ECE 333 Green Electric Energy - Quiz 4

Thursday, October 19, 2017

Duration: 20 minutes

Name: ____ **last 4 digits of your UIN:**_______

Closed book, closed notes, cell phones are not allowed.

Show all you work and always indicate the units, as appropriate.

Problem 1 [100 points]:

Suppose a wind turbine has a cut-in wind speed of 7 *m/s* , a rated wind speed of 15 *m/s* and a cut-out wind speed of 20 *m/s*. Take the air density as 1.225 kg/m^3.

a. [20 points] State the general expression of the Weibull *p.d.f.* $f_V(v)$ of the wind speed

b. [30 points] Determine the average wind speed and the average wind power density if the winds have the Rayleigh statistics with scale parameter c=11

c. [25 points] Determine how many hours per year in average the turbine will be shut down because of excessively high-speed winds

d. [25 points] Compute how much energy per year would be produced for winds blowing at or above 15 *m/s* if the wind turbine has rated power of 1 *MW*

Problem 1 Solution

a. The general expression of the Weibull *p.d.f.* of the wind speed

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^k} \qquad \begin{array}{l} k = shape\ parameter \\ c = scale\ parameter \end{array}$$

b. For $k = 2$, the *Weibull distribution* is called the *Rayleigh p.d.f.*

$$f(v) = \frac{2v}{c^2} e^{-\left(\frac{v}{c}\right)^2}$$

The average wind speed

$$\bar{v} = \int_0^\infty v\, f_V\, dv = 2\int_0^\infty \left(\frac{v}{c}\right)^2 e^{-\left(\frac{v}{c}\right)^2} dv = \frac{\sqrt{\pi}}{2} c = 9.746\ m/s$$

The average wind power density

$$E(p) = E(\frac{1}{2}\rho V^3) = \frac{1}{2}1.225 \cdot E(V^3) \approx \frac{1}{2}1.225 \cdot 1.91 \cdot \bar{v}^3 = \frac{1}{2}1.225 \cdot 1.91 \cdot 9.74^3 = 1082\ W/m^2$$

c. if the turbine is shut down because of excessively high-speed winds, the probability of this event is equal to the probability that the wind speed is bigger than 20 *m/s*

$$F_V\left(V \geq 20\right)\Big|_{Rayleigh} = 1 - F_V\left(V \leq 20\right)\Big|_{Rayleigh} = e^{-\left(\frac{20}{11}\right)^2} = 3.667 \times 10^{-2}$$

the average hours over a year when the wind speed is bigger than 25 m/s

$$8760 \cdot F_V\left(V \geq 20\right)\Big|_{Rayleigh} = 321\ hours$$

thus there are 321 hours per year in average when the turbine will be shut down because of excessively high-speed wind

d. the power output of wind turbine is 1 *MW* when the wind speed is smaller than 20 m/s and bigger than 15 m/s and 0 when the wind speed is larger than 20 m/s. Thus the average hours over a year when the wind speed is smaller than 20 m/s and bigger than 15 m/s:

$$8760 \cdot (F_V\left(V \leq 20\right)\Big|_{Rayleigh} - F_V\left(V \leq 15\right)\Big|_{Rayleigh})$$

$$= 8760 \cdot [(1 - e^{-\left(\frac{20}{11}\right)^2}) - (1 - e^{-\left(\frac{15}{11}\right)^2})]$$

$$= 8760 \cdot 0.1191$$

$$= 1043\ hours$$

Thus the average energy produced by the wind at or above 15 m/s **1043** ***MWh***