

# ECE 330

# POWER CIRCUITS AND ELECTROMECHANICS

## LECTURE 8

## TRANSFORMERS (2)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.

2/12/2018

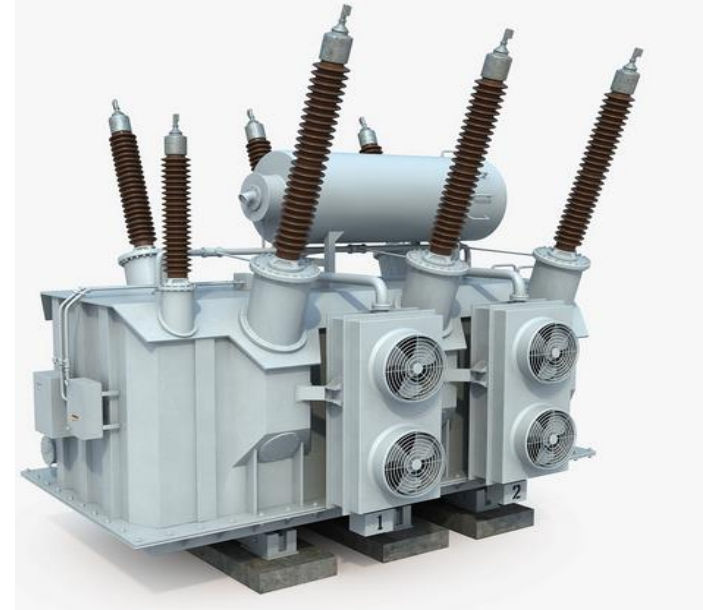
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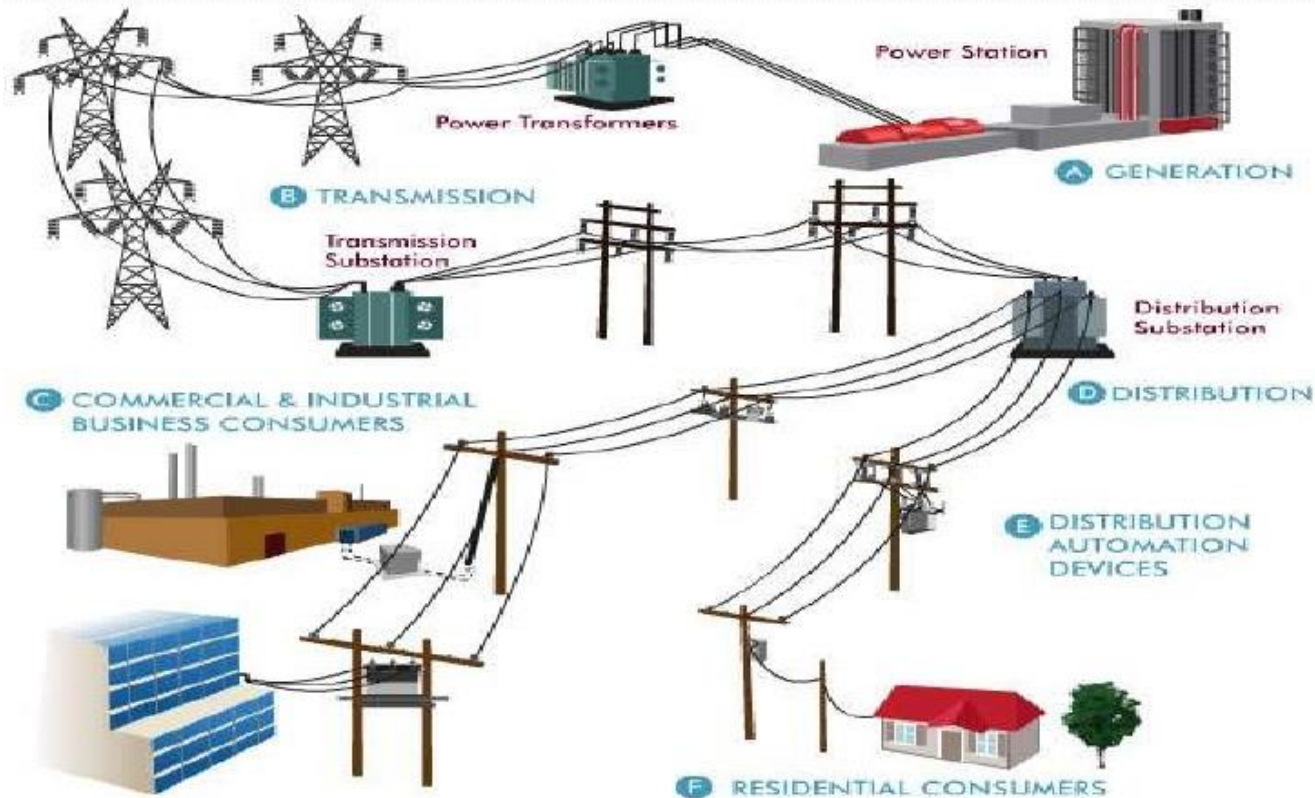
# POWER TRANSFORMER

The power transformer is an essential component of the power system. It steps up the voltage at the generating point, transmits it over long distances, and then steps it down at the sub-transmission and distribution levels for use by both commercial units and individual homes.



Source: [torbosquid.com](http://torbosquid.com)

# POWER SYSTEM

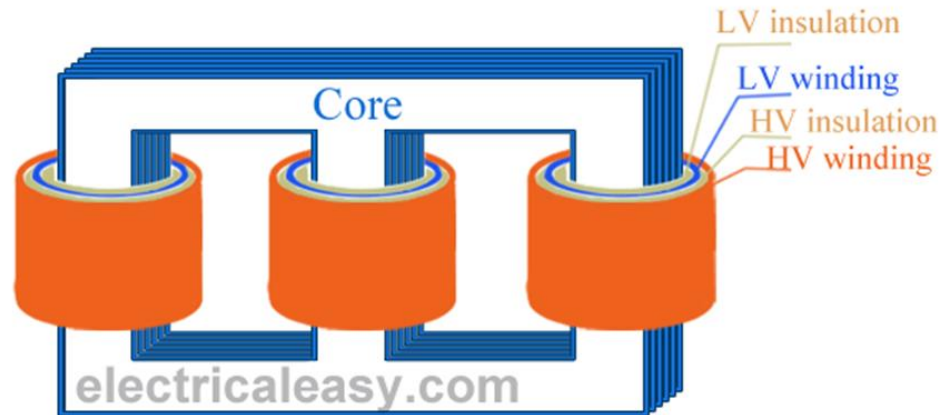


# POWER TRANSFORMER

A physical transformer consists of two windings mounted on a core, generally of a ferromagnetic material of high permeability.



Source: [electronichub.org](http://electronichub.org)



Core type three phase transformer

Source: [electricaleasy.com](http://electricaleasy.com)

# POWER TRANSFORMER

- The construction of the core is such as to keep the leakage flux to a minimum.
- The winding to which power is supplied is called the “primary” winding, and the other one is called the “secondary” winding.
- In the case of power transformers, the terms high-voltage (HV) and low-voltage (LV) windings are also used, depending on the voltage levels.

# POWER TRANSFORMER

We assume it to be an ideal transformer. This implies:

- No leakage flux.
- Winding resistances are neglected.
- Core has infinite permeability. Hence, reluctance of the core is zero and negligible current is required to set up the magnetic field.
- Magnetic core is lossless

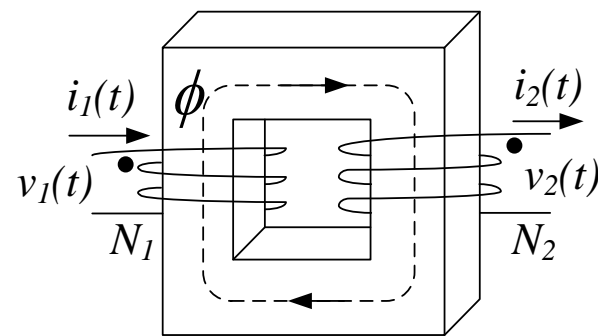
# POWER TRANSFORMER

- Power transformers are usually excited with sinusoidal voltages and currents.
- It is interesting to find the maximum flux, flux density, and current that a transformer can handle.

$$v_1(t) = V_{m1} \cos(\omega t) = N_1 \frac{d\phi}{dt}$$

$$\Rightarrow \phi(t) = \frac{1}{N_1} \int V_{m1} \cos(\omega t) dt = \frac{V_{m1}}{\omega N_1} \sin(\omega t)$$

$$\Rightarrow \phi_{\max} = \frac{V_{m1}}{\omega N_1} = \frac{V_{m1}}{2\pi f N_1}$$



# POWER TRANSFORMER

- Also,  $V_{m1} = 2\pi f N_1 \phi_{\max}$   
$$\Rightarrow V_{1rms} = V_1 = \frac{V_{m1}}{\sqrt{2}} = \sqrt{2}\pi f N_1 \phi_{\max} = 4.44 f N_1 \phi_{\max}$$
- Example: Coil 1 has 100 turns, operating at 60 Hz and  $V_{rms} = 230$  V. The core area is  $50 \text{ cm}^2$  and the core length is 25 cm. Find the maximum flux and flux density.

$$\phi_{\max} = 8.64 \times 10^{-3} \text{ Wb}$$

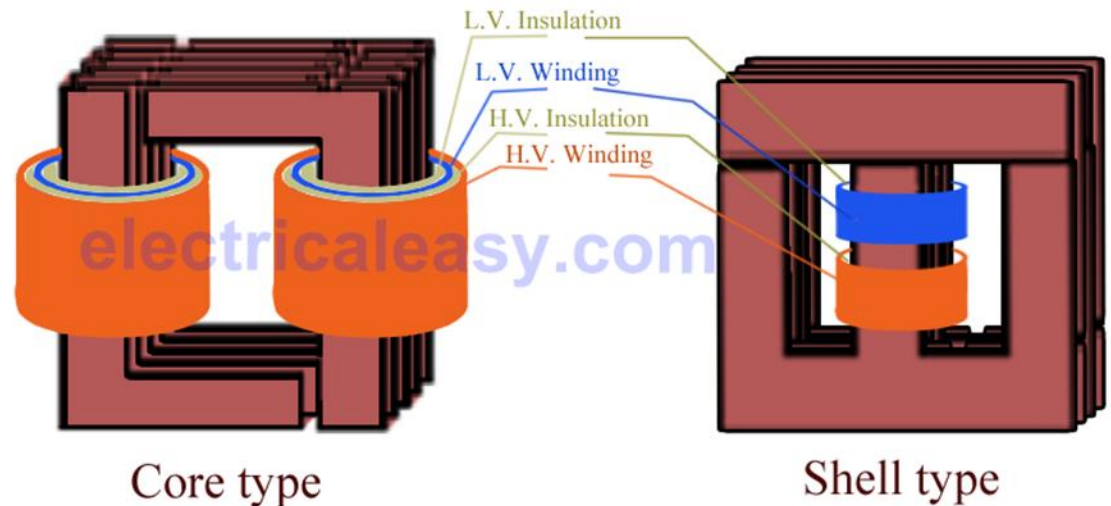
$$B_{\max} = \frac{\phi_{\max}}{A} = 1.728 \text{ Wb} / \text{m}^2$$



# POWER TRANSFORMER

- Given that  $H = 600$  A.turns/m at  $B_{\max}$ , find the maximum current.

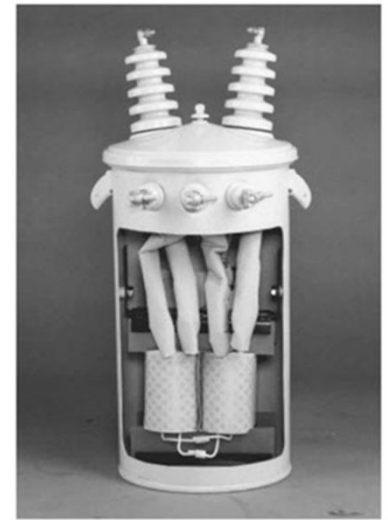
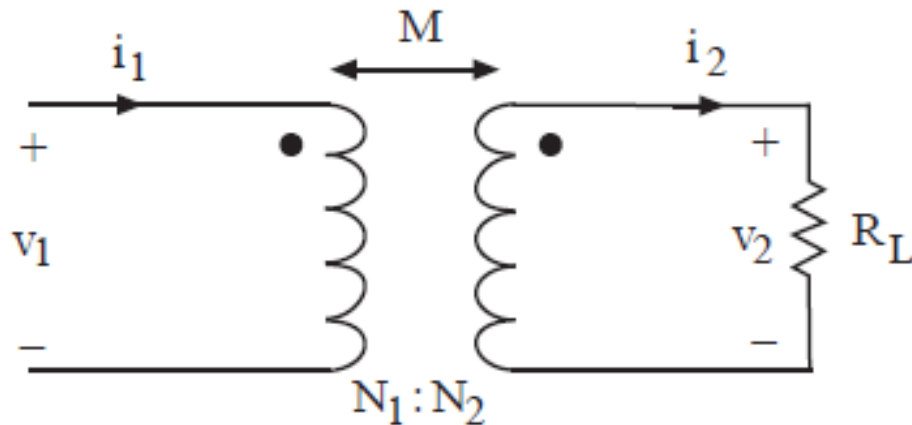
$$i_{\max} = \frac{H \ell}{N} = 1.5 \text{ A}$$



Source: electrical easy.com

# EQUIVALENT CIRCUIT MODEL

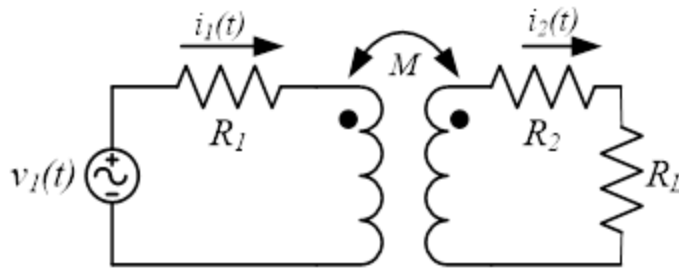
Consider the two-winding transformer. Let  $v_2$  be the voltage across  $R_L$ . Let the resistances of the primary and secondary windings be  $R_1$  and  $R_2$ , respectively. Viewing the two windings as coupled coils, the differential equations are written as



Source: [machineequipmentonline.com](http://machineequipmentonline.com)

# EQUIVALENT CIRCUIT MODEL

- Non-ideal:



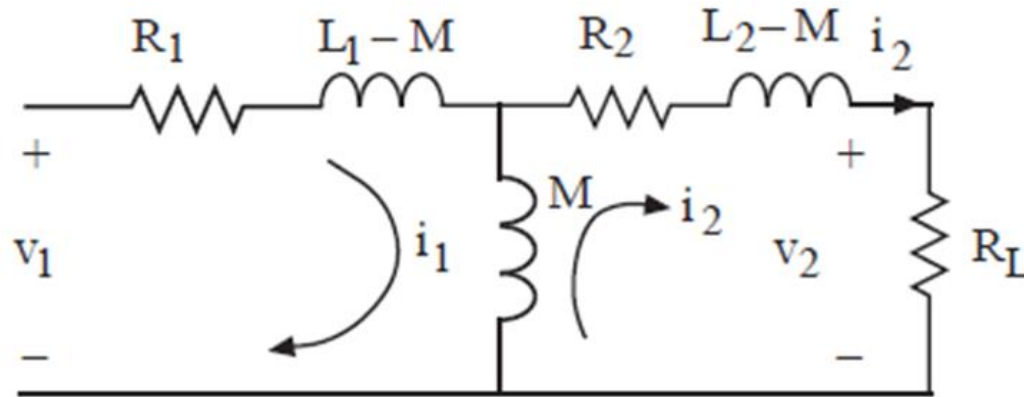
$$\begin{cases} v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \\ 0 = R_2 i_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} + i_2 R_L \\ v_2 = i_2 R_L \end{cases}$$

- Add and subtract  $M di_1/dt$  to the first equation and  $M di_2/dt$  to the second equation:

$$\begin{cases} v_1 = R_1 i_1 + (L_1 - M) \frac{di_1}{dt} + M \frac{di_1}{dt} - M \frac{di_2}{dt} \\ 0 = R_2 i_2 + (L_2 - M) \frac{di_2}{dt} + M \frac{di_2}{dt} - M \frac{di_1}{dt} + i_2 R_L \\ v_2 = i_2 R_L \end{cases}$$

# EQUIVALENT CIRCUIT MODEL

$$\begin{cases} v_1 = R_1 i_1 + (L_1 - M) \frac{di_1}{dt} + M \frac{d}{dt} (i_1 - i_2) \\ 0 = R_2 i_2 + (L_2 - M) \frac{di_2}{dt} + M \frac{d}{dt} (i_2 - i_1) + i_2 R_L \\ v_2 = i_2 R_L \end{cases}$$



What if  $(L_1 - M)$  or  $(L_2 - M) < 0$ ? There is something wrong.

# EQUIVALENT CIRCUIT MODEL

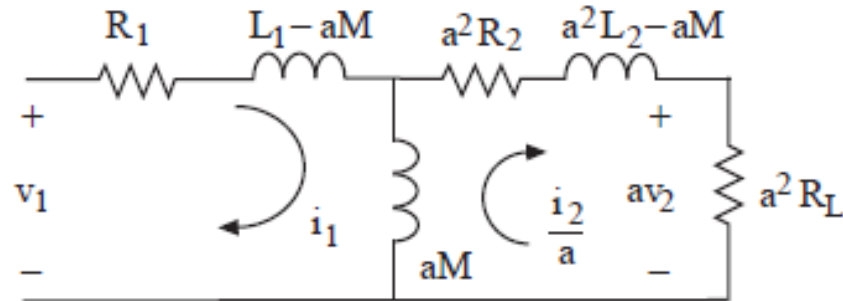
- We need to refer all impedances from side 2 to side 1:

$$i_2' = \frac{i_2}{a}, \quad Z_2' = a^2 Z_2, \quad M' = a M \text{ (since } M \text{ either sees } i_1 \text{ or } i_2).$$

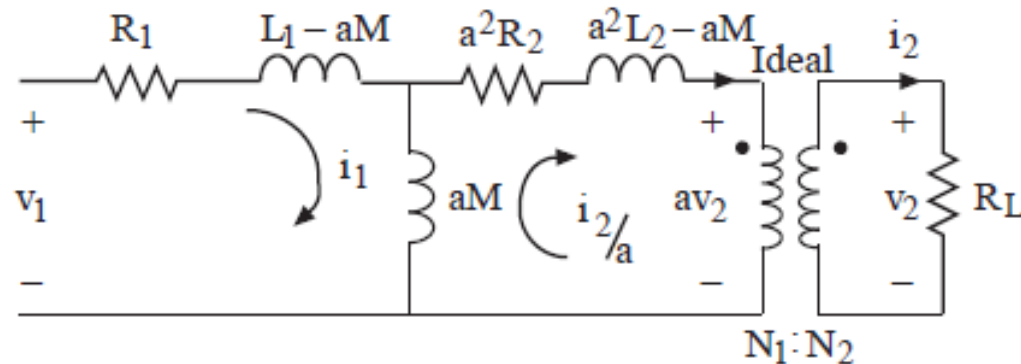
$$\left\{ \begin{array}{l} v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} - aM \frac{d}{dt} \left( \frac{i_2}{a} \right) \\ 0 = a^2 R_2 \left( \frac{i_2}{a} \right) + a^2 L_2 \frac{d}{dt} \left( \frac{i_2}{a} \right) - aM \frac{di_1}{dt} + \left( \frac{i_2}{a} \right) a^2 R_L \\ av_2 = \left( \frac{i_2}{a} \right) a^2 R_L \end{array} \right.$$

# EQUIVALENT CIRCUIT MODEL

- The equivalent circuit becomes:



- If we wish to preserve  $R_L$ , the circuit becomes:



# EQUIVALENT CIRCUIT MODEL

Losses:

Hysteresis losses due to the nonlinear multi-valued nature of the  $\phi-i$  relationship of the actual core.

$$\text{Hysteresis loss } P_h = K_h f B_m^n W$$

- $B_m$  the maximum flux density, and  $n$  (often assumed to be 1.6) and  $K_h$  are constants depending on the material of the core

# EQUIVALENT CIRCUIT MODEL

- Eddy current losses occurring in the laminations, resulting in an -type loss.

$$\text{Eddy current loss } P_e = K_e f^2 B_m^2 \quad W$$

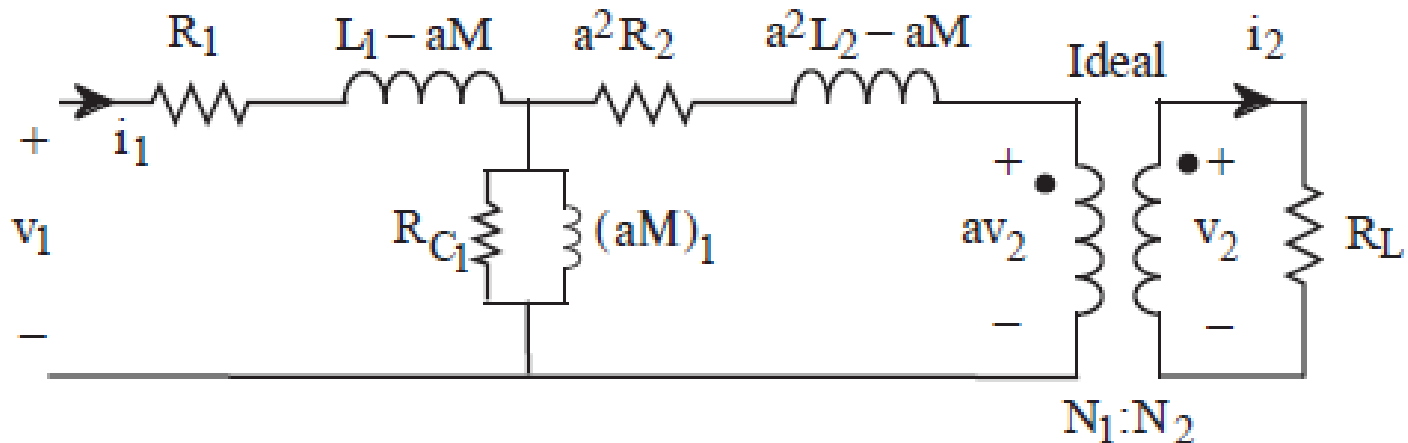
- $K_e$  is a constant that depends on the resistivity of the material of the laminations
- The sum of these two losses represents the constant losses for the transformer and depends only on the value of  $B_m$



# EQUIVALENT CIRCUIT MODEL

- $B_m$  depends on the applied voltage  $V_m$

Losses are represented by means of a resistance  $R_c$  in parallel with the magnetizing inductance  $aM$  .



# EQUIVALENT CIRCUIT MODEL

- What does  $(L_1 - a M)$  mean? (and similarly the secondary term)?

$$L_1 i_1 = N_1 \phi_{11}, \quad M i_1 = N_2 \phi_{21}$$

$$L_1 - aM = \frac{N_1 \phi_{11}}{i_1} - \frac{aN_2 \phi_{21}}{i_1} = \frac{N_1}{i_1} (\phi_{11} - \phi_{21}) = \frac{N_1}{i_1} \phi_{l1}$$

- Therefore,  $(L_1 - a M)$  is a leakage term.

# EQUIVALENT CIRCUIT MODEL

- $\omega(L_1 - aM) = X_{\ell_1}$  Leakage reactance of winding 1
- $\omega(aM) = X_{m_1}$  magnetizing reactance referred to winding 1
- $\omega(L_2 - M/a) = X_{\ell_2}$  Leakage reactance of winding 2
- $\omega(a^2 L_2 - aM) = a^2 X_{\ell_2}$  Leakage reactance of winding 2 referred to side 1

