ECE 333 Green Electric Energy

Fall 2017
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Homework 2 Solutions

7.1 A horizontal-axis wind turbine with a 20-m diameter rotor is 30-% efficient in 10 m/s winds at 1-atm of pressure and 15°C temperature.

a. How much power would it produce in those winds?

b. Estimate the air density on a 2500-m mountaintop at 10° C?

c. Estimate the power the turbine would produce on that mountain with the same windspeed assuming its efficiency is not affected by air density.

SOLN:

a. Power from the turbine would be

$$P = \eta \cdot \frac{1}{2} \rho A v^3 = 0.30 \cdot 0.5 \cdot 1.225 \cdot \frac{\pi}{4} 20^2 \cdot 10^3 = 57,727 W = 57.73 kW$$

b. From (7.17)

$$\rho = \frac{353.1 \exp(-0.0342 z / T)}{T}$$
$$= \frac{353.1 \exp[-0.0342 \cdot 2500 / (10 + 273.15)]}{283.15} = 0.922 \text{ kg/m}^3$$

c. Turbine power is proportional to air density, so

$$P = 57.73 \text{ kW} \cdot \frac{0.922}{1.225} = 43.5 \text{ kW}$$

7.2 An anemometer mounted 10 m above a surface with crops, hedges and shrubs, shows a windspeed of 5 m/s. Assuming 15°C and 1 atm pressure, determine the following for a wind turbine with hub height 80 m and rotor diameter of 80 m:

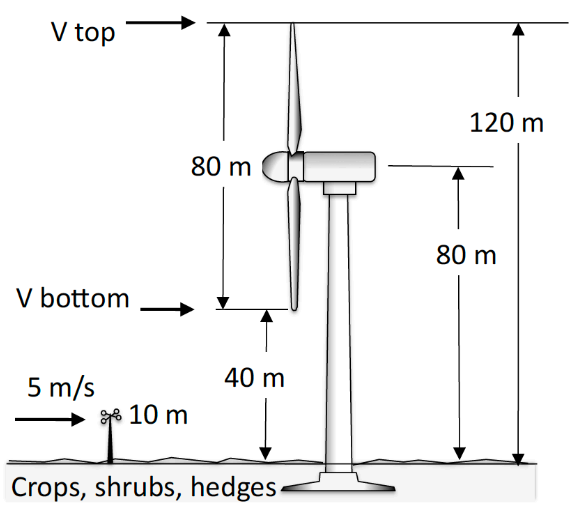

Figure P 7.2

a. Estimate the windspeed and the specific power in the wind (W/m^2) at the highest point that the rotor blade reaches. Assume no air density change over these heights.

b. Repeat (a) at the lowest point at which the blade falls.

c. Compare the ratio of wind power at the two elevations using results of (a) and (b) and compare that with the ratio obtained using (7.20).

d. What would be the power density at the highest tip of the blade if we include the impact of elevation on air density. Assume the temperature is still 15°C. Does air density change seem worth considering in the above analysis?

SOLN:

From Table 7.1, the friction coefficient α for ground with hedges, etc., is estimated to be 0.20. From the 15°C, 1 atm conditions, the air density is ρ = 1.225 kg/m^3.

a. Windspeed at the lowest point of the rotor (40 m) will be:

$$v_{40} = 5\left(\frac{40}{10}\right)^{0.20} = 6.5975 \text{ m/s}$$

and the specific power will be:

$$P_{40} / A = \frac{1}{2}\rho v^3 = 0.5x1.225x6.5975^3 = 175.894 \text{ W/m}^2$$

b. At the highest point (120 m) the rotor will see:

$$v_{40} = 5\left(\frac{120}{10}\right)^{0.20} = 8.2188 \text{ m/s}$$

and the specific power will be:

$$P_{40} / A = \frac{1}{2}\rho v^3 = 0.5x1.225x8.2188^3 = 340.04 \text{ W/m}^2$$

c. The ratio of power top-to-bottom, is:

$$\text{Power Ratio} = \frac{340.04}{175.894} = 1.93$$

OR, using (7.20)

$$\frac{P_{120}}{P_{40}} = \left(\frac{H_{120}}{H_{40}}\right)^{3\alpha} = \left(\frac{120}{40}\right)^{3x0.20} = 1.93 \quad \text{.... the same... good}$$

d. Using (7.17)

$$\rho = \frac{353.1\exp(-0.0342z/T)}{T}$$

$$= \frac{353.1\exp\left[-0.0342\cdot 120/(15+273.15)\right]}{288.15} = 1.2081 \text{ kg/m}^3$$

And the specific power is now

$$P_{40}/A = \frac{1}{2}\rho v^3 = 0.5x1.2081x8.2188^3 = 335.3 \text{ W/m}^2$$

That's only a drop of (340.04-335.3)/340.04 = 1.4%. not a big deal.

7.3 The analysis of a tidal power facility is similar to that for a normal wind turbine. That is, we can still write $P = 1/2\ \rho A v^3$ but now ρ = 1000 kg/m^3 and v is the speed of water rushing toward the turbine. The following graphs assume sinusoidally varying water speed, with amplitude V_{max}. We assume the turbine can accept flows in either direction (as the tide ebbs and floods) so it is only the magnitude of the tidal current that matters.

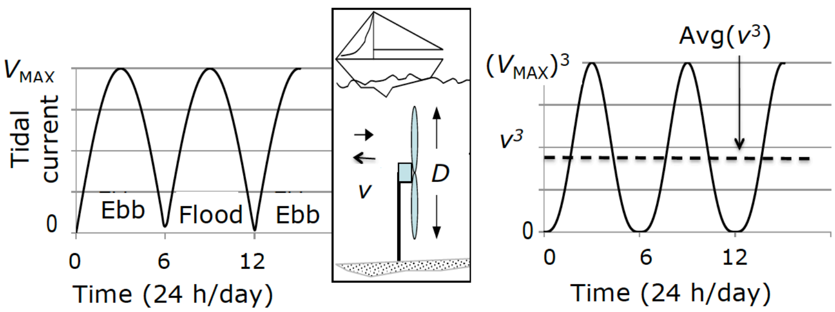

Figure P 7.3

a. What is the average power density (W/m^2) in the tidal current? A bit of calculus gives us the following helpful start:

$$\left(v^3\right)_{avg} = avg\left(V_{\max}\sin v\right)^3 = V_{\max}^3 \frac{\int_0^{\pi/2}\sin^3 v\,dv}{\pi/2} = \frac{4}{3\pi}V_{\max}^3$$

SOLN:

$$\frac{P_{avg}}{A} = \frac{1}{2}\rho\left(v^3\right)_{avg} = 0.5 \text{ x } 1000 \text{ x} \frac{4}{3\pi} \text{ x } 2^3 = 1698 \text{ W/m}^2$$

b. If a 600-kW turbine with 20-m diameter blades has a system efficiency of 30%, how many kWh would it deliver per year in these tides?

SOLN:

$$\text{Energy} = 1698 \text{ W/m}^2 \text{ x } \frac{\pi}{4}(20)^2 \text{ m}^2 \text{ x } 0.30 \text{ x} \frac{1\text{kW}}{1000\text{W}} \text{x } 8760 \text{ h/yr} = 1.40 \text{ x}10^6 \text{ kWh/yr}$$

7.5 An early prototype 10-kW Makani Windpower system consisted of two 5-kW wind turbines mounted on a wing that flies in somewhat vertical circles (like a kite) several hundred meters above ground. A tether attached to the "kite" carries power from the turbines down to the ground. Since the speed of the kite-turbines moving through the air is much faster than the wind speed, much smaller turbine blades can be used than those on conventional ground-mounted wind turbines. Also with no need for a tower, the cost of materials is far lower than for a conventional system.

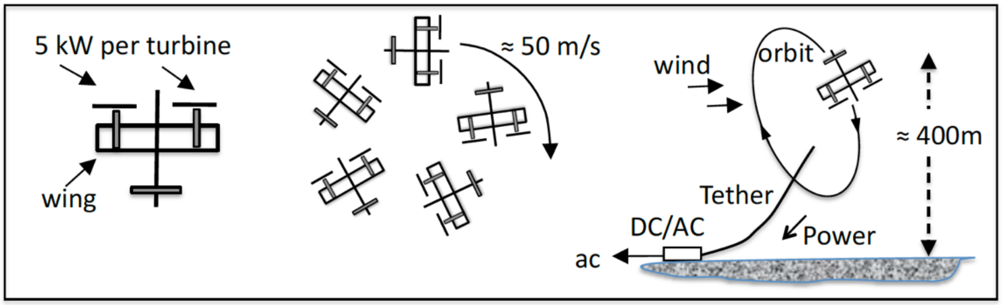

Figure P 7.5

Suppose each wing/turbine is moving through the air at 50 m/s and suppose the overall efficiency is half that of the Betz limit, what blade diameter would be required to deliver 5 kW of power per turbine. Don't bother to correct air density for this altitude.

SOLN:

$$P = C_p \frac{1}{2} \rho A v^3$$

$$5000 \text{ W} = (0.5x0.59)\frac{1}{2}\cdot 1.225\frac{\pi}{4}D^2 50^3 = 17{,}739D^2$$

$$D = \sqrt{\frac{5000}{17{,}739}} = 0.53 \text{ m} = 1.74 \text{ ft}$$