ECE 333 Green Electric Energy

Fall 2017
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Homework 5 Solutions

Quiz Date: Tuesday, November 14, 2017 during class

The quiz is based on the following material: Lecture 10, Lecture 11, Chapter 7 (Sections 7.8 and 7.9) and Appendix A from the textbook, and the problems in Homework 5.

Problem 1 Solution:

a. the present value of this 30-year savings:

$$P_{saving} = \sum_{t=1}^{n} A_t \beta^t = 0.55\sum_{t=1}^{30}\left(\frac{1}{1+0.12}\right)^t = A\beta\left[1+\beta+\beta^2+\ldots+\beta^{n-1}\right] = 4.43\$/ft^2$$

thus the NPV is **4.43-3=1.43** ***\$/ft²***

b. when NPV is zero, the present value of saving is equal to the additional cost of the windows

$$P_{saving} = \sum_{t=1}^{n} A_t \beta^t = 0.55\sum_{t=1}^{30}\left(\frac{1}{1+d}\right)^t = A\beta\left[1+\beta+\beta^2+\ldots+\beta^{n-1}\right] = 3\$/ft^2$$

Then we can solve the above equation and have ***IRR*=0.1821**

c. if the savings escalate at 7% per year due to fueling savings

$$d' = \frac{d-e}{1+e} = \frac{.12-.07}{1+.07} = \frac{.05}{1.07} = 0.046729$$

$$P_{saving} = \sum_{t=1}^{n} W_t \beta'^t = 0.55\sum_{t=1}^{30}\left(\frac{1}{1+0.046729}\right)^t = 8.78\$/ft^2$$

thus the NPV is **8.78-3=5.78** ***\$/ft²***

d. we have the $\boldsymbol{IRR}'$**=0.1821 ,** because

$$P_{saving} = 0.55\sum_{t=1}^{30}\left(\frac{1}{1+IRR'}\right)^t = 3\$/ft^2$$

thus the actual $\boldsymbol{IRR}$ is $\boldsymbol{IRR}'(1+e)+e=0.2649$, because

$$IRR' = \frac{IRR - e}{1+e}$$

Problem 2:

a. the annual saving is:

$$0.07\cdot 60{,}000 + 9\cdot 25\cdot 12 = 6900\ \$$$

b.

$$P_{saving} = \sum_{t=1}^{n} A_t\beta^t = 6900\sum_{t=1}^{30}\left(\frac{1}{1+IRR}\right)^t = 135{,}000\ \$$$

$$IRR = 0.02993$$

c.

$$P_{saving} = \sum_{t=1}^{n} A_t\beta^t = 6900\sum_{t=1}^{30}(1+0.06)^t\left(\frac{1}{1+IRR}\right)^t = 135{,}000\ \$$$

$$IRR = 0.0917$$

Problem 3:

Annual cost is

$$A = P\frac{1-\beta}{\beta(1-\beta^n)} = 15{,}000\frac{1-\frac{1}{1+0.06}}{\frac{1}{1+0.06}[1-(\frac{1}{1+0.06})^{20}]} = 1307.77\$/year$$

Annual energy production is:

$$10 \cdot 8760 \cdot 0.25 = 21900\ kWh$$

The electricity price is

$$\frac{1307.77}{21900} = 5.97 cents/kWh$$

Problem 4:

7.10 The 101-m Siemens turbines in Table 7.5 come with either a 2300-kW or a 3000-kW generator. Using the approach based on (7.63):

a. Find the energy (kWh/yr) each will deliver in an area with 5.7 m/s average wind speed.

SOLN:

$$E(\text{kWh/yr}) = 8760 P_R\left[0.087\overline{V} - \frac{P_R}{D^2}\right]$$

$$E(2300, 5.7) = 8760 \cdot 2300\left[0.087x5.7 - \frac{2300}{101^2}\right] = 5.45\text{x}10^6\ \text{kWh/yr}$$

$$E(3000, 5.7) = 8760 \cdot 3000\left[0.087x5.7 - \frac{3000}{101^2}\right] = 5.30\text{x}10^6\ \text{kWh/yr}$$

So, the smaller generator might actually deliver more energy and would cost less as well.

b. Determine the optimum generator size for these winds. Check to be sure it does better than the standard size generators.

SOLN: Using (7.67)

$$\frac{P_R}{D^2} = 0.0435\overline{V}$$

$$P_R = 0.0435x5.7x101^2 = 2{,}529\ \text{kW}$$

$$E(2529,5.7) = 8760 \cdot 2529\left[0.087x5.7 - \frac{2529}{101^2}\right] = 5.49\text{x}10^6 \text{ kWh/yr}$$

And, yes, it does deliver more energy than either the 2300 or 3000 kW generators.

c. At what wind speed would the 3000-kW generator begin to out-perform the 2300-kW generator? Check to see that the two generator outputs are the same at that windspeed.

SOLN: Setting the energies delivered equal gives

$$E(2300,5.7) = E(3000,5.7)$$

$$2300x8760\left(0.087\bar{V} - \frac{2300}{101^2}\right) = 3000x8760\left(0.087\bar{V} - \frac{3000}{101^2}\right)$$

$$2300x0.087\bar{V} - 3000x0.087\bar{V} = \left(\frac{2300}{101}\right)^2 - \left(\frac{3000}{101}\right)^2$$

$$60.9\bar{V} = 363.69$$

$$\bar{V} = 5.972 \text{ m/s}$$

Check... do they deliver the same output?

$$E(2300,5.972) = 8760 \cdot 2300\left[0.087x5.972 - \frac{2300}{101^2}\right] = 5.925\text{x}10^6 \text{ kWh/yr}$$

$$E(3000,5.972) = 8760 \cdot 3000\left[0.087x5.972 - \frac{3000}{101^2}\right] = 5.925\text{x}10^6 \text{ kWh/yr}$$

Yep, as the winds get higher than this, the 3000 kW begins to outperform the smaller one.

7.11 Consider the design of a home-built wind turbine using a 350-W permanent magnet dc motor used as a generator. The goal is to deliver 70 kWh in a 30-day month.

a. What capacity factor would be needed for the machine?

$$CF = \frac{\text{Energy delivered (kWh/mo)}}{P_R\text{(kW) x 30 day/mo x 24 h/day}} = \frac{70}{0.35x30x24} = 0.2778 = 27.78\%$$

b. If the average wind speed is 5 m/s, and Rayleigh statistics apply, what should the rotor diameter be if the CF correlation of (7.63) is used?

SOLN:

$$CF = 0.087\bar{V} - \frac{P_R}{D^2} = 0.087x5 - \frac{0.35}{D^2} = 0.2778$$

$$D = \sqrt{\frac{0.35}{0.435 - 0.2778}} = 1.49m$$

7.13 The 2013 "Low Wind" turbine pricing in Table 7.6 uses a 1.62 MW turbine with an installed cost of $2025/kW with a 100-m rotor diameter.

a. At a site with 6 m/s Rayleigh winds at 50-m, estimate the energy this turbine would deliver at a hub height of 100 m assuming the usual 1/7th wind-shear factor. Assume 15% losses.

SOLN: First estimate the winds at 100 m

$$\bar{V}_{100} = \bar{V}_{ref}\left(\frac{H}{H_{ref}}\right)^{\alpha} = 6\left(\frac{100}{50}\right)^{1/7} = 6.6245 \text{ m/s}$$

Using (7.63) for CF

$$CF = 0.087\bar{V} - \frac{P_R}{D^2} = 0.087x6.6245 - \frac{1620}{100^2} = 0.4143$$

$$\text{Annual energy} = 1620 \text{ kW x 8760 h/yr x 0.4143 x (1 - 0.15)}$$
$$= 4.998\text{x}10^6 \text{ kWh/yr}$$

b. Assuming a nominal 9% financing charge with a 20-yr term along with annual O&M costs of $60/kW, find the levelized cost of electricity. Does it agree with Figure 7.48?

SOLN: $$CRF(9\%,20) = \frac{0.09(1.09)^{20}}{(1.09)^{20} - 1} = 0.10955 / yr$$

Annual capital cost = \$2025/kW x 1620 kW x 0.10955/yr = \$359,379/yr

Annual O&M cost = \$60/kW-yr x 1620 kW = \$97,200/yr

$$LCOE = \frac{\$359,379 + 97,200 / yr}{4.998x10^{6}\, kWh / yr} = 0.0914 / kWh = 9.14¢ / kWh$$

Looks like it agrees with Figure 7.48.