# ECE 333 – Green Electric Energy

## 10. Energy Economics Concepts

**George Gross**

**Department of Electrical and Computer Engineering**

**University of Illinois at Urbana–Champaign**

# ENERGY ECONOMICS CONCEPTS

- The economic evaluation of a renewable energy resource requires a meaningful quantification of the cost elements
  - fixed costs
  - variable costs
- We use engineering economics notions for this purpose since they provide the means to compare on a consistent basis
  - two different projects; or,
  - the costs with and without a given project

# TIME VALUE OF MONEY

- ☐ Basic underlying notion: a dollar today is not the same as a dollar in a year
- ☐ We represent the time value of money by the standard approach of *discounted cash flows*
- ☐ The notation is

$$P = \textit{principal}$$

$$i = \textit{interest value}$$

- ☐ We use the convention that every payment occurs at the *end of a period*

# SIMPLE EXAMPLE

loan $P$ for $1$ $year$

repay $P + iP = P(1+i)$ at the end of $1$ $year$

year $0$ $P$

year $1$ $P(1+i)$

loan $P$ for $n$ $years$

year $0$ $P$

year $1$ $(1+i)\ P$ repay/reborrow

year $2$ $(1+i)^2 P$ repay/reborrow

year $3$ $(1+i)^3 P$ repay/reborrow

$\vdots$ $\vdots$ $\vdots$

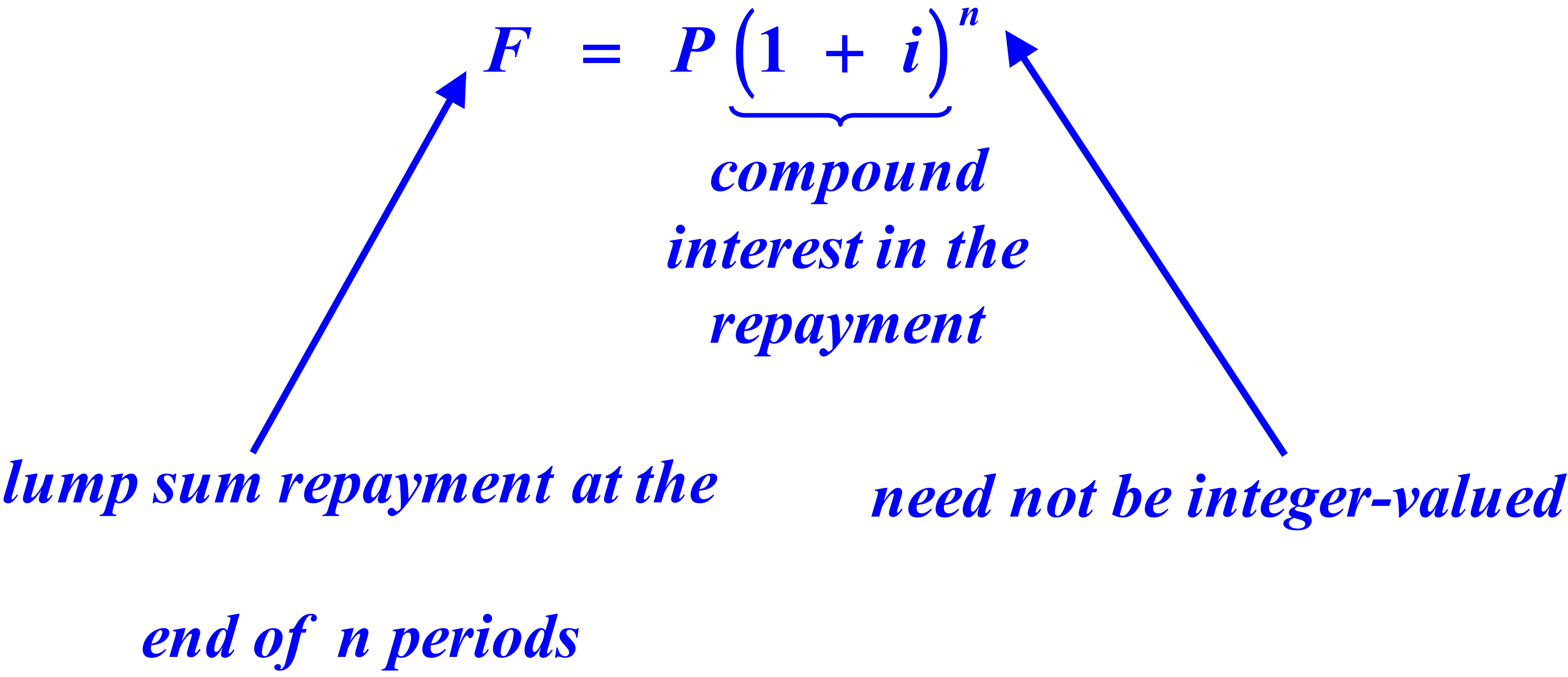
year $n$ $(1+i)^n P$ repay

# COMPOUND INTEREST

| *end of period* | *amount owed* | *interest for next period* | *amount owed at the beginning of the next period* |
|---|---|---|---|
| 0 | $P$ | $P\,i$ | $P + P\,i = P\,(1+i)$ |
| 1 | $P\,(1+i)$ | $P\,(1+i)\,i$ | $P\,(1+i) + P\,(1+i)\,i = P\,(1+i)^2$ |
| 2 | $P\,(1+i)^2$ | $P\,(1+i)^2\,i$ | $P\,(1+i)^2 + P\,(1+i)^2\,i = P\,(1+i)^3$ |
| 3 | $P\,(1+i)^3$ | $P\,(1+i)^3\,i$ | $P\,(1+i)^3 + P\,(1+i)^3\,i = P\,(1+i)^4$ |
| ⋮ | ⋮ | | |
| $n$–1 | $P\,(1+i)^{n-1}$ | $P\,(1+i)^{n-1}\,i$ | $P\,(1+i)^{n-1} + P\,(1+i)^{n-1}\,i = P\,(1+i)^n$ |
| $n$ | $P\,(1+i)^n$ | ▨ | ▨ |

**the value in the last column at the *e.o.p.* (*k*-1) provides the amount in the first column for the *period k***

# TERMINOLOGY



$$F = P\underbrace{(1 + i)}_{\text{compound interest in the repayment}}^{n}$$

*lump sum repayment at the end of n periods*

*need not be integer-valued*

# TERMINOLOGY

- ☐ We call $(1+i)^n$ the single payment compound amount factor
- ☐ We define

$$\beta \triangleq (1+i)^{-1}$$

- ☐ Then,

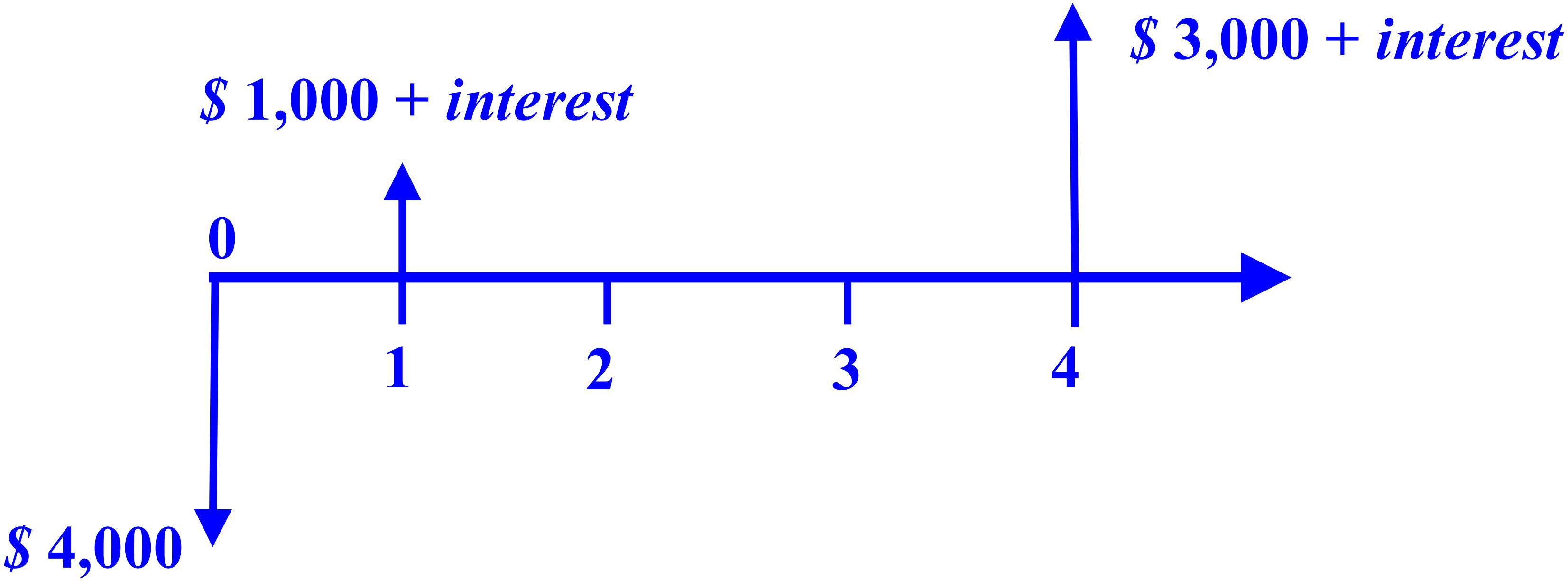
$$\beta^n = (1+i)^{-n}$$

is the single payment present worth factor

- ☐ $F$ denotes the *future worth*; $P$ denotes the *present worth* or *present value* at interest $i$ of a future sum $F$

# EXAMPLE 1

- Consider a loan of *$* 4,000 at 8 % interest to be repaid in two installments
  - *$* 1,000 and interest at the *e.o.y.* 1
  - *$* 3,000 and interest at the *e.o.y.* 4



# EXAMPLE 1

- The cash flows are

  - *e.o.y.* 1: $1{,}000 + 4{,}000\,(.08) = \$\,1{,}320.00$

  - *e.o.y.* 4: $3{,}000\,(1 + .08)^3 = \$\,3{,}779.14$

- Note that the loan is made in year 0 *present $*, but the repayments are in year 1 and year 4 *future $*

# EXAMPLE 2

- ☐ Given

$$P = \$1,000 \qquad \text{and} \qquad i = .12$$

then

$$P\left(1+i\right)^{5} = \$1,000\left(1+.12\right)^{5} = \$1,762.34 = F$$

- ☐ We say that with the cost of money of 12 %, $P$ and $F$ are *equivalent* in the sense that \$ 1,000 today has the same worth as \$ 1,762.34 in 5 years

# EXAMPLE 3

- ☐ Consider an investment that returns

  $\$\,1{,}000$ at the *e.o.y.* 1

  $\$\,2{,}000$ at the *e.o.y.* 2

  $i = 10\%$ ← rate at which money can be freely lent or borrowed

- ☐ We evaluate $P$

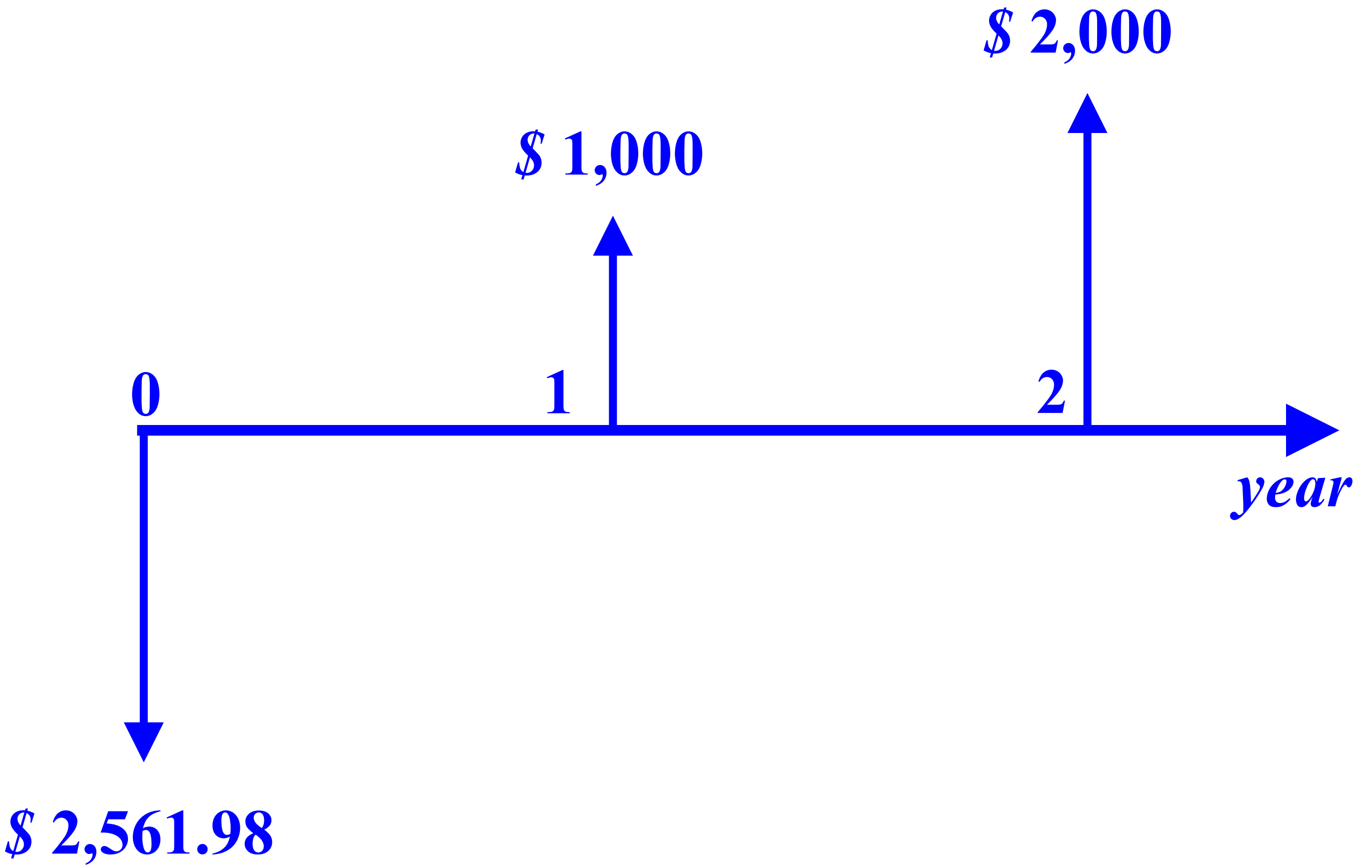
$$P = \$\,1{,}000\underbrace{\left(1+.1\right)^{-1}}_{\beta} + \$\,2{,}000\underbrace{\left(1+.1\right)^{-2}}_{\beta^{2}}$$

$$= \$\,909.9 + \$\,1{,}652.09$$

$$= \$\,2{,}561.98$$

# EXAMPLE 3

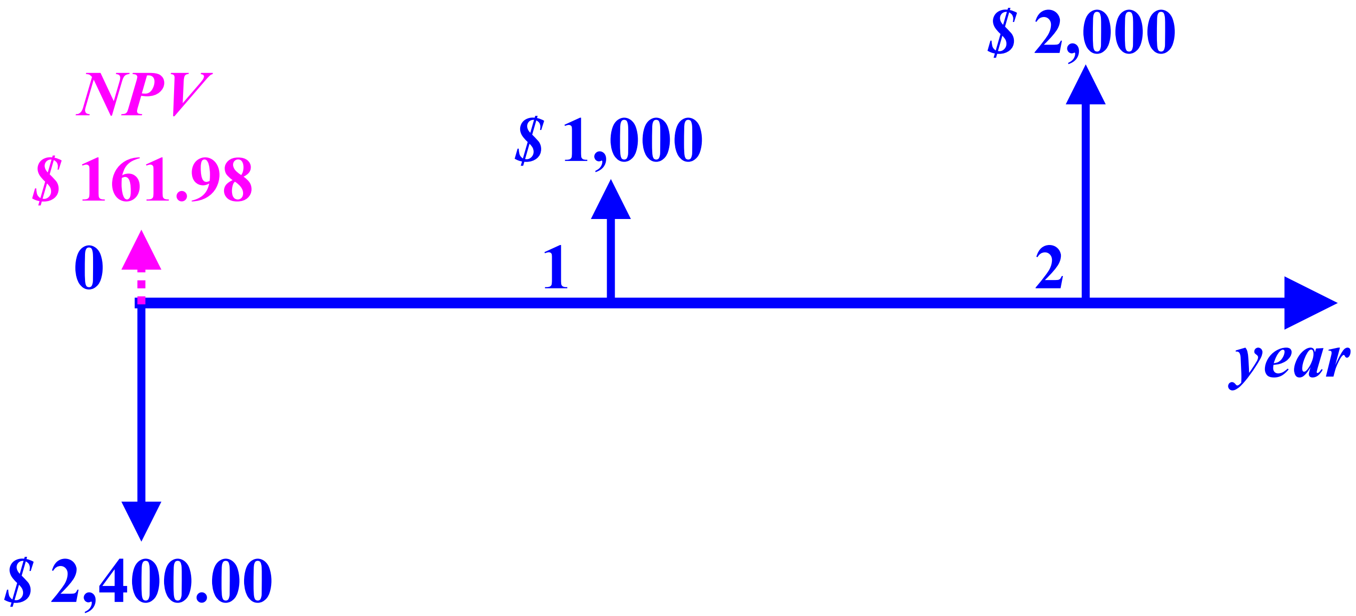
- **We review this example with a *cash–flow diagram***



# EXAMPLE 3

- ☐ Next, suppose that this investment requires \$ 2,400 now and so at 10 % we say that the investment has a *net present value* given by

$$NPV = \$\,2{,}561.98 - \$\,2{,}400 = \$\,161.98$$



# CASH FLOWS

- A *cash–flow* is basically a transfer of an amount $A_t$ from one entity to another at the *e.o.p. t*
- We consider the cash–flow set $\left\{ A_0, A_1, A_2, \ldots, A_n \right\}$
- This set corresponds to the set of the transfers at the end of the periods in $\left\{ 0, 1, 2, \ldots, n \right\}$

# CASH FLOWS

- ☐ We associate the transfer $A_t$ at the *e.o.p.* $t$, $t = 0, 1, 2, \ldots, n$
- ☐ The convention for cash flows is

  $+$ *inflow*

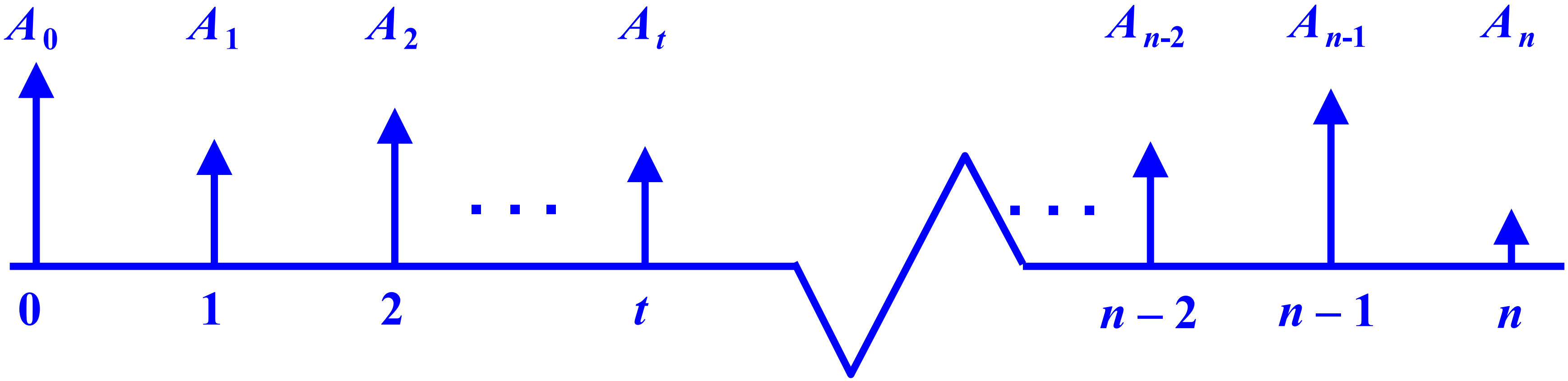
  $-$ *outflow*

- ☐ Each cash flow requires the specification of:
  - ○ amount;
  - ○ time; and,
  - ○ its sign

# CASH FLOWS: FUTURE WORTH

- ☐ Given a cash–flow set $\left\{ A_0, A_1, A_2, \ldots, A_n \right\}$ we define the future worth $F_n$ of the cash flow set at the *e.o.y.* $n$ as

$$F_n = \sum_{t=0}^{n} A_t \left(1 + i\right)^{n-t}$$



# CASH FLOWS : FUTURE WORTH

- ☐ Note that each cash flow $A_t$ in the $(n+1)$ period set contributes differently to $F_n$:

$$
\begin{array}{ccc}
A_0 & \rightarrow & A_0\left(1+i\right)^{n} \\
A_1 & \rightarrow & A_1\left(1+i\right)^{n-1} \\
A_2 & \rightarrow & A_2\left(1+i\right)^{n-2} \\
\vdots & & \vdots \\
A_t & \rightarrow & A_t\left(1+i\right)^{n-t} \\
\vdots & & \vdots \\
A_n & \rightarrow & A_n
\end{array}
$$

# CASH FLOWS : PRESENT WORTH

- ☐ We define the present worth $P$ of the cash–flow set as

$$P = \sum_{t=0}^{n} A_t \beta^t = \sum_{t=0}^{n} A_t (1+i)^{-t}$$

- ☐ Note that

$$P = \sum_{t=0}^{n} A_t (1+i)^{-t}$$

$$= \sum_{t=0}^{n} A_t (1+i)^{-t} \underbrace{(1+i)^{n} (1+i)^{-n}}_{1}$$

# CASH FLOWS

$$= \underbrace{(1+i)^{-n}}_{\beta^n} \underbrace{\sum_{t=0}^{n} A_t (1+i)^{n-t}}_{F_n}$$

$$= \beta^n F_n$$

**or, equivalently,**

$$F_n = (1+i)^n P$$

# UNIFORM CASH – FLOW SET

- ☐ Consider the cash – flow set $\left\{ A_1, A_2, \ldots, A_n \right\}$ with

$$A_t = A \qquad t = 1, 2, \ldots, n$$

- ☐ Such a set is called an *equal payment cash flow set*

- ☐ We compute the present worth at $t = 0$

$$P = \sum_{t=1}^{n} A_t \beta^{t} = A \sum_{t=1}^{n} \beta^{t} = A\beta \left[ 1 + \beta + \beta^{2} + \ldots + \beta^{n-1} \right]$$

# UNIFORM CASH – FLOW SET

- ☐ Now, for $0 < \beta < 1$ , we have the identity

$$\sum_{j=0}^{\infty} \beta^{j} = \frac{1}{1 - \beta}$$

- ☐ It follows that

$$1 + \beta + \ldots + \beta^{n-1} = \sum_{j=0}^{\infty} \beta^{j} - \beta^{n} \overbrace{\left[ 1 + \beta + \beta^{2} + \ldots + \beta^{n-1} + \ldots \right]}^{\sum_{j=0}^{\infty} \beta^{j}}$$

$$= \left( 1 - \beta^{n} \right) \sum_{j=0}^{\infty} \beta^{j}$$

# UNIFORM CASH – FLOW SET

$$= \frac{1-\beta^{n}}{1-\beta}$$

- ❑ **Therefore**

$$P = A\beta \frac{1-\beta^{n}}{1-\beta}$$

- ❑ **But**

$$\beta = (1+d)^{-1} \;,$$

**where $d$ is the interest or discount rate and so**

# UNIFORM CASH – FLOW SET

$$1 - \beta = 1 - \frac{1}{1+d} = \frac{d}{1+d} = \beta d$$

- ☐ **We write**

$$P = A \frac{1 - \beta^{n}}{d}$$

**and we call** $\frac{1 - \beta^{n}}{d}$ **the** ***equal payment series present worth factor***

# EQUIVALENCE

- ☐ **We consider two cash – flow sets**

  $$\left\{A_t^a : t = 0,1,2,\,\ldots\,,n\right\} \text{ and } \left\{A_t^b : t = 0,1,2,\,\ldots\,,n\right\}$$

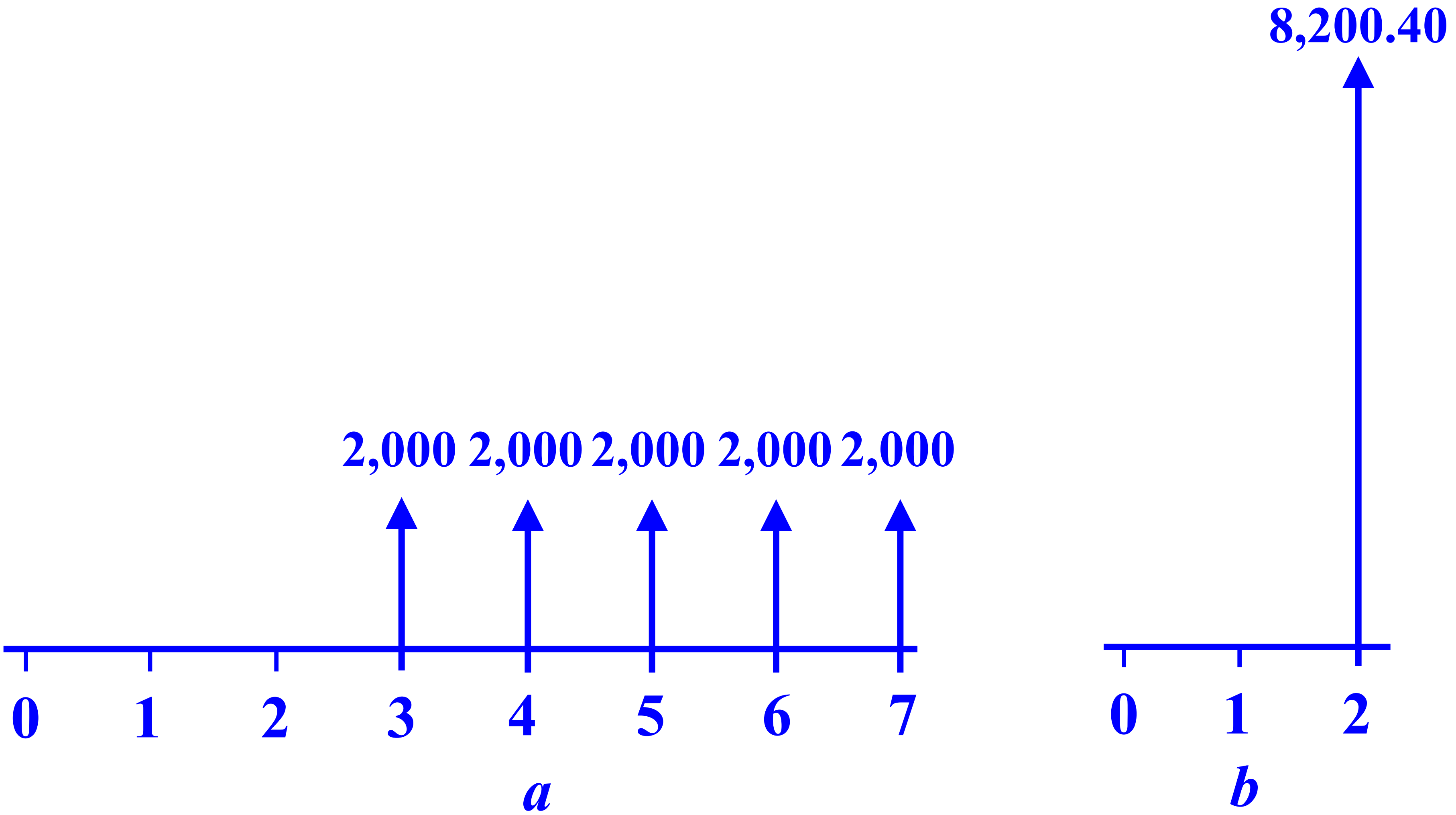
  **under a given discount rate $d$**

- ☐ **We say $\left\{A_t^a\right\}$ and $\left\{A_t^b\right\}$ are *equivalent* cash – flow sets if and only if**

  $$F_m \text{ of } \left\{A_t^a\right\} = F_m \text{ of } \left\{A_t^b\right\} \text{ for every value of } m$$

# EQUIVALENCE EXAMPLE

- Consider the two cash–flow sets under $d = 7\%$



# EQUIVALENCE

- ☐ We compute

$$P^a = 2,000 \sum_{t=3}^{7} \beta^t = 7,162.55$$

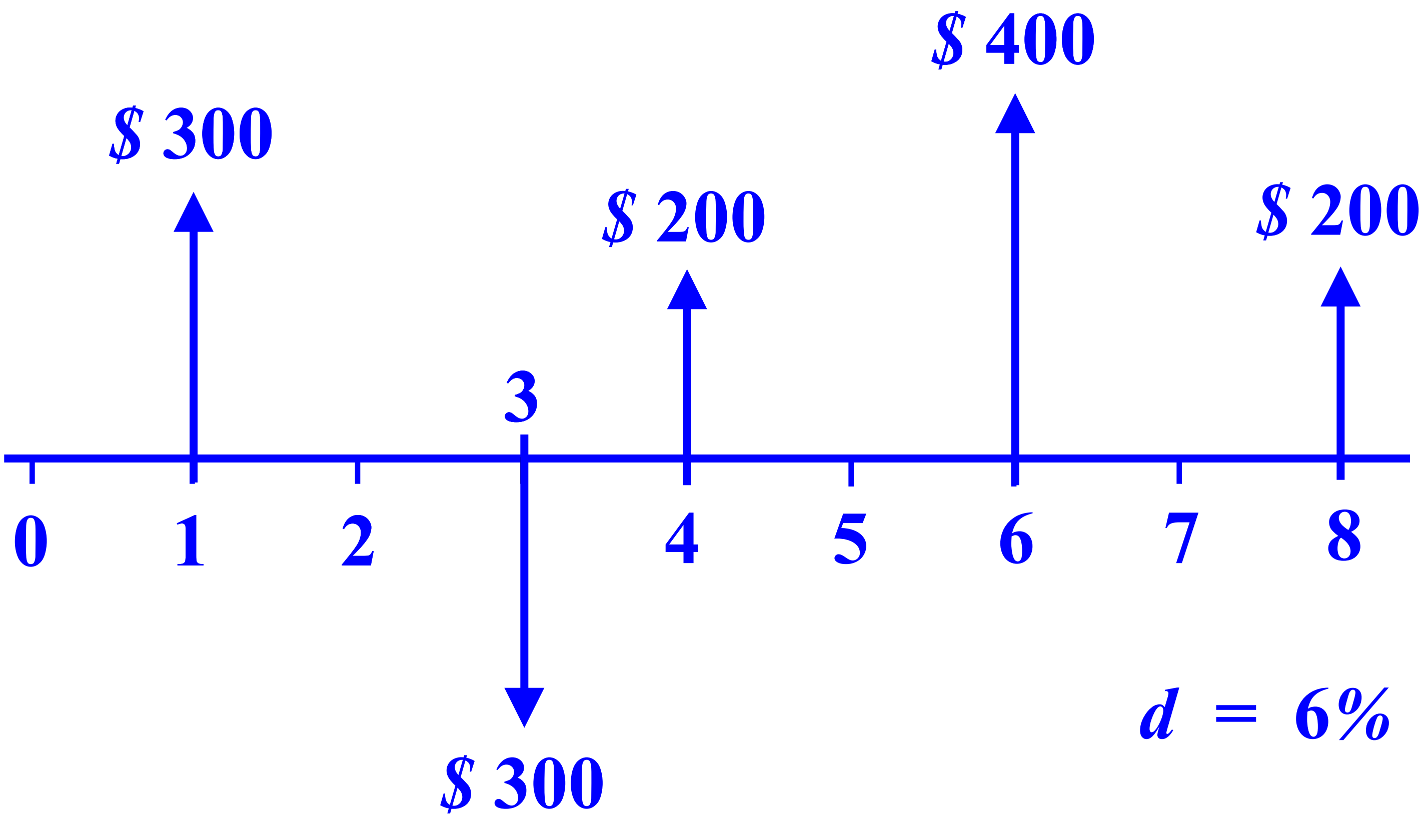
and

$$P^b = 8,200.40 \ \beta^2 = 7,162.55$$

- ☐ Therefore, $\left\{ A_t^a \right\}$ and $\left\{ A_t^b \right\}$ are equivalent cash flow sets under $d = 7\%$

# EXAMPLE

- ☐ Consider the cash–flow set illustrated below



- ☐ We compute $F_8$ at $t = 8$ for $d = 6\ \%$

# EXAMPLE

$$F_8 = 300\left(1+.06\right)^7 - 300\left(1+.06\right)^5 +$$

$$200\left(1+.06\right)^4 + 400\left(1+.06\right)^2 + 200$$

$$= \$951.56$$

☐ **We also compute** $P$

# EXAMPLE

$$P = 300\left(1+.06\right)^{-1} - 300\left(1+.06\right)^{-3} +$$

$$200\left(1+.06\right)^{-4} + 400\left(1+.06\right)^{-6} + 200\left(1+.06\right)^{-8}$$

$$= \$\,597.04$$

- ☐ We check that at $d = 6\%$

$$F_8 = 597.04\left(1+.06\right)^{8} = \$\,951.56$$

# DISCOUNT RATE

- ☐ The interest rate $i$ is, typically, referred to as the *discount rate* and is denoted by $d$
- ☐ In the conversion of the future amount $F$ to the present worth $P$, we view the *discount rate* as the interest rate that may be earned from the best investment alternative
- ☐ A postulated savings of $\$10{,}000$ in a project in 5 years is worth at present

$$P = F_5 \beta^5 = 10{,}000 \left(1 + d\right)^{-5}$$

# DISCOUNT RATE

- For $d = 0.1$

$$P = \$\, 6{,}201,$$

while for $d = 0.2$

$$P = \$\, 4{,}019$$

- In general, for a specified future worth, the *lower the discount factor, the higher the present worth* is

# DISCOUNT RATE

- We may state this notion slightly differently; the lower the discount factor, the more valuable a future payoff becomes

- The present worth of a set of costs under a given discount rate is called the *life–cycle costs*, an important term in economic assessment studies

# EXAMPLE

- ☐ We consider the purchase of two 100–*hp* motors – *a* and *b* – to be used over a 20–*year* period; the given discount rate is 10 %
- ☐ The relative merits of *a* and *b* are

| *motor* | *costs ( $ )* | *load ( kW )* |
|---|---|---|
| *a* | 2,400 | 79.0 |
| *b* | 2,900 | 77.5 |

# EXAMPLE

- ☐ The motor is used 1,600 *hours per year* and electricity costs are constant at 0.08 *$/kWh*
- ☐ We evaluate yearly energy costs for *the two motors*

$$A_t^a = (79.0\ kW)(1600\ h)(.08\ \$/kWh) = \$\ 10{,}112$$

$$t = 1, 2, \ldots, 20$$

$$A_t^b = (77.5\ kW)(1600\ h)(.08\ \$/kWh) = \$\ 9{,}920$$

# EXAMPLE

❑ **We next evaluate the present worth of $a$ and $b$**

$$P^{a} = 2{,}400 + 10{,}112 \boxed{\sum_{t=1}^{20} (1.1)^{-t}} \longleftarrow 8.5136$$

$$= \$\,88{,}489$$

$$P^{b} = 2{,}900 + 9{,}920 \boxed{\sum_{t=1}^{20} (1.1)^{-t}} \longleftarrow 8.5136$$

$$= \$\,87{,}354$$

# EXAMPLE

- The difference

$$P^{a} - P^{b} = 88,489 - 87,354 = \$1,135$$

- Therefore, the purchase of motor *b* results in the savings of $\$ 1,135$ under the specified $10\ \%$ discount rate due to the use of the smaller load consumption motor over the $20$–*year* horizon

# INFINITE HORIZON CASH – FLOW SETS

- ☐ Consider a uniform cash–flow set with $n \to \infty$

$$\left\{ A_t = A : t = 0, 1, 2, \ldots \right\}$$

- ☐ Then,

$$P = A \frac{\left(1 - \beta^n\right)}{d} \xrightarrow[n \to \infty]{} A \frac{1}{d}$$

- ☐ For an infinite horizon uniform cash–flow set

# INFINITE HORIZON CASH – FLOW SETS

$$\frac{A}{P} = d$$

- ☐ We may view $d$ as the *capital recovery factor* with the following interpretation:

  for an initial investment of $P$, the amount

  $$d * P = A$$

  is recovered annually in terms of returns on the investment $A$

# INTERNAL RATE OF RETURN

- ☐ We consider a cash–flow set

$$\left\{ A_t = A : t = 0, 1, 2, \ldots , n \right\}$$

- ☐ The value of $d$ for which

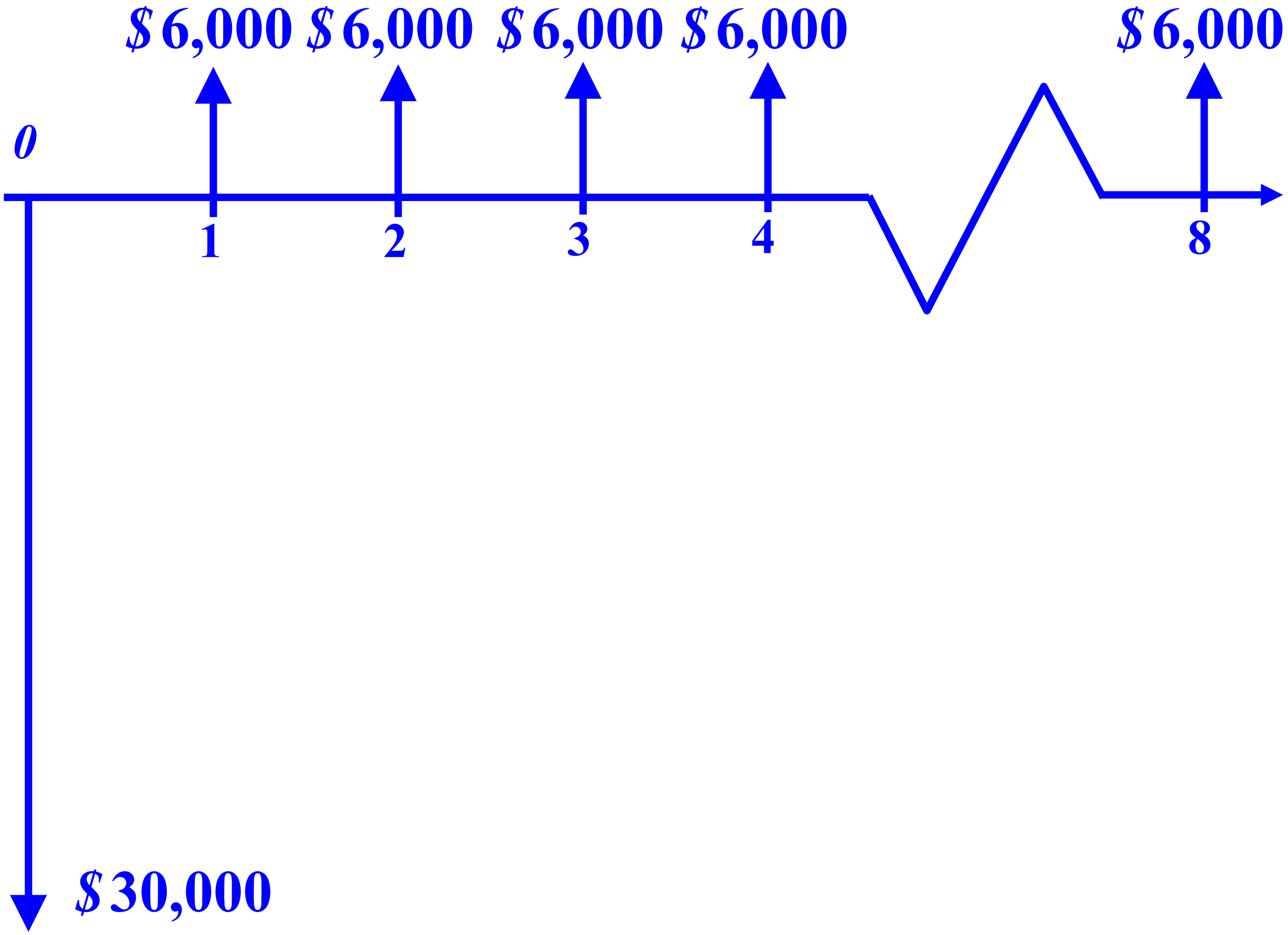
$$P - \sum_{t=0}^{n} A_t \beta^t = 0$$

is called the *internal rate of return* (*IRR*)

- ☐ The *IRR* is a measure of how quickly we recover an investment, or stated differently, the *speed* or *rate* at which the returns recover an investment

# EXAMPLE: INTERNAL RATE OF RETURN

- Consider the following cash – flow set



# INTERNAL RATE OF RETURN

- ☐ The present value

$$P = -30,000 + 6,000 \frac{1-\beta^{8}}{d} = 0$$

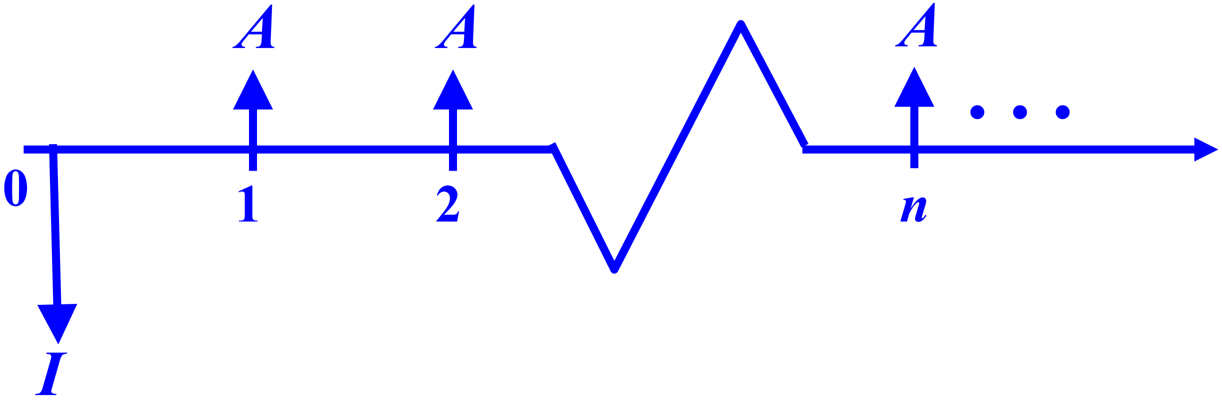
has the solution

$$d \approx 12\%$$

- ☐ The interpretation is that under a 12 % *discount rate*, the *present value* of the cash–flow set is 0 and so $d \approx 12\%$ is the *IRR* for the given cash–flow set

# INTERNAL RATE OF RETURN

- ☐ Consider an *infinite horizon* simple investment



- ☐ Therefore

$$d = \frac{A}{I} \longleftarrow$$

ratio of annual return to initial investment $I$

# INTERNAL RATE OF RETURN

- ☐ Consider

$$I \quad = \; \$\,1{,}000$$

$$A \quad = \quad \$\,200$$

and

$$d = 20\ \%$$

- ☐ We interpret that the returns capture 20 % of the investment each year, or equivalently that we have a *simple payback period* of 5 *years*

# EXAMPLE: EFFICIENT REFRIGERATOR

- ☐ A more efficient refrigerator incurs an investment of additional *$* 1,000 but provides *$* 200 of energy savings annually
- ☐ For a lifetime of 10 years, the *IRR* is computed from the solution of

$$0 = -1,000 + 200 \frac{1-\beta^{10}}{d}$$

or

# EXAMPLE: EFFICIENT REFRIGERATOR

$$\frac{1-\beta^{10}}{d} = 5$$

- ☐ $IRR$ tables show that

$$\left.\frac{1-\beta^{10}}{d}\right|_{d = 15\,\%} = 5.02$$

and so the $IRR$ is approximately 15 %

# INFLATION IMPACTS

- ☐ Inflation is a general *increase* in the level of prices in an economy; equivalently, we may view inflation as a general *decline* in the value of the *purchasing power of money*
- ☐ Inflation is measured using prices: different products may have distinct escalation rates
- ☐ Typically, indices such as the *CPI* – the *consumer price index* – use a market basket of goods and

# INFLATION IMPACTS

services as a proxy for the entire *US* economy

- ○ reference basis is the year 1967 with the price of \$ 100 for the basket $\longrightarrow L_0$
- ○ in the year 1990, the same basket cost \$ 374 $\longrightarrow L_{21}$
- ○ the average inflation rate $j$ is estimated from

$$(1 + j)^{23} = \frac{374}{100} = 3.74$$

and so

$$j = (3.74)^{\frac{1}{23}} - 1 \approx 0.059$$

# INFLATION RATE

- The inflation rate contributes to the *overall market interest rate i*, sometimes called the *combined interest rate*
- We write, using *d* for *i*

$$(1+\underset{\substack{\uparrow\\ \textit{combined}\\ \textit{interest rate}}}{d}) = (1+\underset{\substack{\uparrow\\ \textit{inflation}\\ \textit{rate}}}{j})\ (1+\underset{\substack{\uparrow\\ \textit{real interest}\\ \textit{rate}}}{d'})$$

# INFLATION

- We obtain the following identities

$$d' = \frac{d - j}{1 + j}$$

and

$$j = \frac{d - d'}{1 + d'}$$

# CASH – FLOWS INCORPORATING INFLATION

- ☐ We express the cash flow in then current dollars

  in the set $\left\{ A_t : t = 0,1,2, \ldots ,n \right\}$

- ☐ The following is synonymous terminology

  $$current \equiv then\ current \equiv inflated \equiv after\ inflation$$

- ☐ An *indexed* or *constant–worth* cash–flow is one that

  does **not explicitly** take inflation into account, i.e.,

# CASH – FLOWS INCORPORATING INFLATION

whatever amount in current inflated dollars will buy the same goods and services as in the reference year, typically, the year 0

- ☐ The following terms are synonymous

$$constant \equiv indexed \equiv inflation\ \ free \equiv before\ inflation$$

and we use them interchangeably

# CASH – FLOWS INCORPORATING INFLATION

- ☐ We define the set of constant currency flows

$$\left\{ W_t : t = 0,1,2, \dots ,n \right\}$$

corresponding to the set

$$\left\{ A_t : t = 0,1,2, \dots ,n \right\}$$

with each element $A_t$ given in period $t$ currency

# CASH – FLOWS INCORPORATING INFLATION

- ☐ We use the relationship

$$A_t = W_t \left(1 + j\right)^t$$

or equivalently

$$W_t = A_t \left(1 + j\right)^{-t}$$

with $W_t$ expressed in reference year 0 (today's) dollars

# CASH – FLOWS INCORPORATING INFLATION

- ☐ **We have**

$$P = \sum_{t=0}^{n} A_t \beta^t$$

$$= \sum_{t=0}^{n} W_t (i+j)^t (i+d)^{-t}$$

$$= \sum_{t=0}^{n} W_t (i+j)^t (i+j)^{-t} (i+d')^{-t}$$

$$= \sum_{t=0}^{n} W_t (i+d')^{-t}$$

# CASH – FLOWS INCORPORATING INFLATION

- ☐ Therefore, the *real interest rate* $d'$ is used to discount the indexed cash flows

- ☐ In summary,

we discount current *dollar* cash flow at $d$

we discount indexed *dollar* cash flow at $d'$

# CASH FLOWS INCORPORATING INFLATION

- ☐ Whenever inflation is taken into account, it is convenient to carry out the analysis in *present worth* rather than future worth or on a *cash–flow basis*

- ☐ Under inflation $\left( j > 0 \right)$ , it follows that a uniform set of cash flows $\left\{ A_t = A : t = 1, 2, \dots , n \right\}$ implies a real decline in the cash flows

# EXAMPLE: INFLATION CALCULATIONS

- ☐ We consider an annual inflation rate of $j = 4\ \%$ ;

  the cost for a piece of equipment is assumed

  constant for the next 3 years in terms of today's $\$$

$$W_0 = W_1 = W_2 = W_3 = \$\,1{,}000$$

- ☐ The corresponding cash flows in current $\$$ are

$$A_0 = \$\,1{,}000$$

$$A_1 = 1{,}000\left(1+.04\right) = \$\,1{,}040$$

# EXAMPLE: INFLATION CALCULATIONS

$$A_2 = 1,000(1+.04)^2 = \$\,1,081.60$$

$$A_3 = 1,000(1+.04)^3 = \$\,1,124.86$$

☐ The interpretation of $A_3$ is that under 4 % inflation, \$ 1,125 in 3 years will have the same value as \$ 1,000 today; it must **not** be confused with the present worth calculation

# MOTOR ASSESSMENT EXAMPLE

- ☐ For the motor $a$ or $b$ purchase example, we consider the escalation of electricity at an annual rate of $j = 5\ \%$
- ☐ We compute the $NPV$ taking into account the inflation (price escalation of 5 %) and $d = 10\ \%$
- ☐ Then,

$$d' = \frac{d - j}{1 + j} = \frac{.10 - .05}{1 + .05} = \frac{.05}{1.05} = 0.04762$$

# MOTOR ASSESSMENT

- ☐ The savings of $\$\,192$ per year are in constant dollars

$$P_{savings} = \sum_{t=1}^{20} W_t \left(1 + d'\right)^{-t} \longleftarrow 0.04762$$

and so

$$P_{savings} = \$\,2,442$$

- ☐ The total savings are

$$P = -\,500 + P_{savings} = \$\,1,942$$

which are larger than those of $\$\,1,135$ without electricity price escalation

# EXAMPLE: *IRR* FOR *HVAC* RETROFIT WITH INFLATION

- ☐ An energy efficiency retrofit of a commercial site reduces the *HVAC* load consumption to 0.8 *GWh* from 2.3 *GWh* and the peak demand by 0.15 *MW*
- ☐ Electricity costs are 60 *\$/MWh* and demand charges are 7,000 *\$/(MW–mo)* and these prices escalate at an annual rate of $j = 5\ \%$
- ☐ The retrofit requires a *\$* 500,000 investment today and is planned to have a 15 – *year* lifetime

# EXAMPLE: *IRR* FOR *HVAC* RETROFIT WITH INFLATION

- ☐ We evaluate the *IRR* for this project

- ☐ The annual savings are

$$energy \ : \left(2.3 - 0.8\right) GWh \left(60 \ \$ / MWh\right) = \$ \ 90,000$$

$$demand : \left(.15 \ MW\right)\left(7000 \ \$ / (MWh - mo)\right) 12 mo = \$ \ 12,600$$

$$total \ : 90,000 + 12,600 = \$ \ 102,600$$

- ☐ The *IRR* is the value of $d'$ that results in

# EXAMPLE: *IRR* FOR *HVAC* RETROFIT WITH INFLATION

$$0 = -500,000 + 102,600 \frac{1-\left(\beta'\right)^{15}}{d'}$$

☐ The table look up produces the $d'$ of 19 % and

with inflation factored in, we have

$$\left(1+d\right) = \left(1+j\right)\left(1+d'\right)$$

$$= \left(1.05\right)\left(1.19\right)$$

$$= 1.25$$

resulting in a combined *IRR* of 25 %

# ANNUALIZED INVESTMENT

☐ A capital investment, such as a renewable energy project, requires funds, either borrowed from a bank, or obtained from investors, or taken from the owner’s own accounts

☐ Conceptually, we may view the investment as a loan that converts the investment costs into a series of equal annual payments to pay back the loan with the interest

# ANNUALIZED INVESTMENT

- ☐ For this purpose, we use a uniform cash – flow set and use the relation

$$P \quad = \quad A \;\underbrace{\frac{1 - \beta^{n}}{d}}$$

present worth

equal payment term

equal payment series present worth factor

# ANNUALIZED INVESTMENT

- Therefore, the equal payment is given by

$$A = P \boxed{\frac{d}{1-\beta^{n}}} \longleftarrow \text{capital recovery factor}$$

- The capital recovery factor measures the speed

  with which the initial investment is repaid

# EXAMPLE: EFFICIENT AIR CONDITIONER

- ☐ An efficiency upgrade of an air conditioner incurs a $\$ \, 1{,}000$ investment and results in annual savings of $\$ \, 200$
- ☐ The $\$ \, 1{,}000$ is obtained as a $10 - year$ loan repaid at $7 \, \%$ interest
- ☐ The repayment on the loan is done as a uniform cash flow

$$A = 1{,}000 \, \frac{0.07}{1 - \beta^{10}} = \$ \, 142.38$$

# EXAMPLE: EFFICIENT AIR CONDITIONER

- The annual net savings are

$$200 - 142.38 \; = \; \$\,57.62$$

and not only are the savings sufficient to pay back the loan in 10 *years*, they also provide a yearly surplus of $\$\,57.62$

- The *benefits/costs ratio* is

$$\frac{200}{142.38} = 1.4$$

# EXAMPLE: PV SYSTEM

- ☐ We consider a $3 - kW\ PV$ system whose capacity factor $\kappa = 0.25$
- ☐ The investment incurred $\$\,10{,}000$ and the funds are obtained as a $20 -$ year $6\ \%$ loan
- ☐ The annual loan repayments are

$$A = 10{,}000\ \frac{0.06}{1 - \beta^{20}} = 10{,}000\left(0.0872\right) = \$\ 872$$

# EXAMPLE: PV SYSTEM

- ☐ The annual energy generated is

$$\left(3\right)\left(0.25\right)\left(8,760\right) = 6,570 \; kWh$$

- ☐ We can compute the unit costs of electricity for break–even operation to be

$$\frac{872}{6,570} = 0.133 \;\; \$ / kWh$$

# LEVELIZED BUS – BAR COSTS

- The comparison of various alternatives must be carried out on a consistent basis taking into account
  - inflation impacts
  - fixed investment costs
  - variable costs
- The customary approach for cost valuation consists of the following steps:

# LEVELIZED BUS – BAR COSTS

- ○ present worthing of all the cash – flow
- ○ determining the equal amount of an *equivalent* annual uniform cash – flow set
- ○ determination of the yearly total generation

❑ The ratio of the equal amount to the total generation is called the *levelized bus – bar* costs of energy

# EXAMPLE: MICROTURBINE ENGINE

- We consider the economics of a microturbine with the characteristics given in the table below
- We calculate
  - annualized fixed costs
  - initial year variable costs
  - inflation impacts

# EXAMPLE: MICROTURBINE ENGINE

| *characteristic* | *value* | *units* |
|---|---|---|
| *investment costs* | **850** | *$ / kW* |
| *heart rate* | **12,500** | *Btu / KWh* |
| *capacity factor* | **0.7** | — |
| *fuel costs ( year 0 )* | **$4.00 \times 10^{-6}$** | *$ / Btu* |
| *annual fuel escalation rate* | **6** | *%* |
| *variable O&M costs* | **0.002** | *$ / kWh* |
| *annual investor discount rate* | **10** | *%* |
| *fixed charge rate* | **12** | *%* |
| *life time* | **20** | *y* |

# EXAMPLE: MICROTURBINE ENGINE

- ☐ The annualized fixed costs are

$$\frac{\left(850\,\$/kW\right)\left(12\,\%\right)}{\left(8760\,h\right)\left(0.70\right)} = 0.0166 \quad \$/kWh$$

- ☐ The initial year variable costs are

$$A_0 = \left(12.500\,Btu/kWh\right)\left(4\times10^{-6}\ \$/Btu\right) + 0.002 \quad \$/kWh$$

$$= 0.052 \quad \$/kWh$$

- ☐ We next account for inflation and we compute

$$d' = \frac{d - j}{1 + j} = \frac{0.1 - 0.06}{1 + 0.06} = 0.037736$$

# EXAMPLE: MICROTURBINE ENGINE

- The constant uniform cash – flow set with fuel escalation incorporated is

$$A_0 \cdot \frac{1-(\beta')^{20}}{d'} = 0.052\left(\frac{1-\left(\frac{1}{1.037736}\right)^{20}}{0.0037736}\right)$$

and the levelized annual costs are

# EXAMPLE: MICROTURBINE ENGINE

$$0.052 \left( \frac{1 - \left( \frac{1}{1.037736} \right)^{20}}{0.0037736} \right) \left( \frac{0.10}{1 - \left( \frac{1}{1.1} \right)^{20}} \right) = 0.0847 \; \$/kWh$$

- ☐ The levelized bus – bar costs are, therefore,

$$0.0166 + 0.0847 = 0.1013 \; \$/kWh$$