ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

LECTURE 19
SYNCHRONOUS MACHINES (3)

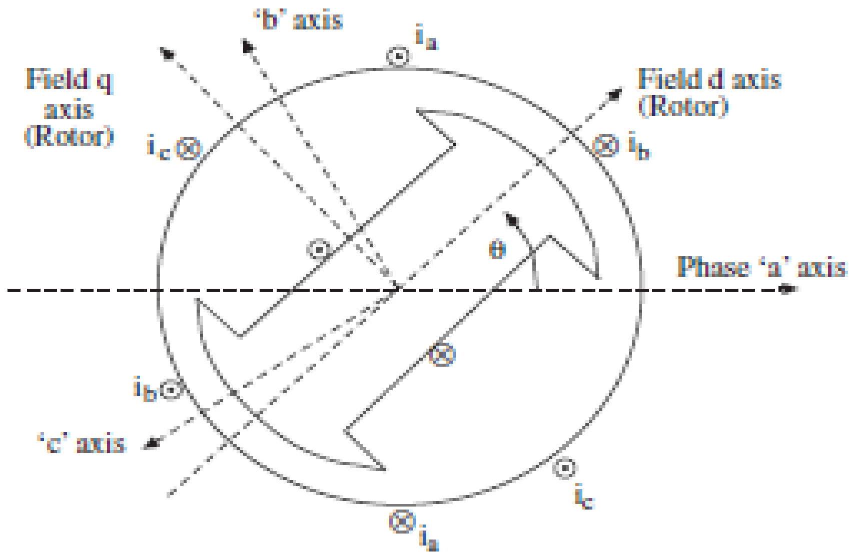
Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.

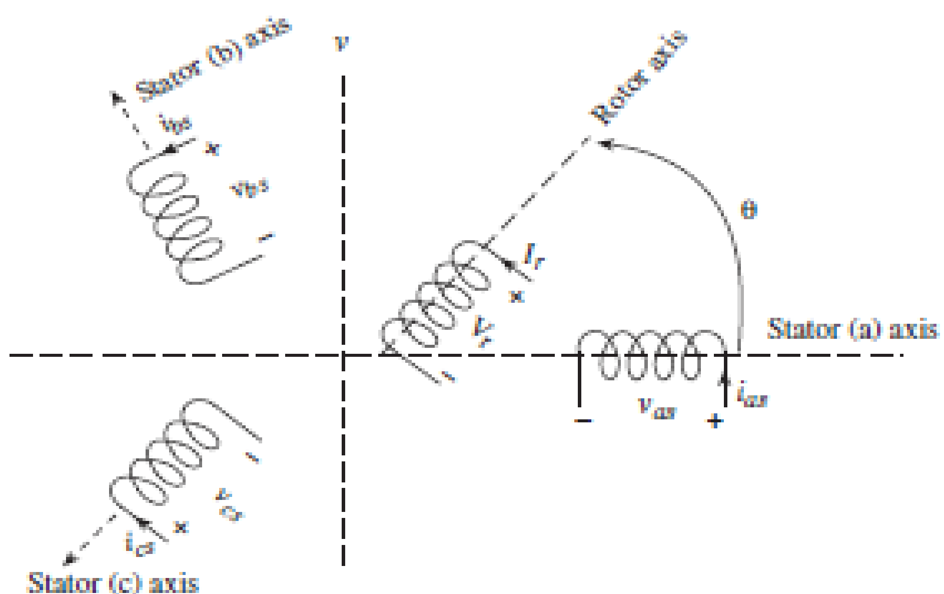
SALIENT POLE THREE-PHASE SYNCHRONOUS MACHINE

- First we consider a salient pole three-phase machine as opposed to a round rotor machine
- Salient pole machines are used in hydro-generators and low-power single-phase synchronous motors.
- We consider a two-pole machine.
- There are three stator coils distributed so that each coil creates a sinusoidal mmf around the periphery.
- The rotor has a field coil that carries a constant current i_r.

SALIENT POLE THREE-PHASE SYNCHRONOUS MACHINE



(a)



(b)

SALIENT POLE THREE-PHASE SYNCHRONOUS MACHINE

The stator coils are separated mechanically. The flux current relations can be derived as

$$\begin{bmatrix}\lambda_a\\ \lambda_b\\ \lambda_c\\ \lambda_r\end{bmatrix}=\begin{bmatrix} L_0+L_2\cos 2\theta & -M_0+M_2\cos 2(\theta-60^\circ) & -M_0+M_2\cos 2(\theta+60^\circ) & M\cos\theta \\ -M_0+M_2\cos 2(\theta-60^\circ) & L_0+L_2\cos 2(\theta-120^\circ) & -M_0+M_2\cos 2(\theta-180^\circ) & M\cos(\theta-120^\circ) \\ -M_0+M_2\cos 2(\theta+60^\circ) & -M_0+M_2\cos 2(\theta-180^\circ) & L_0+L_2\cos 2(\theta+120^\circ) & M\cos(\theta+120^\circ) \\ M\cos\theta & M\cos(\theta-120^\circ) & M\cos(\theta+120^\circ) & L_r \end{bmatrix}\begin{bmatrix} i_a\\ i_b\\ i_c\\ i_r\end{bmatrix}$$

SALIENT POLE THREE-PHASE SYNCHRONOUS MACHINE

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_r \end{bmatrix} = \begin{bmatrix} L_a & M_{ab} & M_{ac} & M_{ar} \\ M_{ab} & L_b & M_{bc} & M_{br} \\ M_{ac} & M_{bc} & L_c & M_{cr} \\ M_{ar} & M_{br} & M_r & L_r \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_r \end{bmatrix}$$

- inductance matrix is symmetric.
- all inductances except L_r are functions of θ
- Also $M_o = \frac{L_o}{2}$ and $L_2 = M_2$.
- All windings have the same number of turns and there is no leakage flux.

SALIENT POLE THREE-PHASE SYNCHRONOUS MACHINE

$$W'_m = \frac{1}{2}L_a i_a^2 + \frac{1}{2}L_b i_b^2 + \frac{1}{2}L_c i_c^2 + \frac{1}{2}L_r i_r^2$$
$$+ i_a i_b M_{ab} + i_a i_c M_{ac} + i_b i_c M_{bc}$$
$$+ i_a i_r M_{ar} + i_b i_r M_{br} + i_c i_r M_{cr}$$

$$T^e = \frac{\partial W'_m}{\partial \theta} = \frac{i_a^2}{2}\frac{dL_a}{d\theta} + \frac{i_b^2}{2}\frac{dL_b}{d\theta} + \frac{i_c^2}{2}\frac{dL_c}{d\theta}$$
$$+ i_a i_b \frac{dM_{ab}}{d\theta} + i_a i_c \frac{dM_{ac}}{d\theta} + i_b i_c \frac{dM_{bc}}{d\theta}$$
$$+ i_a i_r \frac{dM_{ar}}{d\theta} + i_b i_r \frac{dM_{br}}{d\theta} + i_c i_r \frac{dM_{cr}}{d\theta}$$

ROUND ROTOR THREE-PHASE SYNCHRONOUS MACHINE

For round rotor case $L_2 = M_2 = 0$ and $M_0 = \dfrac{L_0}{2}$

$$L(\theta) = \begin{bmatrix} L_0 & -M_0 & -M_0 & M\cos\theta \\ -M_0 & L_0 & -M_0 & M\cos(\theta - 120^\circ) \\ -M_0 & -M_0 & L_0 & M\cos(\theta + 120^\circ) \\ M\cos\theta & M\cos(\theta - 120^\circ) & M\cos(\theta + 120^\circ) & L_r \end{bmatrix}$$

ROUND ROTOR THREE-PHASE SYNCHRONOUS MACHINE

$$W_m' = \frac{1}{2}L_0 i_a^2 - M_0 i_a i_b + \frac{1}{2}L_0 i_b^2 - M_0 i_a i_c$$

$$-M_0 i_b i_c + \frac{1}{2}L_0 i_c^2 + i_a i_r M \cos\theta$$

$$+i_b i_r M \cos(\theta - 120^\circ) + i_c i_r M \cos(\theta + 120^\circ) + \frac{1}{2}L_r i_r^2$$

ROUND ROTOR THREE-PHASE SYNCHRONOUS MACHINE

Since $L_a = L_b = L_c = M_{ab} = M_{ac} = M_{bc} =$ constant

$$T^e = \frac{\partial W_m'}{\partial \theta} = i_a i_r \frac{dM_{ar}}{d\theta} + i_b i_r \frac{dM_{br}}{d\theta} + i_c i_r \frac{dM_{cr}}{d\theta}$$

$$= -i_a i_r M \sin\theta - i_b i_r M \sin(\theta - 120^\circ) - i_c i_r M \sin(\theta + 120^\circ)$$

In steady-state AC conditions and in terminal conditions.

$$p_m = T^e \omega_m$$

Whatever happens in phase a, happens in phase b , 120 deg.

later and in phase c, 240 deg. later.

ROUND ROTOR THREE-PHASE SYNCHRONOUS MACHINE

Assume a balanced set of currents in the stator

$$i_a = I_m \cos \omega_s t$$

$$i_b = I_m \cos(\omega_s t - 120^\circ)$$

$$i_c = I_m \cos(\omega_s t + 120^\circ)$$

$$i_r = I_r = constant$$

ROUND ROTOR THREE-PHASE SYNCHRONOUS MACHINE

$$T^e = -I_m I_r M[\sin\theta\cos\omega_s t + \sin(\theta - 120^\circ)\cos(\omega_s t - 120^\circ)$$

$$+\sin(\theta + 120^\circ)\cos(\omega_s t + 120^\circ)]$$

$$= -I_m I_r M\left[\frac{\sin(\theta+\omega_s t)+\sin(\theta-\omega_s t)}{2}\right.$$

$$+\frac{\sin(\theta+\omega_s t-240^\circ)+\sin(\theta-\omega_s t)}{2}$$

$$\left.+\frac{\sin(\theta+\omega_s t+240^\circ)+\sin(\theta-\omega_s t)}{2}\right]$$

ROUND ROTOR THREE-PHASE SYNCHRONOUS MACHINE

Using the identity

$$\sin(\theta+\omega_s t)+\sin(\theta+\omega_s t-240^\circ)+\sin(\theta+\omega_s t+240^\circ)=0$$

We get,

$$T^e=\frac{-I_m I_r M 3\sin(\theta-\omega_s t)}{2}$$

$$I_m=\sqrt{2}I_a \quad , \quad I_a = rms \quad , \quad \theta=\omega_m t+\gamma$$

$$T^e=-\sqrt{2}\,\frac{I_a I_r}{2}3M\sin(\omega_m t+\gamma-\omega_s t)$$

ROUND ROTOR THREE-PHASE SYNCHRONOUS MACHINE

For T^e to have an average value, $\omega_m = \omega_s$, which is called the *synchronous speed*.

$$T^e = \frac{-3}{\sqrt{2}} I_a I_r M \sin\gamma$$

The synchronous speed ω_m is equal to the electrical frequency ω_s in radians per second, $\omega_m = \frac{2\pi N_s}{60} = 2\pi f$

where $N_s =$ synchronous speed in rpm.

For two pole machine $N_s = f \times 60 = 3600$ rpm.

VOLTAGE IN STEADY-STATE(ROUND ROTOR CASE)

Compute v_a in steady state

$$\theta = \omega_m t + \gamma = \omega_s t + \gamma$$

$$v_a = \frac{d\lambda_a}{dt} = L_a \frac{di_a}{dt} + M_{ab}\frac{di_b}{dt} + M_{ac}\frac{di_c}{dt} + I_r \frac{dM_{ar}}{dt}$$

$$= -L_0 I_m \omega_s \sin\omega_s t + \frac{L_0}{2} I_m \omega_s \sin(\omega_s t - 120^\circ)$$

$$+ \frac{L_0}{2} I_m \omega_s \sin(\omega_s t + 120^\circ) - I_r M\, \omega_s \sin(\omega_s t + \gamma)$$

Adding and subtracting $-\frac{L_0}{2} I_m \omega_s \sin\omega_s t$

VOLTAGE IN STEADY-STATE(ROUND ROTOR CASE)

$$v_a = -\frac{3L_0}{2} I_m \omega_s \sin \omega_s t + \frac{L_0}{2} I_m \omega_s [\sin \omega_s t + \sin(\omega_s t - 120^\circ)$$

$$+\sin(\omega_s t + 120^\circ)] - I_r M \omega_s \sin(\omega_s t + \gamma)$$

$$v_a = -\frac{3}{2} L_0 \sqrt{2} I_a \omega_s \sin \omega_s t - \sqrt{2}\frac{MI_r}{\sqrt{2}} \omega_s \sin(\omega_s t + \gamma)$$

$$v_a = Re[\sqrt{2}\,\overline{V}_a e^{j\omega_s t}] = Re\left[\sqrt{2}\,\frac{3}{2} L_0 \omega_s \overline{I}_a e^{j\pi/2} e^{j\omega_s t}\right]$$

$$+Re\left[\sqrt{2}\,\frac{M\ I_r}{\sqrt{2}} \omega_s e^{j\gamma} e^{j\omega_s t} e^{j\frac{\pi}{2}}\right]$$

VOLTAGE IN STEADY-STATE(ROUND ROTOR CASE)

$$\bar{I}_a = I_a \angle 0^o \quad , \quad e^{j\frac{\pi}{2}} = j$$

$$\bar{V}_a = j\frac{3}{2}L_0\omega_s\bar{I}_a + j\frac{MI_r}{\sqrt{2}}\omega_s e^{j\gamma}$$

$$\frac{3}{2}L_0\omega_s \triangleq x_s \ (synchronous\ reactance)$$

$$j\frac{\omega_s M\ I_r e^{j\gamma}}{\sqrt{2}} = \frac{\omega_s M\ I_r}{\sqrt{2}}\angle\left(\frac{\pi}{2}+\gamma\right)$$

$$\bar{V}_a = jx_s\bar{I}_a + \bar{E}_{ar}$$

$$\bar{E}_{ar}\ (voltage\ phasor\ proportional\ to\ field\ (rotor)\ current\ I_r)$$

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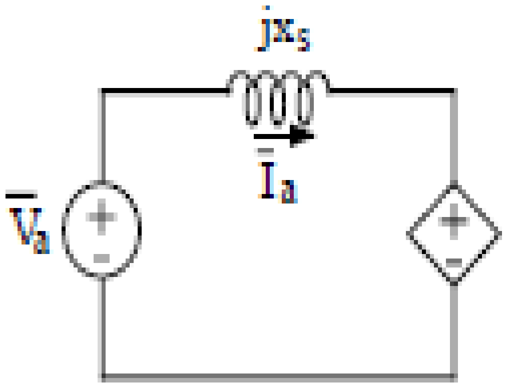
VOLTAGE IN STEADY-STATE (ROUND ROTOR CASE)

$$\bar{V}_a = jx_s \bar{I}_a + \bar{E}_{ar}$$

The equivalent circuit



$$\bar{E}_{ar} = \frac{\omega_s M I_r}{\sqrt{2}} \angle \tfrac{\pi}{2} + \gamma$$

$$i_a = I_m \cos \omega_s t = \sqrt{2} I_a \cos \omega_s t$$

$\bar{I}_a = I_a \angle 0^\circ$ Reference phasor.

Real power into the source $P_a = Re[\bar{E}_{ar} \bar{I}_a^*]$

$$P_a = Re\left[\left(\frac{\omega_s M I_r}{\sqrt{2}} \angle\left(\frac{\pi}{2} + \gamma\right)\right) I_a\right]$$

$$= -\frac{\omega_s M I_r}{\sqrt{2}} I_a \sin \gamma$$

VOLTAGE IN STEADY-STATE(ROUND ROTOR CASE)

Power in three phases: $P_T = 3P_a$

$$P_T = \frac{-3}{\sqrt{2}} \omega_s M I_r I_a \sin\gamma$$

The mechanical power output: $P_m = T^e \omega_m$.

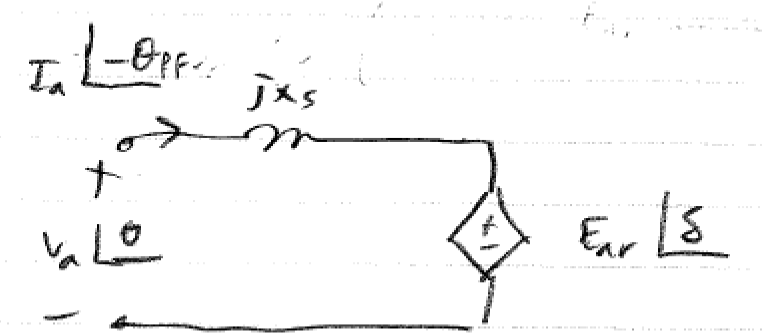
Since we have DC field excitation, $\omega_r = 0$.

from the frequency condition $\omega_m = \omega_s - \omega_r$, we have $\omega_m = \omega_s$.

For a conservative system, $P_T = P_m$

$$T^e = \frac{-3}{\sqrt{2}} M I_a I_r \sin\gamma$$

POWER IN TERMS OF VOLTAGES



$$S_{IN\,3\phi} = 3\overline{V}_a \overline{I}_a^{*} = 3V_a I_{a\angle\theta_{P.F}}$$

$$\overline{S}_{IN\,3\phi} = 3V_a\angle 0^\circ \left(\frac{V_a\angle 0^\circ - E_{ar}\angle\delta}{jX_S}\right)^{*}$$

$$\overline{S}_{IN\,3\phi} = \frac{3V_a^{2}\angle 90^\circ}{X_S} - \frac{3V_a E_{ar}\angle 90^\circ - \delta}{X_S}$$

$$P_{IN\,3\phi} = -\frac{3V_a E_{ar}}{X_S}\cos(90^\circ - \delta) = -\frac{3V_a E_{ar}}{X_S}\sin\delta$$

POWER IN TERMS OF VOLTAGES

$$T^{e} = \frac{P_{IN\,3\phi}}{\omega_s} = -\frac{3V_a E_{ar}}{\omega_s X_S}\sin\delta$$

$\delta < 0$ Motor

$\delta > 0$ Generator

$$Q_{IN\,3\phi} = \frac{3V_a^{\,2}}{X_S} - \frac{3V_a E_{ar}}{X_S}\cos\delta$$

$Q_{IN\,3\phi} < 0$ Overexcitation $\left(\mathrm{E}_{ar}\cos\delta > V_a\right)$

$Q_{IN\,3\phi} > 0$ Underexcitation $\left(\mathrm{E}_{ar}\cos\delta < V_a\right)$

Note: $Q_{IN\,3\phi}$ is determind by E_{ar} (excitation)

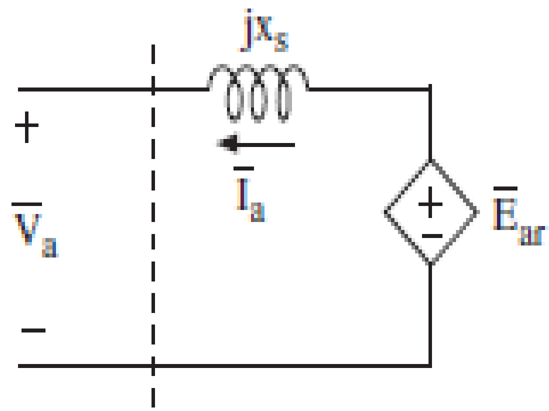
CONVENIENT PHASOR NOTATIONS

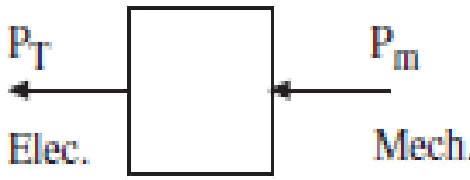
$$i_{a(g)} = -i_a$$

$$\bar{V}_a = -jx_s \bar{I}_a + \bar{E}_{ar}$$

$$\bar{E}_{ar} = jx_s \bar{I}_a + \bar{V}_a$$

$$\bar{I}_a = \frac{\bar{E}_{ar} - \bar{V}_a}{jx_s}$$





$$P_{T3\phi} = \frac{3V_a E_{ar}}{X_S} \sin\delta$$

$$Q_{T3\phi} = \frac{3V_a E_{ar}}{X_S} \cos\delta - \frac{3V_a^2}{X_S}$$

POWER IN TERMS OF VOLTAGES

$\delta < 0$ Motor

$\delta > 0$ Generator

$Q_{IN\ 3\phi} > 0$ Overexcitation $\left(E_{ar}\cos\delta > V_a\right)$

$Q_{IN\ 3\phi} < 0$ Underexcitation $\left(E_{ar}\cos\delta < V_a\right)$

Note: $Q_{IN\ 3\phi}$ is determined by E_{ar} (excitation)

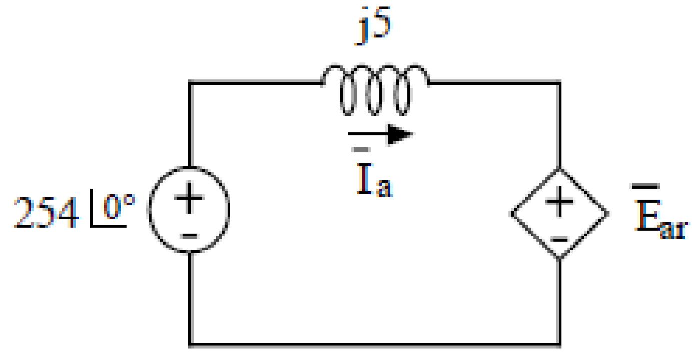
EXAMPLE 6.1

A three-phase wye-connected 60 Hz synchronous machine with two poles has synchronous reactance $x_s = 5.0\Omega$ /phase. Operating as a motor: 30A, 254 V, PF= 0.8 leading, windage, friction, and core losses = 400 W

Find: E_{ar} and T^e , useful shaft torque, efficiency

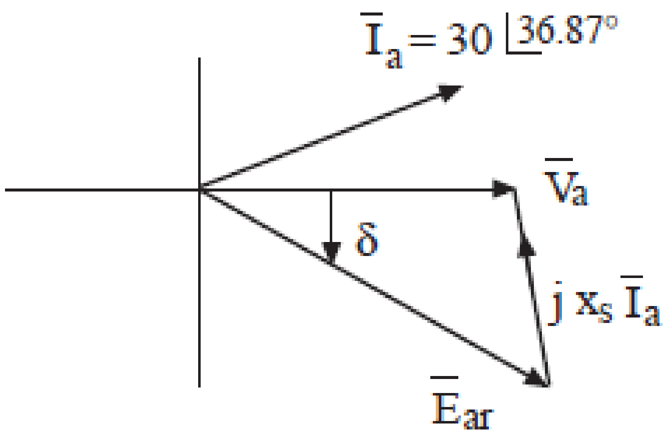
EXAMPLE 6.1

The equivalent circuit



$$\bar{V}_a = 254\angle 0^\circ$$

$$\cos\theta = 0.8 \Rightarrow \theta = 36.87^\circ$$



$$\bar{I}_a = 30\angle 36.87^\circ \ A$$

$$\bar{E}_{ar} = \bar{V}_a - jx_s\bar{I}_a$$

$$\bar{E}_{ar} = 254\angle 0^\circ - (j5)30\angle 36.87^\circ$$

$$= 364.3\angle -19.23^\circ \ V$$

$Q_{IN\ 3\phi}$

E_{ar}

EXAMPLE 6.1

The torque angle is $\delta = -19.23^\circ$ and $\omega_m = \omega_s = 377 \; rad/sec$

$$P_T = \frac{-3(364.3)(254)\sin(-19.23^\circ)}{5} = 18286\,W$$

The torque of electric origin is

$$T^e = \frac{P_T}{\omega_m} = \frac{18286}{377} = 48.5 \; N-m$$

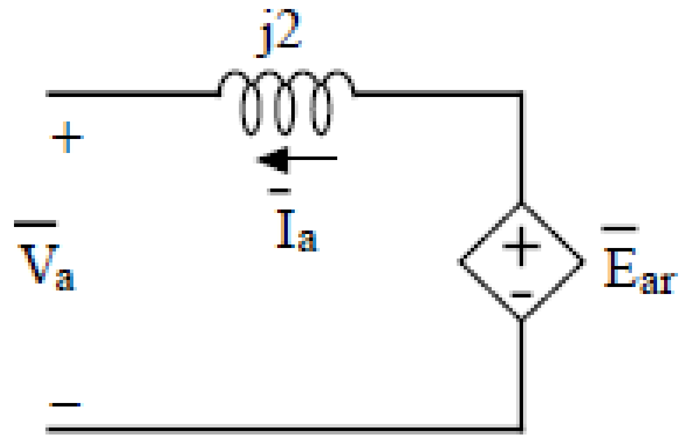
Overall efficiency $= \dfrac{18286-400}{18286} = 1788618286 = 97.8\%$

Useful shaft torque = $\dfrac{17886}{377} = 47.44$ N-m

EXAMPLE 6.4

A two-pole, three-phase, 60 Hz, wye-connected synchronous machine, $x_s = 2\Omega$, is operating as a generator 1905 V per phase, 350 A , PF of the load is 0.8 lagging.

Find , E_{ar} , δ and the torque of electric origin



$$\overline{V}_a = 1905\angle 0^\circ \quad V$$

$$\overline{I}_a = 350\angle -36.87^\circ \qquad A$$

$$\cos\theta = 0.8 \Rightarrow \theta = 36.87^\circ$$

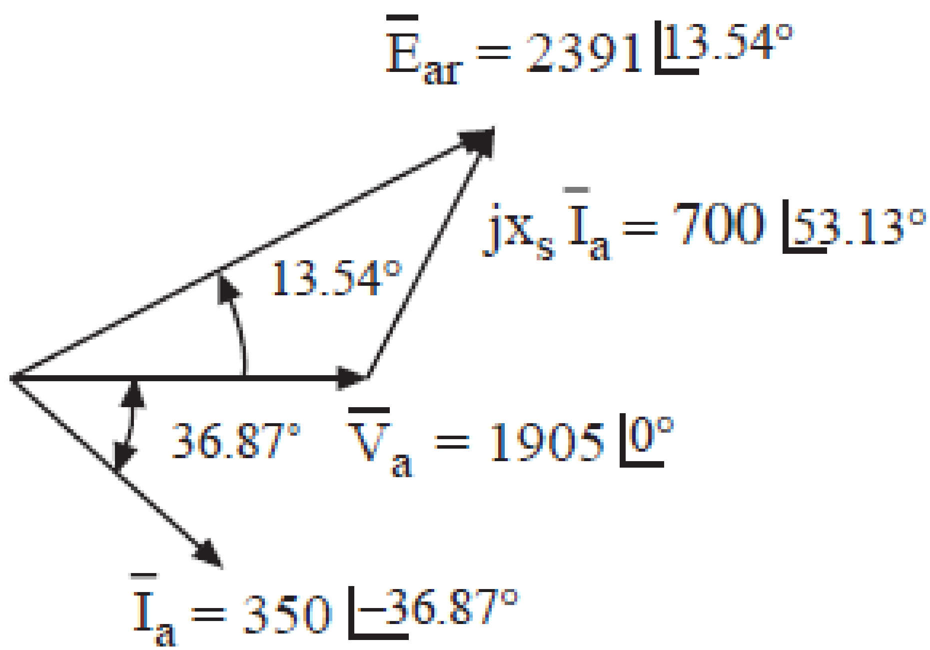
EXAMPLE 6.4

$$\overline{E}_{ar} = \overline{V}_a + jx_s\overline{I}_a = 1905\angle 0^\circ + j2(350\angle -36.87^\circ)$$

$$= 2325 + j560 = 2391\angle 13.54^\circ\ V \qquad \delta = 13.54^\circ$$

$$P_T = \frac{3E_{ar}V_a \sin\delta}{x_s} = \frac{3(2391)(1905)(.23416)}{2} = 1600,000\,W \quad = 1.6\ MW$$

$$P_m = T^e\omega_m = 1600,000\ W$$



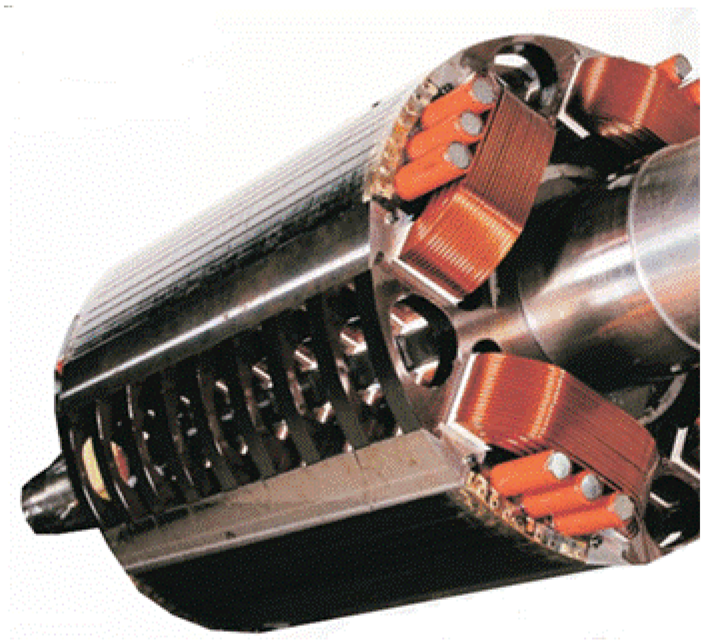
$$\omega_m = \omega_s \quad two - pole\ machine$$

$$= 377\ rads/sec$$

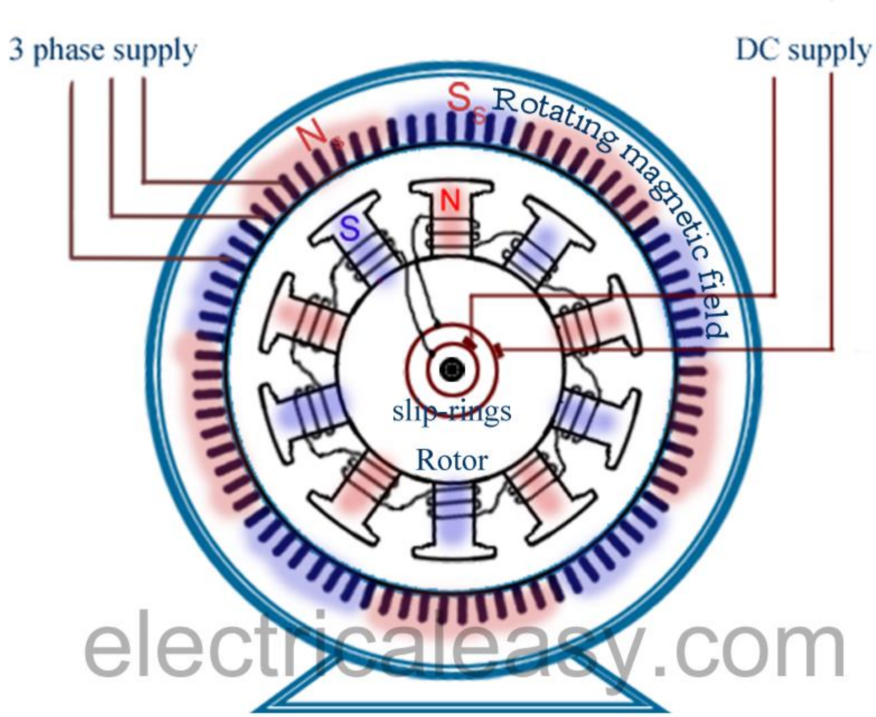
$$T^e = \frac{1600,000}{377} = 42440\ N-m$$

MULTI-POLE MACHINES

The number of poles in a machine is defined by the configuration of the magnetic field pattern.



Source: machineryequipmentonline.com



Source: electricaleasy.com

MULTI-POLE MACHINES

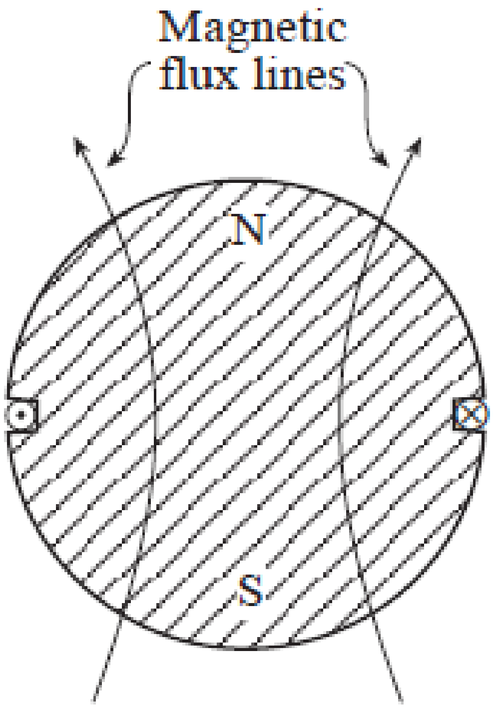
When the instantaneous current is

in the direction indicated, the resulting

flux lines effectively create

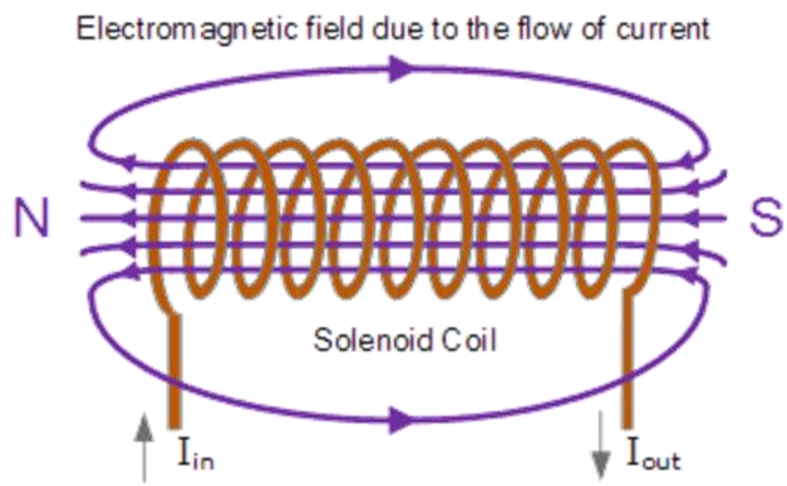
an electromagnetic field with

North (N) and the South (S) poles

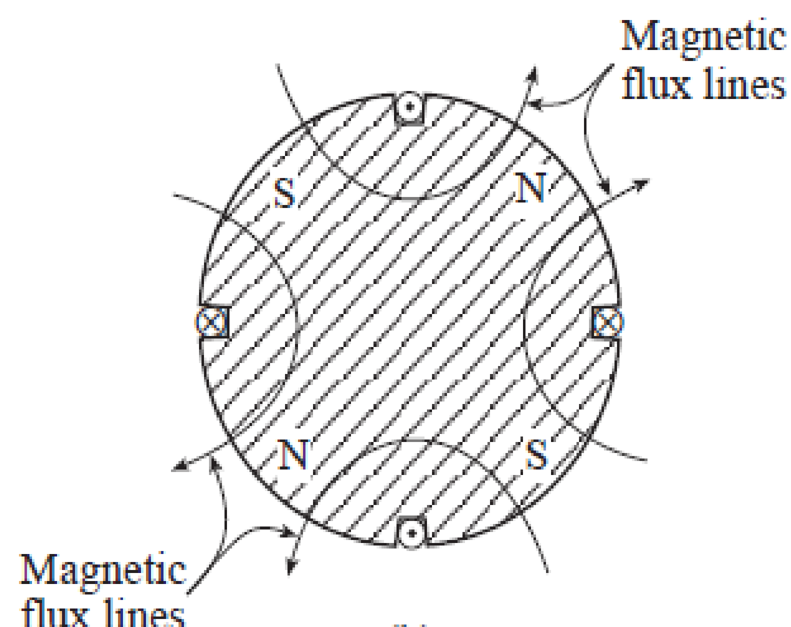


MULTI-POLE MACHINE

For four slots carrying coils connected in series with polarities indicated by dots and crosses. It can be one phase of a three-phase winding. In this, we effectively have a four-pole machine



Source: electronics-tutorial.ws



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MULTI-POLE MACHINES

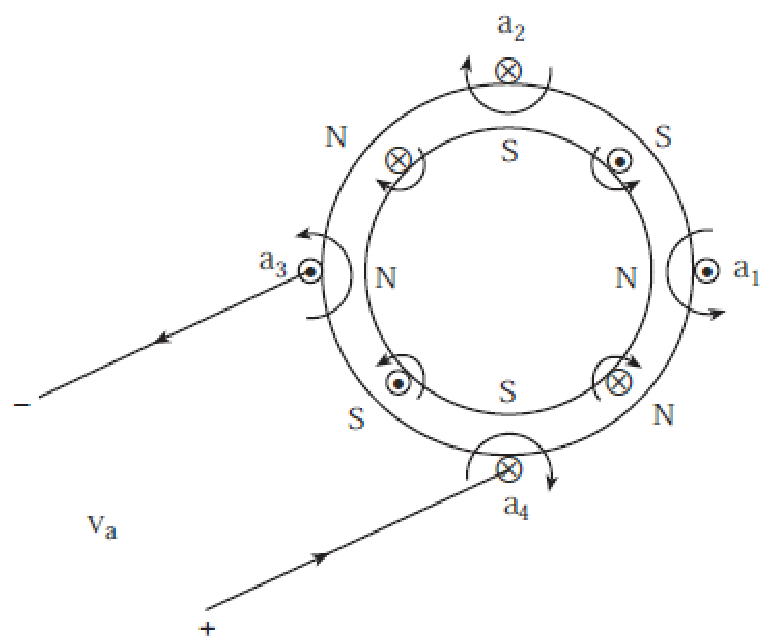
For four-pole, three-phase machines the rotating field completes two cycles (720 degrees. Electrical) in one mechanical revolution of 360 degrees.

$$\theta_{elec} = 2\theta_{mech}$$

$$Since \quad \frac{d\theta}{dt} = \omega \Rightarrow \omega_{elec} = 2\omega_{mech}$$

$$\omega_{elec} = \frac{p\omega_{mech}}{2}$$

$$\omega_s = \frac{p\omega_m}{2}, \qquad \omega_s = 2\pi f = 120\pi$$



MULTI-POLE MACHINES

$$\omega_m = \frac{2\pi N_s}{60} \quad , \quad \omega_s = \frac{p\omega_m}{2}$$

$$f = \frac{pN_s}{120}$$

Poles (p)	ω_m (mech.)	Speed
2	120π rad/sec	3600 rpm
4	60π rad/sec	1800 rpm
6	40π rad/sec	1200 rpm
8	30π rad/sec	900 rpm
10	24π rad/sec	720 rpm

In mathematical terms, the relationship between the number of poles and synchronous speed is reflected in mutual inductance. $M\cos\theta$ becomes $M\cos\frac{p\theta}{2}$ and the frequency condition becomes $\omega_m = (\pm\omega_s \pm \omega_r)/p$

$$\omega_r = 0 \quad , \quad \omega_s = \frac{p\omega_m}{2}$$

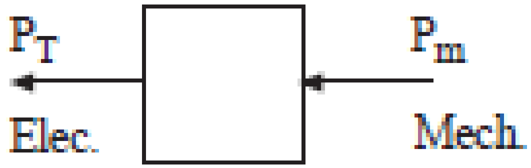
SUMMARY FOR P POLE MACHINE

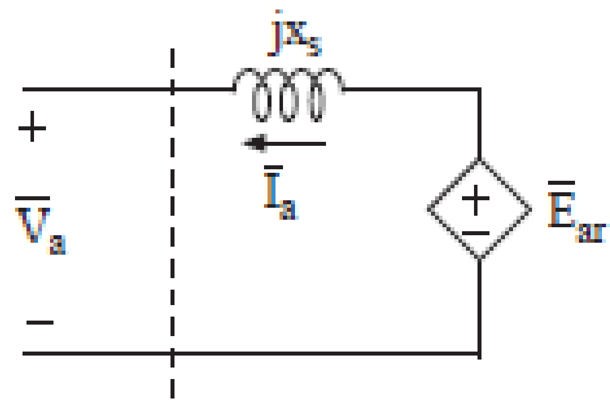
Generator action

With the generator convention for current, $P_T = P_m > 0$

$$P_T = \frac{3E_{ar}V_a \sin\delta}{x_s} = P_m$$

$$\overline{E}_{ar} = jX_s\overline{I}_a + \overline{V}_a$$





Per-phase equivalent circuit

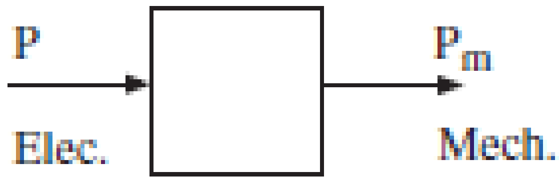
In the phasor diagram $\delta > 0$

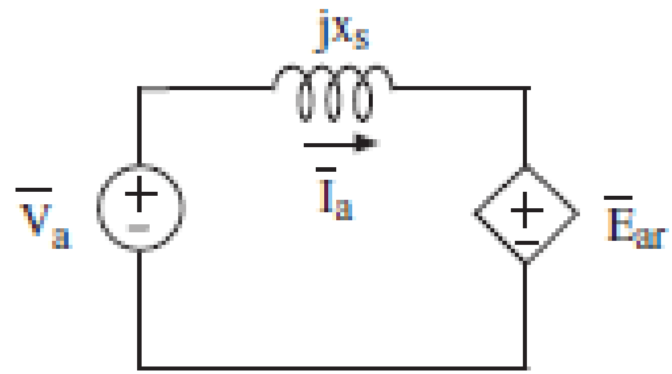
SUMMARY FOR P POLE MACHINE

Motor action

With the motor convention $P_T = P_m > 0$

$$P_T = \frac{-3E_{ar}V_a \sin\delta}{x_s} = P_m$$





Per-phase equivalent circuit

$$\overline{V}_a = jx_s\overline{I}_a + \overline{E}_{ar}$$

In the phasor diagram $\delta < 0$

SUMMARY FOR P POLE MACHINE

Both for motor and generator $\omega_s = \frac{p}{2}\omega_m$

ω_s is the supply frequency

ω_m is the synchronous speed in mechanical radians per second. the synchronous speed $N_s = \frac{120f}{p}$ in rpm

The power factor is cosine of the angle between terminal voltage $\bar{V}_a$ and terminal current $\bar{I}_a$. It is called *leading PF* when $\bar{I}_a$ leads $\bar{V}_a$. It is called *lagging PF* when $\bar{I}_a$ lags $\bar{V}_a$. δ is called the *torque angle* and is the angle from $\bar{V}_a$ to $\bar{E}_{af}$. It is *positive* for generator action and *negative* for motor action. $\bar{E}_{af}$ is a dependent voltage source proportional to field current I_f.

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