ECE 333 – GREEN ELECTRIC ENERGY

9. Wind Data Analysis

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WIND POWER MEASUREMENT AND DATA

- The collection of sufficient wind data for the estimation of the generation is an essential task in the assessment of a wind project at a specified site
- Various measurement devices – cup, sonic detection and ranging (*SODAR*), and light detection and ranging (*LIDAR*) anemometers – provide the ability to measure wind speed, its direction and other relevant metrics of interest

WIND POWER MEASUREMENT AND DATA

- Wind is a highly uncertain phenomenon with high variability and wide changes over a brief period of time; thus, wind speed exhibits much volatility and randomness
- While wind speed is a continuous variable, wind speed data are collected on a sampled basis: values are measured on a periodic basis, such as hourly, every 10 minutes, or every minute

WIND POWER MEASUREMENT AND DATA

- **Wind data for wind analysis requires the** ***collection around–the–clock of wind speed measurements at the altitude of interest*** **and with a frequency commensu–rate with the nature and scope of the analysis**
- **The measurement scheme requires the specifica–tion of the smallest indecomposable unit of time:**

WIND POWER MEASUREMENT AND DATA

- for planning evaluation and assessment, the collection of data on an *hourly or half–hourly basis* is, typically, adequate
- for the analysis of dynamic phenomena such as stability, the collection has to be at a much finer resolution than hourly to capture the short time constants of such phenomena

WIND POWER DATA

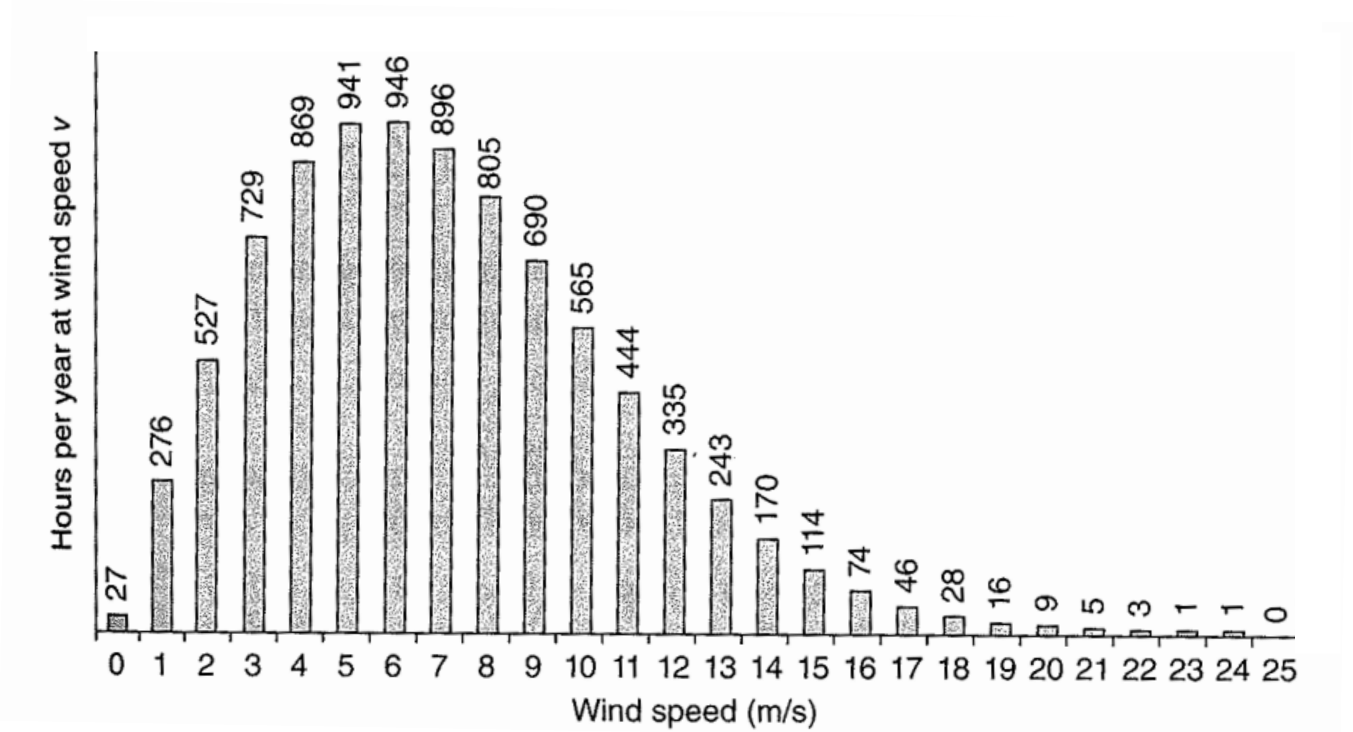
- The wind data collected may be used to approxi–mate the probability distribution of wind at a specified site
- We make use of such approximations under the assumption that natural phenomena, such as wind, continue to behave in the future in a way similar to their past behavior

WIND SPEED HISTOGRAM

- Suppose we wish to *probabilistically* characterize the wind speed at a given site and at a specified altitude: for that purpose, we collect hourly measurements over a long period of time and construct a *histogram* of the measured values
- We discretize the speed axis – we use the integer values of wind speed from 0 to 25 *m/s* – and we create 26 "buckets" of wind speed values

WIND SPEED HISTOGRAM

- We place each hourly measured value in the appropriate "bucket" and we construct a *histogram* of the historical data such as shown below



INTERPRETATION OF THE HISTOGRAM

- We interpret the height of each bar at wind speed value v in the histogram as the number of hours with wind speed value v
- We *normalize* the vertical axis values by dividing the number of hours of each bar by the total number of hours to obtain the fraction of the total hours at a particular wind speed v
- Clearly, each bar has a value < 1 and the sum of all the bars must be exactly 1

INTERPRETATION OF THE HISTOGRAM

- In effect, we obtain a probability mass function of the wind speed
- To understand the probability interpretation, we view that wind speed is a random variable $(r.v.)$ $\underset{\sim}{V}$ and that the *normalized histogram* provides the probability associated with each of its possible discrete–valued outcomes or realizations

INTERPRETATION OF HISTOGRAM

- The bar of the mass density function at the wind speed v provides

$$\mathbb{P}\left\{ \underset{\sim}{V} = v \right\} = \textit{probability of wind speed at } v \; m/s$$

- We discretized the values of $\underset{\sim}{V}$ by creating the 26 discrete buckets 0, 1, 2, …, 25 but **in reality, wind speed does not take discrete values** since it is a continuously–valued variable
- Alternatively, we may consider to make use of an increasingly finer resolution grid so as to capture the fact that $\underset{\sim}{V}$ is a **continuous** *r.v.*

PROBABILITY DENSITY

- We associate with the continuous *r.v.* $\underset{\sim}{V}$ a *probability density function* (*p.d.f.*) $f_{\underset{\sim}{V}}(v)$ with the following properties

  - $f_{\underset{\sim}{V}}(v) \geq 0 \qquad \forall\, v \geq 0$

  - $\int_0^{\infty} f_{\underset{\sim}{V}}(v)\, dv = 1$

PROBABILITY DENSITY

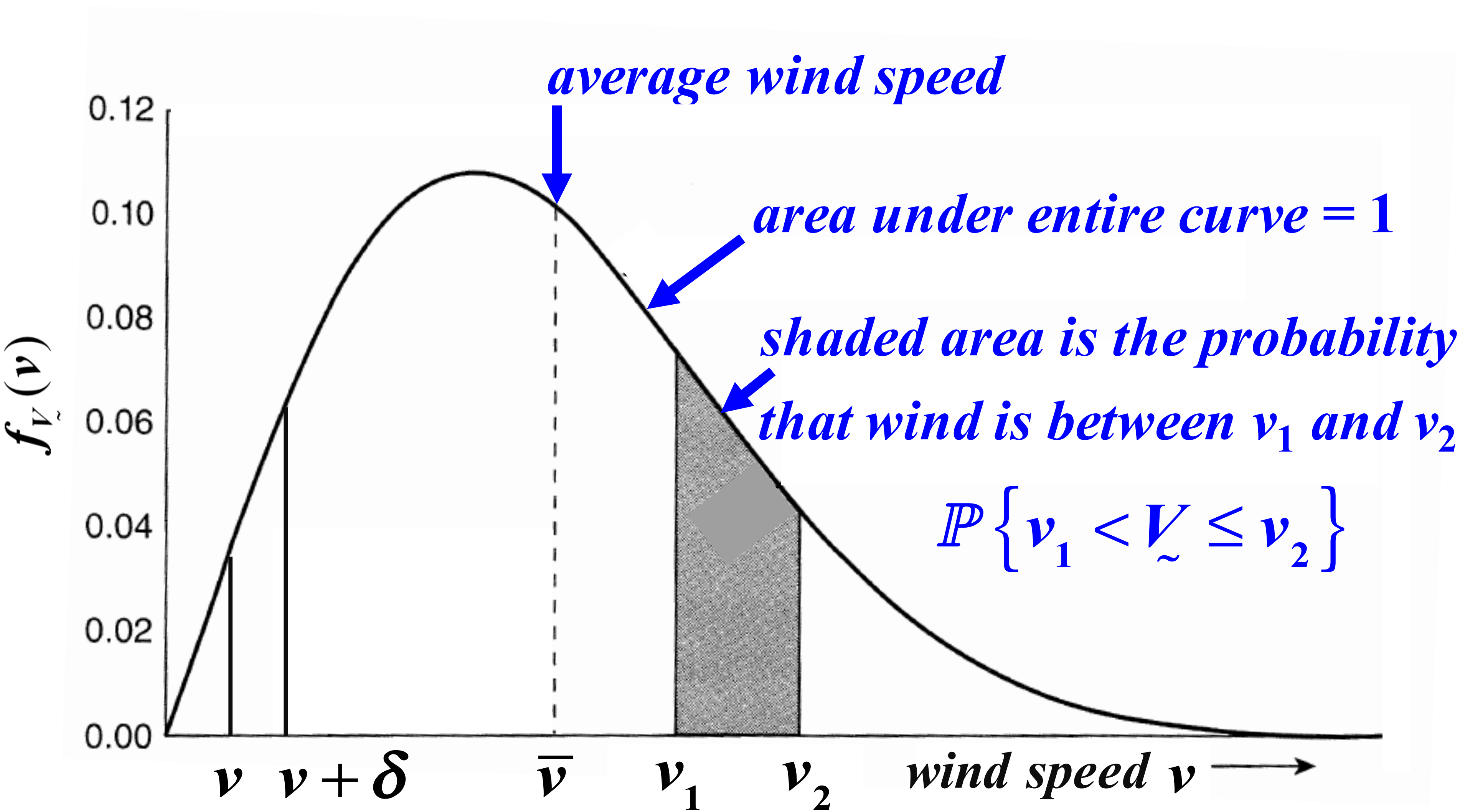
- for an infinitesimally small $\delta > 0$

$$\mathbb{P}\left\{ v < \underset{\sim}{V} \leq v + \delta \right\} \approx f_{\underset{\sim}{V}}(v)\,\delta$$

$$\mathbb{P}\left\{ v_1 < \underset{\sim}{V} \leq v_2 \right\} = \int_{v_1}^{v_2} f_{\underset{\sim}{V}}(v)\, dv$$

- The *p.d.f.* $f_{\underset{\sim}{V}}(\bullet)$ provides a complete analytic characterization of the continuous *r.v.* $\underset{\sim}{V}$

PROBABILITY DENSITY



PROBABILITY DENSITY

- We may readily compute any function of $\underset{\sim}{V}$,
  - average wind speed:

$$\overline{v} = \int_0^\infty v \; f_{\underset{\sim}{V}}(v)\, dv$$

  - wind speed cubed:

$$E\left\{\underset{\sim}{V}^3\right\} = \int_0^\infty v^3 \; f_{\underset{\sim}{V}}(v)\, dv$$

PROBABILITY DENSITY

- number of annual hours $v_1 < \underset{\sim}{V} \leq v_2$: we define an indicator function $i(x)$ with the property

$$i(x) = \begin{cases} 1 & v_1 < x \leq v_2 \\ 0 & otherwise \end{cases}$$

and compute

$$8{,}760 \int_0^{\infty} i(v)\, f_{\underset{\sim}{V}}(v)\, dv = 8{,}760 \int_{v_1}^{v_2} (1)\, f_{\underset{\sim}{V}}(v)\, dv$$

WEIBULL DISTRIBUTION

- The general Weibull distribution given by

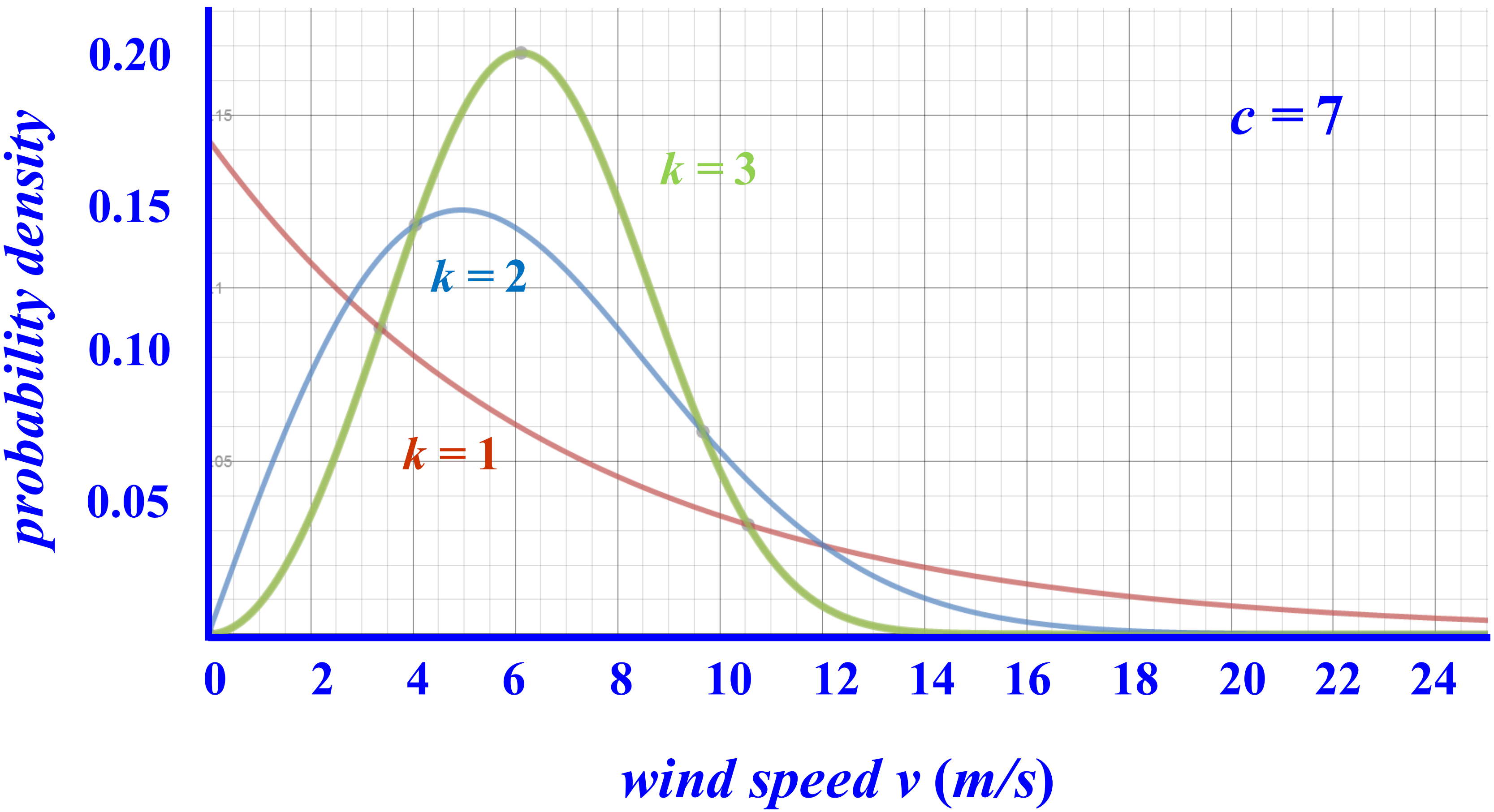
$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^{k}}$$

k = *shape parameter*

c = *scale parameter*

is often used to approximate the *p.d.f.* of $\underset{\sim}{V}$

WEIBULL DISTRIBUTION



WEIBULL DISTRIBUTION

- **For $k = 2$, the *Weibull distribution* is called the *Rayleigh p.d.f.***

$$f(v) = \frac{2v}{c^2} e^{-\left(\frac{v}{c}\right)^2} \quad \textit{Rayleigh p.d.f.}$$

- **The Rayleigh distribution is widely used in the analytic characterization of wind**

WEIBULL DISTRIBUTION

- Note that for $\underset{\sim}{V} \sim Rayleigh$, the mean is given by

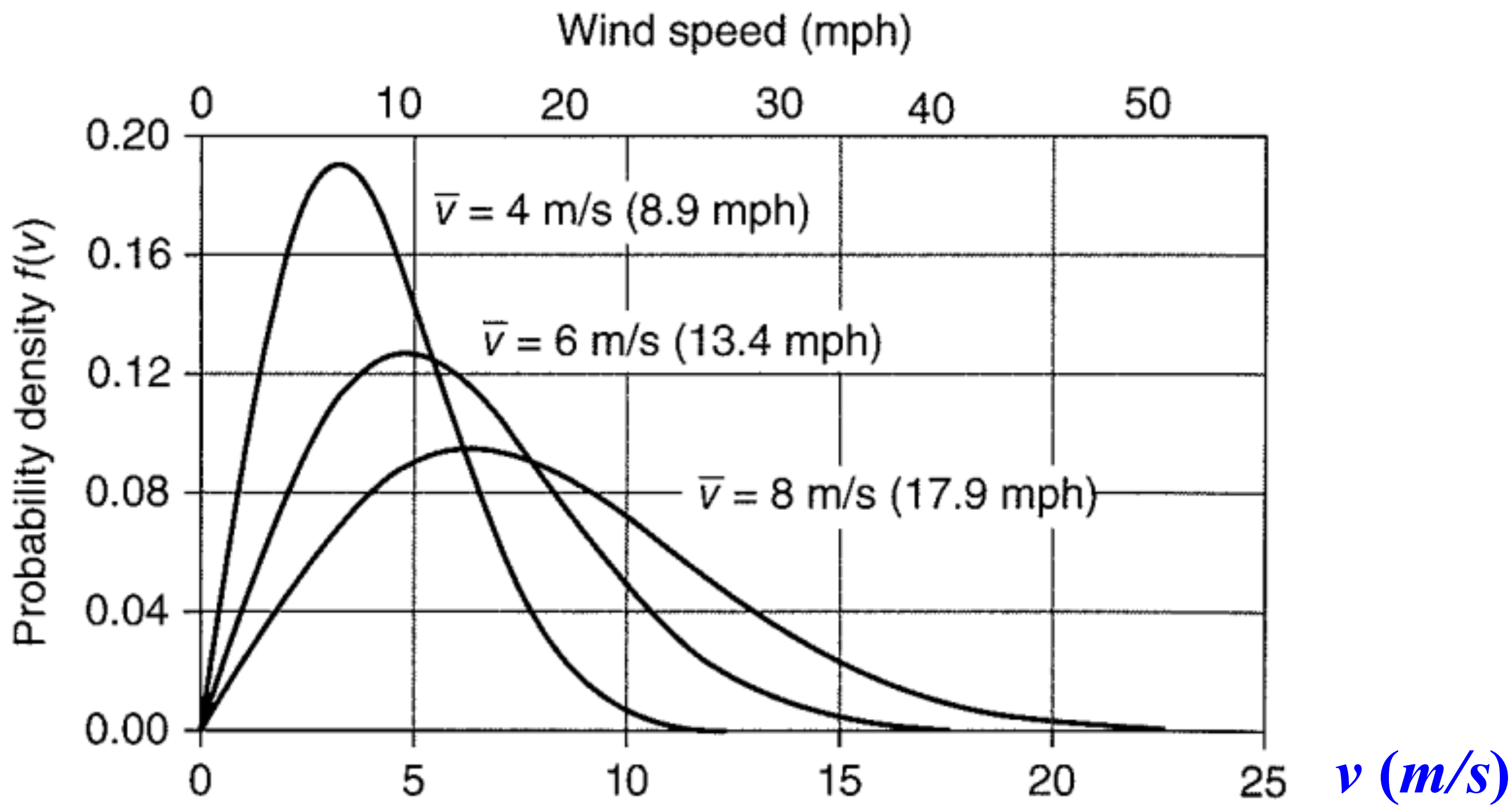
$$\bar{v} = \int_0^{\infty} v \, f_{\underset{\sim}{V}} \, dv = 2\int_0^{\infty} \left(\frac{v}{c} \right)^2 e^{-\left(\frac{v}{c} \right)^2} dv = \frac{\sqrt{\pi}}{2} c$$

and so we may restate the expression for $f_{\underset{\sim}{V}}(\bullet)$ as

$$f_{\underset{\sim}{V}}(v) = \frac{v\,\pi}{2\left(\bar{v}\right)^2} e^{-\frac{\pi}{4}\left(\frac{v}{\bar{v}} \right)^2}$$

WEIBULL DISTRIBUTION

- **As $\overline{v}$ increases, $f_{\underset{\sim}{V}}(\bullet)$ becomes flatter and shifts to the right, as shown below**



RAYLEIGH–DISTRIBUTION–BASED CALCULATIONS

- The wide use of Rayleigh distribution is in light of the good approximations it provides for the average wind power $\bar{v}$
- We have that

$$\bar{v} = \frac{\sqrt{\pi}}{2} c$$

and so we evaluate

$$E\left(\underset{\sim}{V}^{3}\right) = \int_{0}^{\infty} v^{3} \frac{\pi v}{2\left(\bar{v}\right)^{2}} e^{-\left[\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^{2}\right]} dv = \frac{6}{\pi}\left(\bar{v}\right)^{3} \approx 1.91\left(\bar{v}\right)^{3}$$

RAYLEIGH–DISTRIBUTION–BASED CALCULATIONS

- This closed–form solution for Rayleigh–based wind distribution allows us to calculate the average power in wind

$$\overline{p} = \frac{1}{2}\rho a\left(\overline{v}\right)^{3}\left(1.91\right)$$

and therefore, it becomes very clear that we cannot simply use $\left(\overline{v}\right)^{3}$ directly to evaluate $\overline{p}$ but need to also explicitly include the $\frac{6}{\pi} \approx 1.91$ factor

WIND POWER OUTPUT DISTRIBUTION

- Wind power output is a function of the *r.v.* $\underset{\sim}{V}$ and therefore wind power output is itself a *r.v.*, i.e.,

$$\underset{\sim}{P} = g\left(\underset{\sim}{V}\right) = \frac{1}{2}\rho a\left(\underset{\sim}{V}\right)^{3}$$

- For wind *r.v* $\underset{\sim}{V} \sim Weibull\ \ p.d.f.$ with

$$f_{\underset{\sim}{V}}\left(v\right) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^{k}}$$

the *cumulative distribution function* is given by

$$F_{\underset{\sim}{V}}\left(v\right) = \mathbb{P}\left\{\underset{\sim}{V} \leq v\right\} = \int_{0}^{v} \frac{k}{c}\left(\frac{\xi}{c}\right)^{k-1} e^{-\left(\frac{\xi}{c}\right)^{k}} d\xi$$

WIND POWER OUTPUT DISTRIBUTION

- Since, we can introduce a change of variables, we set

$$u = \left(\frac{\xi}{c} \right)^{k} \quad \text{and} \quad du = \frac{k}{c} \left(\frac{\xi}{c} \right)^{k-1} d\xi$$

so that

$$F_{\underset{\sim}{V}}\left(v\right) = \int_{0}^{\left(\frac{v}{c}\right)^{k}} e^{-u}\, du = 1 - e^{-\left(\frac{v}{c}\right)^{k}}$$

WIND POWER OUTPUT DISTRIBUTION

- For the special case of *Rayleigh p.d.f.*

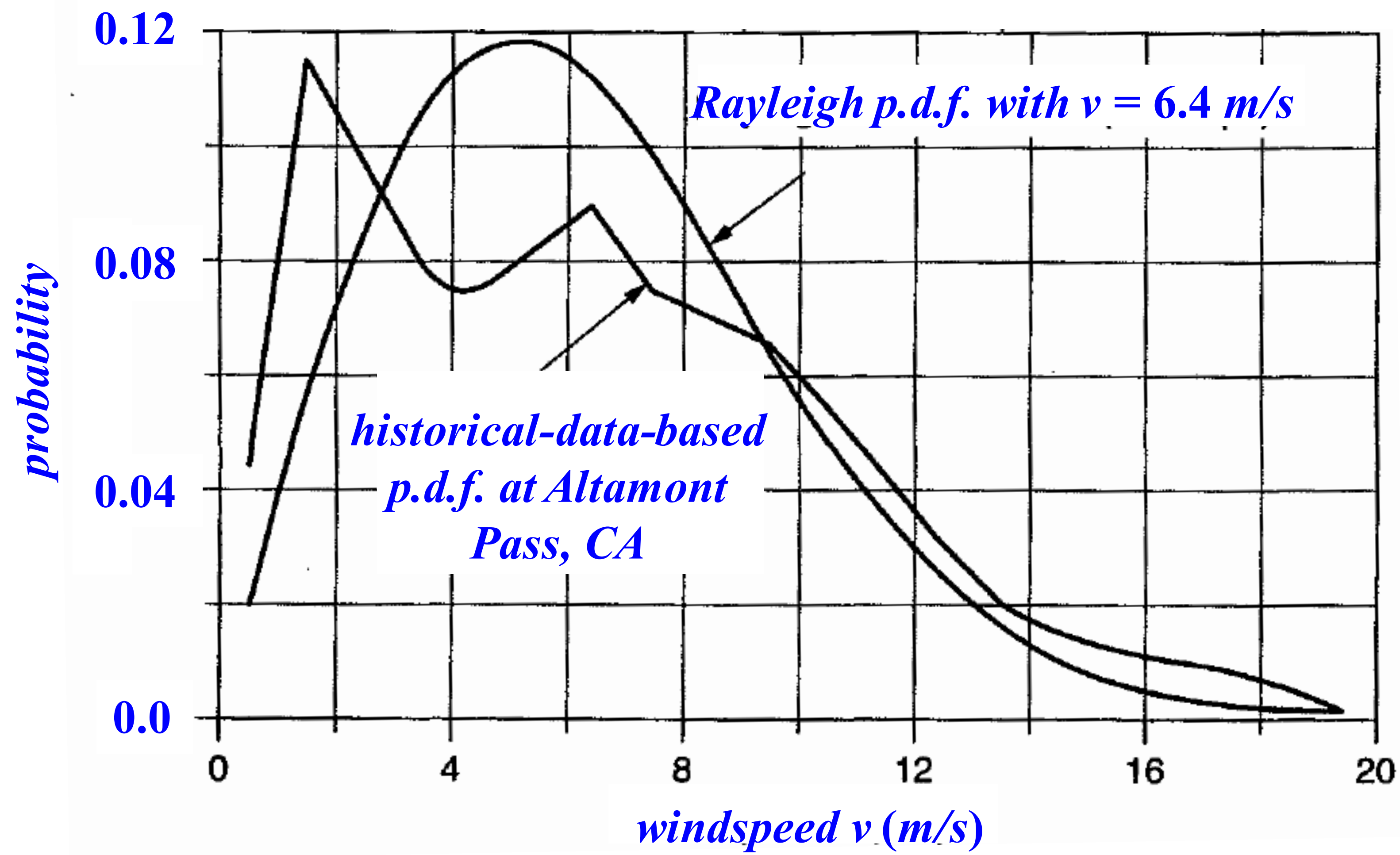
$$F_{\underset{\sim}{V}}(v)\Big|_{Rayleigh} = 1 - e^{-\left[\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^{2}\right]}$$

- Note that the probability that *Rayleigh* wind

  exceeds the value v is

$$\mathbb{P}\left\{\underset{\sim}{V} > v\right\} = 1 - F_{\underset{\sim}{V}}(v)\Big|_{Rayleigh} = e^{-\left[\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^{2}\right]}$$

ALTAMONT PASS, CA: HISTORICAL DATA *vs.* RAYLEIGH *p.d.f.*s



EXAMPLE: AVERAGE POWER IN THE WIND

- Based on data from a standard anemometer at a height of $10\ m$, $\bar{v}(10) = 6\ m/s$
- The plan is to erect a $50\ m$ tower to place the nacelle and we need to estimate the average power under the assumptions
  - Hellman exponent $\alpha = \dfrac{1}{7}$
  - $\rho = 1.225 \dfrac{kg}{m^3}$
  - Rayleigh distribution may be used

EXAMPLE: AVERAGE POWER IN THE WIND

- The first step is to compute $\bar{v}(50)$

$$\bar{v}(50) = \bar{v}(10)\left(\frac{50}{10}\right)^{\frac{1}{7}} = 7.55\ \frac{m}{s}$$

- Since Rayleigh distribution holds

$$\frac{\bar{p}(50)}{a} = \frac{6}{\pi}\cdot\frac{1}{2}\rho\left[\bar{v}(50)\right]^3 = 1.91\cdot\frac{1}{2}\cdot 1.225\cdot(7.55)^3 = 504\frac{W}{m^2}$$

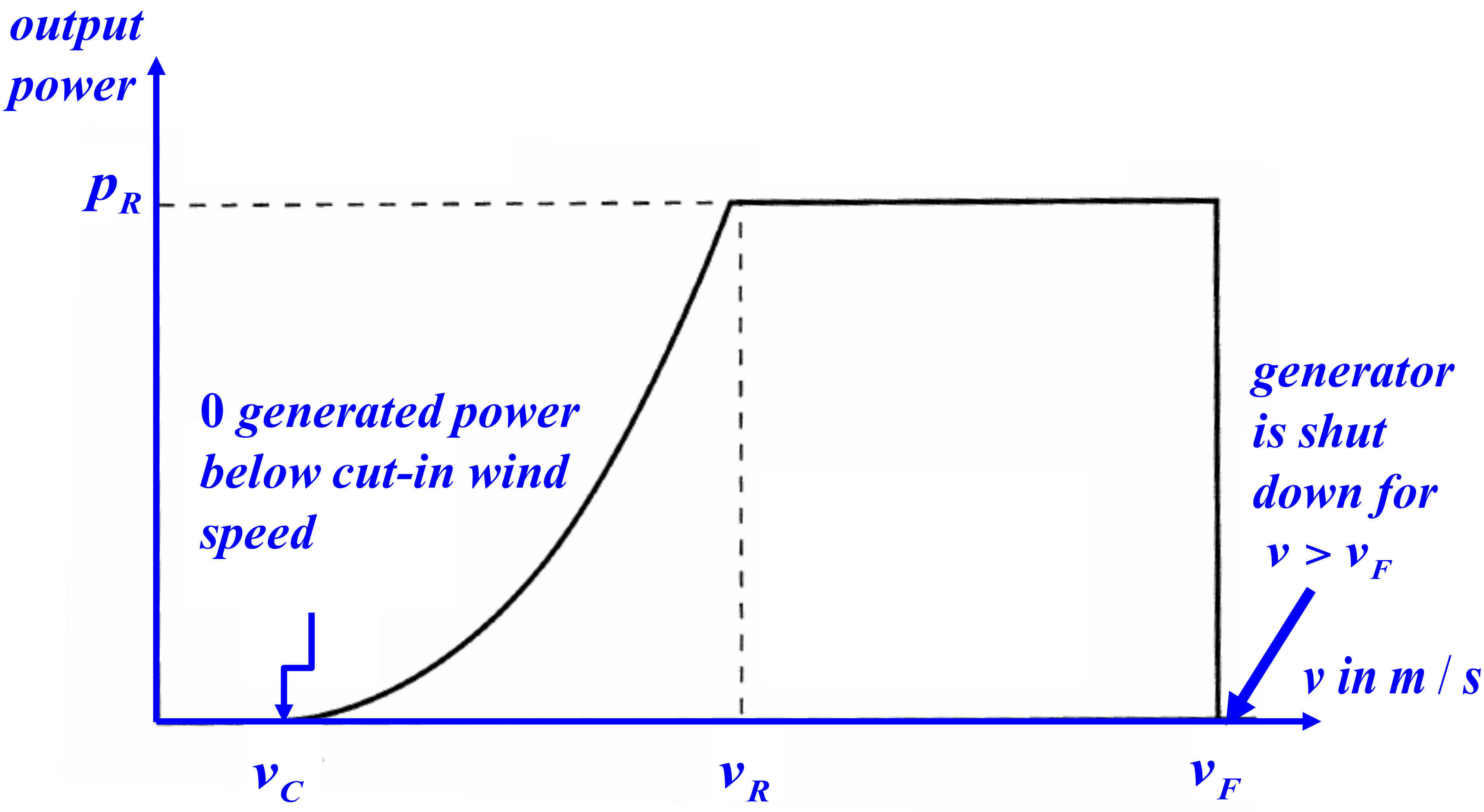
- Sensitivity case for an $80\text{–}m$ tower:

$$\frac{\dfrac{\bar{p}(80)}{a}}{\dfrac{\bar{p}(50)}{a}} = \left(\frac{80}{50}\right)^{\frac{3}{7}} \qquad \frac{\bar{p}(80)}{a} = 504\left(\frac{80}{50}\right)^{\frac{3}{7}} = 616\frac{W}{m^2}$$

THE IDEALIZED WIND TURBINE POWER CURVE

- **Each turbine manufacturer provides a plot of the electrical power output of the entire system – the blades, the gearbox, the generator, and the other components – as a function of wind speed**
- **Such a plot is called an *idealized wind turbine power curve***
- **The typical shape of an idealized wind turbine power curve is given below**

THE IDEALIZED WIND TURBINE POWER CURVE



THE IDEALIZED WIND TURBINE POWER CURVE

- **At low speeds, wind has insufficient energy to overcome friction in the turbine drive train, even if the generator rotor is spinning: below the *cut–in wind speed* v_C, the power output is 0**
- **Above v_C, the power output is a cubic function of v; at the rated wind speed v_R, the generator delivers its rated power p_R**

THE IDEALIZED WIND TURBINE POWER CURVE

- At $v > v_R$, controls are deployed to shed some of the wind so as not to exceed p_R
- When wind speed reaches the *cut–out value* v_F – sometimes called by the sailing term furling wind speed – the machine is shut down and the mechanical brakes lock down the rotor shaft above v_F wind speeds and the output power is 0

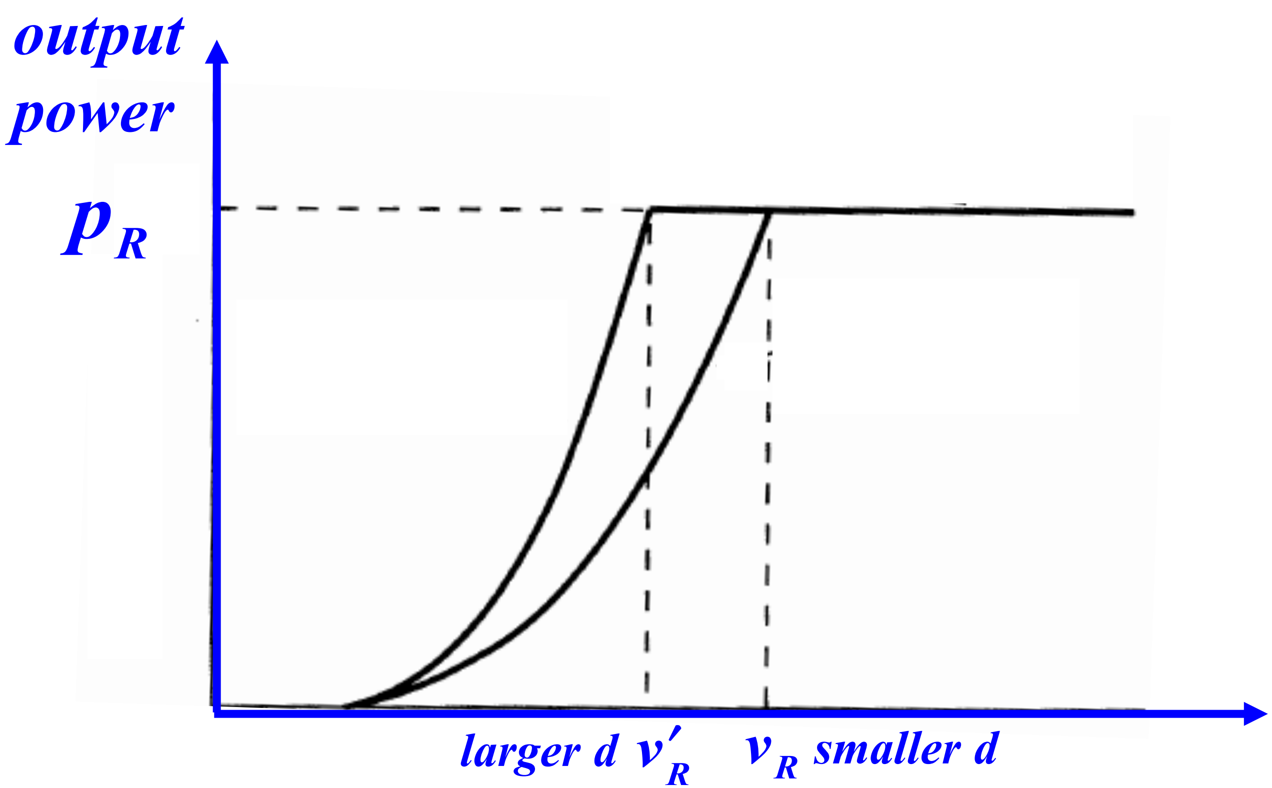
IMPACTS OF DESIGN PARAMETERS

- We can assess the impact of two key design parameters:
  - the diameter d of the blade rotor
  - the rated generator capacity

  on the power output determined via the idealized power curve
- The power output $p \propto d^{2}$ since d^{2} determines the area swept by the blades

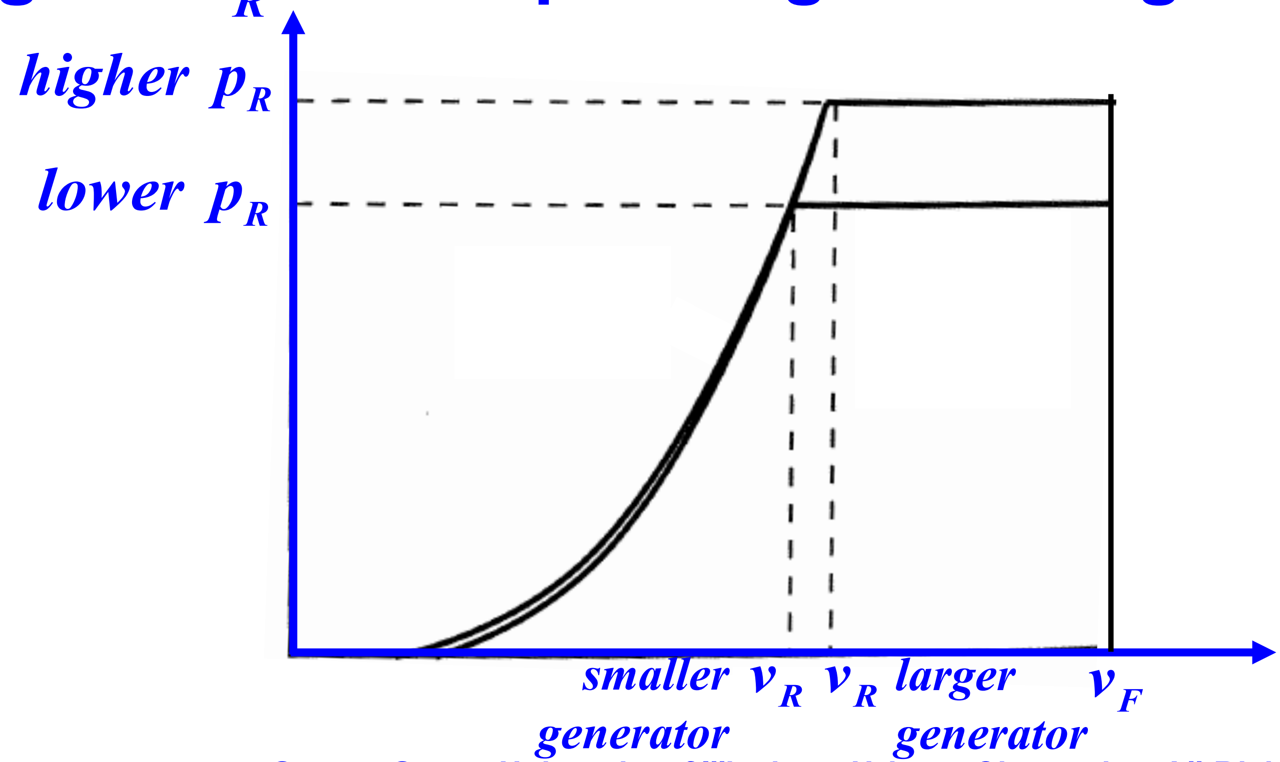
IMPACTS OF DESIGN PARAMETERS

- For a generator with rated power p_R, an increase in d produces a shift in the power curve to the left and the output p_R is reached at a lower speed



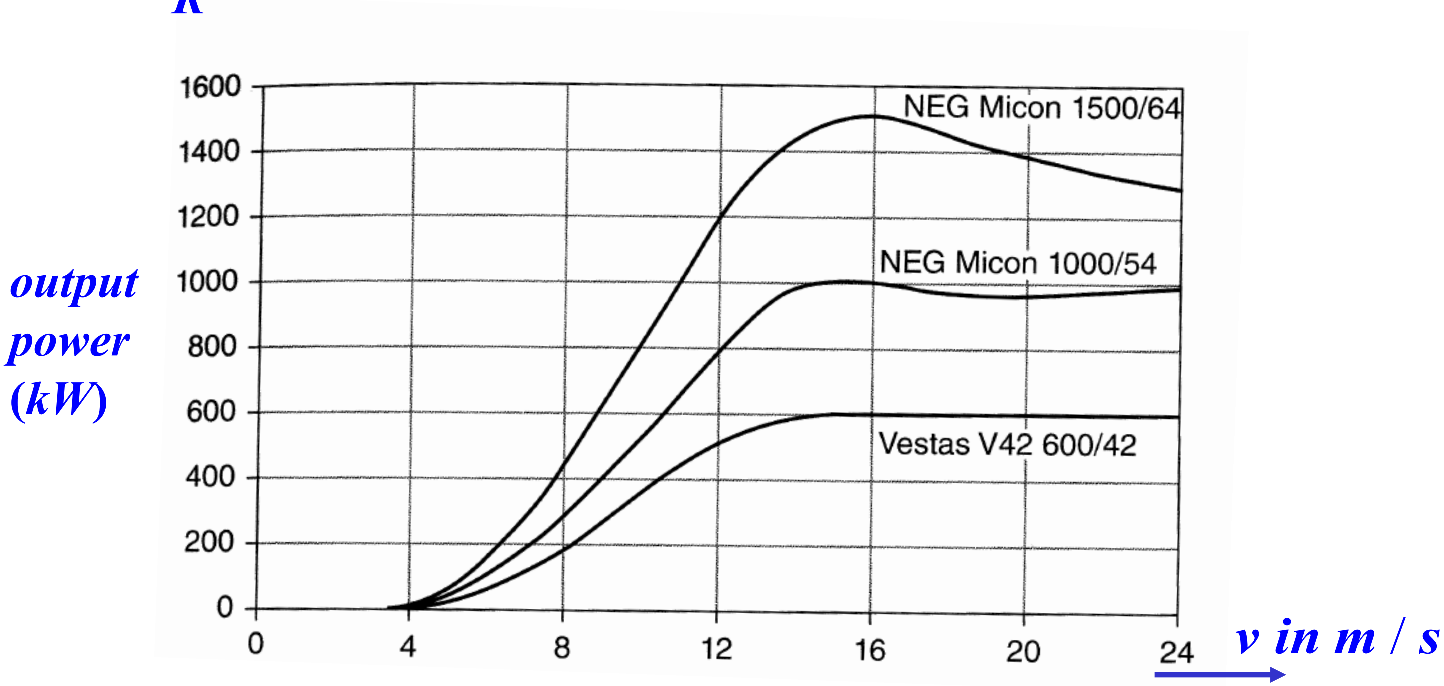
IMPACTS OF DESIGN PARAMETERS

- For a fixed rotor diameter d, an increase in the generator rated capacity may be accommodated by the continuation of the power curve up to the higher v_R corresponding to the higher p_R



IMPACTS OF DESIGN PARAMETERS

- Actual power curves do not veer too far from the idealized ones with much of the variance due to the *inability of wind shedding techniques to control the power outputs at speeds* $v > v_R$; in certain cases, the value of v_R is difficult to determine

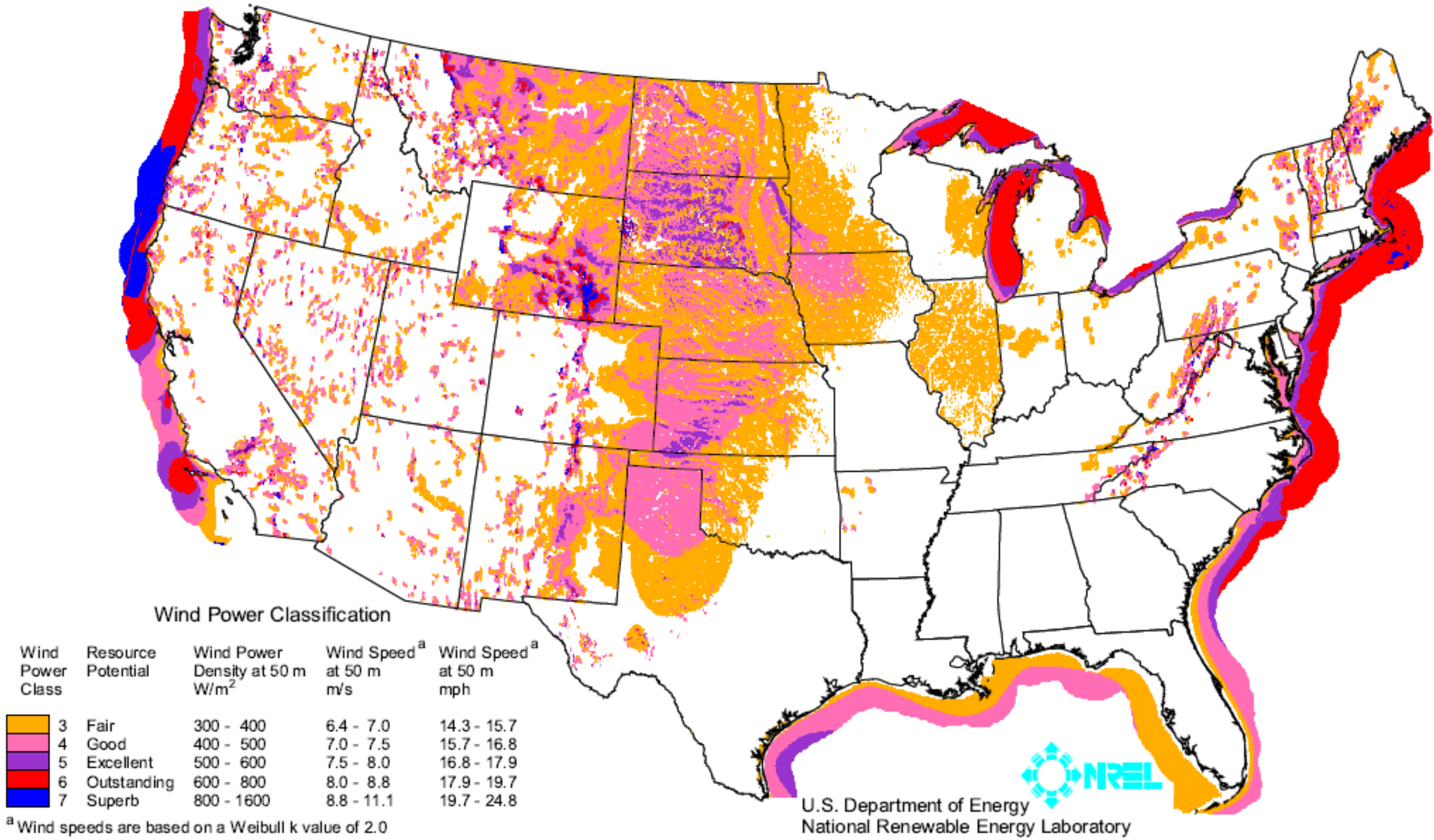


classes of wind power density at 10 m and 50 m

	10 *m* (33 *ft*)		50 *m* (164 *ft*)	
wind power class	*wind power density* (W/m^2)	*speed m/s (mph)*	*wind power density* (W/m^2)	*speed m/s (mph)*
1	< 100	< 4.4 (9.8)	< 200	< 5.6 (12.5)
2	100 - 150	4.4 (9.8)/5.1 (11.5)	200 - 300	5.6 (12.5)/6.4 (14.3)
3	150 - 200	5.1 (11.5)/5.6 (12.5)	300 - 400	6.4 (14.3)/7.0 (15.7)
4	200 - 250	5.6 (12.5)/6.0 (13.4)	400 - 500	7.0 (15.7)/7.5 (16.8)
5	250 - 300	6.0 (13.4)/6.4 (14.3)	500 - 600	7.5 (16.8)/8.0 (17.9)
6	300 - 400	6.4 (14.3)/7.0 (15.7)	600 - 800	8.0 (17.9)/8.8 (19.7)
7	> 400	> 7.0 (15.7)	> 800	> 8.8 (19.7)

Source: http://www.awea.org/faq/basicwr.html

WIND POWER EQUI – DENSITY CONTOURS AT 50 *m*



Source: http://www.windpoweringamerica.gov/pdfs/wind_maps/us_windmap.pdf

ESTIMATES OF WIND TURBINE ENERGY

- It is not possible to extract 100 % of the power in the wind as the rotor spills high-speed winds and the little energy at low-speed winds is lost
- The energy generated depends on rotor, gearbox, generator, tower, controls, terrain, and the wind

wind

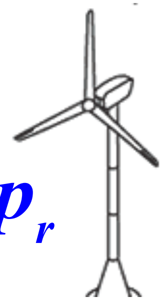
p_w

power in the wind

$\longrightarrow$

η_r

rotor efficiency



p_r

power extracted by rotor blades

$\longrightarrow$

η_g

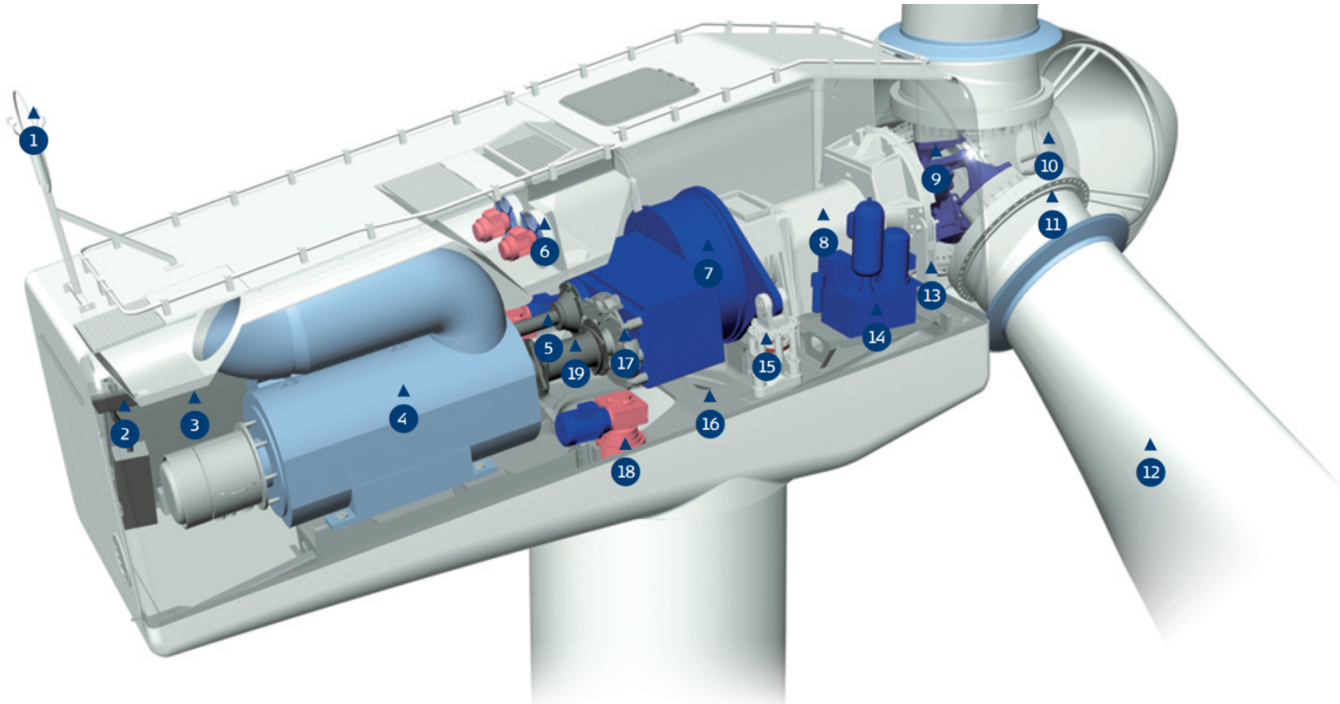
gearbox & generator efficiency

P_E

electric power delivered to grid

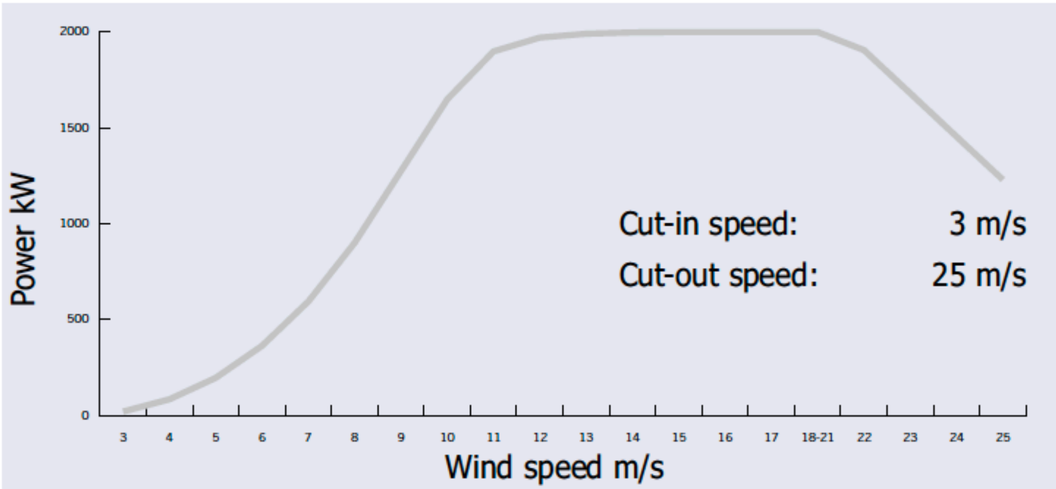
- Overall conversion efficiency $\eta_r \, \eta_g$ is around 30 %

VESTAS *V52 850 kW* WIND TURBINE COMPONENTS

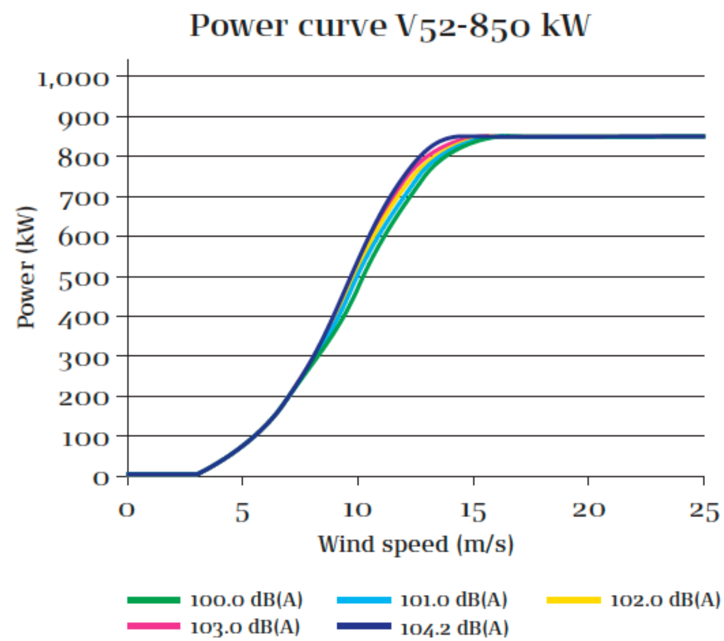


1. Ultrasonic wind sensor
2. Service crane
3. VMP-Top controller with converter
4. OptiSpeed® Generator
5. Pitch cylinder
6. Oil and water coolers
7. Gearbox
8. Main shaft
9. Pitch system
10. Blade hub
11. Blade bearing
12. Blade
13. Rotor lock system
14. Hydraulic unit
15. Torque arm
16. Machine foundation
17. Mechanical disc brake
18. Yaw gear
19. Composite disc coupling

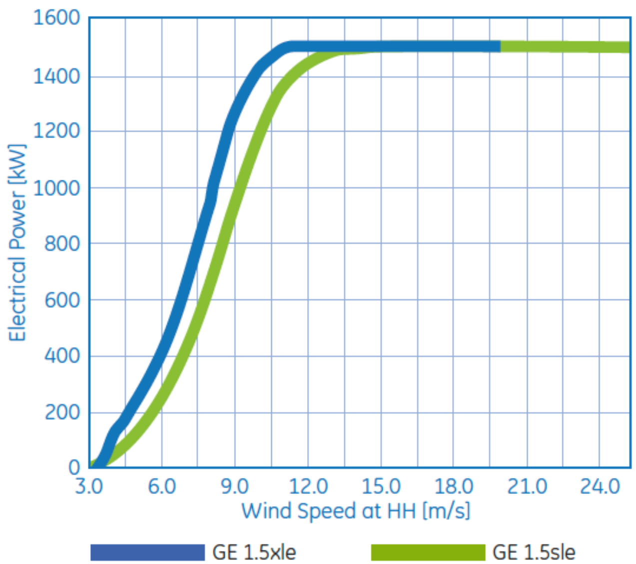
MANUFACTURER POWER CURVES



Gamesa G90-2.0 MW



Vestas V52-850 kW



GE 1.5sle/xle-1.5 MW

