


ECE330: Power Circuits & Electromechanics
Lecture 5. Wye-Delta Conversion

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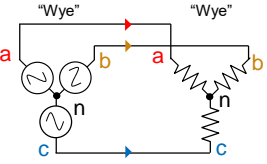
Schedule

- Fri 1/31: Three-phase power ← Ready for HW2
- Mon 2/3: Wye-Delta conversion ← Ready for HW3
- **Wed 2/5: Quiz 2 + Review** ← Ready for HW3
- Fri 2/7: Inductors

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Important

Last time: 3φ models



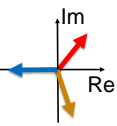
Assumption 1 (Balanced)

- Same phase-neutral voltage
- Same phase current

Assumption 2 (Positive sequence)

- Phase b lags phase a by 120°
- Phase c leads phase a by 120°

$\bar{V}_{an} = V \angle \theta_v$ $\bar{I}_a = I \angle \theta_i$
 $\bar{V}_{bn} = V \angle (\theta_v - 120^\circ)$ $\bar{I}_b = I \angle (\theta_i - 120^\circ)$
 $\bar{V}_{cn} = V \angle (\theta_v + 120^\circ)$ $\bar{I}_c = I \angle (\theta_i + 120^\circ)$



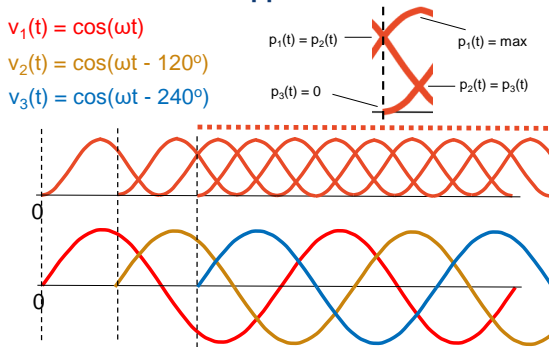
Implications (Assumption 1)

- Zero neutral current
- Identical load per phase
- Total power = **3x** power per phase

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Last time: Power ripple cancellation

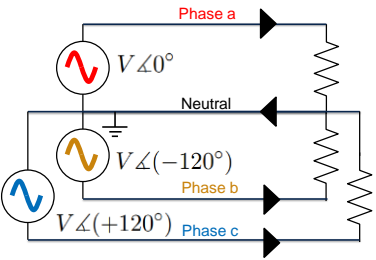
$v_1(t) = \cos(\omega t)$
 $v_2(t) = \cos(\omega t - 120^\circ)$
 $v_3(t) = \cos(\omega t - 240^\circ)$



$p_1(t) = p_2(t)$ $p_1(t) = \max$
 $p_2(t) = 0$ $p_2(t) = p_3(t)$

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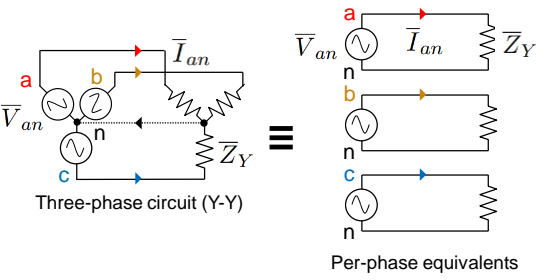
Last time: Neutral current cancellation



$\bar{S}_{tot} = \bar{S}_a + \bar{S}_b + \bar{S}_c = 3 \cdot \bar{S}_a$
 $|\bar{I}_n| = |\bar{I}_a + \bar{I}_b + \bar{I}_c| = 0$

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Last time: Analyze 3φ using tools of 1φ



Three-phase circuit (Y-Y) Per-phase equivalents

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Today: Wye and Delta

Assumption 1 (Balanced)

- Same phase-neutral voltage
- Same phase current

Assumption 2 (Positive sequence)

- Phase b lags phase a
- Phase c leads phase a

$$\bar{V}_{an} = V \angle \theta_v$$

$$\bar{V}_{bn} = V \angle (\theta_v - 120^\circ)$$

$$\bar{V}_{cn} = V \angle (\theta_v + 120^\circ)$$

$$\bar{I}_a = I \angle \theta_i$$

$$\bar{I}_b = I \angle (\theta_i - 120^\circ)$$

$$\bar{I}_c = I \angle (\theta_i + 120^\circ)$$

Why Delta?

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Goal: Analyze Delta using tools of Wye

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Today

- Review: Line voltage & line current
- Why delta? (Wye?)
- Delta to Wye equivalence
- Example problems

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Specifying 3-p voltages

Important

Phase-to-phase ("Line voltage") $|\bar{V}_{ab}| = |\bar{V}_{bc}| = |\bar{V}_{ca}|$ Δ

Phase-to-neutral $|\bar{V}_{an}| = |\bar{V}_{bn}| = |\bar{V}_{cn}|$

Ratio $|\bar{V}_{ab}| = \sqrt{3} |\bar{V}_{an}|$ (PP is bigger)

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Specifying 3-p currents

Important

Phase-to-phase $|\bar{I}_{ab}| = |\bar{I}_{bc}| = |\bar{I}_{ca}|$

Phase-to-neutral ("Line current") $|\bar{I}_{an}| = |\bar{I}_{bn}| = |\bar{I}_{cn}|$ $\Delta \Delta \Delta$

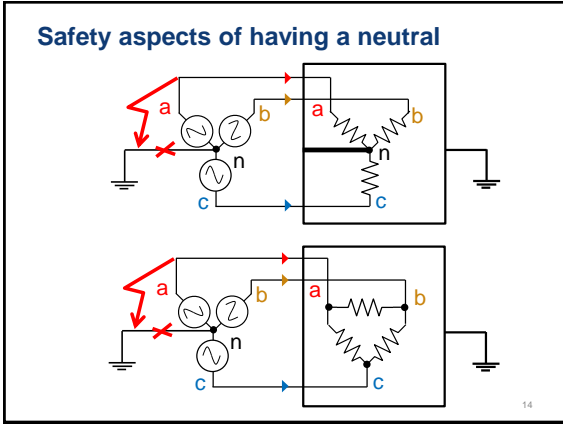
Ratio $|\bar{I}_{an}| = \sqrt{3} |\bar{I}_{ab}|$ (PN is bigger)

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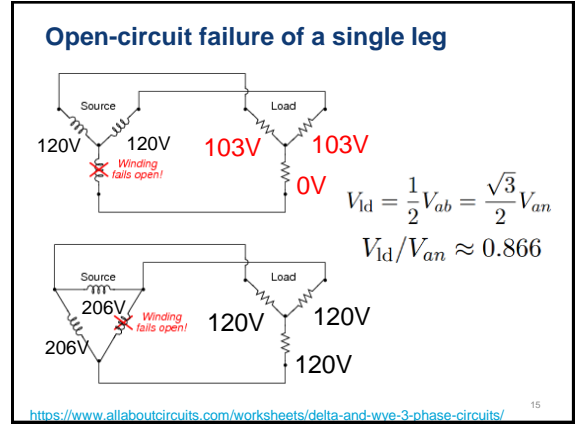
Today

- Review: Line-voltage & line current
- Why delta? (Wye?) ← Good to know
Won't be tested
- Delta to Wye equivalence
- Example problems

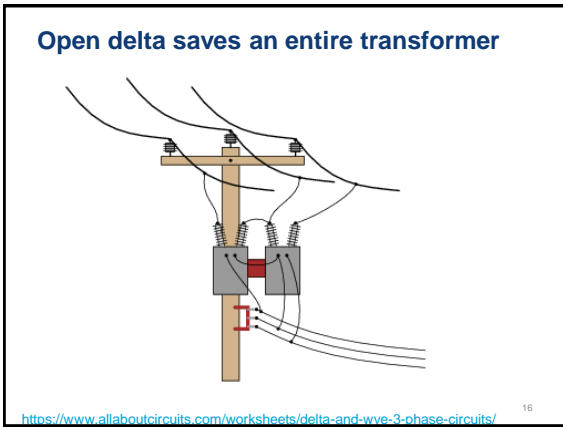
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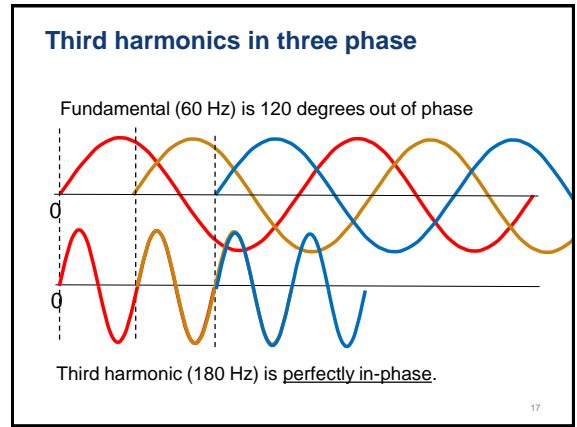
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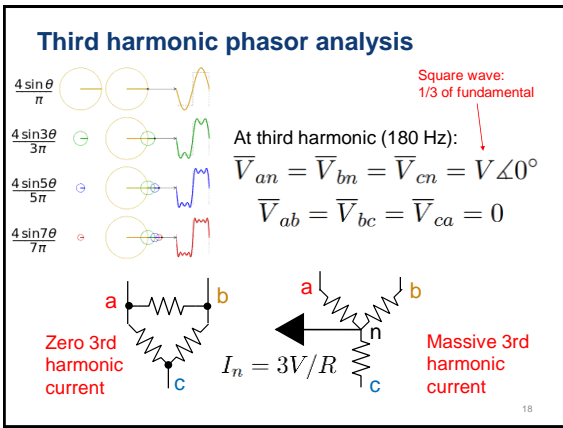
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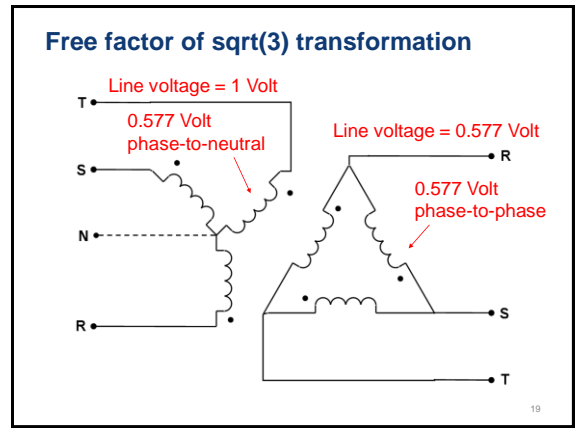
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Source-load practical considerations

		Load	
		wye	delta
Source	Wye	Wye – wye (Yy)	Delta – wye (Dy)
	Delta	Wye – delta (Wd)	Delta – delta (Dd)

Delta

- Robust
- Low-cost when balanced
- 3rd harmonic cancelation

Wye

- Safe
- Imbalance gracefully
- Flexible voltages

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Today

- Review: Line voltage & line current
- Why delta? (Wye?)
- Delta to Wye equivalence
- Example problems

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Notion of equivalence

Important

Indistinguishable

Indistinguishable

Objective: Identical 3-port characteristics
Method: Match complex power

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Theorem. Wye-Delta equivalence

Important

\bar{S}_Δ

\bar{S}_Y

$\bar{S}_\Delta = \bar{S}_Y$

Power

\bar{I}_Δ

\bar{I}_Y

$\sqrt{3}|\bar{I}_\Delta| = |\bar{I}_Y|$

Current

PowFac_Δ = PowFac_Y

\bar{Z}_Δ

\bar{Z}_Y

$\bar{Z}_\Delta = 3\bar{Z}_Y$

Impedance

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Proof of power equivalence

\bar{S}_Δ

\bar{S}_Y

Power

$\bar{S}_\Delta = \bar{S}_Y$

Proof.
 Total power is 3x per-phase power. If same per-phase power, then same total power. □

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Proof of current equivalence

\bar{I}_Δ

\bar{I}_Y

Current

$\sqrt{3}|\bar{I}_\Delta| = |\bar{I}_Y|$

PowFac_Δ = PowFac_Y

Proof. Want same per-phase power $\bar{S}_\Delta = \bar{S}_Y$.
 $\bar{S}_\Delta = \bar{V}_\Delta \bar{I}_\Delta^* = V_\Delta I_\Delta \angle \phi_\Delta$
 $\bar{S}_Y = \bar{V}_Y \bar{I}_Y^* = V_Y I_Y \angle \phi_Y$
 But $V_\Delta = \sqrt{3}V_Y$
 Hence $\sqrt{3}I_\Delta = I_Y, \phi_\Delta = \phi_Y$ □

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Proof of impedance equivalence

Impedance
 $\bar{Z}_\Delta = 3\bar{Z}_Y$

Proof. Want same per-phase power $\bar{S}_\Delta = \bar{S}_Y$.
 $\bar{S}_\Delta = |\bar{V}_\Delta|^2 / \bar{Z}_\Delta$ $\bar{S}_Y = |\bar{V}_Y|^2 / \bar{Z}_Y$
 But $V_\Delta = \sqrt{3}V_Y$
 Hence $\bar{Z}_\Delta = 3\bar{Z}_Y$ \square

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Theorem. Wye-Delta equivalence Important

Power
 $\bar{S}_\Delta = \bar{S}_Y$

Current
 $\sqrt{3}|\bar{I}_\Delta| = |\bar{I}_Y|$
 PowFac $_\Delta$ = PowFac $_Y$

Impedance
 $\bar{Z}_\Delta = 3\bar{Z}_Y$

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Today

- Review: Line voltage & line current
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- Delta to Wye equivalence
- Example problems

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Warm-up

What is the equivalent per-phase Wye impedance?

$$2j + \left(\frac{1}{-2j} + \frac{1}{j} \right)^{-1} = 4j$$

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Example 1

2.16) A 345 kV three-phase line supplies 750 MVA at 0.8 PF lagging to a three-phase load which is delta connected.

- Find the complex impedance per phase. "equivalent Delta current"
- Find the magnitudes of the line and phase currents.
- Compute real and reactive power per phase.
- Compute the total complex power.

- 1 → Draw and label the 3-phase circuit diagram
- 2 → Draw and label the equivalent wye-wye diagram
- 3 → Draw and label the equivalent 1-phase diagram
- 4 → Solve the 1-phase problem
- 5 → Convert back to wye-wye, then back to orig 3-phase

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Example 2

2.19) A three-phase wye-connected load draws 120 kW at a PF of 0.85 lagging from a 440 V (line-line) three-phase system. Three capacitors are connected in delta across the load. The kVAR of each capacitor is 50/3.

- Find the line current before and after the capacitors are added.
- The new PF.
 (Answer: $I_{new} = 160.6$ A; PF = 0.98 lagging)

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