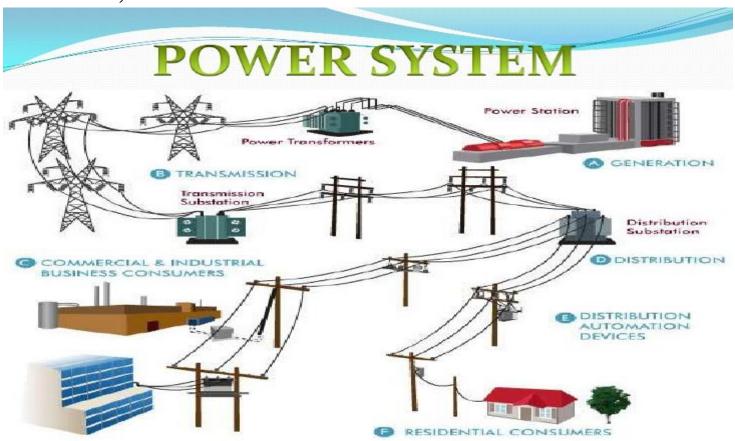
## **Objectives:**

To build cognitive skills, such as analytical thinking and problem solving. The course integrates fundamentals of advanced math and science to enhance the ability to design a system and meet desired needs.

#### **COURSE OUTLINE**

- Review of phasors, complex power
- Three-phase circuits, three-phase power, wye-delta conversion
- Magnetic circuits, self and mutual inductance, Maxwell's equations
- Ideal transformers, practical transformers, equivalent circuits
- Electromechanical systems, energy, co-energy, energy cycles, computation of forces and torques.
- Dynamic equations, numerical integration of electromechanical systems
- Equilibrium points, linearization, stability
- Synchronous machines
- Induction machines
- Other machines

• Power system's main components: generation, transmission, distribution, and loads.



Source: image.slidesharecdn.com

## Generating System:

- The generating system consists of a fuel source such as coal, water, natural gas or nuclear power.
- Hydropower accounts for about 8%, and nuclear power 20%, renewable energy 14% of electric energy production in the US.
- The turbine acting as the prime-mover converts mechanical energy into electric energy through a three-phase synchronous machine.
- The three-phase voltages of the generators range from 13.8 kV to 24 kV.

## Transmission System:

- A three-phase transformer at each generator steps up the voltage to a high value ranging from 138 kV to 765 kV.
- The transmission lines carry power over these lines to load centers and, where appropriate, step it down to lower voltages up to 34 kV at the bulk-power substations.
- Some industrial customers are supplied from these substations. This is known as the transmission—subtransmission system.

## Distribution System:

- Transformers step down the voltage to a range of 2.4 kV to 69 kV.
- Power is carried by main feeders to specific areas where there are lateral feeders to step it down further to customer levels, such as 208, 240, or 600 volts.
- The distribution transformers serve anywhere from 1 to 10 customers.

The classes of entities in the electricity market are:

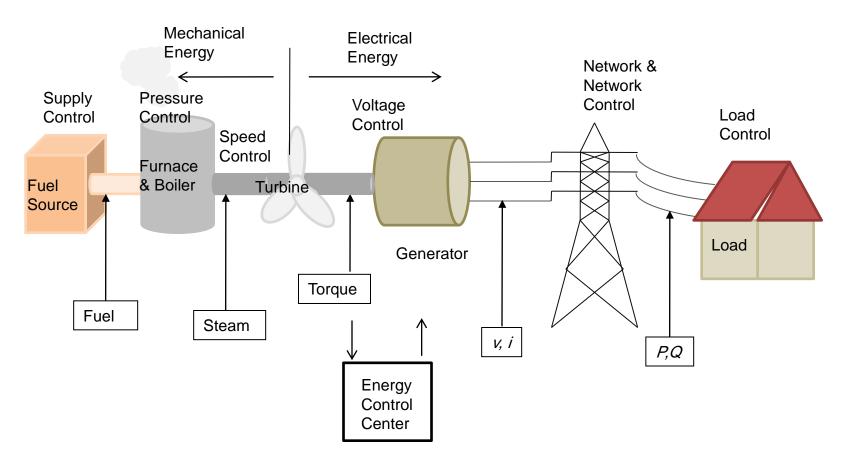
- Generator companies (GENCOs), also called independent power producers (IPPs).
- Transmission companies TRANSCOs,
   Their primary responsibility is to transport power from generators to customers and make the transmission system available to all.

- Distribution companies DISCOs,
   owning and operating the local distribution network.
- Independent system operator (ISO)
   The ISO is charged with ensuring the reliability and security of the entire system.

#### DYNAMICS OF POWER SYSTEMS AND COMPONENTS

- The power system is a dynamic one, it is described by a set of vector of differential equations  $\dot{x} = f(x + u)$
- The time scales in the response of this equation range from milliseconds, in the case of electromagnetic transients, to a few seconds in the control of frequency, and a few hours in the case of boiler dynamics.
- Therefore, we analyze such equations for time-domain response, steady-state sinusoidal behavior, equilibrium points, stability, etc.

#### DYNAMICS OF POWER SYSTEMS AND COMPONENTS



Power system dynamic structure

#### DYNAMICS OF POWER SYSTEMS AND COMPONENTS

- Diagram does not show all the complex dynamic interaction between components and their controls.
- Electrical side contains mechanical dynamics(TCUL)
- Mechanical side contains components with electrical dynamics (electrical valves)
- After the model is derived we put it in the state-space form.

- Phasors represent quantities with magnitude and angle with respect to a reference and are commonly used in energy systems.
- Example:  $v(t) = V_m \cos(\omega t + \theta_v)$

Euler's expansion:  $e^{j\alpha} = \cos(\alpha) + j\sin(\alpha)$ 

Then, v(t) can be written as

$$v(t) = \text{Re}\{V_m e^{j(\omega t + \theta_v)}\} = \text{Re}\{V_m e^{j\omega t} e^{j\theta_v}\} = \text{Re}\{\sqrt{2}V e^{j\omega t}\}$$
where  $V = \frac{V_m}{\sqrt{2}}e^{j\theta_v} = V_{ms} \angle \theta_v$  is the RMS phasor with

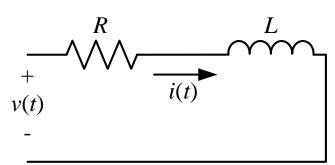
cosine reference.

• Recall: RMS (root-mean square) where v is a function of time, t<sub>0</sub> is the initial time, and *T* is the period of *v*.

$$V_{ms} = \sqrt{\frac{1}{T}} \int_{t_o}^{t_o + T} v^2(t) dt = \sqrt{\frac{1}{T}} \int_{t_o}^{t_o + T} V_m^2 \cos^2(\omega t + \theta_v) dt = \frac{V_m}{\sqrt{2}}$$

- How do phasors apply in electric circuits?
- Example:

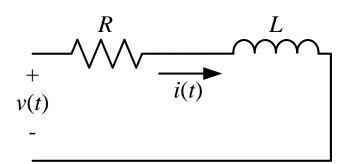
Find 
$$v(t)$$
 for  $i(t) = I_m \sin(\omega t)$  A



## • Time-domain approach:

$$v(t) = Ri(t) + L \frac{di(t)}{dt}$$

$$v(t) = RI_{m} \sin(\omega t) + L\omega I_{m} \cos(\omega t)$$



$$v(t) = \sqrt{(RI_m)^2 + (L\omega I_m)^2} \cos\left(\omega t - \tan^{-1}\left(\frac{RI_m}{L\omega I_m}\right)\right)$$

For  $R = 2\Omega$ , L = 1/377H,  $\omega = 377$  rad/s, and  $I_m = 10$ A,

$$v(t) = \sqrt{(2 \times 10)^2 + \left(\frac{1}{377} \times 377 \times 10\right)^2} \cos \left(377t - \tan^{-1}\left(\frac{2 \times 10}{\frac{1}{377} \times 377 \times 10}\right)\right)$$

$$v(t) = 22.36\cos(377t - 63.4^{\circ}) V$$

• Frequency-domain approach (phasors)

Time-domain → Frequency-domain

$$v(t) \to V$$

$$i(t) \to \overline{I}$$

$$L \to j \omega L = jX_{L} \quad (X_{L} = \omega L)$$

$$C \to \frac{1}{j \omega C} = \frac{-j}{\omega C} = jX_{C} \quad (X_{C} = \frac{-1}{\omega C})$$

• In frequency-domain  $V = Z \bar{I}$  (Ohm's law)where Z can be a series-parallel combination of R,  $X_C$ , and/or  $X_L$ .

Frequency-domain approach:

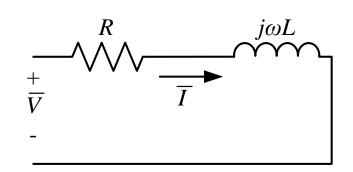
$$\overline{V} = (R + j\omega L)\overline{I}$$
 where  $\overline{I} = \frac{10}{\sqrt{2}} \angle -90^{\circ}$  A (RMS)

$$\overline{V} = \left(2 + j377 \frac{1}{377}\right) \frac{10}{\sqrt{2}} \angle -90^{\circ}$$

$$\overline{V} = \left\lceil \sqrt{2^2 + 1^2} \angle \left( \tan^{-1} \left( \frac{1}{2} \right) \right) \right\rceil \frac{10}{\sqrt{2}} \angle -90^\circ$$

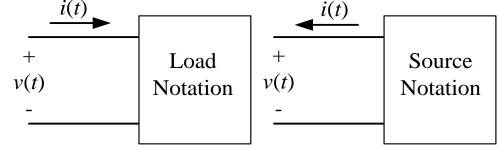
$$\overline{V} = \left(\frac{10\sqrt{5}}{\sqrt{2}}\right) \angle \left(26.56^{\circ} - 90^{\circ}\right)$$

$$\overline{V} = 15.81 \angle \left(-63.4^{\circ}\right) \text{ V(RMS)}$$



#### TWO-TERMINAL NETWORK

• A two-terminal electrical network has voltage at its terminals and current flowing in and out of its terminals. i(t)



- The instantaneous power is p(t) = v(t)i(t).
- For  $i(t) = I_m \cos(\omega t + \theta_i)$  A and  $v(t) = V_m \cos(\omega t + \theta_v)$  V we get

$$p(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

#### TWO-TERMINAL NETWORK

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$p(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i) \quad \mathbf{W}$$

The first term is time-independent, while the second term is a sinusoid at double frequency.

#### TWO-TERMINAL NETWORK

The average power is thus

$$P = \frac{1}{T} \int_{0}^{T} P(t) d(t) \qquad , \qquad T = \frac{2\pi}{\omega}$$

$$P_{in} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i).$$

- This is called the active or real power and its unit is watts (W).
- The power factor is the cosine of the phase angle between v(t) and i(t).

#### POWER FACTOR

- The power factor (*P.F.*) is thus  $P.F. = \cos(\theta_v \theta_i)$ .
- The power factor can be:

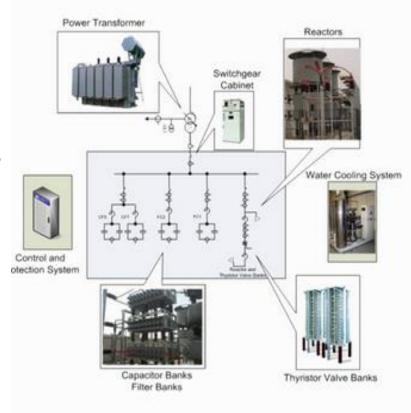
- Lagging: 
$$0^{\circ} < \theta_{v} - \theta_{i} < 90^{\circ}$$

- Leading: 
$$-90^{\circ} < \theta_{v} - \theta_{i} < 0^{\circ}$$

- Unity: 
$$\theta_{v} - \theta_{i} = 0$$

Therefore,  $0 \le P.F. \le 1$ ,

and the highest real power exists when *P.F.*=1.



Source: grupovision.com

#### APPARENT POWER AND REACTIVE POWER

- The apparent power is  $S = \frac{V_m I_m}{2}$
- The apparent power unit is volt-amps (VA).
- The reactive power is  $Q_{in} = \frac{V_m I_m}{2} \sin(\theta_v \theta_i)$ .
- The reactive power unit is volt-amps-reactive

(VARs).

#### **COMPLEX POWER**

The instantaneous power is

$$p(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i) \quad \mathbf{W}$$

• The time varying component

$$\frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i) = \frac{V_m I_m}{2} \left\{ \cos\left[ (2\omega t + 2\theta_i) + (\theta_v - \theta_i) \right] \right\}$$

$$\frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i) = \frac{V_m I_m}{2} \left\{ \cos\left[ (2\omega t + 2\theta_i) + (\theta_v - \theta_i) \right] \right\}$$

$$= \frac{V_m I_m}{2} \cos(2\omega t + 2\theta_i) \cos(\theta_v - \theta_i) - \frac{V_m I_m}{2} \sin(2\omega t + 2\theta_i) \sin(\theta_v - \theta_i)$$

#### **COMPLEX POWER**

Define

$$Q_{in} = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i), \qquad \text{(Reactive power)}$$

$$p(t) = P_{in} + P_{in} \cos(2\omega t + 2\theta_i) - Q_{in} \sin(2\omega t + \theta_i)$$

$$= P_{in} (1 + \cos(2\omega t + \theta_i)) - Q_{in} \sin(2\omega t + 2\theta_i)$$

The real power can be written as

$$P_{in} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) = V_{ms} I_{ms} \cos(\theta_v - \theta_i)$$

#### PHASOR REPRESENTATION

$$P_{in} = \text{Re}\{\frac{V_{m}I_{m}}{2}e^{j\theta_{v}}e^{-j\theta_{i}}\} = \text{Re}\{V_{rms}e^{j\theta_{v}}I_{rms}e^{-j\theta_{i}}\}$$

• The reactive power can be written as

$$Q_{in} = \operatorname{Im} \{ \frac{V_m I_m}{2} e^{j\theta_v} e^{-j\theta_i} \} = \operatorname{Im} \{ V_{rms} e^{j\theta_v} I_{rms} e^{-j\theta_i} \}$$

• The voltages and currents can be written as phasors:

$$\begin{split} V_{\mathit{rms}} e^{j\theta_{\mathit{v}}} = & \overline{V} \text{ and } I_{\mathit{rms}} e^{j\theta_{\mathit{i}}} = \overline{I} \,. \\ P_{\mathit{in}} = & \operatorname{Re}(\overline{V} \ \overline{I}^*) = V_{\mathit{ms}} \, I_{\mathit{ms}} \, \cos(\theta_{\mathit{v}} - \theta_{\mathit{i}}) \\ Q_{\mathit{in}} = & \operatorname{Im}(\overline{V} \ \overline{I}^*) = V_{\mathit{ms}} \, I_{\mathit{ms}} \, \sin(\theta_{\mathit{v}} - \theta_{\mathit{i}}) \end{split}$$

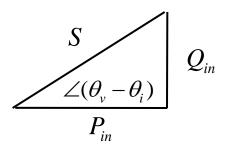
# Complex Power

- Define the complex power as  $\overline{S} = P_{in} + jQ_{in}$
- Then  $\overline{S}$  can be written as  $\overline{S} = \overline{V} \overline{I}^*$
- The quantity  $\overline{I}^*$  is the complex conjugate of I.
- $\overline{S}$  can also be written as

$$\overline{S} = S \angle (\theta_{v} - \theta_{i})$$

Note that

$$S = \frac{V_m I_m}{2} = \sqrt{P_{in}^2 + Q_{in}^2}$$



#### ALTERNATE FORMS OF COMPLEX POWER

• If the load is  $\overline{Z} = R + jX$ , connected across the source  $V^{\overline{I}}$ By Ohm's law:  $V^{\overline{I}} = \overline{Z} \overline{I}$ , but  $\overline{S} = V^{\overline{I}} \overline{I}^*$ 

Then  $\overline{S}$  can be written as  $\overline{S} = I^2R + jI^2X$  Also,

$$P = I^2 R$$
 and  $Q = I^2 X$ ,  $\overline{Z}$  and  $P.F. = \cos(angle(\overline{Z}))$ .

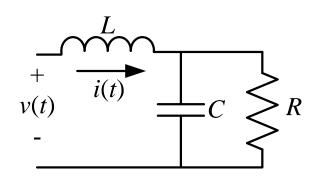
- Thus, Q > 0 when  $\bar{Z}$  is inductive,  $X = \omega L$  and Q < 0 when is capacitive,  $X = -\frac{1}{\omega C}$
- $\overline{S}$  and  $\overline{Z}$  are not phasors but complex quantities.

## EXAMPLE: LC FILTER AND R LOAD

- The circuit shown is commonly used as an LC filter to supply a load, which is resistive in this case.
- Find the current, real, reactive, and complex powers, and the P.F. for  $v(t) = \sqrt{2}V_{rms}\cos(377t)$

$$\bar{Z} = j\omega L + \left(\frac{R}{\sqrt{\frac{-j}{\omega C}}}\right)$$

$$\bar{Z} = \frac{\omega L + j(\omega^2 RLC - R)}{\omega RC - j}$$



#### EXAMPLE: LC FILTER AND R LOAD

Let

$$V_{rms} = 120V, L = 1$$
mH,  $C = 6.8$ mF, and  $R = 10\Omega$ .  
 $\overline{Z} = 0.0197 \angle -39.41^{\circ} = 0.0152 - j0.0125\Omega$ 

$$\overline{I} = \frac{\overline{V}}{\overline{Z}} = \frac{120\angle 0^{\circ}}{0.0197\angle -39.41^{\circ}} = 6091.4\angle 39.41^{\circ} A$$

$$i(t) = 6091.4\sqrt{2}\cos(377t + 39.41^{\circ})$$

$$\overline{S} = \overline{V} \overline{I}^* = 731 \angle -39.41^{\circ} \text{kVA}$$

$$P_{in} = 731\cos(-39.41^{\circ}) = 564.8$$
kW

$$Q_{in} = 731\sin(-39.41^{\circ}) = -464.1\text{kVAR}$$

P.F.=
$$\cos(-39.41^{\circ}) = 0.773 \text{ leading } (\theta_{v} - \theta_{i} = -39.41^{\circ})$$

