ECE 333 Green Electric Energy

Fall 2017
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Homework 4 Solutions

7.6 Consider the following probability density function for wind speed:

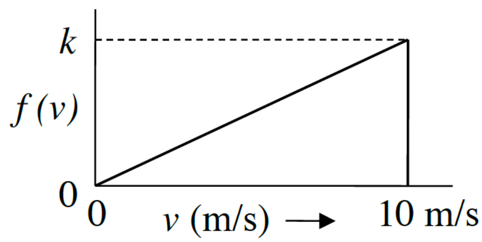


Figure P 7.6

a. What is an appropriate value of k for this to be a legitimate pdf?

SOLN: The area under the curve must equal 1. So, k = 0.2.

b. What is the average power in these winds (W/m²) under standard temperature and pressure conditions (1 atm, 15°C)?

SOLN:

With k = 0.2, $f(v) = 0.02v$ ($0 \le v \le 10$) and we need $P_{avg} = 1/2\ \rho A \left(v^3\right)_{avg}$

From (7.42)

$$\left(v^3\right)_{avg} = \int_0^\infty v^3 f(v)\, dv$$

$$= \int_0^{10} v^3 \cdot 0.02v\, dv = 0.02 \left.\frac{v^5}{5}\right|_0^{10} = 0.02 \cdot \frac{10^5}{5} = 400$$

$$P_{avg} / A = 1/2\ \rho \left(v^3\right)_{avg} = 0.5 \cdot 1.225 \cdot 400 = 245\ \text{W/m}^2$$

7.7 Suppose a wind turbine has a cut-in windspeed of 5 m/s and a furling windspeed of 25 m/s. If the winds the turbine sees have Raleigh statistics with an average windspeed of 9 m/s,

a. For how many hours per year will the turbine be shut down because of excessively high speed winds?

SOLN: From (7.59)

$$\text{Prob } (v > V_F) = \exp\left[-\frac{\pi}{4}\left(\frac{V_F}{v_{AVG}}\right)^2\right]$$

$$\text{Prob } (v > 25) = \exp\left[-\frac{\pi}{4}\left(\frac{25}{9}\right)^2\right] = 0.002334$$

Hrs ($v > 25$) = 8760 x 0.002334 = 20.4 hrs/yr

b. For how many hours per year will the turbine be shut down because winds are too low?

SOLN: From (7.56):

$$\text{Prob } (v < V_{\text{Cut-in}}) = 1 - \exp\left[-\frac{\pi}{4}\left(\frac{V_{\text{Cut-in}}}{v_{AVG}}\right)^2\right]$$

$$\text{Prob } (v \le 5) = 1 - \exp\left[-\frac{\pi}{4}\left(\frac{5}{9}\right)^2\right] = 0.215$$

Hrs ($v > 25$) = 8760 x 0.215 = 1886 hrs/yr = 78.6 days

c. If this is a 1-MW turbine, how much energy (kWh/yr) would be produced for winds blowing at or above the rated wind speed of 12 m/s?

SOLN: From (7.59)

$$\text{Prob } (v > V_R) = \exp\left[-\frac{\pi}{4}\left(\frac{V_R}{v_{AVG}}\right)^2\right]$$

$$\text{Prob } (v > 12) = \exp\left[-\frac{\pi}{4}\left(\frac{12}{9}\right)^2\right] = 0.2475$$

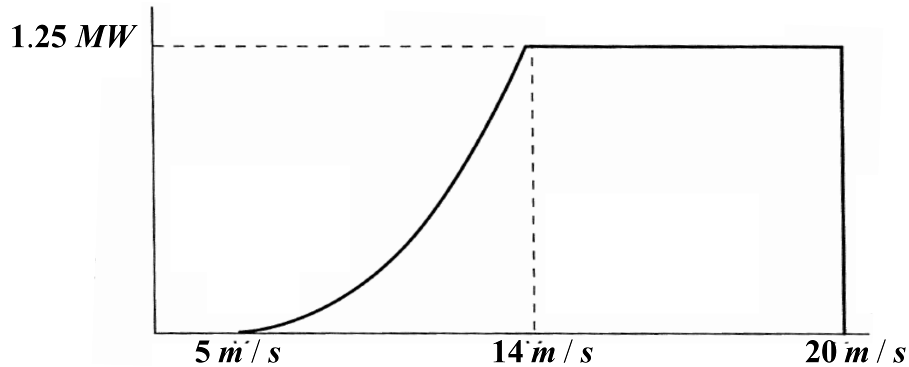
Hrs ($v > 12$) = 8760 x 0.2475 = 2168 h/yr

Hrs (≥ 12 v ≤ 25) = 2168 - 20.4= 2,148 h/yr

Energy = 2148 h/yr x 1 MW = 2148 MWh/yr

Problem 2:

a.



b. if the wind blows continuously between 15 and 20 m/s all day, the power output is 1.25 *MW* all day. Thus the total energy is 1.25×24=30 *MWh/day*

c. No. Consider the question on Quiz 2.

Problem 3:

For $k = 2$, the Weibull distribution is called the Rayleigh *p.d.f.*

$$f(v) = \frac{2v}{c^2} e^{-\left(\frac{v}{c}\right)^2}$$

$$\bar{v} = \int_0^\infty v \, f_{\underset{\sim}{V}} \, dv = 2\int_0^\infty \left(\frac{v}{c}\right)^2 e^{-\left(\frac{v}{c}\right)^2} dv = \frac{\sqrt{\pi}}{2} c$$

$$F_{\underset{\sim}{V}}\left(\underset{\sim}{V} \le v\right)\Big|_{Rayleigh} = 1 - e^{-\left[\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2\right]}$$

Since the average wind speed is 20 *m/s*

$$c = \frac{2}{\sqrt{\pi}} \bar{v} = \frac{2}{\sqrt{\pi}} 20 = 22.53$$

a.

when temperature is 15 ^oC

$$\rho_{10m,15^oC} = \frac{353.1}{T}\exp(-0.0342\frac{z}{T}) = \frac{353.1}{273.15+15}\exp(-0.0342\frac{10}{273.15+15}) = 1.224\ kg/m^3$$

$$E(\underset{\sim}{p}) = E(\frac{1}{2}\rho\underset{\sim}{V}^3) = \frac{1}{2}1.224\cdot E(\underset{\sim}{V}^3) \approx \frac{1}{2}1.224\cdot 1.91\cdot \bar{V}^3 = \frac{1}{2}1.224\cdot 1.91\cdot 20^3 = 9351\ W/m^2$$

when temperature is -5 ^oC

$$\rho_{10m,15^oC} = \frac{353.1}{T}\exp(-0.0342\frac{z}{T}) = \frac{353.1}{273.15-5}\exp(-0.0342\frac{10}{273.15-5}) = 1.315\ kg/m^3$$

$$E(\underset{\sim}{p}) = E(\frac{1}{2}\rho\underset{\sim}{V}^3) = \frac{1}{2}1.315\cdot E(\underset{\sim}{V}^3) \approx \frac{1}{2}1.315\cdot 1.91\cdot \bar{V}^3 = \frac{1}{2}1.315\cdot 1.91\cdot 20^3 = 10047\ W/m^2$$

b.

when temperature is 15 ^oC

$$annual\ energy = 8760\cdot\eta E(\underset{\sim}{P}) = 8760\cdot 0.3\cdot E(\underset{\sim}{p})\cdot A = 8760\cdot 0.3\cdot 9351\cdot\pi(\frac{60}{2})^2 = 69447\quad MWh$$

when temperature is -5 ^oC

$$annual\ energy = 8760\cdot\eta E(\underset{\sim}{P}) = 8760\cdot 0.3\cdot E(\underset{\sim}{p})\cdot A = 8760\cdot 0.3\cdot 10047\cdot\pi(\frac{60}{2})^2 = 74616\quad MWh$$