ECE 340
Solid State Electronic Devices

M,W,F 12:00-12:50 (X), 2015 ECEB
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Office Hours: Wednesday 13:00 – 14:00

ECE Illinois
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

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Announcements

- Hour Exam II, Thursday 4/12, 7:30-8:30 pm
  - 1002 ECEB (Grainger Auditorium)
  - Conflict: Contact J. Dallesasse Before Monday 4/9
- Per Syllabus: Section 4.4 (including 4.4.1, 4.4.2, 4.4.3, and 4.4.4), Chapters 5 & 8 (assigned sections)

Today's Discussion

- Review
- Assignments
- Topics for Next Lecture

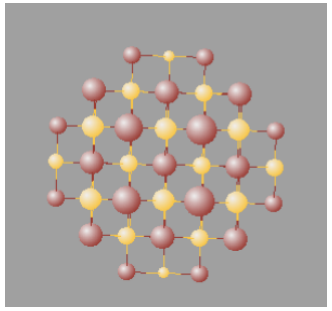
Tentative Schedule [3]

APR 2 LEDs and Diode Lasers	APR 4 Metal-semiconductor junctions	APR 6 MIS-FETs: Basic operation, ideal MOS capacitor
APR 9 MOS capacitors: flatband & threshold voltage	APR 11 Review, discussion, problems (4/12 exam)	APR 13 MOS capacitors: C-V analysis
APR 16 MOSFETs: Output & transfer characteristics	APR 18 MOSFETs: small signal analysis, amps, inverters	APR 20 Narrow-base diode
APR 23 BJT fundamentals	APR 25 BJT specifics	APR 27 BJT normal mode operation
APR 30 BJT common emitter amplifier and current gain	MAY 2 (LAST LECTURE) Review, discussion, problem solving	FINAL EXAM **Date & time to be announced**

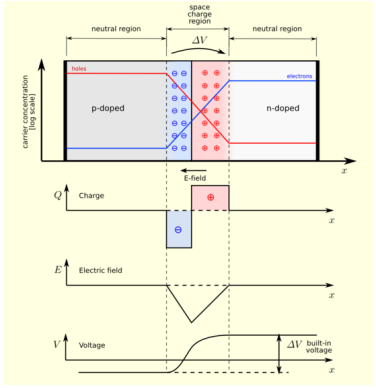
Subject to Change

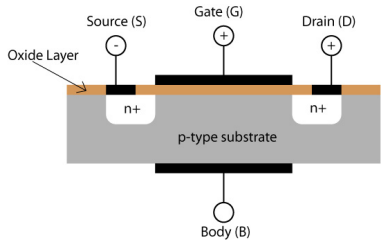
Course Purpose & Objectives

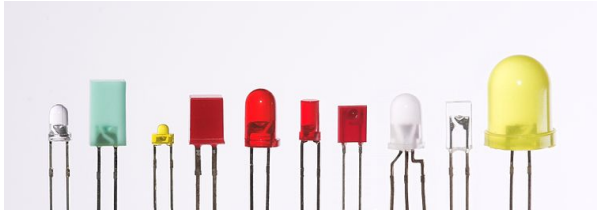
- Introduce key concepts in semiconductor materials
- Provide a basic understanding of p-n junctions
- Provide a basic understanding of light-emitting diodes and photodetectors
- Provide a basic understanding of field effect transistors
- Provide a basic understanding of bipolar junction transistors

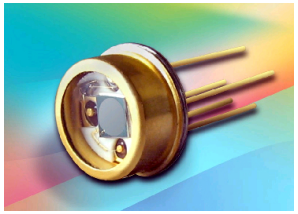


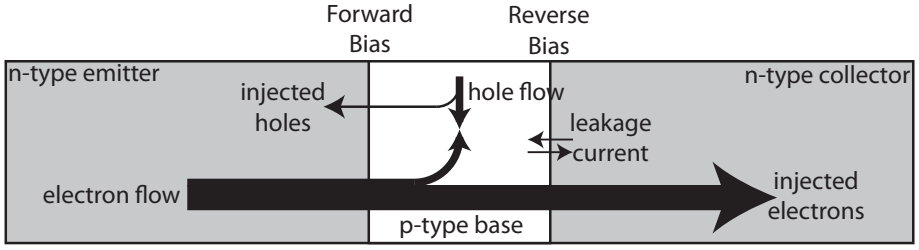












Instructional Objectives (1)

By the time of exam No. 1 (after 17 lectures), the students should be able to do the following:

✔1. Outline the classification of solids as metals, semiconductors, and insulators and distinguish direct and indirect semiconductors.

✔2. Determine relative magnitudes of the effective mass of electrons and holes from an E(k) diagram.

✔3. Calculate the carrier concentration in intrinsic semiconductors.

✔4. Apply the Fermi-Dirac distribution function to determine the occupation of electron and hole states in a semiconductor.

✔5. Calculate the electron and hole concentrations if the Fermi level is given; determine the Fermi level in a semiconductor if the carrier concentration is given.

✔6. Determine the variation of electron and hole mobility in a semiconductor with temperature, impurity concentration, and electrical field.

✔7. Apply the concept of compensation and space charge neutrality to calculate the electron and hole concentrations in compensated semiconductor samples.

✔8. Determine the current density and resistivity from given carrier densities and mobilities.

✔9. Calculate the recombination characteristics and excess carrier concentrations as a function of time for both low level and high level injection conditions in a semiconductor.

✔10. Use quasi-Fermi levels to calculate the non-equilibrium concentrations of electrons and holes in a semiconductor under uniform photoexcitation.

✔11. Calculate the drift and diffusion components of electron and hole currents.

✔12. Calculate the diffusion coefficients from given values of carrier mobility through the Einstein's relationship and determine the built-in field in a non-uniformly doped sample.

Plus continuity equation, steady-state carrier injection, and diffusion length

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Instructional Objectives (2)

By the time of Exam No.2 (after 32 lectures), the students should be able to do all of the items listed under A, plus the following:

13. Calculate the contact potential of a p-n junction.
14. Estimate the actual carrier concentration in the depletion region of a p-n junction in equilibrium.
15. Calculate the maximum electrical field in a p-n junction in equilibrium.
16. Distinguish between the current conduction mechanisms in forward and reverse biased diodes.
17. Calculate the minority and majority carrier currents in a forward or reverse biased p-n junction diode.
18. Predict the breakdown voltage of a p+-n junction and distinguish whether it is due to avalanche breakdown or Zener tunneling.
19. ~~Calculate the charge storage delay time in switching p-n junction diodes.~~
20. Calculate the capacitance of a reverse biased p-n junction diode.
21. Calculate the capacitance of a forward biased p-n junction diode.
22. Predict whether a metal-semiconductor contact will be a rectifying contact or an ohmic contact based on the metal work function and the semiconductor electron affinity and doping.
23. Calculate the electrical field and potential drop across the neutral regions of wide base, forward biased p+-n junction diode.
24. Calculate the voltage drop across the quasi-neutral base of a forward biased narrow base p+-n junction diode.
25. Calculate the excess carrier concentrations at the boundaries between the space-charge region and the neutral n- and p-type regions of a p-n junction for either forward or reverse bias.

Plus optoelectronic devices and #35 (next slide)

Instructional Objectives (3)

By the time of the Final Exam, after 44 class periods, the students should be able to do all of the items listed under A and B, plus the following:

26. Calculate the terminal parameters of a BJT in terms of the material properties and device structure.
27. Estimate the base transport factor "B" of a BJT and rank-order the internal currents which limit the gain of the transistor.
28. Determine the rank order of the electrical fields in the different regions of a BJT in forward active bias.
29. Calculate the threshold voltage of an ideal MOS capacitor.
30. Predict the C-V characteristics of an MOS capacitor.
31. Calculate the inversion charge in an MOS capacitor as a function of gate and drain bias voltage.
32. Estimate the drain current of an MOS transistor above threshold for low drain voltage.
33. Estimate the drain current of an MOS transistor at pinch-off.
34. Distinguish whether a MOSFET with a particular structure will operate as an enhancement or depletion mode device.
35. Determine the short-circuit current and open-circuit voltage for an illuminated p/n junction solar cell. ***Exam 2***

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Major Topics

- PN junctions at equilibrium, contact potential, and electrostatics
- Injection currents in pn junctions
- Reverse bias, breakdown, and junction capacitance
- Optoelectronic devices
- Metal-semiconductor junctions

Exam: Problem Topics

- pn Junctions at Equilibrium, Contact Potentials, and Junction Electrostatics (Including Metal-Semiconductor Junctions)
- Forward and Reverse Bias, Breakdown, and Junction Capacitance
- Optoelectronic Devices

Steady-State Carrier Injection

Diffusion Length

Diffusion Equation Derivation (Recap)

Continuity Equation

$$\frac{\partial p(x,t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}$$

Current Density Equation

$$J_p(x) = -qD_p\frac{dp(x)}{dx} = -qD_p\frac{\partial \delta p}{\partial x} \text{ (no drift)}$$

Diffusion Equation

$$\frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial}{\partial x}\left(-qD_p\frac{\partial \delta p}{\partial x}\right) - \frac{\delta p}{\tau_p} = D_p\frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_p}$$

$$\frac{\partial \delta p}{\partial t} = D_p\frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_p}$$

Diffusion Length: Steady State

Steady-State Diffusion Equations

$$\frac{\partial \delta p}{\partial t} = D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_p}$$

In steady state, $\frac{\partial \delta p}{\partial t} = 0$

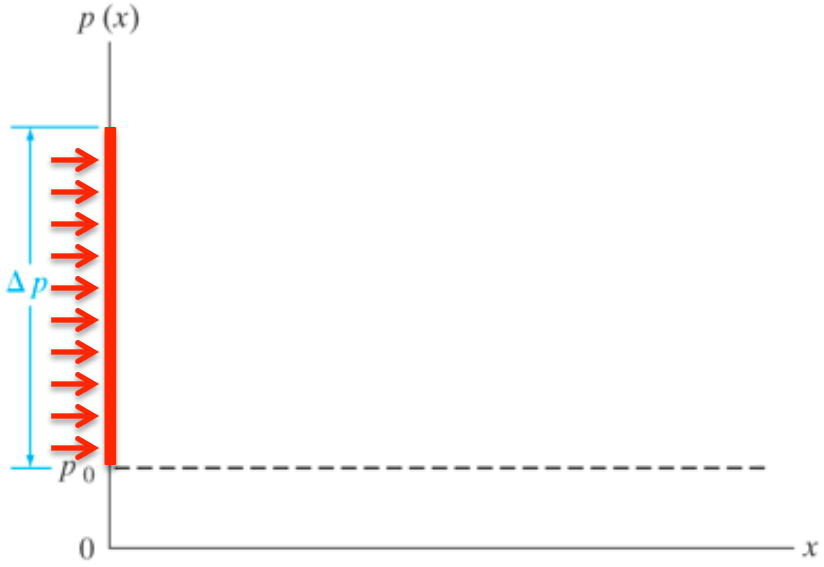
$$\therefore\ D_p \frac{\partial^2 \delta p}{\partial x^2} = \frac{\delta p}{\tau_p} \text{ or } \boxed{\frac{\partial^2 \delta p}{\partial x^2} = \frac{\delta p}{D_p \tau_p} \equiv \frac{\delta p}{L_p^2}}$$

$$\frac{\partial \delta n}{\partial t} = D_n \frac{\partial^2 \delta n}{\partial x^2} - \frac{\delta n}{\tau_n}$$

In steady state, $\frac{\partial \delta n}{\partial t} = 0$

$$\therefore\ D_n \frac{\partial^2 \delta n}{\partial x^2} = \frac{\delta n}{\tau_n} \text{ or } \frac{\partial^2 \delta n}{\partial x^2} = \boxed{\frac{\delta n}{D_n \tau_n} \equiv \frac{\delta n}{L_n^2}}$$

Hole Injection at x=0



(The injection mechanism is not specified.)

L_p is the average distance a hole travels before recombining.
L_n is the average distance an electron travels before recombining.

$$\boxed{L_n \equiv \sqrt{D_n \tau_n} \text{ and } L_p \equiv \sqrt{D_p \tau_p}}$$

Excess Hole Distribution and Current

Solution to Hole Diffusion Equation

$$\frac{d^2\delta p}{dx^2} = \frac{\delta p}{L_p^2} \qquad \frac{\partial \delta p}{\partial t} = 0$$

Solution:

$$\delta p(x) = C_1 e^{x/L_p} + C_2 e^{-x/L_p}$$

Boundary Condition:

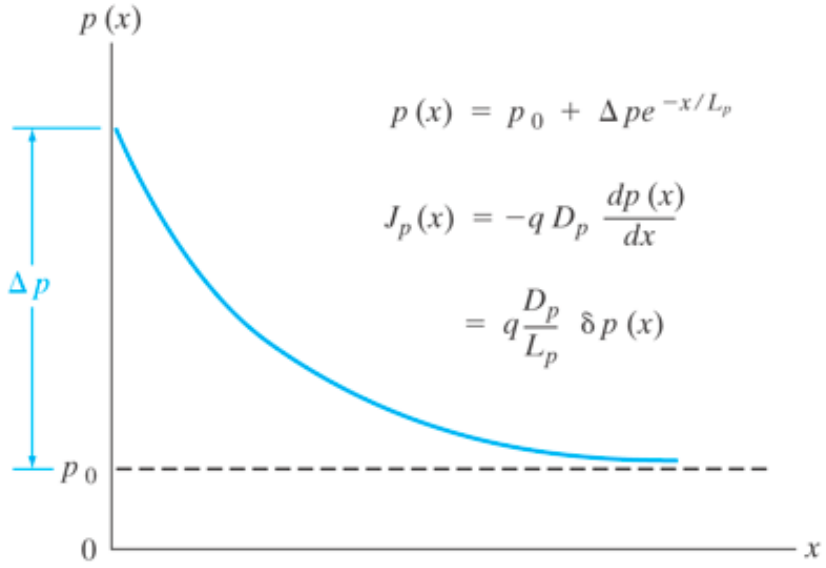
$$\delta p(x=0) = \Delta p = C_1 + C_2$$

$$\delta p(x=\infty) = 0 = C_1 e^{x/L_p}$$

$$C_1 = 0, \quad C_2 = \Delta p$$

$$\delta p(x) = \Delta p \bullet e^{-x/L_p}$$

Hole Injection at x=0



$$J_p(x) = -qD_p \frac{dp}{dx} = -qD_p \frac{d(p_o + \delta p)}{dx} = -qD_p \frac{d\delta p}{dx}$$

$$= q\frac{D_p}{L_p}\Delta p e^{-x/L_p} = q\frac{D_p}{L_p}\delta p(x)$$

Current Flow and Charge Loss

Current Density and Current

$$\delta p(x) = \Delta p \bullet e^{-x/L_p}$$

$$J_p(x) = -qD_p \frac{dp(x)}{dx} = -qD_p \frac{d\delta p(x)}{dx} = q\frac{D_p}{L_p}\delta p(x)$$

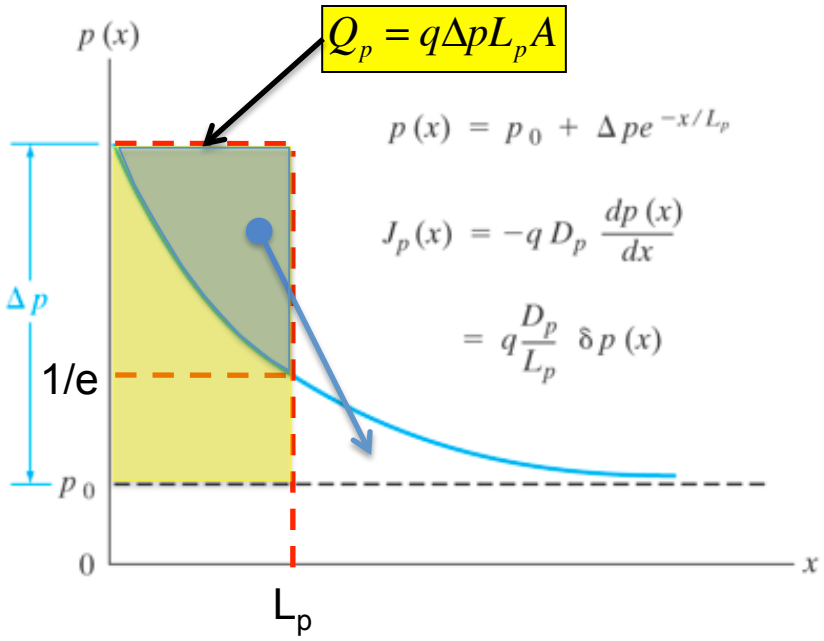
$$J_p(0) = q\frac{D_p}{L_p}\Delta p = q\left(\frac{L_p^2}{\tau_p}\right)\frac{1}{L_p}\Delta p = \frac{qL_p\Delta pA}{\tau_p A} = \frac{Q_p}{\tau_p A}$$

$$J_p(0) = \frac{Q_p}{\tau_p A} \quad , \quad I_p(0) = \frac{Q_p}{\tau_p}$$

- The current flow into the device at x=0 is equal to the charge loss due to recombination

Note:

$$\frac{d\delta p(x)}{dx} = -\frac{1}{L_p}\Delta p e^{-x/L_p}$$



Note, the total number of excess carriers is:

$$A\times\int_0^\infty \Delta pe^{-x/L_p}\,dx = -A\Delta pL_pe^{-x/L_p}\Big|_0^\infty = -A\Delta pL_p\left(e^{-\infty/L_p} - e^0\right)$$

$$= -A\Delta pL_p\left(e^{-\infty/L_p} - e^0\right)$$

$$= A\Delta pL_p$$

Contact Potential

Derivation of Contact Potential

In Equilibrium, there is no net current flow:

$$J_p(x) = q\mu_p p(x)\mathcal{E}(x) - qD_p \frac{dp(x)}{dx} = 0$$

$$\text{so } \frac{\mu_p}{D_p}\mathcal{E}(x) = \frac{1}{p(x)}\frac{dp(x)}{dx}$$

$$\text{but since } \frac{D}{\mu} = \frac{kT}{q} \text{ and } \mathcal{E}(x) = -\frac{d\mathcal{V}(x)}{dx}$$

$$\text{we have } -\frac{q}{kT}\frac{d\mathcal{V}(x)}{dx} = \frac{1}{p(x)}\frac{dp(x)}{dx}$$

Potential on p-side

Hole concentration on p-side

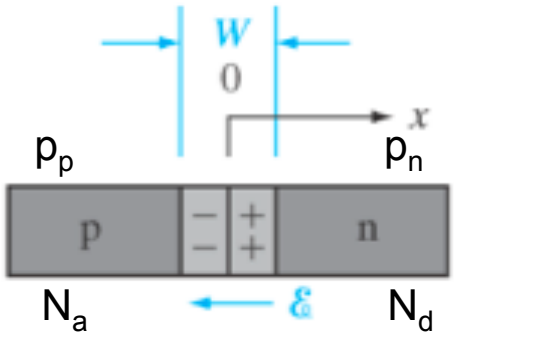
This is integrated over the limits $\mathcal{V}_p$ to $\mathcal{V}_n$ and p_p to p_n

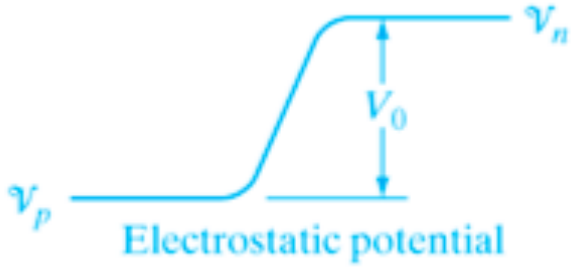
across the space charge region:

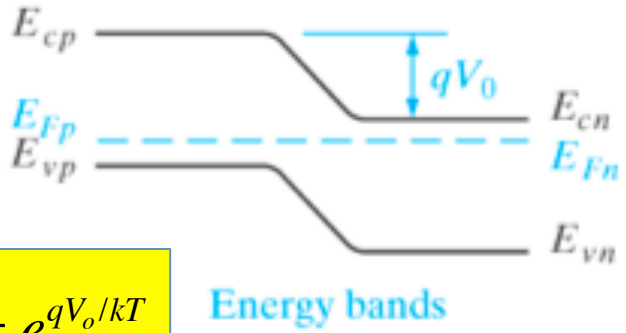
$$-\frac{q}{kT}\int_{\mathcal{V}_p}^{\mathcal{V}_n} d\mathcal{V} = \int_{p_p}^{p_n}\frac{1}{p}dp \text{ so } -\frac{q}{kT}\left(\mathcal{V}_n - \mathcal{V}_p\right) = \ln p_n - \ln p_p = \ln\frac{p_n}{p_p}$$

$$\text{so } \mathcal{V}_n - \mathcal{V}_p = V_o = -\frac{kT}{q}\ln\frac{p_n}{p_p} = \frac{kT}{q}\ln\frac{p_p}{p_n} = \frac{kT}{q}\ln\frac{N_a}{\left(n_i^2/N_d\right)} = \frac{kT}{q}\ln\frac{N_aN_d}{n_i^2}$$

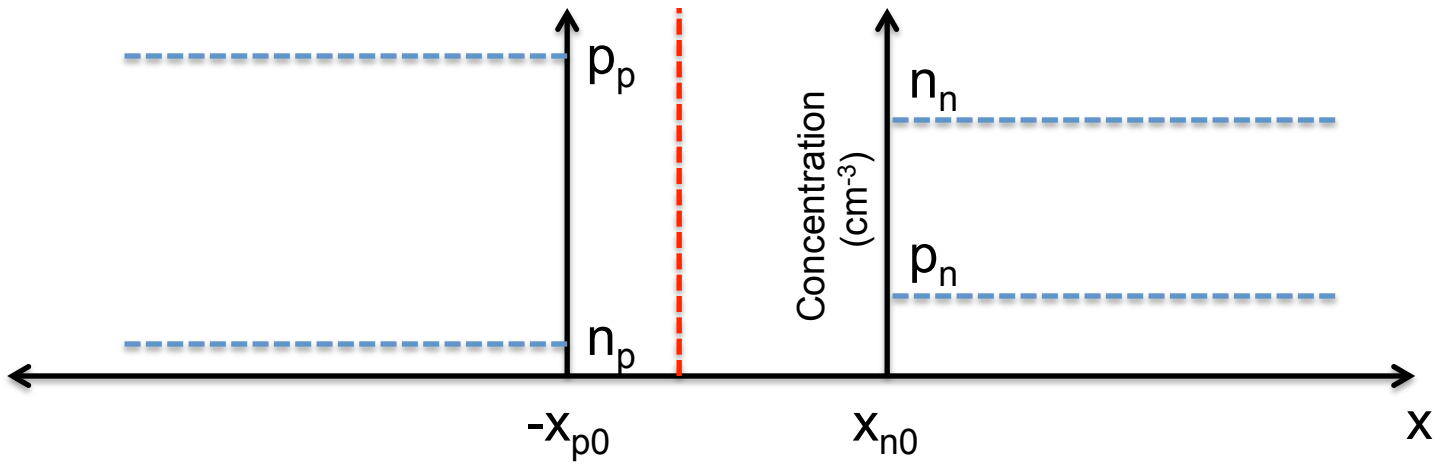
$$\text{Since } V_o = \frac{kT}{q}\ln\frac{p_p}{p_n} \text{ and } p_pn_p = n_i^2 = p_nn_n, \quad \boxed{\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{qV_o/kT}}$$







Carrier Concentrations



$$\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{qV_o/kT}$$

$$p_p = p_n e^{qV_o/kT} \quad \text{or} \quad p_n = p_p e^{-qV_o/kT}$$

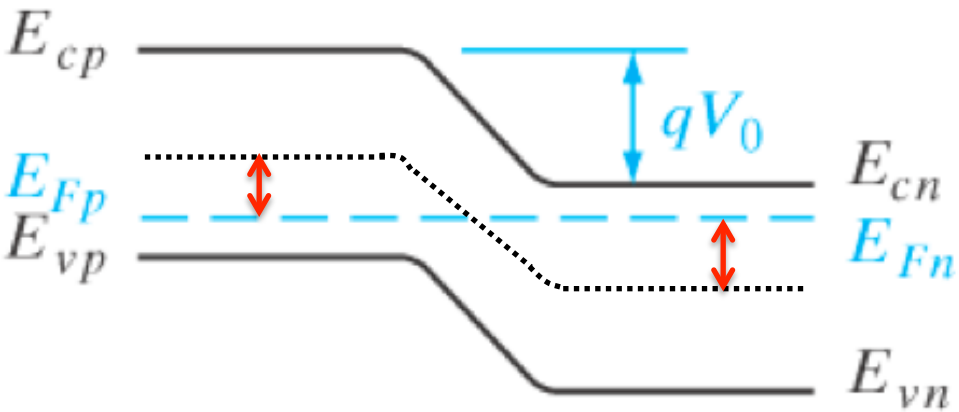
$$n_p = n_n e^{-qV_o/kT} \quad \text{or} \quad n_n = n_p e^{qV_o/kT}$$

Calculation of Contact Potential

Method 1:

- Use the relationship we derived last time

$$V_0 = \frac{kT}{q}\ln\frac{p_p}{p_n} = \frac{kT}{q}\ln\frac{N_a N_d}{n_i^2}$$



Method 2:

- Calculate the separation between the Fermi level and the intrinsic level on both sides of the junction, and sum the results

$$E_{ip} - E_F = kT\ln\frac{p_p}{n_i}$$

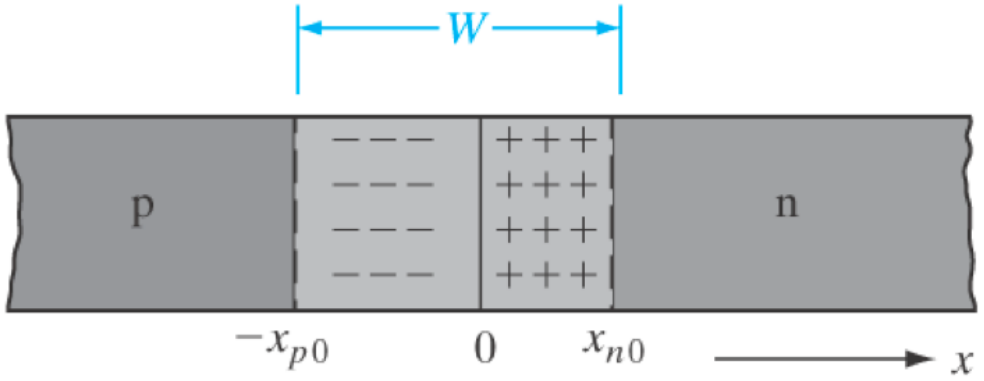
$$E_F - E_{in} = kT\ln\frac{n_n}{n_i}$$

$$V_0 = \left(E_{ip} - E_F\right) + \left(E_F - E_{in}\right)$$

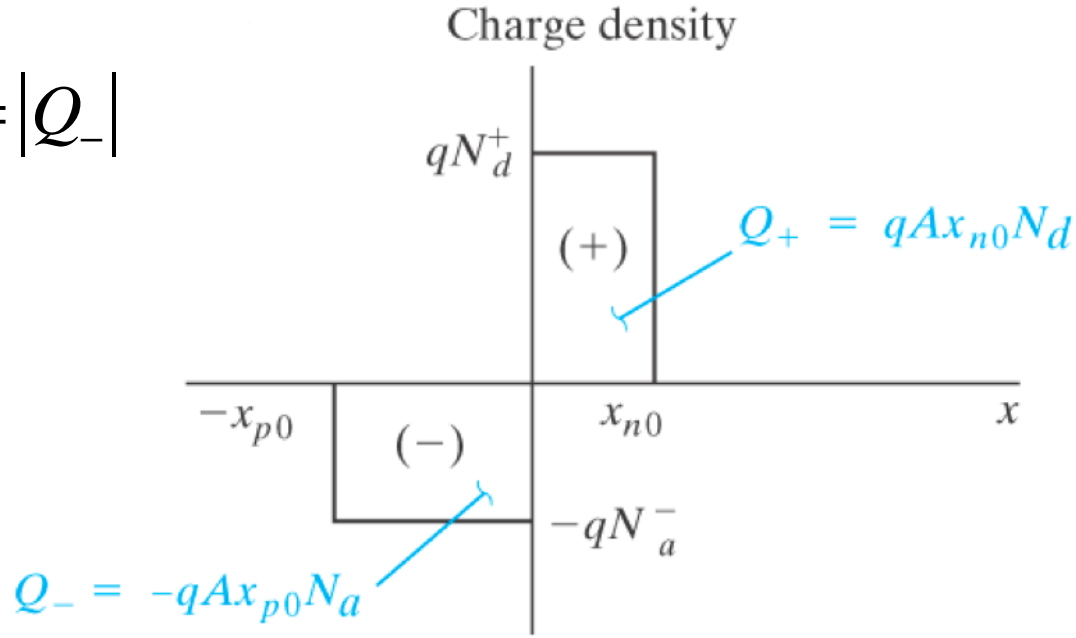
$$= kT\ln\frac{p_p}{n_i} + kT\ln\frac{n_n}{n_i}$$

Space Charge at the Junction

Space Charge



$$|Q_+| = |Q_-|$$



Depletion Approximation

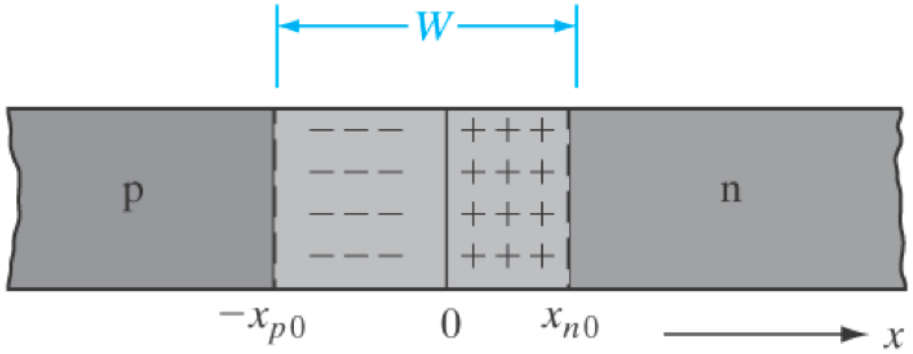
Depletion Approximation:

$$Q^+ = \left|Q^-\right|$$

$$qN_d^+ Ax_{no} = qN_a^- Ax_{po}$$

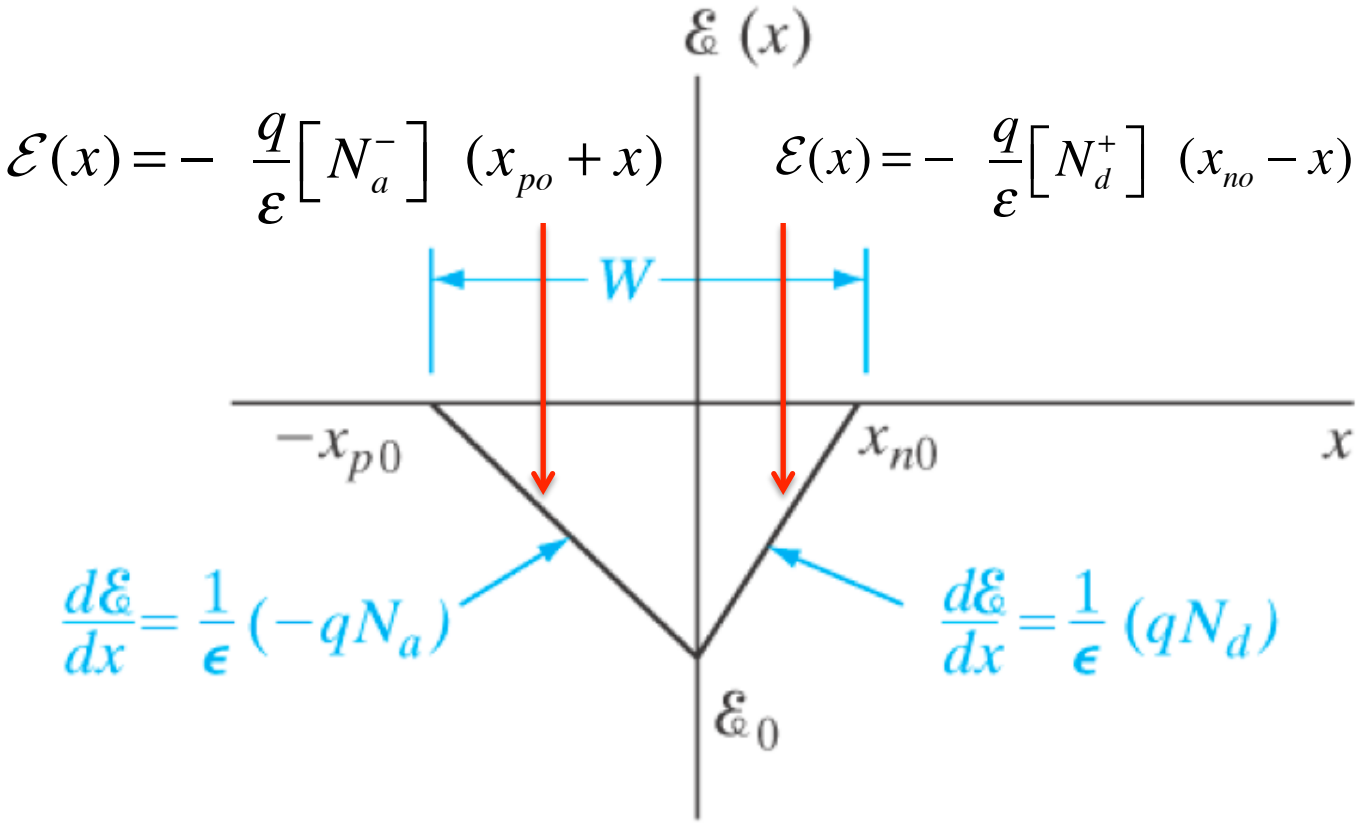
$$N_d^+ x_{no} = N_a^- x_{po}$$

$$x_{no} + x_{po} = W$$



- The depletion approximation states that carriers are depleted within the depletion width “W” and that charge neutrality applies outside of the depletion region

Junction Electric Field: Summary



$$\mathcal{E}_o = -\frac{q}{\varepsilon} N_d x_{n0} = -\frac{q}{\varepsilon} N_a x_{p0}$$

From Integrating Poisson's Equation

Calculation of Depletion Width

From the previous slide:

$$V_o = -\int_{-x_{po}}^{x_{n0}} \mathcal{E}(x)\,dx$$

So, the negative of the contact potential is the area under the $\mathcal{E}$(x) versus x triangle:

$$V_o = -\frac{1}{2}\mathcal{E}_o \cdot W = \frac{1}{2}\frac{q}{\varepsilon}N_d x_{no} W = \frac{1}{2}\frac{q}{\varepsilon}N_a x_{po} W$$

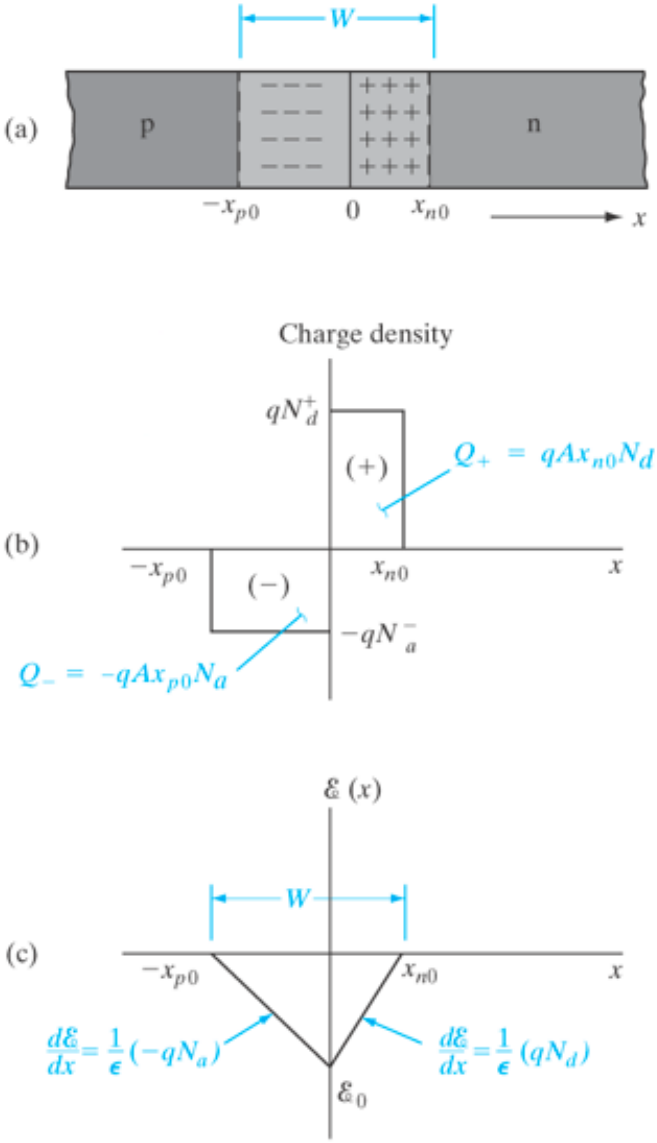
Balance of Charge Requirement: $Q_+ = |Q_-| \Leftrightarrow N_d x_{no} = N_a x_{po}$

$$x_{po} = x_{no}\frac{N_d}{N_a} \text{ and } W = x_{no} + x_{po} = x_{no}\left(1 + \frac{N_d}{N_a}\right),$$

$$\text{so } x_{no} = W\left(\frac{N_a}{N_a + N_d}\right)$$

$$\text{and } V_o = \frac{1}{2}\mathcal{E}_o W = \frac{1}{2}\frac{q}{\varepsilon}\frac{N_a N_d}{N_a + N_d}W^2$$

$$W = \left[\frac{2\varepsilon V_o}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2} = \left[\frac{2\varepsilon V_o}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}$$

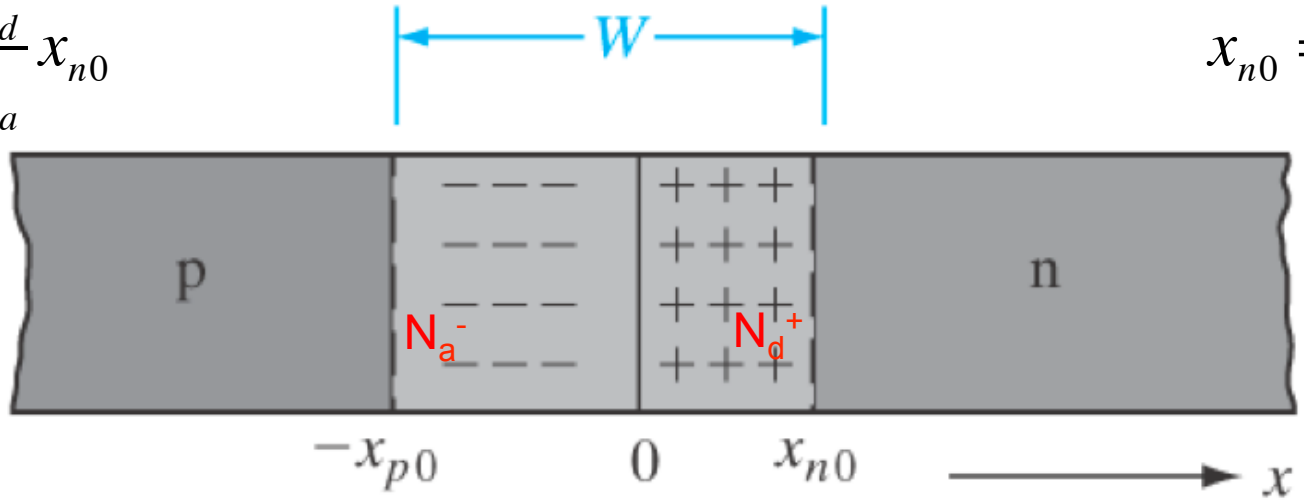


Depletion Width and Contact Potential

$$W = \left[\frac{2\varepsilon V_0}{q} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2}$$

$$x_{p0} = \frac{N_d}{N_a} x_{n0}$$

$$x_{n0} = \frac{N_a}{N_d} x_{p0}$$



$$x_{p0} = \left\{ \frac{2\varepsilon V_0}{q} \left[\frac{N_d}{N_a (N_a + N_d)} \right] \right\}^{1/2}$$

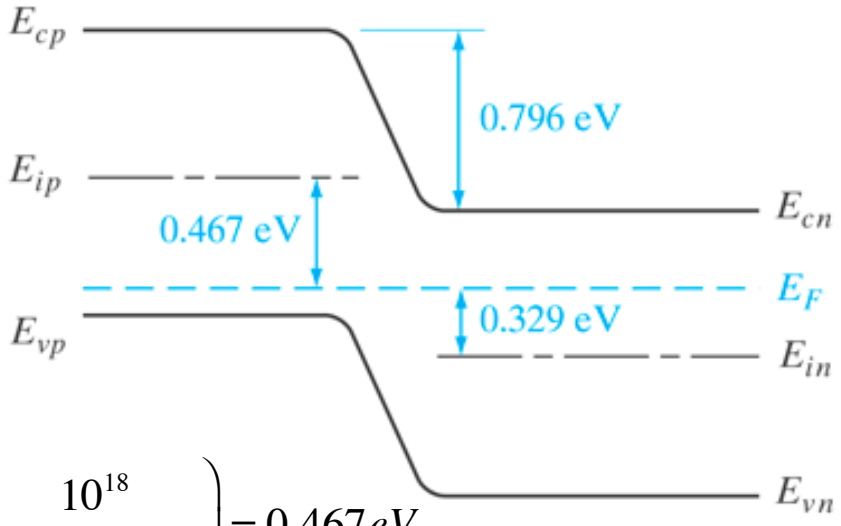
$$x_{n0} = \left\{ \frac{2\varepsilon V_0}{q} \left[\frac{N_a}{N_d (N_a + N_d)} \right] \right\}^{1/2}$$

Abrupt Junctions

Example: Abrupt Junction

Problem:

An abrupt junction in Si has $N_a=10^{18}$ cm^{-3} acceptors on one side and $N_d=5\times10^{15}$ donors on the other.



(a) Calculate the Fermi level positions at 300 K:

$$p_p = n_i e^{(E_i-E_F)/kT} \text{ so } E_{ip}-E_F = kT\ln\left(\frac{p_p}{n_i}\right) = 0.0259\ln\left(\frac{10^{18}}{(1.5\times10^{10})}\right) = 0.467eV$$

$$n_n = n_i e^{(E_F-E_i)/kT} \text{ so } E_F-E_{in} = kT\ln\left(\frac{n_n}{n_i}\right) = 0.0259\ln\left(\frac{5\times10^{15}}{(1.5\times10^{10})}\right) = 0.329eV$$

(b) Determine the contact potential:

$qV_o = 0.467+0.329 = 0.796eV$, so $V_o = 0.796V$

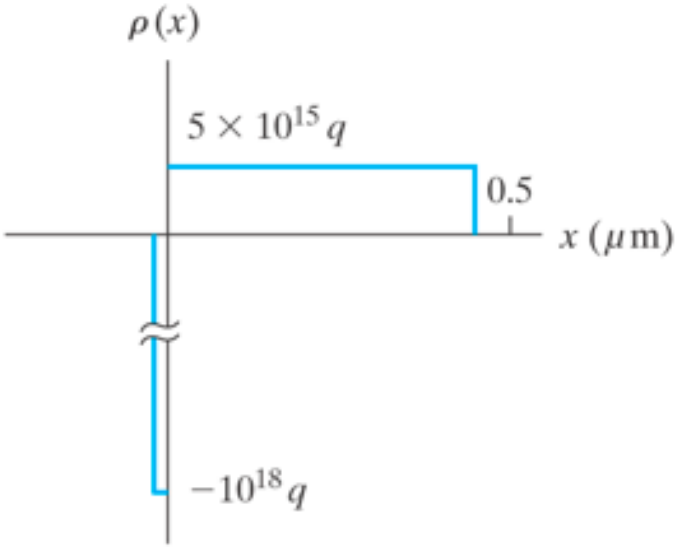
(c) Calculate the contact potential directly:

$$qV_o = kT\ln\frac{p_p}{p_n} = kT\ln\frac{N_aN_d}{n_i^2} = 0.0259\ln\left(\frac{(1\times10^{18})(5\times10^{15})}{(1.5\times10^{10})^2}\right) = 0.796eV, \text{ so } V_o = 0.796V$$

Example: Abrupt Junction [2]

Problem:

An abrupt junction in Si has $N_a=10^{18}$ cm^{-3} on one side and $N_d=5\times10^{15}$ on the other. If the junction has a diameter of 10μm:



Calculate the junction area:

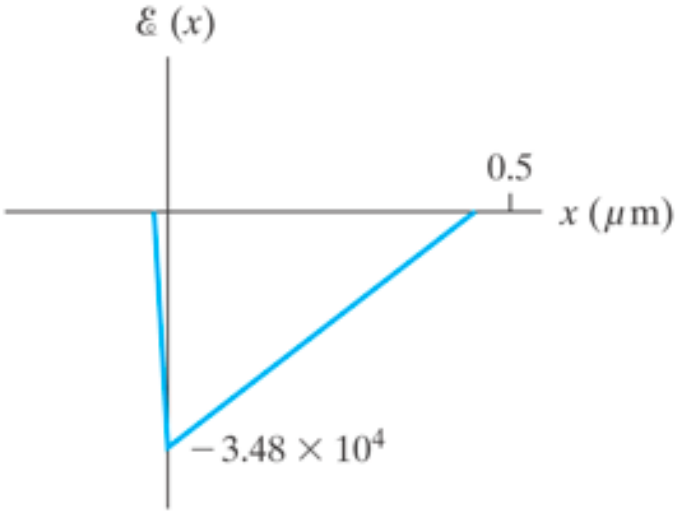
$$A=\pi(5\times10^{-4})^2=7.85\times10^{-7}\,cm^2$$

Assuming a junction in equilibrium at 300K, calculate the total depletion width:

$$W=\left[\frac{2\varepsilon V_o}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)\right]^{1/2}$$

$$=\left[\frac{2(11.8)(8.85\times10^{-14})(0.796)}{1.6\times10^{-19}}\left(10^{-18}+2\times10^{-16}\right)\right]^{1/2}$$

$$=0.457\,\mu m$$



Example: Abrupt Junction [3]

Problem:

An abrupt junction in Si has $N_a=10^{18}$ cm^{-3} on one side and $N_d=5\times10^{15}$ on the other. If the junction has a diameter of 10μm:

Calculate the depletion width on the n-side (x_{no}) and p-side (x_{po}):

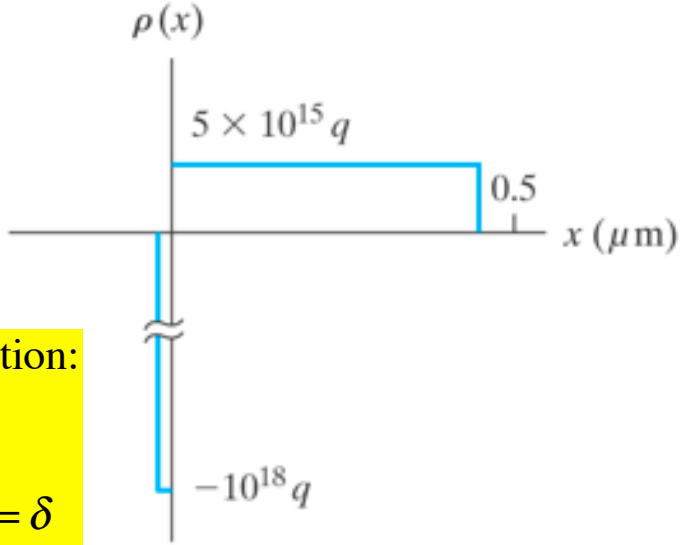
$$x_{no} = \frac{W}{1+N_d/N_a} = \frac{0.457}{1+5\times10^{-3}} = 0.455\,\mu m$$

$$x_{po} = \frac{W}{1+N_a/N_d} = \frac{0.457}{1+200} = 2.27\times10^{-3}\,\mu m$$

Abrupt Junction:

$$x_{no} \sim W$$

$$x_{po} \sim W\frac{N_a}{N_d} = \delta$$



Calculate the charge Q_+ and plot the charge density:

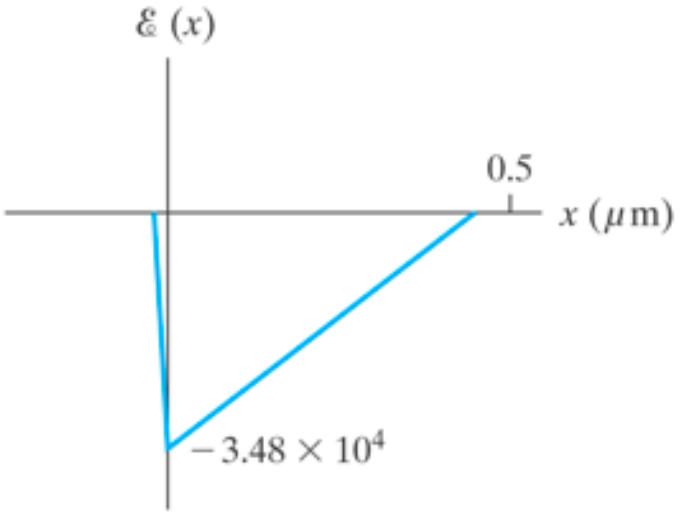
$$Q_+ = qAx_{no}N_d = qAx_{po}N_a$$

$$= (1.6\times10^{-19})(7.85\times10^{-7})(2.27\times10^{11}) = 2.85\times10^{-14}\,C$$

Calculate and plot the electric field:

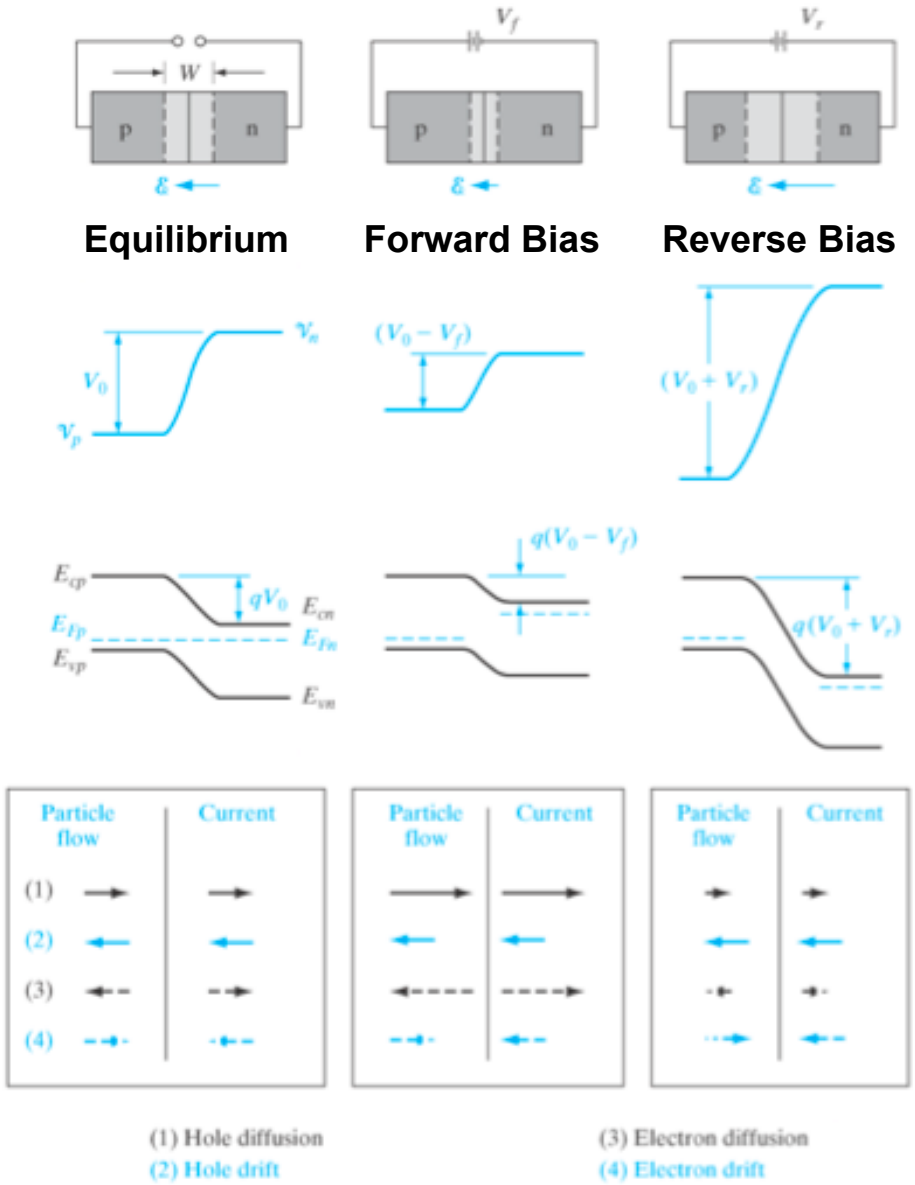
$$E_o = -\frac{q}{\varepsilon}x_{no}N_d = -\frac{q}{\varepsilon}x_{po}N_a$$

$$= \frac{1.6\times10^{-19}}{(11.8)(8.85\times10^{-14})}\left(2.27\times10^{11}\right) = -3.48\times10^{4}\,V/cm$$



pn Junctions Under Bias

Junction Under Bias

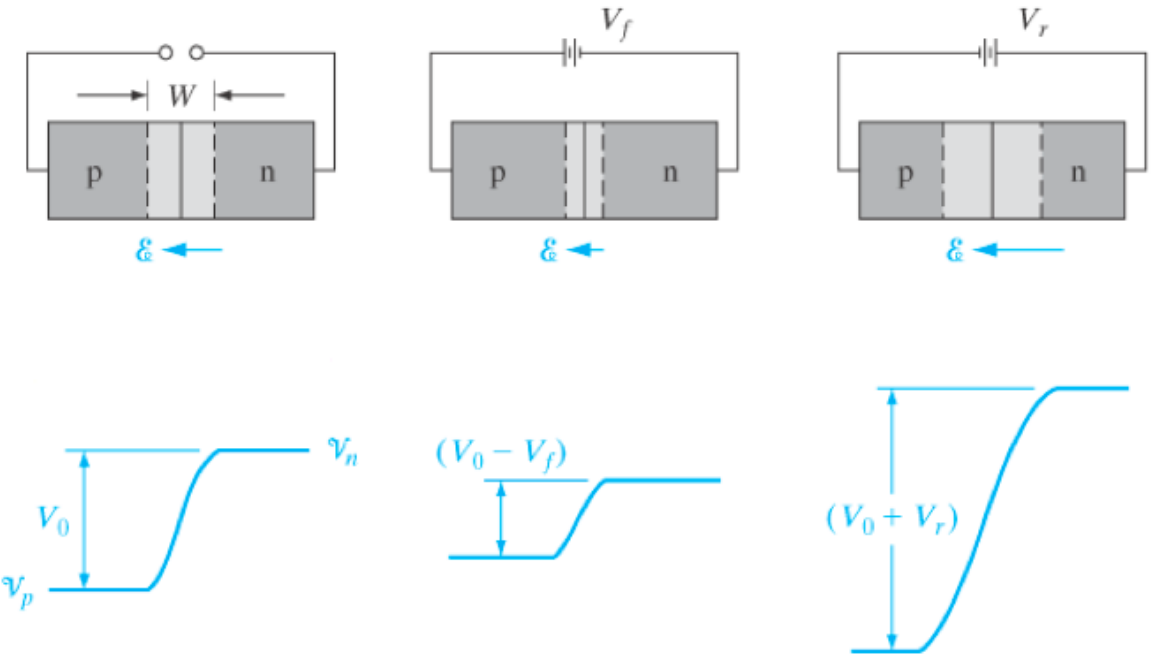


Bias and Field

- An applied bias reduces or increases the electrostatic potential barrier at the junction
- The magnitude of the maximum value of the electric field is lower in a junction under forward bias
- The magnitude maximum value of the electric is higher in a junction under reverse bias

$$\mathcal{E}(x) = -\frac{d\mathcal{V}(x)}{dx}$$

$$V_o - V = -\int_{-x_p}^{x_n} \mathcal{E}(x)\,dx$$

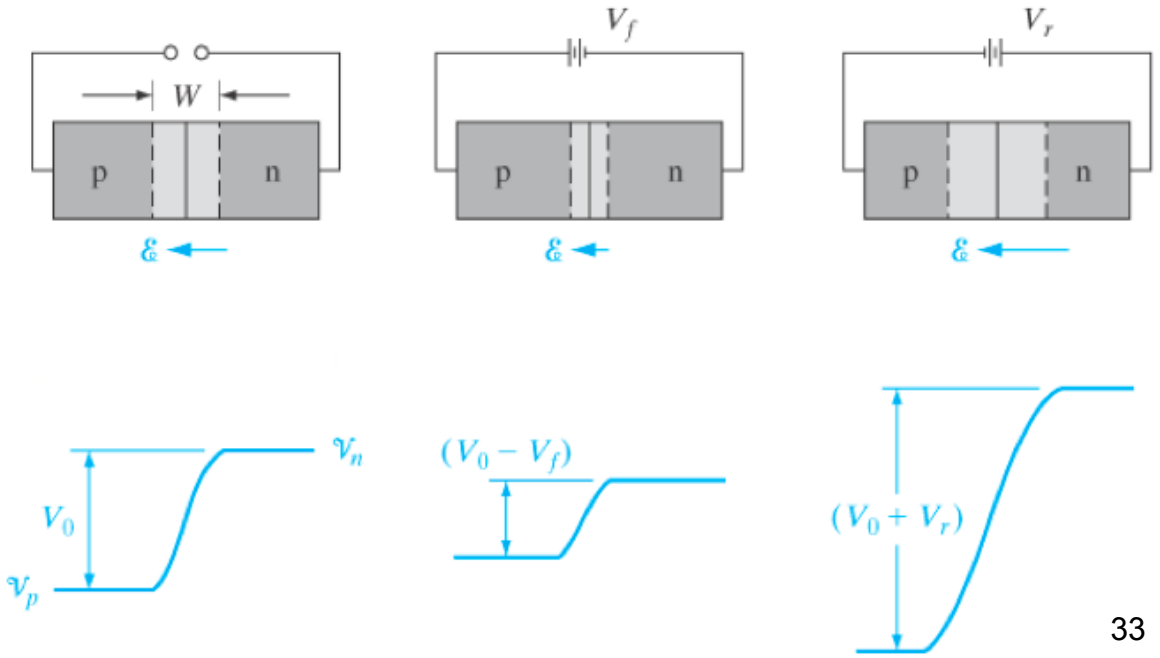


Bias and Depletion Width

- Field is produced by charge, a smaller field implies less charge so the depletion width under forward bias (lower field) is smaller
- The depletion width under reverse bias (higher field) is larger

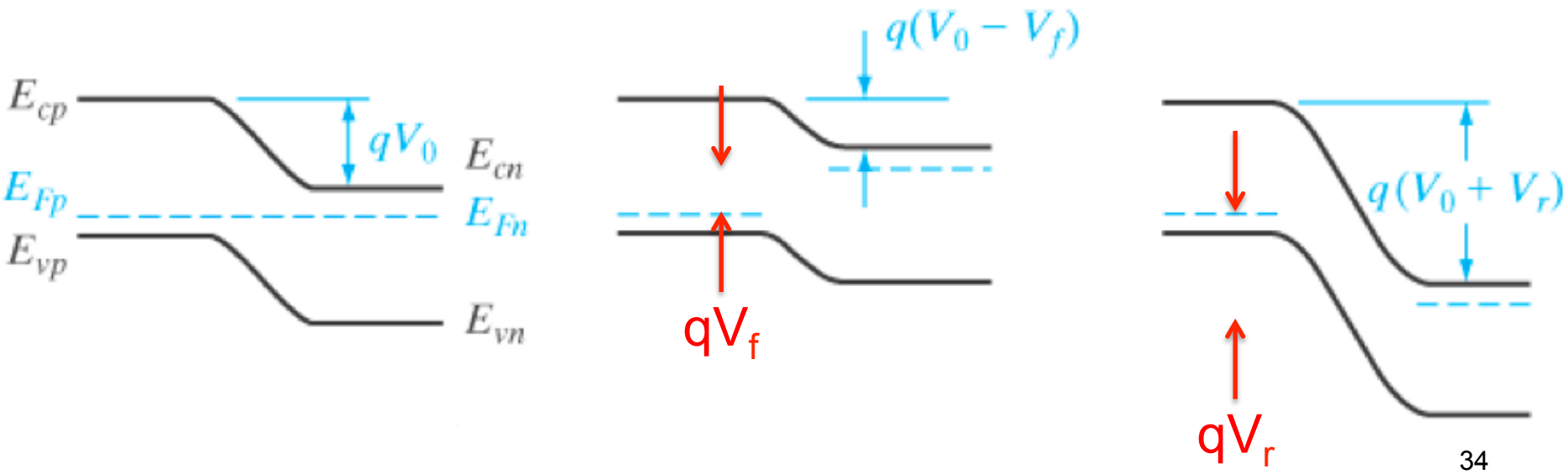
$$W = \left[\frac{2\varepsilon(V_o - V)}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2\varepsilon(V_o - V)}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}$$



Bias and Fermi Level Separation

- When we apply voltage we are no longer in equilibrium
- The application of a voltage creates a difference in the electrochemical potential across the junction boundary
- The Fermi levels become separated by an energy of “q” times the applied voltage
- The band separation is $q(V_0-V)$



Carrier Injection

Minority Carrier Injection: Forward Bias

From the previous slide:

$$\frac{p(x_{no})}{p_n} = e^{qV/kT} \quad \text{so:}$$

$$p(x_{no}) = p_n e^{qV/kT}$$

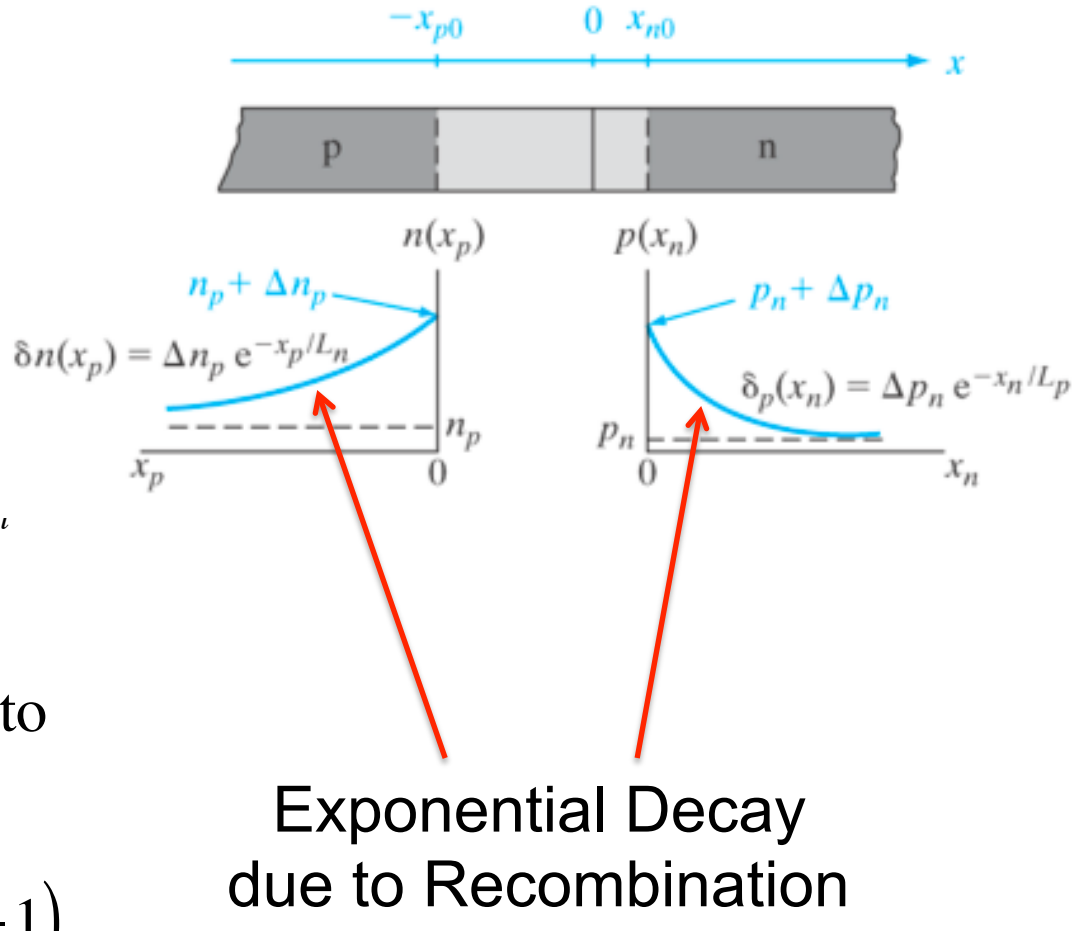
Therefore:

$$\Delta p_n = p(x_{no}) - p_n = p_n e^{qV/kT} - p_n$$

$$= p_n\left(e^{qV/kT} - 1\right)$$

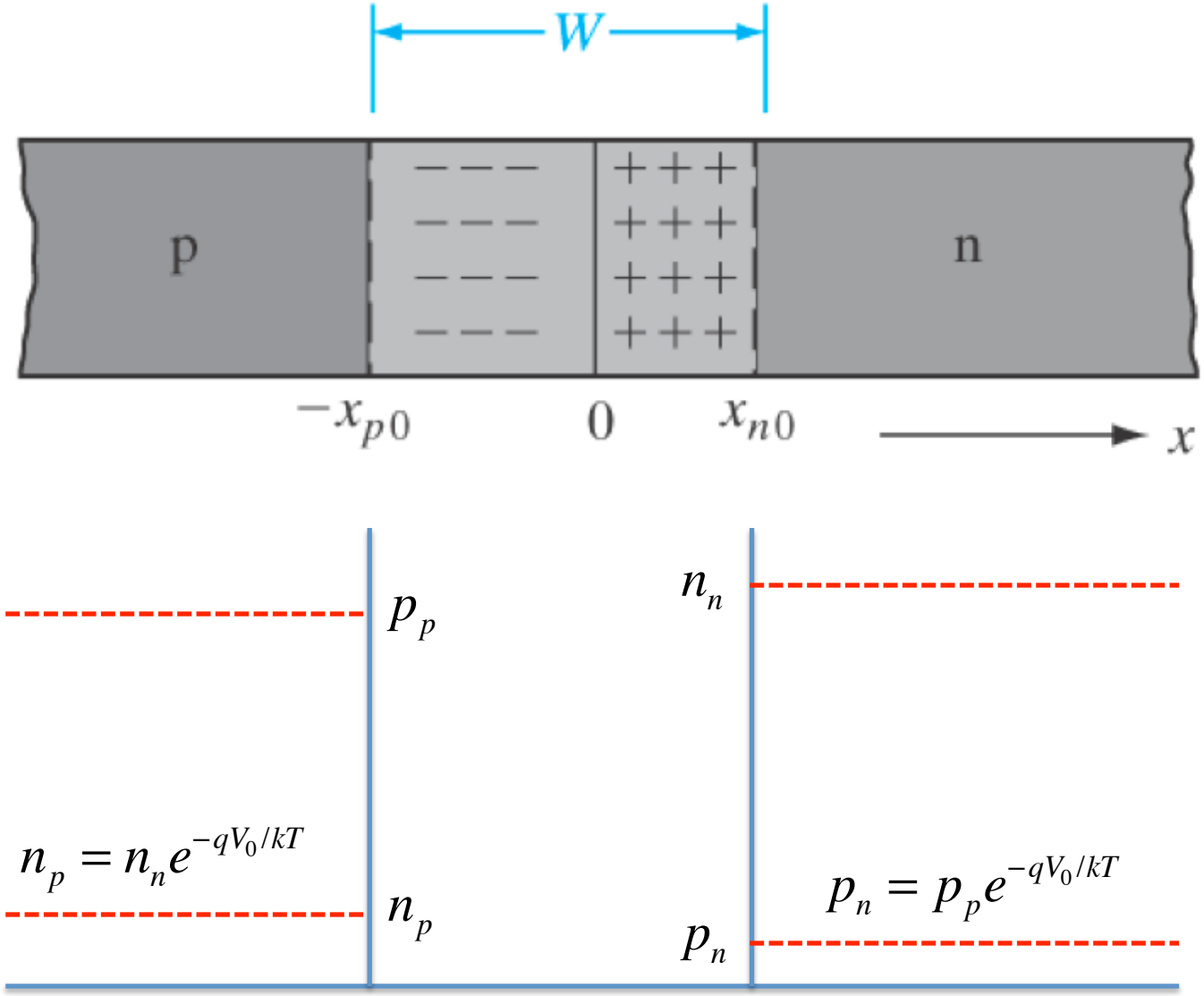
A similar approach can be used to derive the expression for Δn_p :

$$\Delta n_p = n(-x_{po}) - n_p = n_p\left(e^{qV/kT} - 1\right)$$

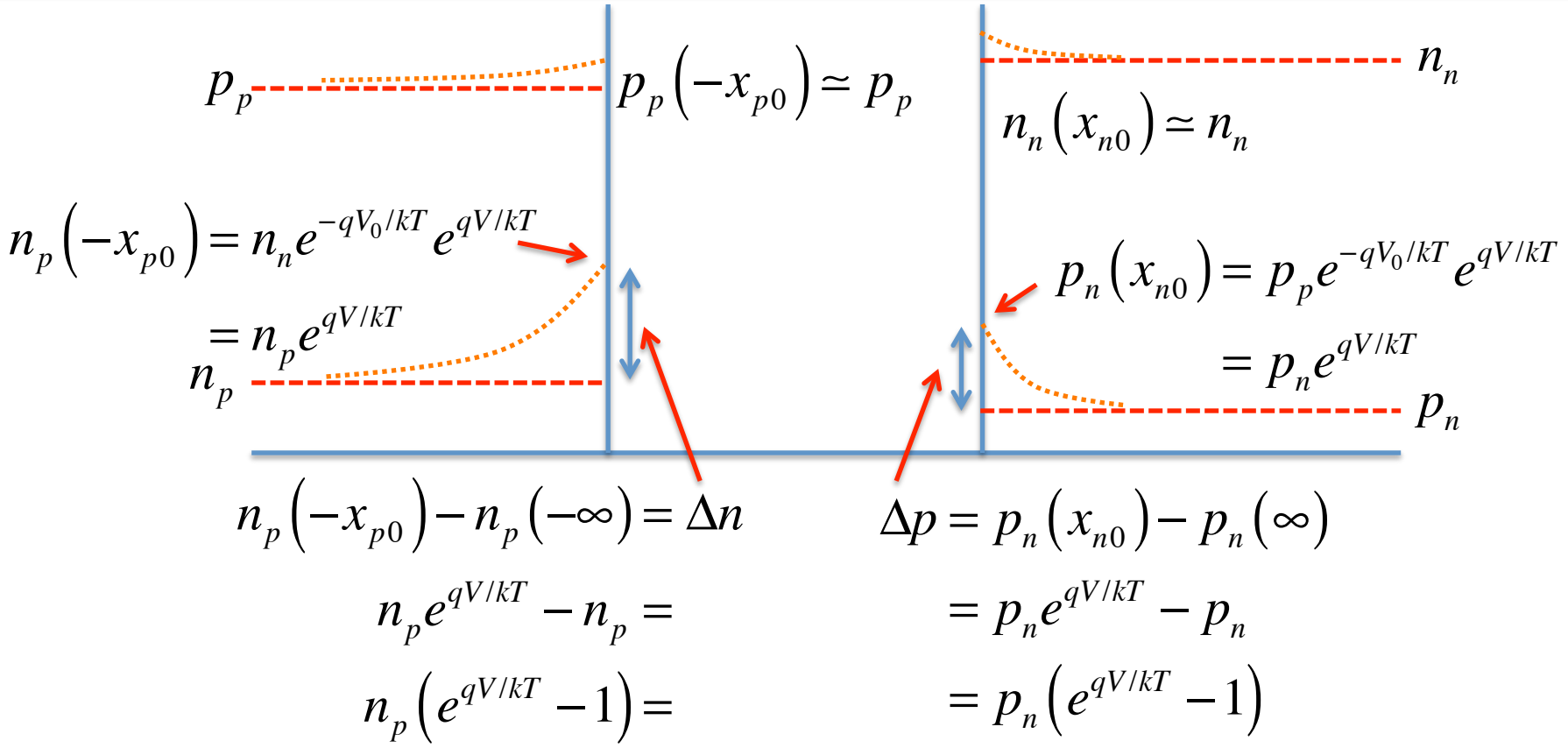


Exponential Decay due to Recombination

Equilibrium Carrier Concentrations



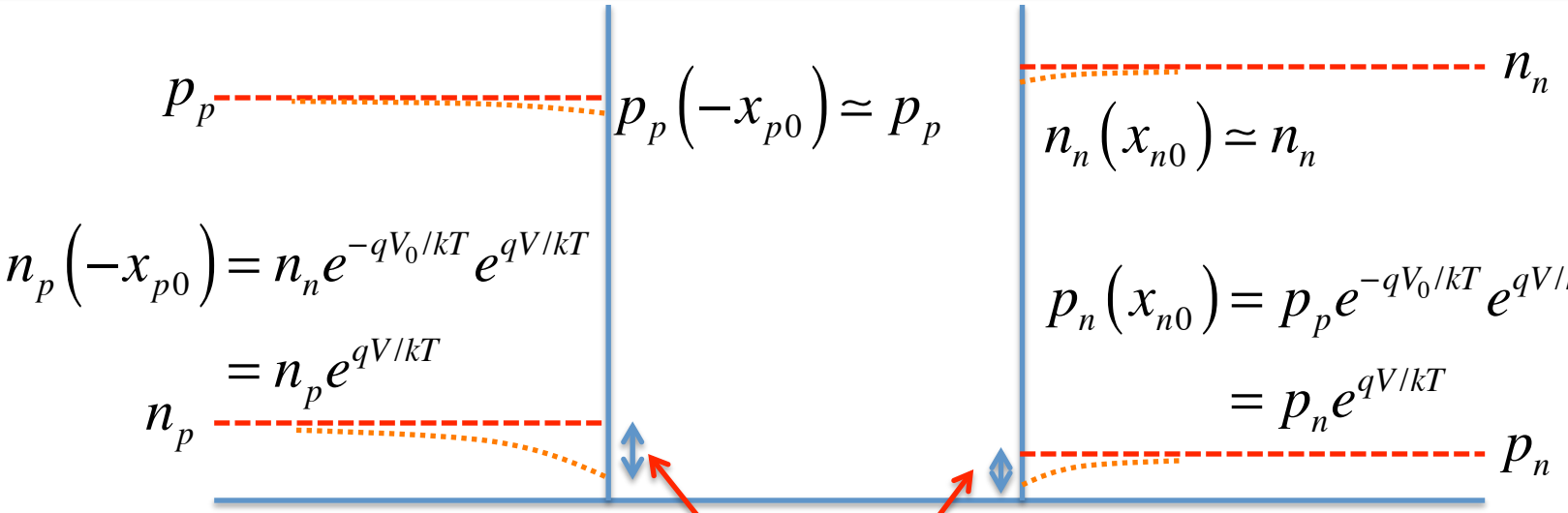
Forward Bias



Space Charge Neutrality, Low - Level Injection :

$$p_p(-x_{p0}) = p_p(-\infty) + \Delta n = p_p + n_p(e^{qV/kT} - 1) \simeq p_p$$

Reverse Bias



$$n_p(-x_{p0}) - n_p(-\infty) = \Delta n$$

$$n_p e^{qV/kT} - n_p =$$

$$n_p\left(e^{qV/kT} - 1\right) =$$

$$-n_p \simeq$$

$$\Delta p = p_n(x_{n0}) - p_n(\infty)$$

$$= p_n e^{qV/kT} - p_n$$

$$= p_n\left(e^{qV/kT} - 1\right)$$

$$\simeq -p_n$$

Minority Carrier Distribution

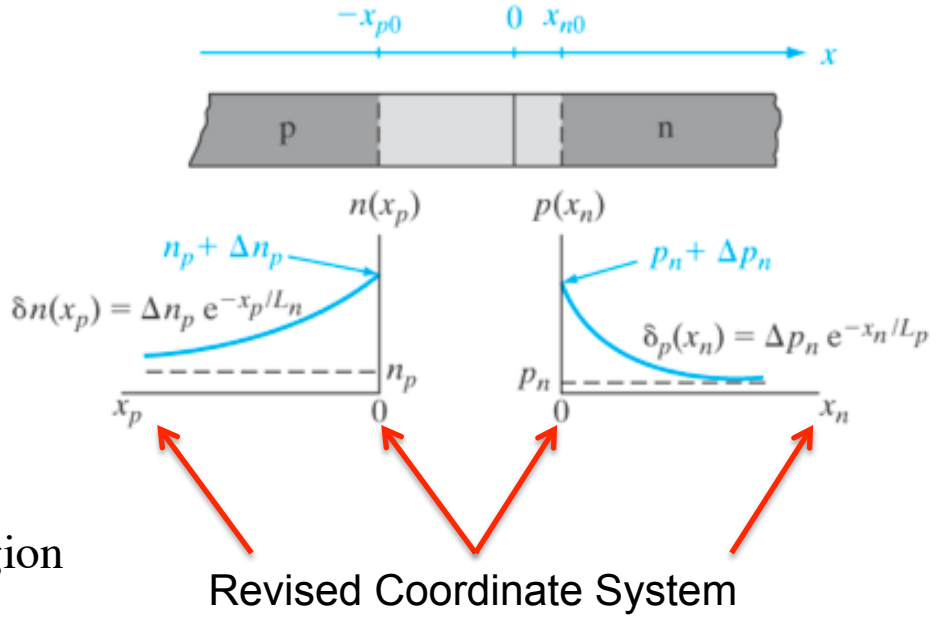
Define new coordinate systems referenced to edges the of the depletion region (x_{no} and $-x_{po}$). The diffusion equation for steady-state injection can be written:

$$\delta p(x_n) = \Delta p e^{-x_n/L_p} = p_n\left(e^{qV/kT} - 1\right)e^{-x_n/L_p}$$

similarly, for electrons as the minority carrier:

$$\delta n(x_p) = \Delta n e^{-x_p/L_n} = n_p\left(e^{qV/kT} - 1\right)e^{-x_p/L_n}$$

where x_n is referenced to the edge of depletion region on the n-side and extends in the +x direction, and x_p is referenced to the edge of the depletion region on the p-side and extends in the -x direction



Revised Coordinate System

Coordinate Transformation:

$x_n = x - x_{n0}$

$x_p = -x - x_{p0}$

x_{n0} and x_{p0} are the n and p-side depletion widths

The Ideal Diode Equation

Hole Current

Recall that:

$$J_p(x) = q\frac{D_p}{L_p}\delta p(x)$$

Therefore:

$$I_p(x_n = 0) = \frac{qAD_p}{L_p} p_n\left(e^{qV/kT} - 1\right)$$

Electron Current

Recall that:

$$J_n(x) = -q\frac{D_n}{L_n}\delta n(x)$$

Therefore:

$$I_n(x_p = 0) = -\frac{qAD_p}{L_p} n_p\left(e^{qV/kT} - 1\right)$$

Relative to the transformed coordinate system

Total Current

Neglecting recombination in the transition region:

$$\left|I_p(x_n = 0)\right| = \left|I_p(x_p = 0)\right| \text{ and } \left|I_n(x_p = 0)\right| = \left|I_n(x_n = 0)\right|$$

Since x_p is defined in the -x direction:

$$I_n(x_n = 0)_{x_n axis} = -I_n(x_p = 0)_{x_p axis}$$

The total current is the sum of the electron and hole currents:

$$I = I_p(x_n = 0) + I_n(x_n = 0)_{x_n axis} = I_p(x_n = 0) - I_n(x_p = 0)_{x_p axis} \text{ so:}$$

$$I = \frac{qAD_p}{L_p}\Delta p_n + \frac{qAD_n}{L_n}\Delta n_p = qA\left(\frac{D_p}{L_p} p_n + \frac{D_n}{L_n} n_p\right)\left(e^{qV/kT} - 1\right)$$

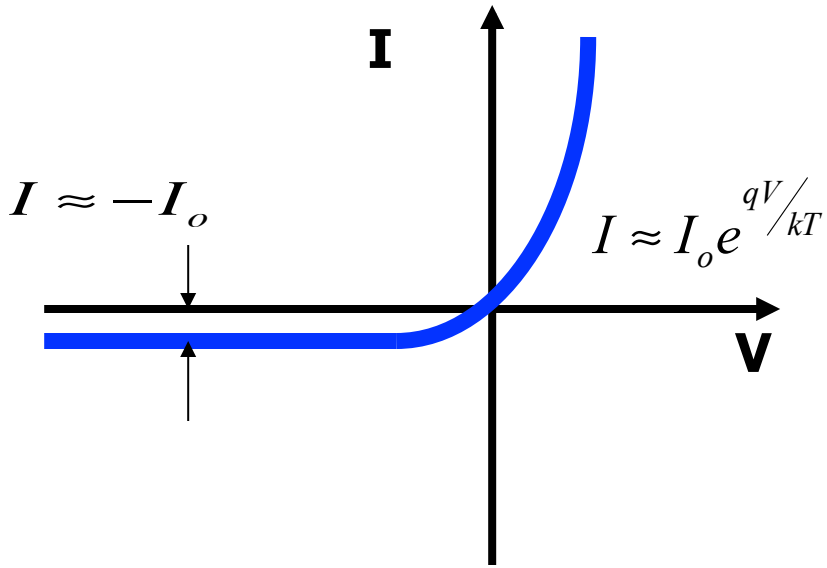
A Few Points

$$I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)\left(e^{qV/kT} - 1\right) = I_o\left(e^{qV/kT} - 1\right)$$

n-side current p-side current

- The dominant current contribution comes from injection from the more heavily doped side into the more lightly doped side
- Reducing the doping level on either side of the junction increases the minority carrier concentrations p_n and n_p which would tend to increase the current for a given voltage
- Increasing the doping level will cause the diffusion length to decrease, which would tend to cause current to increase for a given voltage
- An asymmetrical junction (p^+n or pn^+) will both increase the minority carrier concentration and reduce the diffusion length

Diode Equation: Various Conditions



$$I = I_o(e^{qV/kT} - 1)$$

$$I \approx I_o e^{qV/kT} \text{ for } V = V_f$$

$$I \approx -I_o \text{ for } V = -V_r$$

$$I = 0 \text{ for } V = 0$$

$$I = qA\left(\frac{D_p P_{on}}{L_p} + \frac{D_n n_{op}}{L_n}\right)(e^{qV/kT} - 1) = I_o(e^{qV/kT} - 1)$$

$(a)\ p - n\ Junction$

$$I = qA\left(\frac{D_p P_{on}}{L_p} + \frac{D_n n_{op}}{L_n}\right)(e^{qV/kT} - 1)$$

$(b)\ p^+ - n\ Junction$

$$p_{op} >> n_{on} \Rightarrow n_{op} << p_{on}$$

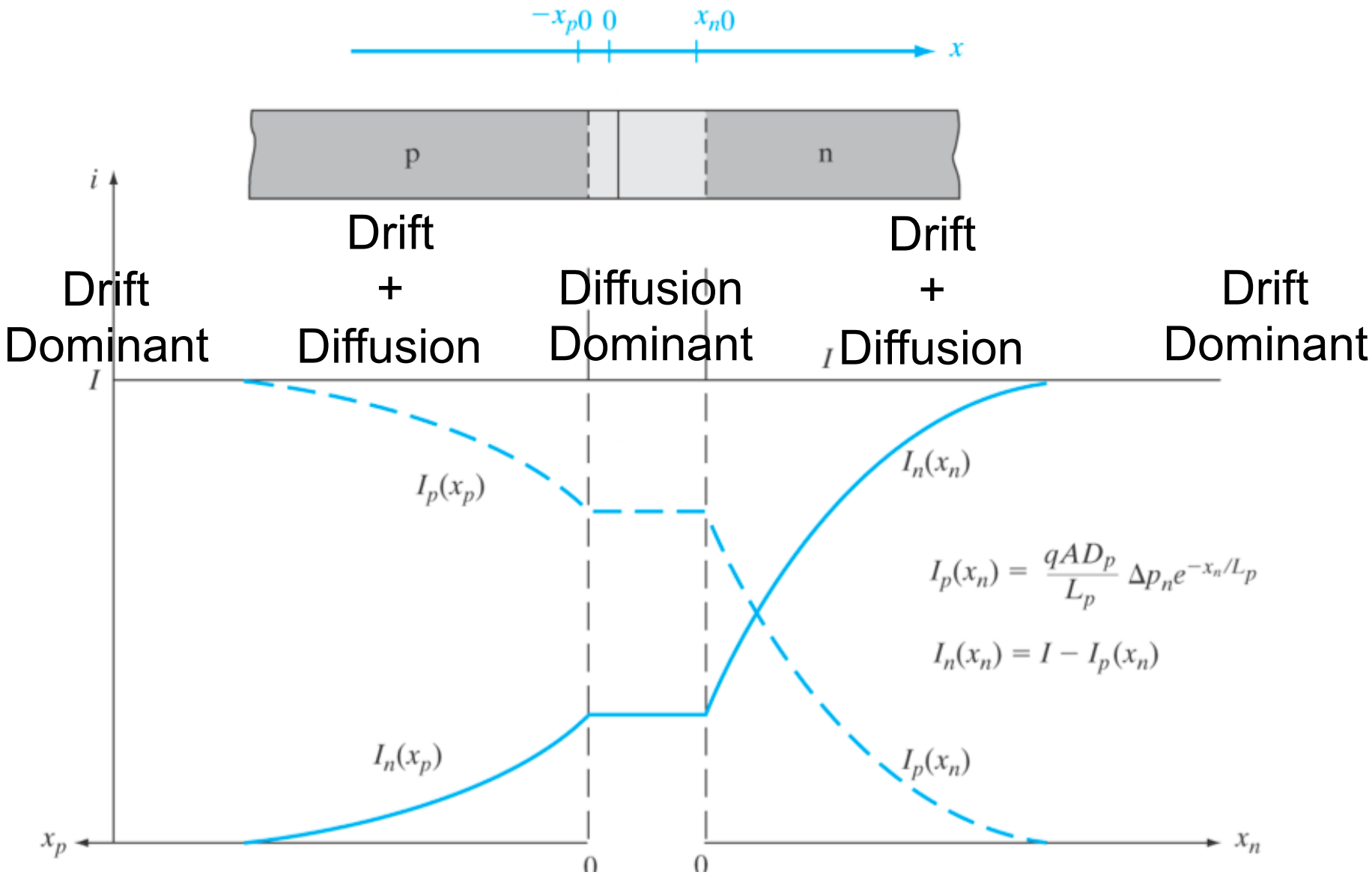
$$I = qA\left[\frac{D_p p_{on}}{L_p}\right](e^{qV/kT} - 1)$$

$(c)\ p - n^+\ Junction$

$$p_{op} << n_{on} \Rightarrow n_{op} >> p_{on}$$

$$I = qA\left[\frac{D_n n_{op}}{L_n}\right](e^{qV/kT} - 1)$$

Drift & Diffusion in Forward Bias



Quasi Fermi Levels in the Forward Biased Junction

Quasi Fermi Levels in the Forward-Biased Junction

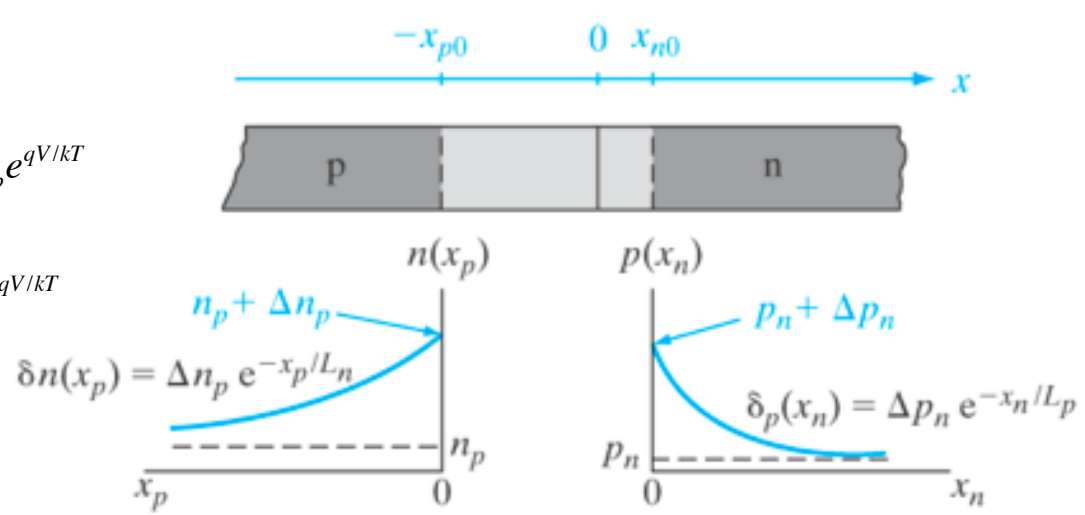
$$p(x_p = 0) \sim p_p(x_p = \infty) = p_p$$

$$n(x_p = 0) = n_p(x_p = \infty) + \Delta n$$

$$= n_p\left(e^{qV/kT} - 1\right) \sim n_p e^{qV/kT}$$

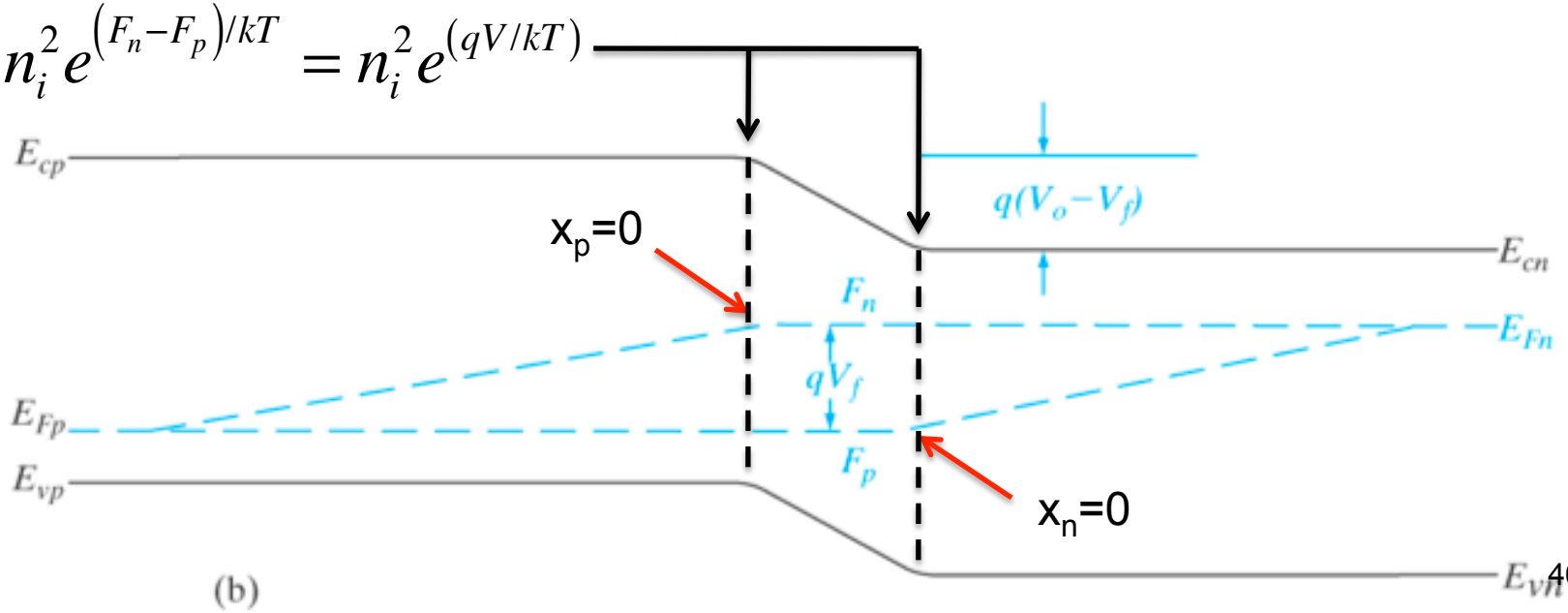
so:

$$pn(x_p = 0) = p_p n_p e^{qV/kT} = n_i^2 e^{qV/kT}$$



(a)

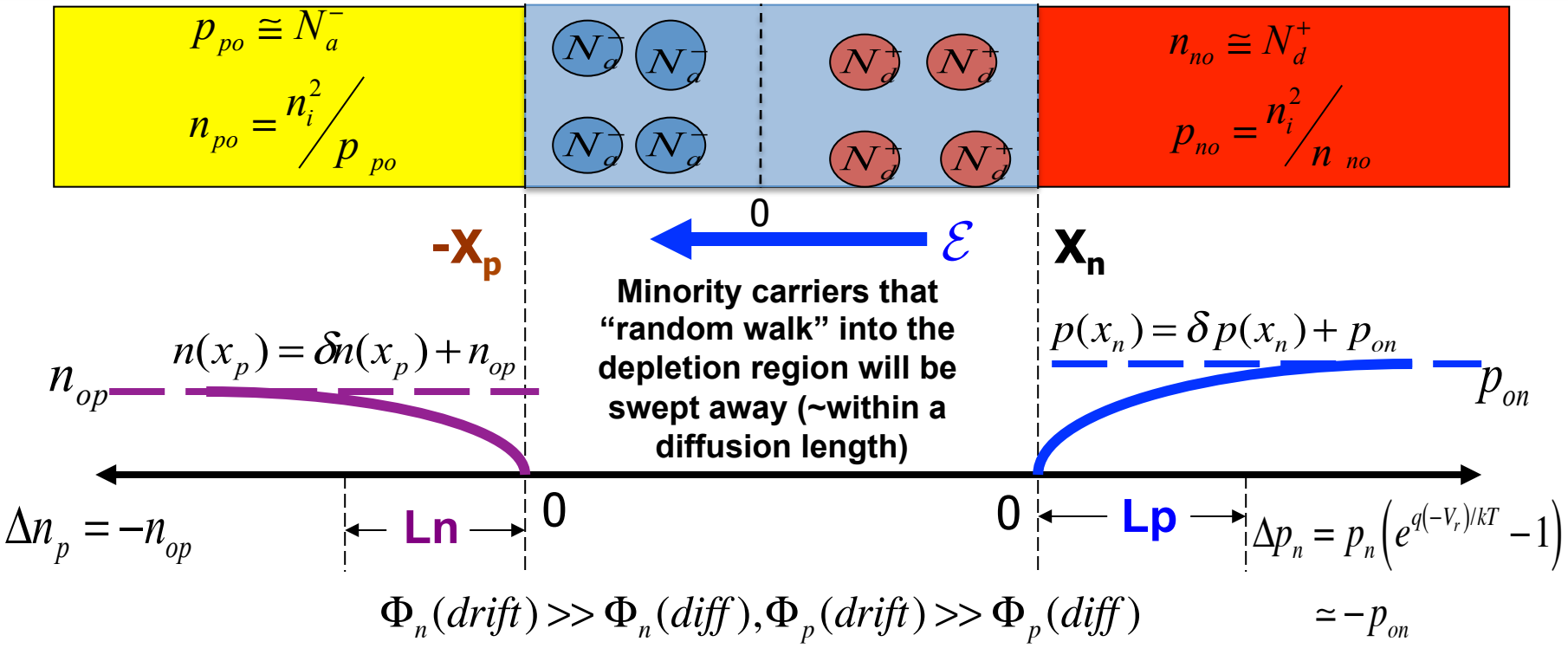
$$pn = n_i^2 e^{(F_n - F_p)/kT} = n_i^2 e^{(qV/kT)}$$



(b)

Reverse Bias

Reverse Bias: Drift Term Dominant



$$\Delta n_p = n_p(-x_p) - n_{op}$$

$$\Delta n_p = 0 - n_{op} = -n_{op}$$

$$\delta n(x_p) = -n_{op} e^{-x_p/Ln}$$

$$I_n(x_p) = +qAD_n \frac{d\delta n(x_p)}{dx_p}$$

$$I_n(x_p) = -qA \frac{D_n}{L_n} \delta n(x_p)$$

Combining Terms:

$$I = I_p(x_n = 0) + I_n(x_n = 0)_{x_n axis} = I_p(x_n = 0) - I_n(x_p = 0)_{x_p axis} \text{ so:}$$

$$I = \frac{qAD_p}{L_p}\Delta p_n + \frac{qAD_n}{L_n}\Delta n_p = qA\left(\frac{D_p}{L_p}(-p_{no}) + \frac{D_n}{L_n}(-n_{po})\right)$$

$$= -qA\left(\frac{D_p}{L_p}(p_{no}) + \frac{D_n}{L_n}(n_{po})\right) = -I_0$$

$$\Delta p_n = p_n(x_n) - p_{on}$$

$$\Delta p_n = 0 - p_{on} = -p_{on}$$

$$\delta p(x_n) = \Delta p_n e^{-x_n/L_p} = -p_{no} e^{-x_n/L_p}$$

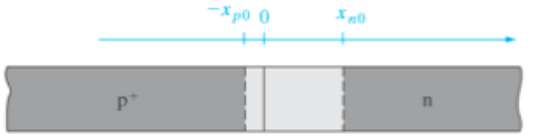
$$I_p(x_n) = -qAD_p \frac{d\delta p(x_n)}{dx_n}$$

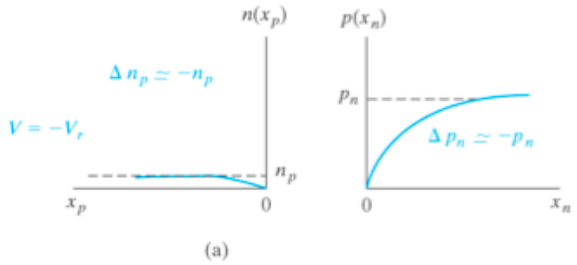
$$I_p(x_n) = qA \frac{D_p}{L_p} \delta p(x_n)$$

Reverse Bias Quasi Fermi Levels

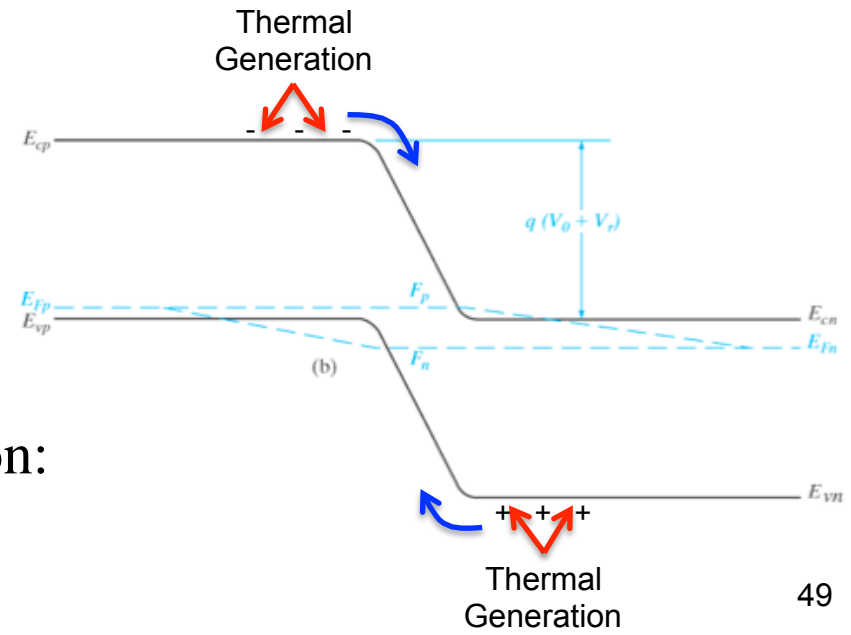
$$\Delta p_n = p_n\left(e^{q(-V_r)/kT} - 1\right) \simeq -p_n \quad \text{for} \quad V_r \gg kT/q$$

Similarly: $\Delta n_p \simeq -n_p$





- Minority carrier concentration approaches zero at the edge of the depletion region
- This effect is termed “minority carrier extraction”
- The reverse saturation current is a drift-dominated current, but the minority carriers arrive at the depletion region by way of diffusion



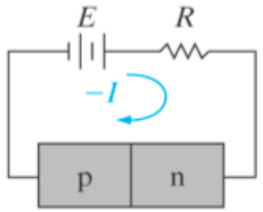
Reverse Bias, Edge of Depletion Region:

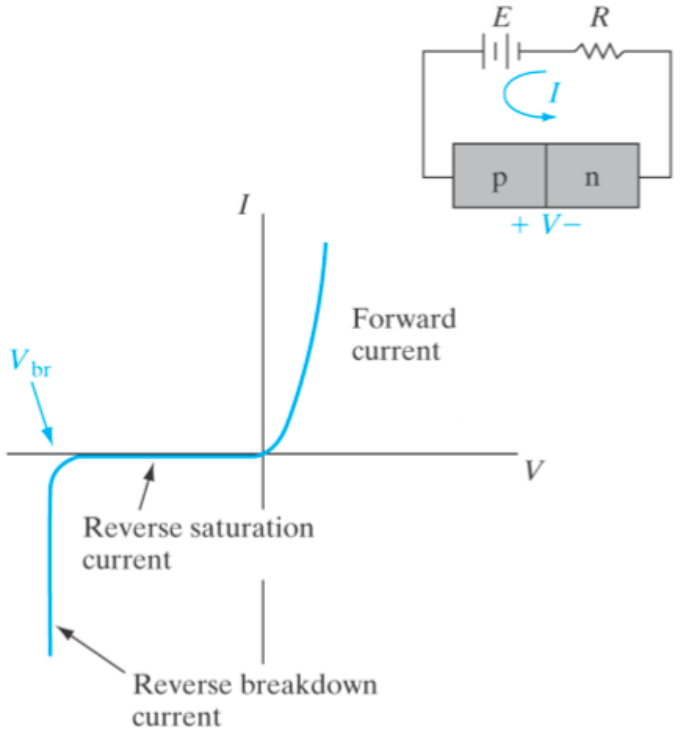
$$pn = n_i^2 e^{(F_n - F_p)/kT} \approx 0$$

Reverse Breakdown

Reverse Breakdown

- Reverse “breakdown” is not a destructive process (despite the name) – as long as current is limited externally
- The two types of breakdown discussed here are Zener effect breakdown and avalanche breakdown
- Zener breakdown occurs at a few volts
- Avalanche breakdown occurs at higher voltages





Zener Effect and Zener Breakdown

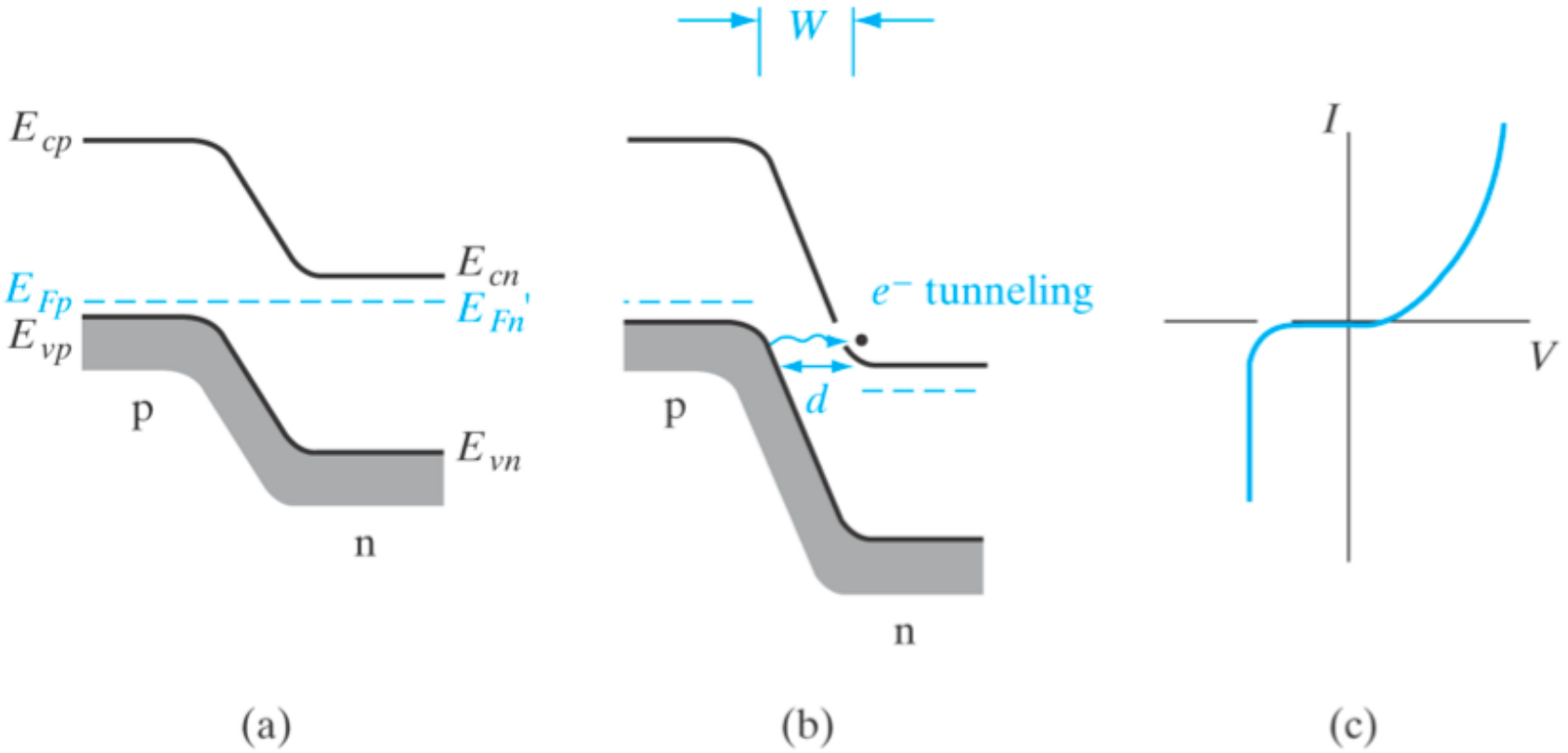
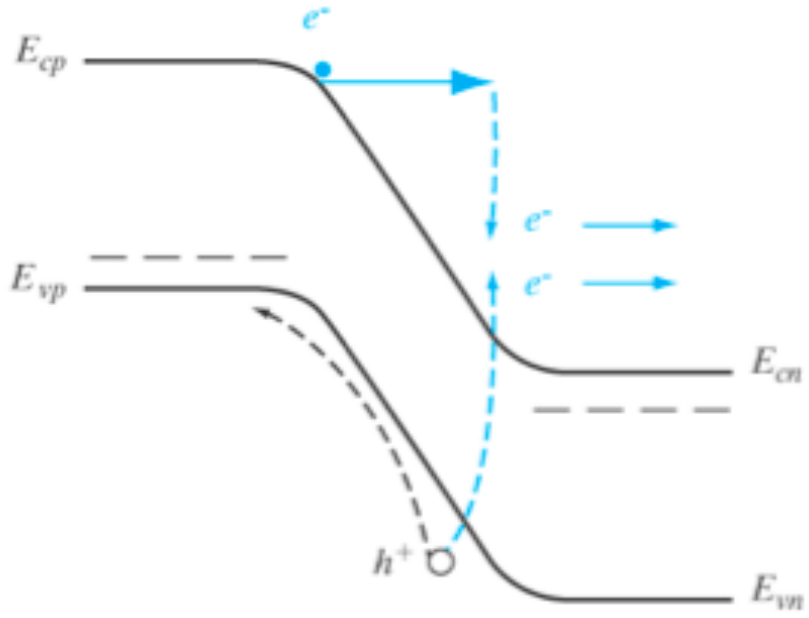


Figure 5.20

The Zener effect: (a) heavily doped junction at equilibrium; (b) reverse bias with electron tunneling from p to n; (c) *I–V* characteristic.

Impact Ionization and Avalanche Breakdown

Impact Ionization



- An electron entering the depletion region is accelerated under high field conditions
- If enough energy is gained, an ionizing collision with the lattice can produce an electron-hole pair ("impact ionization")
- Carrier multiplication occurs ($1\ e^- \rightarrow 2\ e^- + 1\ h^+$)

Capacitance of p-n Junctions

Reminder: Calculation of Depletion Width

The negative of the contact potential is the area under the $\mathcal{E}$(x) versus x triangle:

$$V_o = -\frac{1}{2}\mathcal{E}_o \cdot W = \frac{1}{2}\frac{q}{\varepsilon}N_d x_{no} W = \frac{1}{2}\frac{q}{\varepsilon}N_a x_{po} W$$

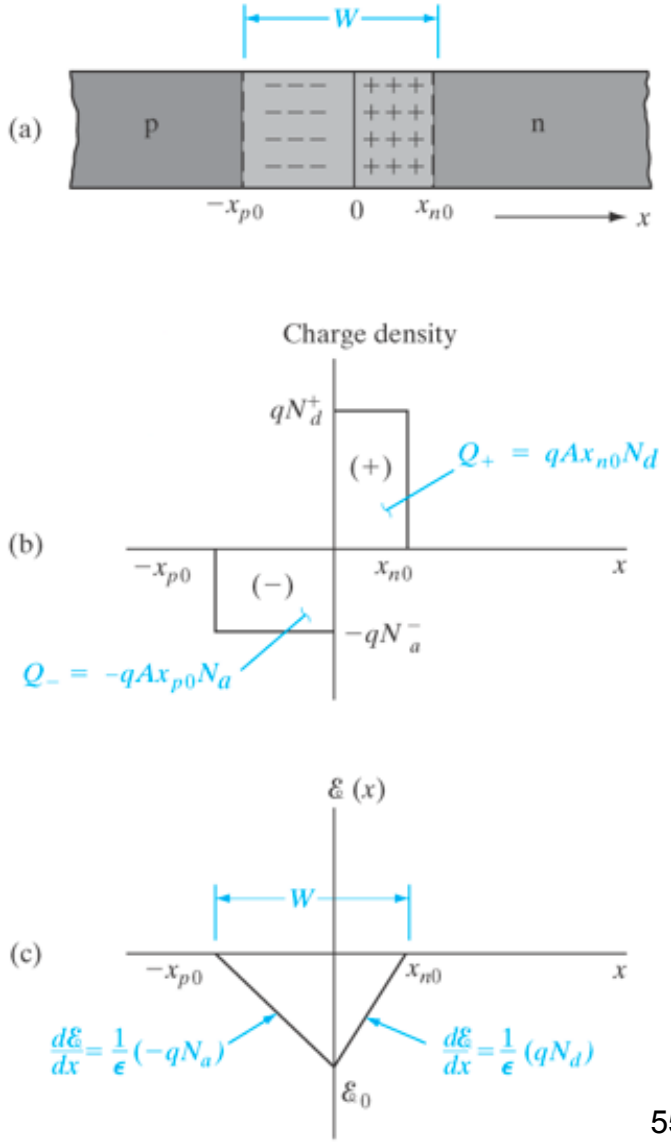
Balance of Charge Requirement: $Q_+ = |Q_-| \Leftrightarrow N_d x_{no} = N_a x_{po}$

$$x_{po} = x_{no}\frac{N_d}{N_a} \text{ and } W = x_{no} + x_{po} = x_{no}\left(1 + \frac{N_a}{N_d}\right),$$

$$\text{so } x_{no} = W\left(\frac{N_a}{N_a + N_d}\right)$$

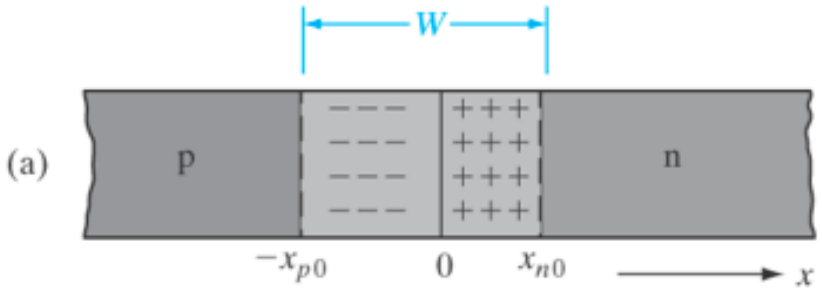
$$\text{and } V_o = \frac{1}{2}\frac{q}{\varepsilon}N_d x_{no} W = \frac{1}{2}\frac{q}{\varepsilon}N_d\left(W\left(\frac{N_a}{N_a + N_d}\right)\right)W = \frac{1}{2}\frac{q}{\varepsilon}\frac{N_a N_d}{N_a + N_d}W^2$$

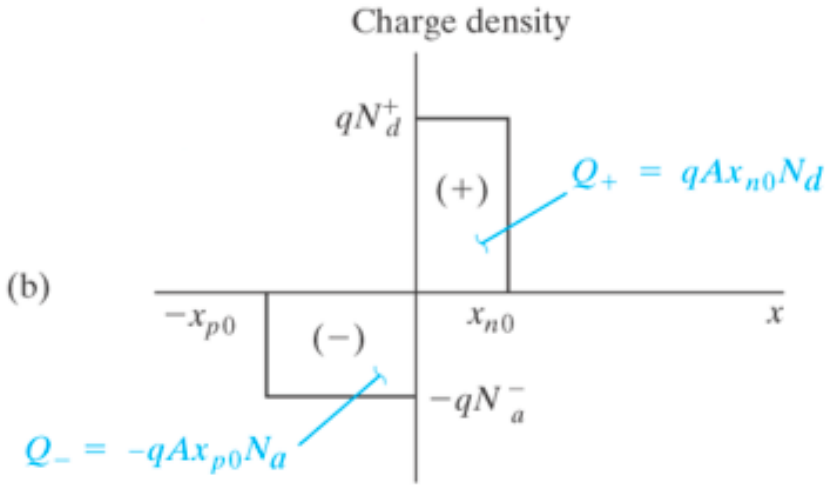
$$W = \left[\frac{2\varepsilon V_o}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2} = \left[\frac{2\varepsilon V_o}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}$$



Junction Capacitance

- Junction Capacitance: due to the dipole in the transition region
  - Dominant in reverse bias
- Charge Storage Capacitance: due to lagging of voltage as current changes due to charge storage effects
  - Dominant in forward bias
- More complex than C=Q/V





$$C = \left|\frac{dQ}{dV}\right|$$

Under bias: $$W = \left[\frac{2\varepsilon(V_o - V)}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2}$$

$$|Q| = qAx_{no}N_d = qAx_{po}N_a$$

$$x_{no} = \frac{N_a}{N_a + N_d}W \text{ and } x_{po} = \frac{N_d}{N_a + N_d}W \text{ so:}$$

$$|Q| = qA\frac{N_d N_a}{N_d + N_a}W = A\left[2q\varepsilon(V_o - V)\left(\frac{N_d N_a}{N_d + N_a}\right)\right]^{1/2}$$

Junction Capacitance [2]

$$|Q| = qA\frac{N_d N_a}{N_d + N_a}W = A\left[2q\varepsilon\left(V_o - V\right)\left(\frac{N_d N_a}{N_d + N_a}\right)\right]^{1/2} \quad \text{define } V' = V_0 - V$$

$$\frac{dV'}{dV} = \frac{d\left(V_0 - V\right)}{dV} = -1 \text{ so } dV' = -dV$$

$$C_j = \left|\frac{dQ}{dV}\right| = \left|\frac{dQ}{-dV'}\right| = \left|\frac{dQ}{-d\left(V_o - V\right)}\right| = \frac{A}{2}\left[\frac{2q\varepsilon}{\left(V_o - V\right)}\left(\frac{N_d N_a}{N_d + N_a}\right)\right]^{1/2}$$

Key Point: A p-n junction has a voltage-variable capacitance

Varactor: device that makes use of this property

$$\text{Note: } C = \frac{A}{2}\left(\frac{4\varepsilon^2}{4\varepsilon^2}\right)^{1/2}\left[\frac{2q\varepsilon}{\left(V_o - V\right)}\left(\frac{N_d N_a}{N_d + N_a}\right)\right]^{1/2} = \varepsilon A\left[\frac{q}{2\varepsilon\left(V_o - V\right)}\left(\frac{N_d N_a}{N_d + N_a}\right)\right]^{1/2} = \frac{\varepsilon A}{W}$$

$\frac{1}{W}$

For a p^+n junction, $N_a \gg N_d$ and $x_{no} \simeq W$ while x_{po} is negligible:

$$C_j = \frac{A}{2}\left[\frac{2q\varepsilon}{\left(V_o - V\right)}N_d\right]^{1/2} \quad \leftarrow \quad p^+n$$

Capacitance can be used to measure doping density!

Charge Storage (Diffusion) Capacitance

$$C_{sp} = \frac{q}{kT} Q_p = \frac{q}{kT} I_p \tau_p$$

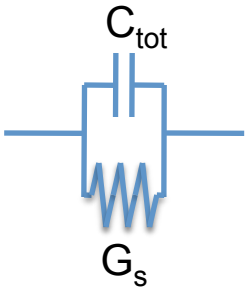
$$C_{sn} = \frac{q}{kT} Q_n = \frac{q}{kT} I_n \tau_n$$

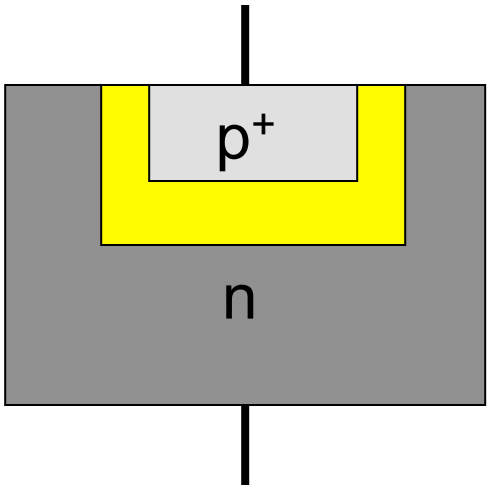
$$I = \frac{Q}{\tau} = I_p + I_n = \frac{Q_p}{\tau_p} + \frac{Q_n}{\tau_n}$$

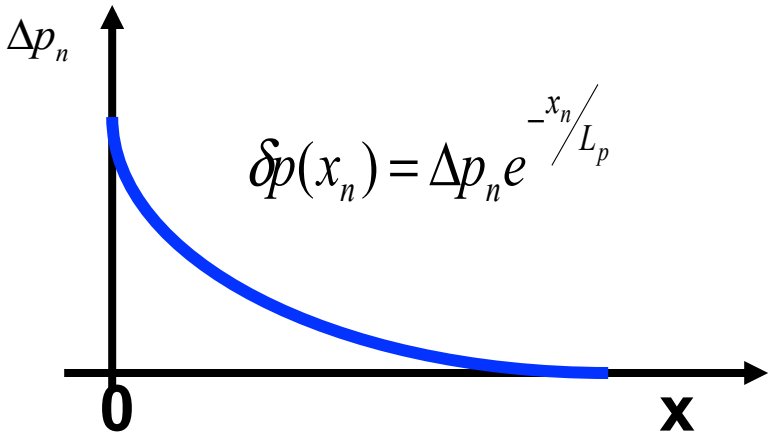
$C_s = C_{sp} + C_{sn}$ for a p-n junction

$C_s = C_{sp}$ for a p^+-n junction

$C_s = C_{sn}$ for a p-n^+ junction







$$C_s = \left| \frac{dQ_p}{dV} \right|$$

$$Q_p = qAL_p \Delta p_n = qAL_p p_n (e^{qV/kT} - 1)$$

$$C_s = \left| \frac{dQ_p}{dV} \right| = \frac{q^2}{kT} AL_p p_n e^{qV/kT} = \frac{q}{kT} Q_p$$

$$C_s = \left| \frac{dQ_p}{dV} \right| = \frac{q}{kT} Q_p = \frac{q}{kT} I \tau_p$$

$$I = \frac{Q_p}{\tau_p} = \frac{qAL_p}{\tau_p} p_n (e^{qV/kT} - 1)$$

But also: $G_s = \frac{dI}{dV} = \frac{q}{kT} I$

So we see: $C_s = G_s \tau_p$

$$i(t) = G_s v(t) + C_s \frac{dv(t)}{dt}$$

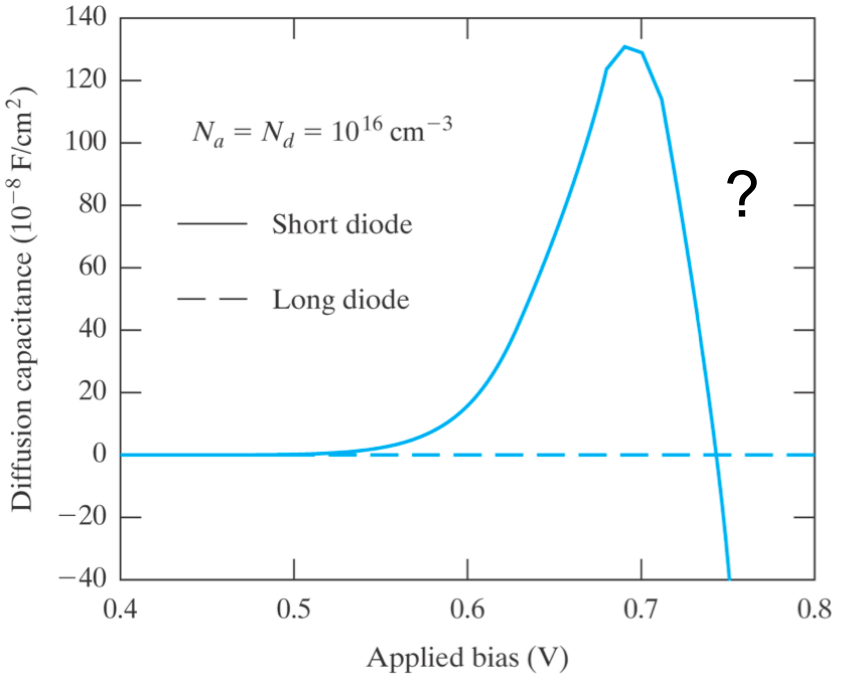
$$f_t = \frac{G_s}{2\pi C_s}$$

The charge storage capacitance can be a serious limitation for forward biased p-n junction in high frequency circuits, where f_t is cut off frequency.

Diffusion Capacitance

From Streetman:

- Under forward bias, the charge storage (diffusion capacitance) C_s can become dominant
- Long diodes – dimensions larger than diffusion lengths: no diffusion capacitance (typical case for laser diodes, direct-gap materials)
- Short diodes – reclaimable charge is 2/3 of the total stored charge (typical case for Si)



Note: There is debate regarding this material - possibly incorrect!

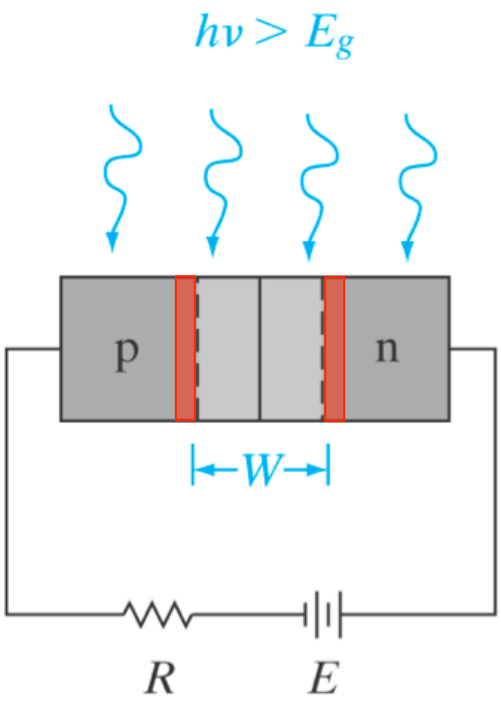
Short Diode Capacitance: $C_s = \frac{dQ_p}{dV} = \frac{1}{3}\frac{q^2}{kT} Acp_n e^{qV/kT}$

A-C Conductance: $G_s = \frac{dI}{dV} = \frac{qAL_p p_n}{\tau_p}\frac{d}{dV}\left(e^{qV/kT}\right) = \frac{q}{kT} I$

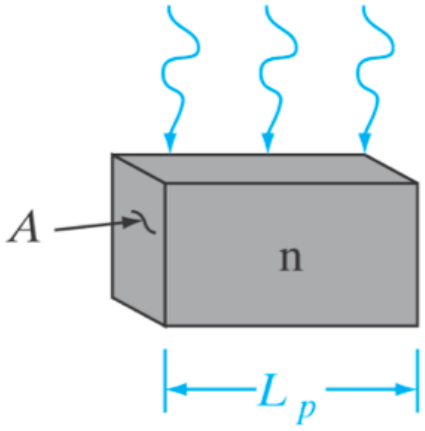
Current and Voltage in Illuminated Junctions

Optical Generation of Carriers

Absorption of Light



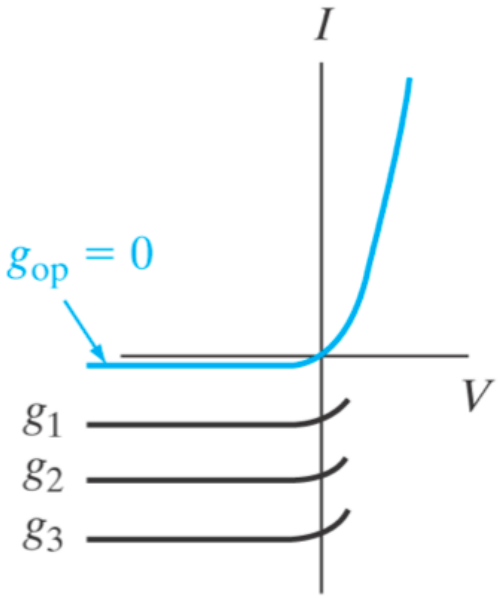
Current from EHP Generated Within a Diffusion Length



$$\delta p_{op} = g_{op}\tau_p$$

$$I_{op} = q\,A L_p g_{op}$$

I-V Characteristic of Illuminated Junction

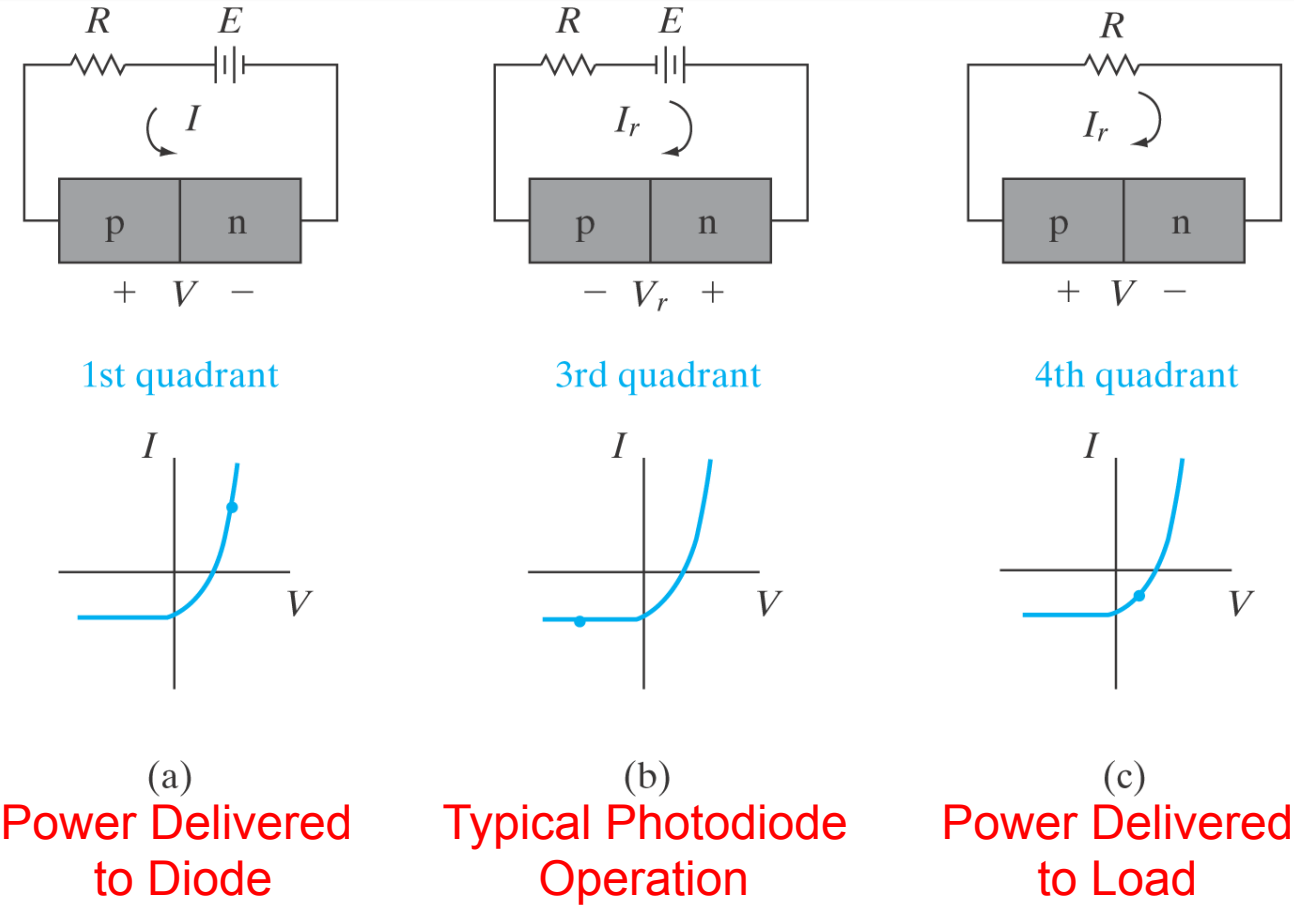


$g_3 > g_2 > g_1$

For both p-side and n-side contributions: $I_{op} = qAg_{op}\left(L_p + L_n + W\right)$

Optical excitation creates an additional term g_{op}

Photodiode Operating Regions

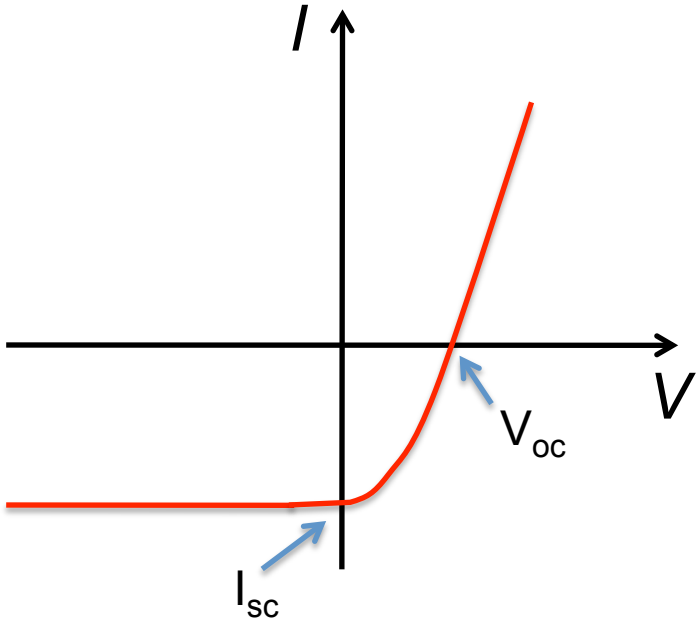


Operation of an illuminated junction in the various quadrants of its *I–V* characteristic; in (a) and (b), power is delivered to the device by the external circuit; in (c) the device delivers power to the load.

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V_{oc} and I_{sc}

- I_{sc} is the **short circuit current**: if the device is shorted under illumination, a current flows from n to p
  - From the diode equation under illumination, $I=-I_{op}$
- Under open circuit conditions, there is a voltage across the device that can be measured: V_{oc} (**open circuit voltage**)
- As the minority carrier concentration increases, the carrier lifetime decreases
  - Since $g_{th}=p_n/\tau_n$ we see g_{th} increases
- V_{oc} cannot increase indefinitely – it is limited by the contact potential V_o
- ***Photovoltaic effect:*** appearance of a voltage across an illuminated junction

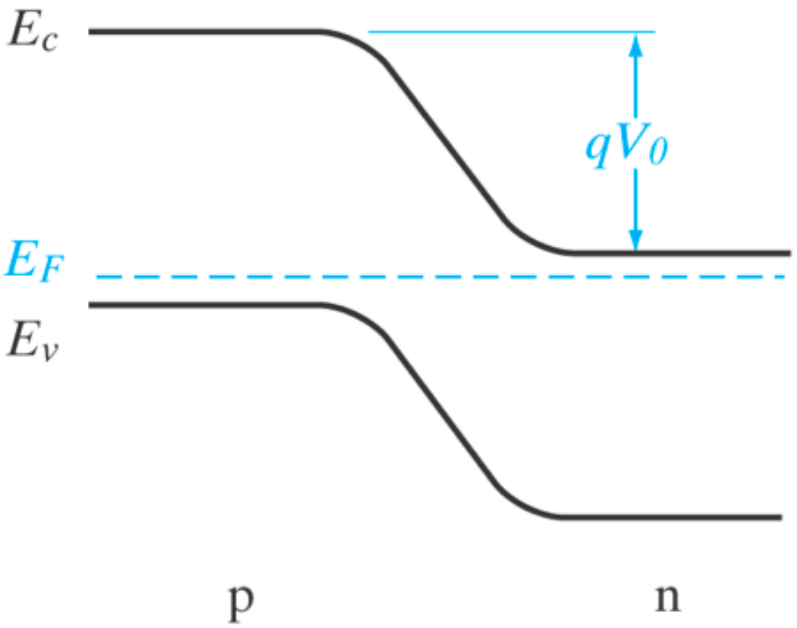


$$V_{oc} \simeq \frac{kT}{q} \ln \frac{g_{op}}{g_{th}}$$

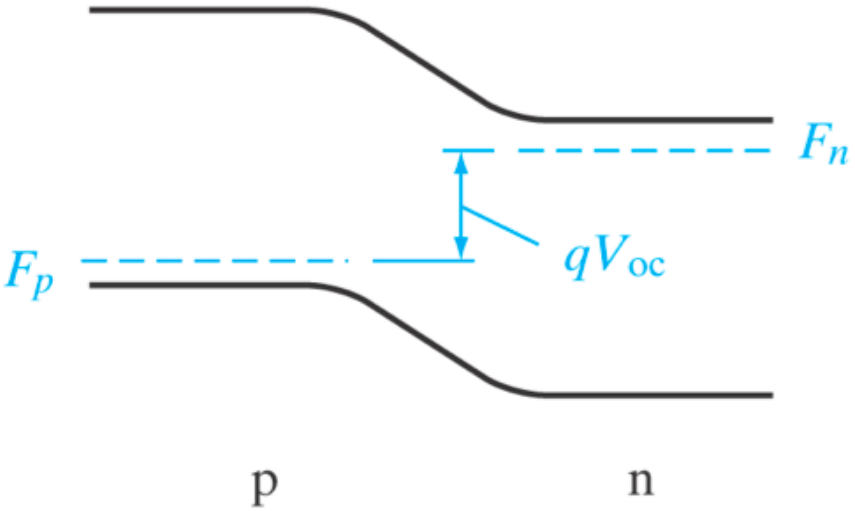
To Be Derived Shortly

V_{oc} Under Illumination

Equilibrium Junction



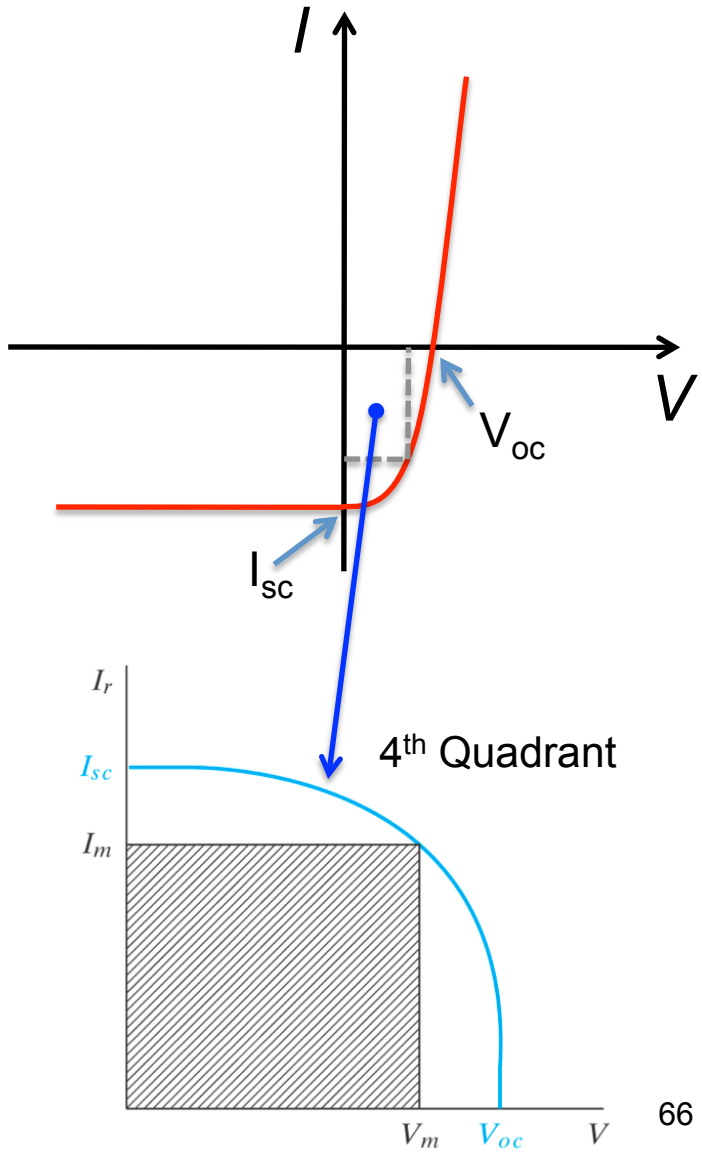
Junction Under Illumination



Solar Cells

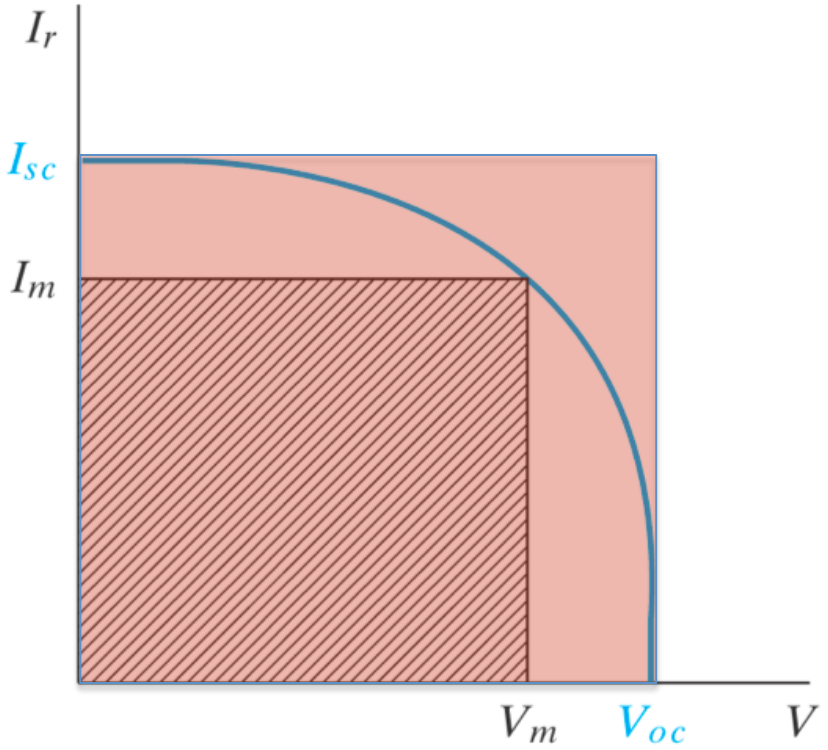
Solar Cell I-V

- Solar cells operate in the 4th quadrant
- The voltage output is limited to be less than the contact potential, which in turn is generally less than the bandgap voltage E_g/q
  - Si solar cell: V_{oc}<1V
  - Current depends upon illuminated area, but typically I_{op} ~10-100 mA for a 1 cm^2 cell
  - To increase voltage, cells are connected in series, to increase current, cells are connected in parallel
- An optimal solar cell has a large-area junction located near the surface
  - Junction depth less than L_p or L_n
  - Holes/electrons generated near surface can diffuse to the junction



Fill Factor

- V_{oc} and I_{sc} determined by solar cell characteristics, solar spectra, and solar intensity
- Maximum power is delivered when the product $V \bullet I_r$ is maximized
- A key solar cell figure of merit is the "fill factor"
- A second key figure of merit is efficiency
- For terrestrial use, the solar intensity is ~1000 W/m^2
- The power produced by the cell under peak conditions is ~$\eta \bullet$1000 W/m^2



Example:

For a Si solar cell with $I_{sc} = 100\text{mA}$ and $V_{oc} = 0.8\text{V}$, assuming a fill factor of 0.7 what is the max power?

$$P_{\max} = (f.f.) I_{sc} V_{oc} = (0.7)(100)(0.7) = 56 \text{ mW}$$

Fill Factor:

$$f.f. = \frac{I_m V_m}{I_{sc} V_{oc}}$$

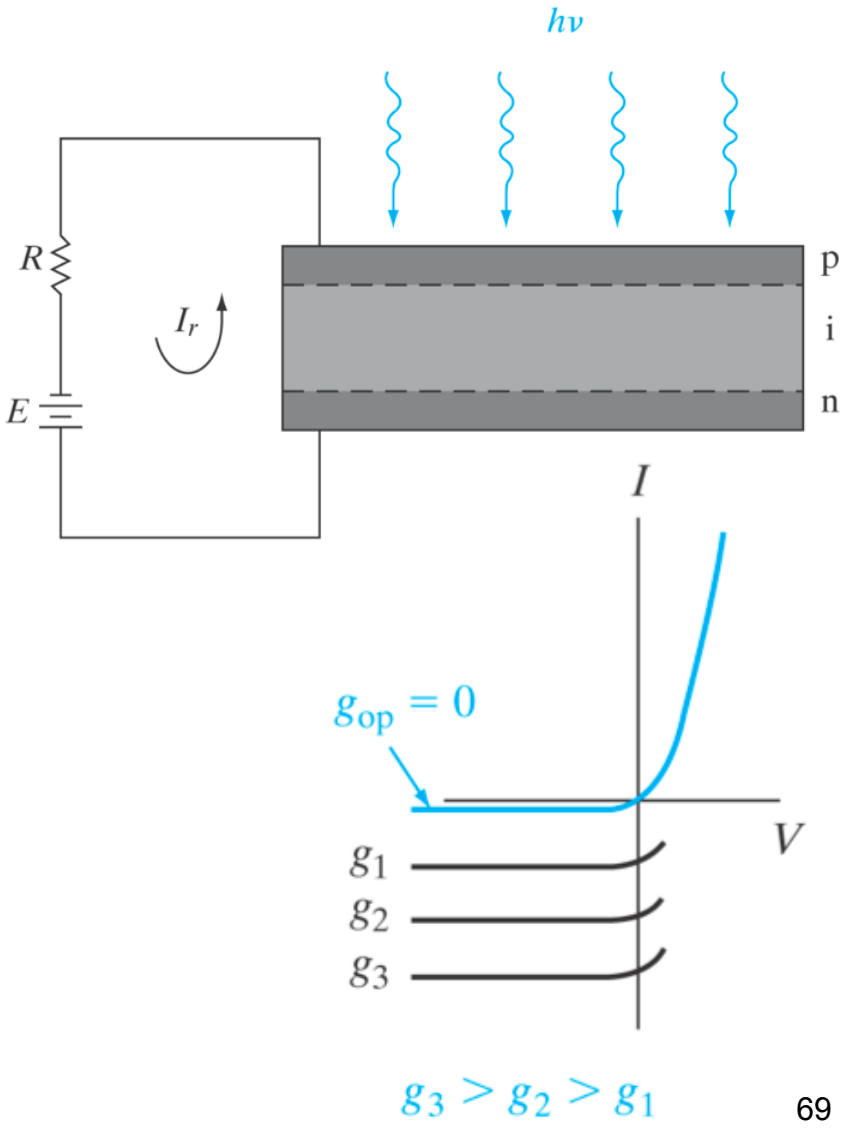
Photodetectors

PIN Illustration

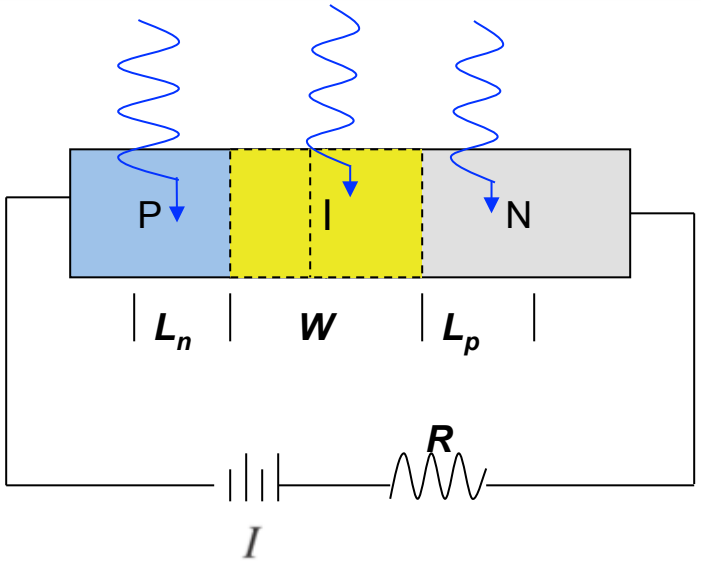
- In the 3rd quadrant, the current is independent of voltage and proportional to the optical generation rate
- In most applications, a key figure of merit is the photodiode bandwidth
- Carrier diffusion is slow, but drift is fast – absorption in the depletion region is preferred "depletion layer photodiode"
- There is also a design trade-off between sensitivity and speed – if the depletion width is too wide bandwidth is reduced

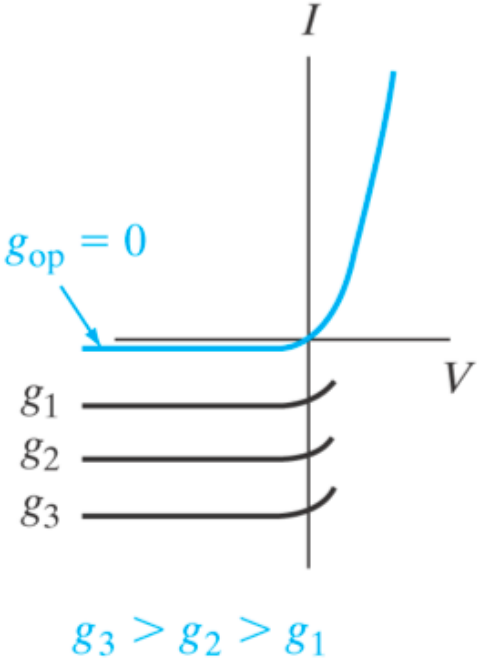
External Quantum Efficiency:

$$\eta_Q = \frac{\text{electrons out}}{\text{photons in}} = \frac{\left(J_{op}/q\right)}{\left(P_{op}/h\nu\right)}$$



Photodetectors: PIN Diodes





$g_3 > g_2 > g_1$

$$I = I_{th}\left(e^{qV/kT} - 1\right) - I_{op}$$

$$I_{op} = qAg_{op}\left(L_p + L_n + W\right)$$

To improve PIN bandwidth,

we need to reduce the photogeneration in

the diffuson layers L_n and L_p and optimize W.

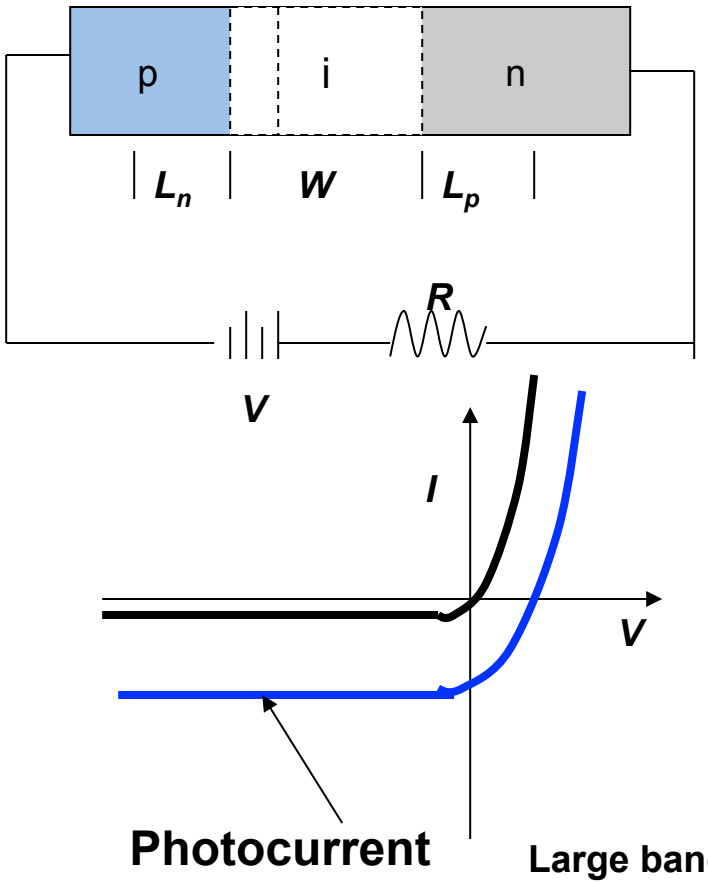
$$I_{op} = qAg_{op}(W) = qA\frac{\delta n}{\tau}W = q\eta\left(\frac{P_{abs}}{h\nu}\right)$$

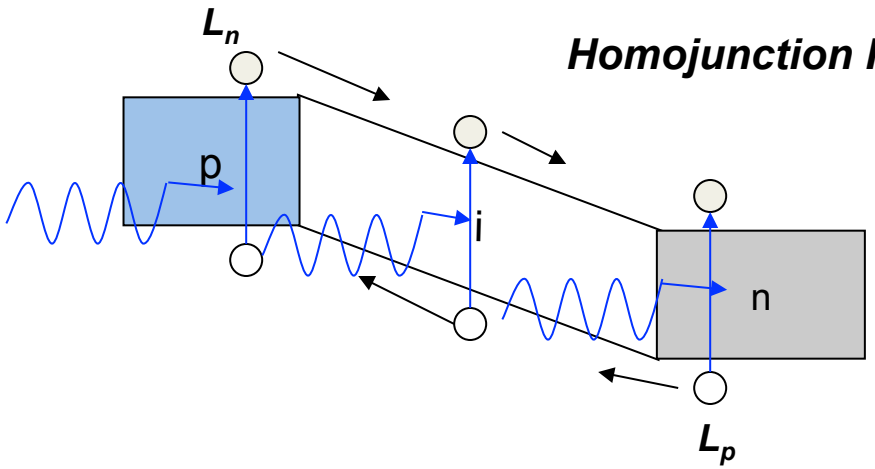
$$\eta = \frac{I_{op}/q}{P_{abs}/h\nu} = \frac{I_{op}/q}{P_{inc}(1-e^{-\alpha\ell})/h\nu}$$

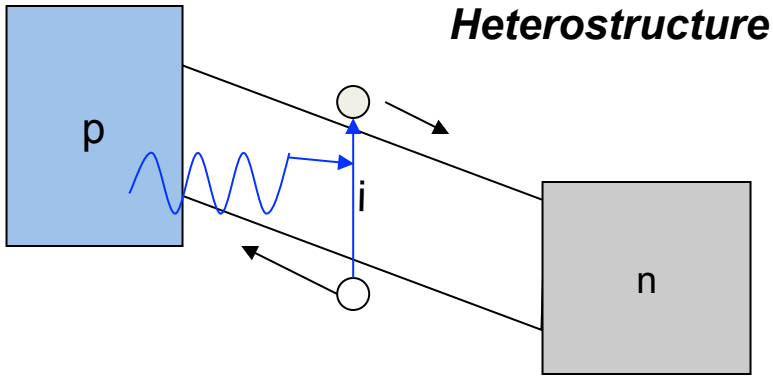
$$I_p = qA\delta n v_d = q\eta\left(\frac{P_{abs}}{h\nu}\right)\left(\frac{\tau v_d}{W}\right) = I_{op}\left(\frac{\tau}{t_r}\right)$$

Drift is fast, diffusion is slow

Homojunction and Heterostructure PINs



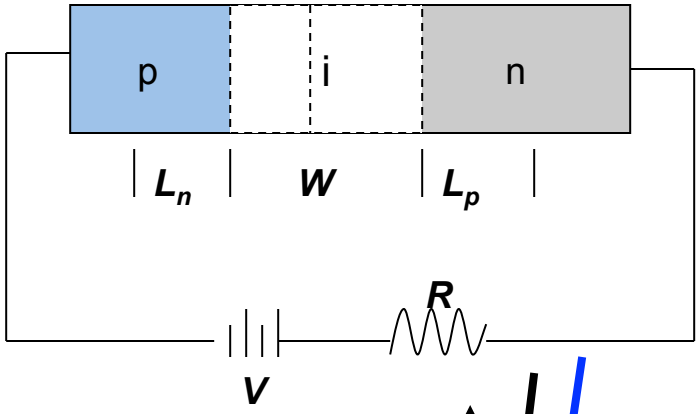


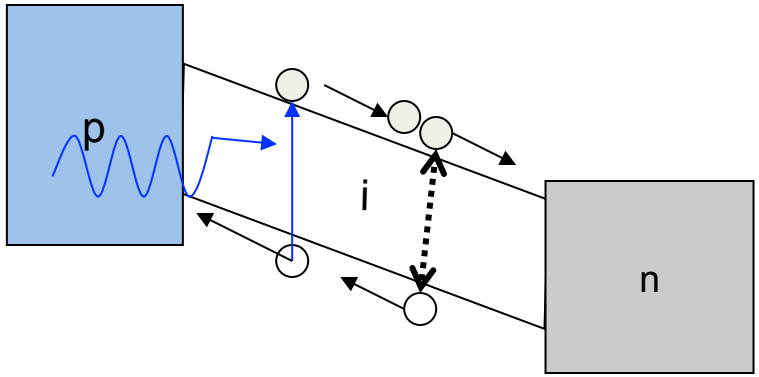


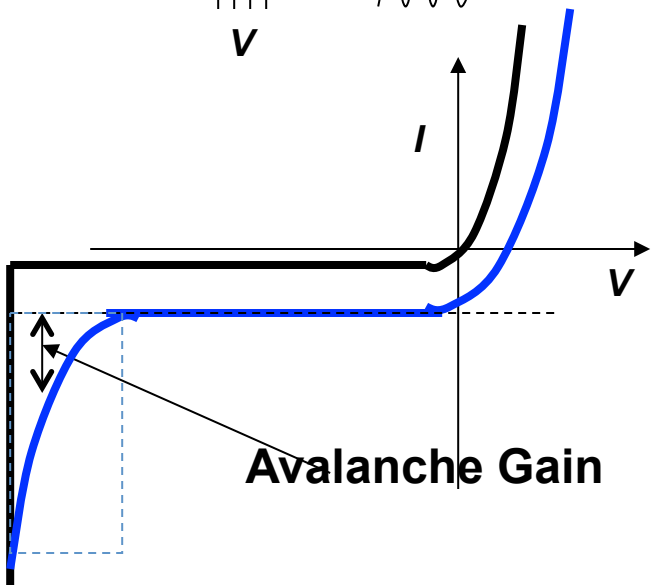
Large bandgap (*Eg* >*hv*) of p-layer or/and n-layer to eliminate EHP generation in any region except the "I" region

Avalanche Photodiodes

An electron or a hole with high energy in "I" layer can break chemical or ionic bonds in crystal to create an EHP







- An avalanche photodiode (APD) is operated under a reverse-bias voltage that is sufficient to enable avalanche multiplication
- An electron or a hole with high energy in the "i" layer can break chemical or ionic bonds in crystal to create an EHP
- The multiplication results in internal current gain

$$Gain \equiv \frac{\left(I_p - I_d\right)_{V_{op}}}{\left(I_p - I_d\right)_{V_{low}}}$$

Light Emitting Diodes

LED: Comments and Definitions

- Injection electroluminescence: light given off by direct-gap diode under forward bias
- Photon "color" determined by bandgap
- Internal efficiency reduced by nonradiative recombination
- External efficiency reduced by photon extraction
- Optical isolator: provides complete electrical isolation by coupling a signal through an "optoelectronic pair" (light emitter – photodiode)

- Useful Expressions:

$$I = I_{dr}\exp\left[\frac{q(V-IR_s)}{kT}\right] + I_{nr}\exp\left[\frac{q(V-IR_s)}{2kT}\right]$$

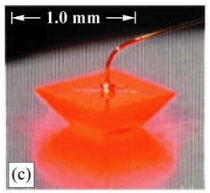
I_{dr}: diffusion radiative saturation current

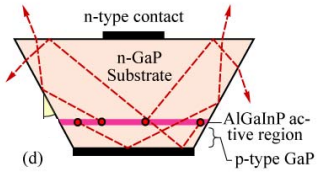
I_{nr}: nonradiative recombination saturation current

R_s: device series resistance

$$\sin\theta_C = \frac{n_1}{n_2},\ n_2 > n_1$$

n_2 incident media





Maximum Eye Sensitivity at $\lambda = 0.555\ \mu$m

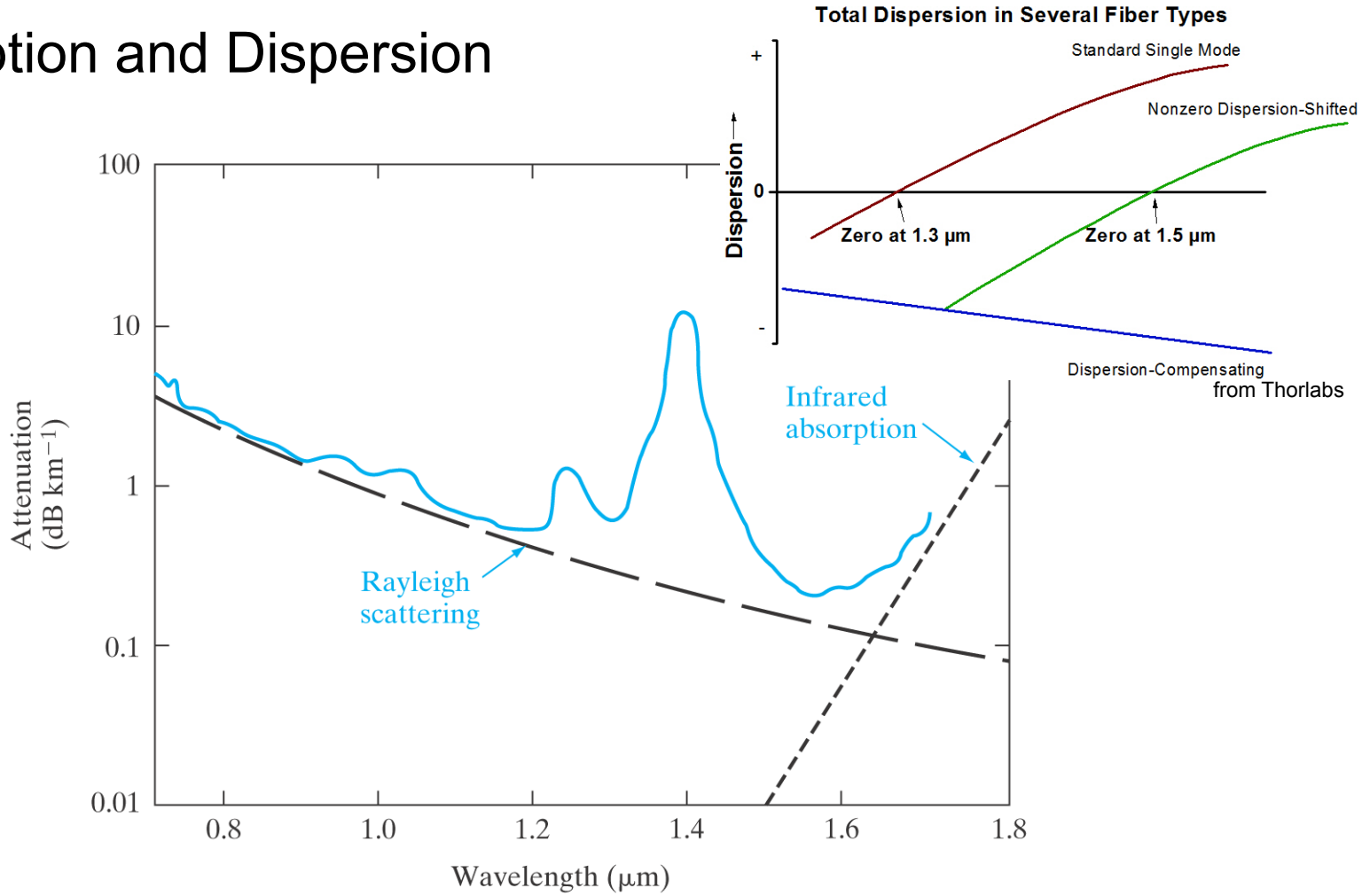
$E_g = 2.234eV(\lambda_m = 0.555\mu m) \Leftrightarrow$ Yellow/Green

$E_g = 1.8eV(\lambda_g = 0.7\mu m) \Leftrightarrow$ Red

$E_g = 2.88eV(\lambda_g = 0.43\mu m) \Leftrightarrow$ Blue

Fiber Optic Communications

SMF Absorption and Dispersion



$$I(x) = I_0 e^{-\alpha x}$$

Figure 8.13

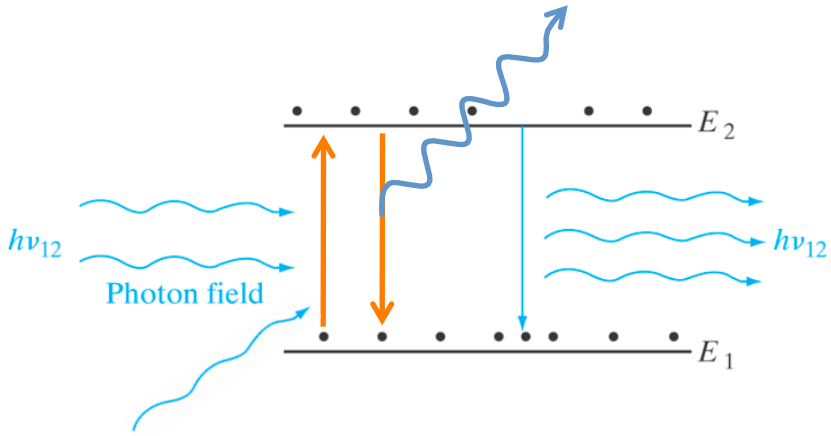
Typical plot of attenuation coefficient α vs. wavelength λ for a fused silica optical fiber. Peaks are due primarily to OH^- impurities.

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Lasers

Transitions in a 2-State System

LASER: Light Amplification through Stimulated Emission of Radiation



3 EHP-Photon Processes:

- ***Spontaneous Emission:*** Random transition from E_2 to E_1
- ***Absorption:*** Transition from E_1 to E_2 caused by interaction with and annihilation of a photon
- ***Stimulated Emission:*** Transition caused by presence of photon field
  - Stimulated photons have the same energy and phase as the stimulating photon field (monochromatic, coherent)

Two State System

Consider a two state system with the following properties:

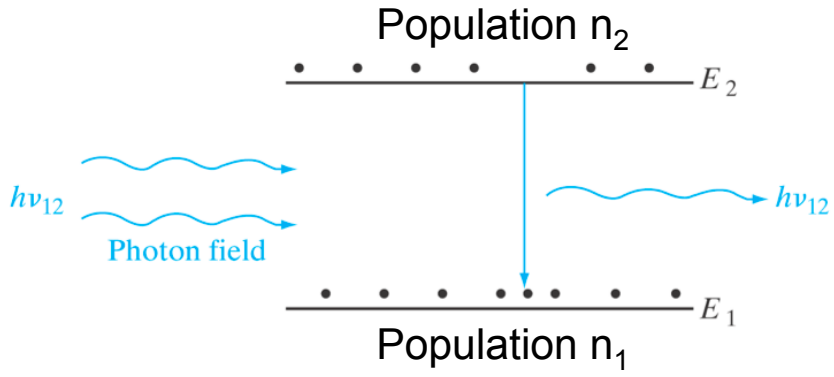
An upper state energy E_2

An upper state electron density (population) n_2

A lower state energy E_1

A lower state electron density (population) n_1

A photon energy $E_2 - E_1 = h\nu_{12}$ between the levels



From Boltzmann Statistics, at **Thermal Equilibrium :**

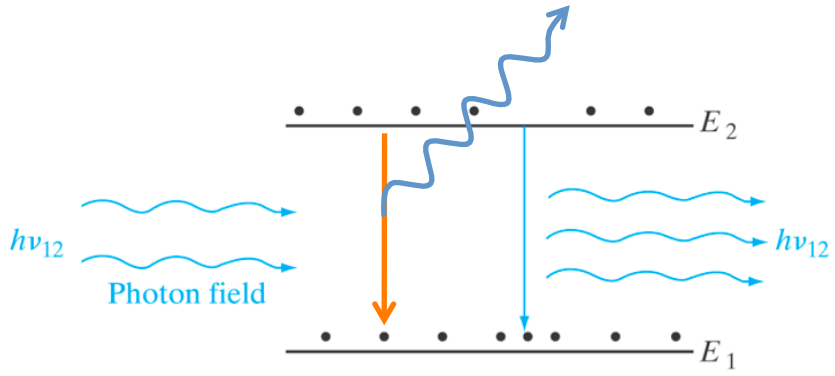
$$\frac{n_2}{n_1} = e^{-(E_2-E_1)/kT} = e^{-h\nu_{12}/kT}$$

so in ***thermal equilibrium*** $n_2 \ll n_1$

What would happen if T were negative?

Spontaneous Emission

The rate of spontaneous emission is proportional only to the number of electrons in the upper state:



$$R_{spon} = -\left.\frac{dn_2}{dt}\right|_{spon} = A_{21} n_2$$

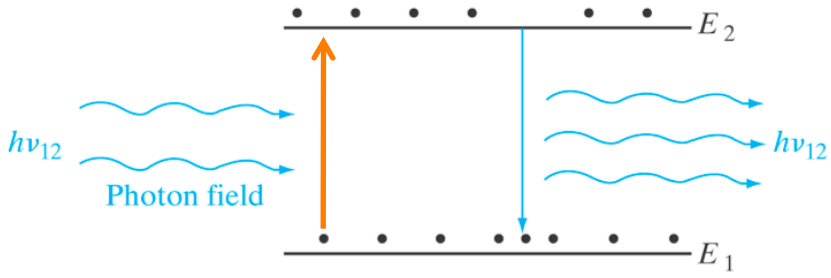
Spontaneous decay time constant: $\tau_{spon} = \left(A_{21}\right)^{-1}$

Note: Don't confuse the two-state system with band-to-band processes discussed earlier where we found:

$R_n = R_p = \alpha_r np$

Absorption

The rate of absorption is proportional to the number of electrons in the lower state and the energy density.



Where:

$R_{abs} \equiv$ Rate of absorption

$\rho(\nu_{12}) \equiv$ System with energy density

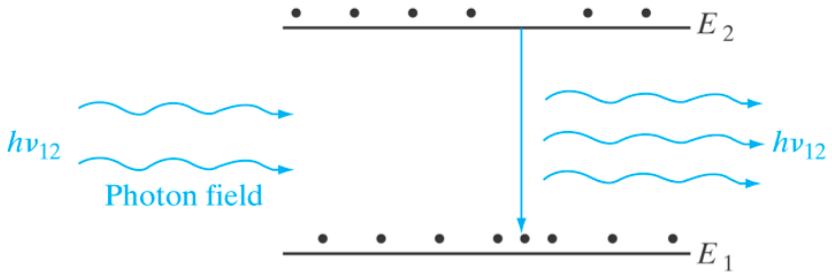
$n_1 \equiv$ Electron population in the lower level

The rate of absorption is given by:

$$R_{abs} = \left.\frac{dn_2}{dt}\right|_{abs} = -\left.\frac{dn_1}{dt}\right|_{abs} = B_{12} n_1 \rho(\nu_{12})$$

Stimulated Emission

The rate of stimulated emission is proportional to the number of electrons in upper level and the energy density.



Where:

$R_{stim} \equiv$ Rate of stimulated emission

$\rho(\nu_{12}) \equiv$ System with energy density

$n_2 \equiv$ Electron population in the upper level

The rate of stimulated emission is given by:

$$R_{stim} = \frac{dn_1}{dt}\bigg| = -\frac{dn_2}{dt}\bigg| = B_{21} n_2 \rho(\nu_{12})$$

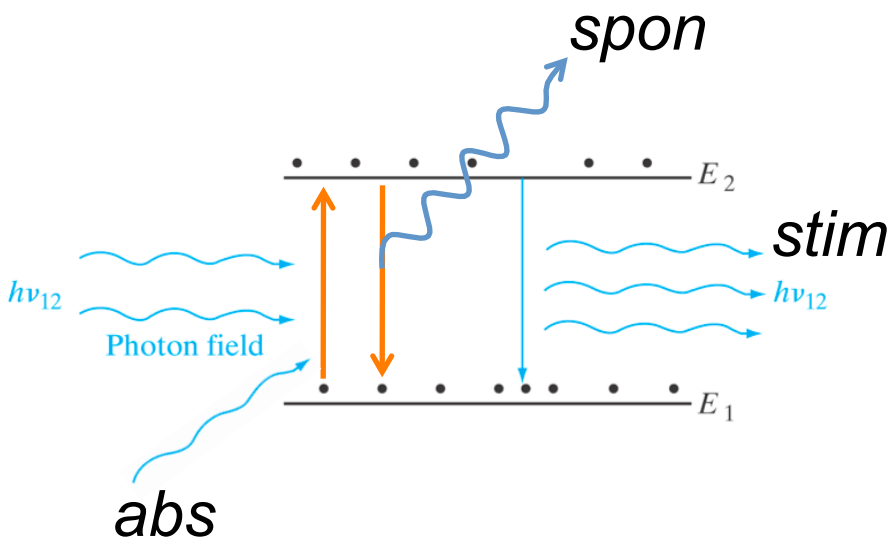
Steady State Condition

For :

$$R_{stim} = B_{21} n_2 \rho(\nu_{12})$$

$$R_{abs} = B_{12} n_1 \rho(\nu_{12})$$

$$R_{spon} = A_{21} n_2$$



In steady state, the emission rates must balance the absorption rate:

$$\left.\frac{dn_2}{dt}\right|_{abs} + \left.\frac{dn_2}{dt}\right|_{spon} + \left.\frac{dn_2}{dt}\right|_{stim} = 0 \quad \text{or}$$

$$B_{12} n_1 \rho(\nu_{12}) = A_{21} n_2 + B_{21} n_2 \rho(\nu_{12})$$

$$[\text{Absorption}] = [\text{Spontaneous Emission}] + [\text{Stimulated Emission}]$$

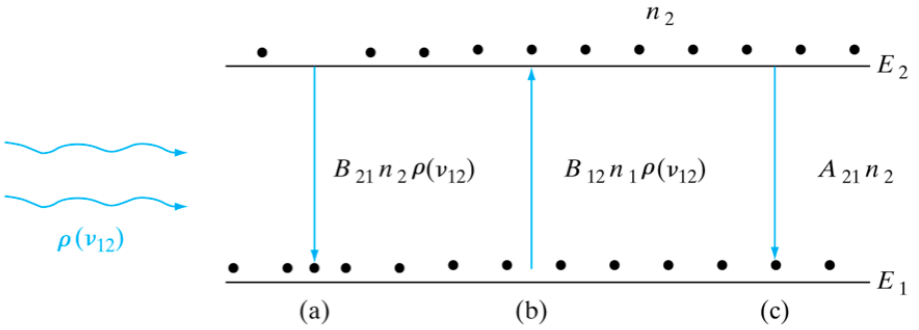
B_{12}, A_{21}, and B_{21} are the Einstein Coefficients

In steady state, the absorption is the sum of spontaneous and stimulated emission

Conditions for Lasing

$$\frac{\text{Stimulated emission rate}}{\text{Spontaneous emission rate}} = \frac{B_{21} n_2 \rho(\nu_{12})}{A_{21} n_2} = \frac{B_{21}}{A_{21}} \rho(\nu_{12})$$

A large photon field energy density enhances the stimulated emission rate (optical resonant cavity).



$$\frac{\text{Stimulated emission rate}}{\text{Absorption rate}} = \frac{B_{21} n_2 \rho(\nu_{12})}{B_{12} n_1 \rho(\nu_{12})} = \frac{B_{21}}{B_{12}} \frac{n_2}{n_1}$$

For stimulated emission to exceed absorption, $n_2 > n_1$

Population Inversion needed for stimulated emission to exceed absorption ("***negative temperature***" concept).

$$\frac{n_2}{n_1} = e^{-(E_2 - E_1)/kT} = e^{-h\nu_{12}/kT}$$

if $n_2 > n_1$ then T is "negative"

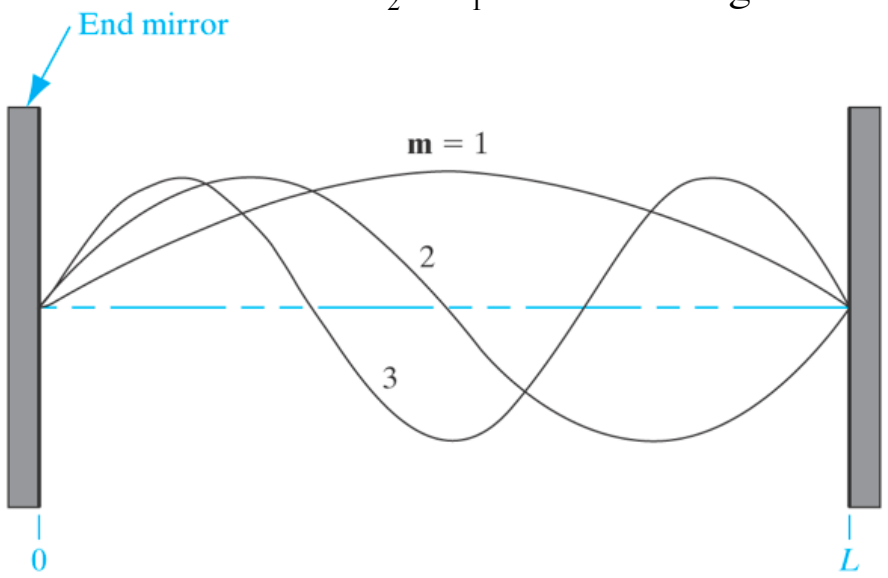
Optical Resonant Cavity: Fabry-Perot Modes

$$L = m\left(\frac{\lambda}{2}\right)$$

λ is the wavelength in the cavity. For a refractive index n:

$$\lambda = \frac{\lambda_o}{n}$$

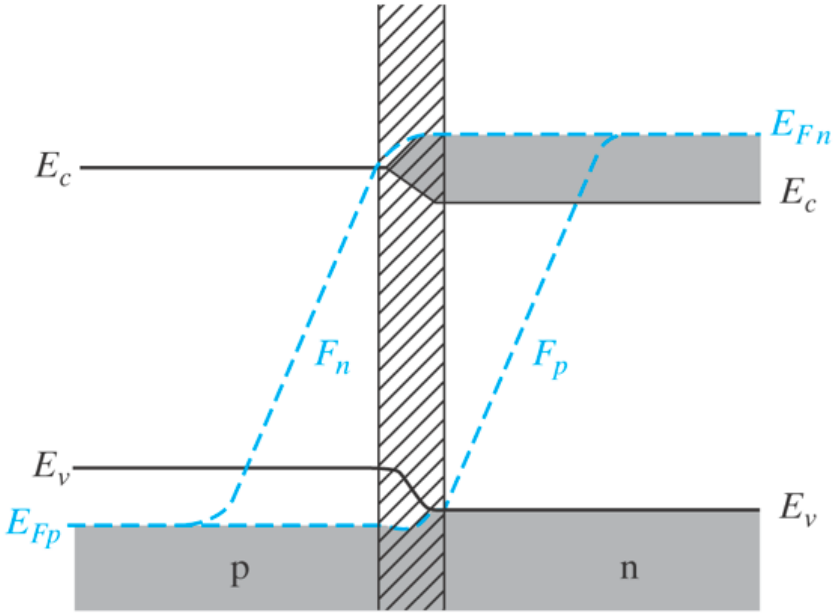
where λ_o is the wavelength in free space.



Semiconductor Lasers

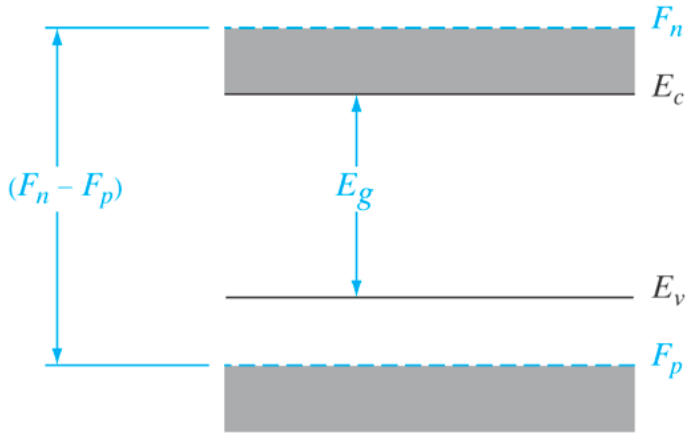
Junction Under Large Bias

- If the bias is such that electrons and holes are injected into and across the transition region in high concentration, an ***inversion region*** is created
- The population inversion is described using quasi-Fermi levels
- The separation of the quasi-Fermi levels is a measure of the population inversion
- Direct bandgap material required



$$n = N_c e^{-(E_c - F_n)/kT} = n_i e^{(F_n - E_i)/kT}$$

$$p = N_v e^{-(F_p - E_v)/kT} = n_i e^{(E_i - F_p)/kT}$$



Lasing Condition (Population Inversion):

$(F_n - F_p) > h\nu$ or, for band-edge transitions

$(F_n - F_p) > E_g$

Inversion Region Variation with Bias

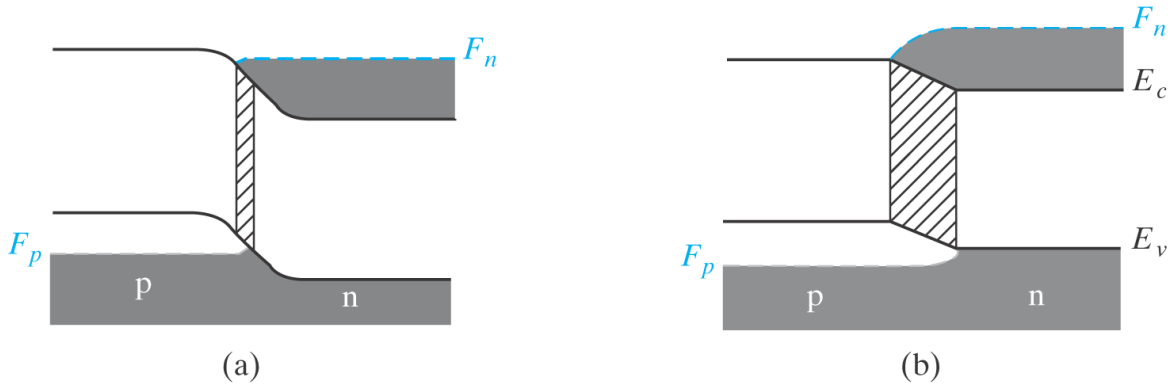
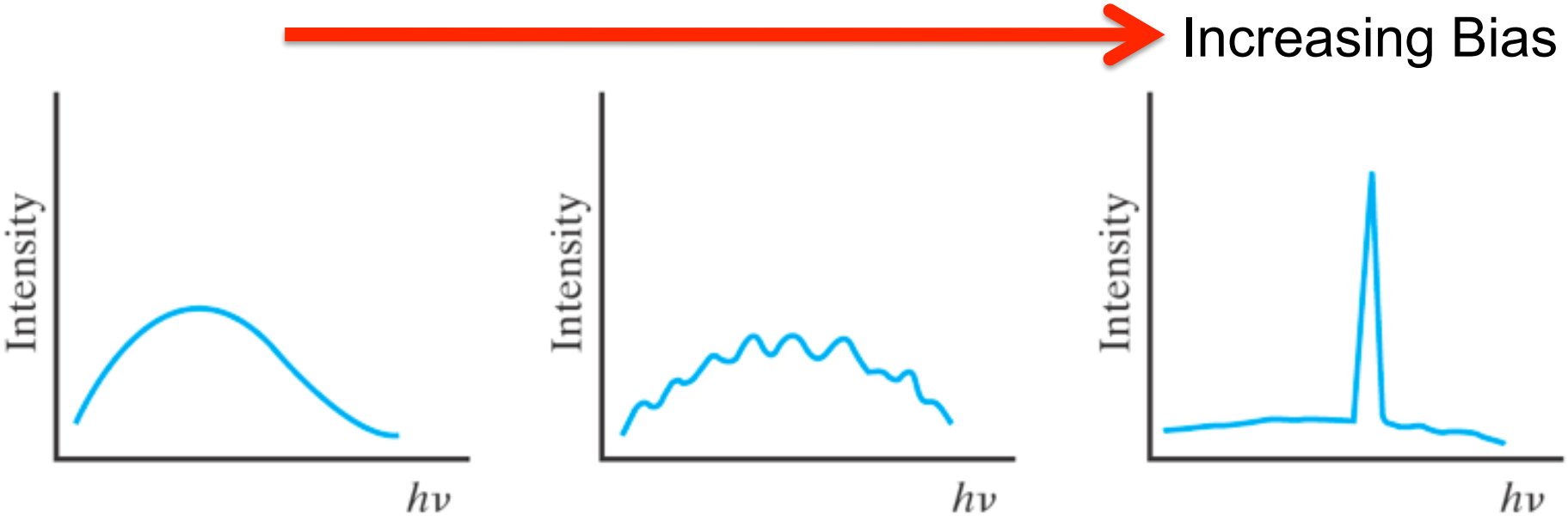


Figure 8.20

Variation of the inversion- region width with forward bias: $V(a) < V(b)$.

From *Solid State Electronic Devices,* Sixth Edition, by Ben G. Streetman and Sanjay Kumar Banerjee.
ISBN 0-13-149726-X. © 2006 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved.

Laser Mode Spacing



$$L = m\left(\frac{\lambda_o}{2n}\right) \text{ so } m = \frac{2Ln}{\lambda_o} \text{ and } \frac{dm}{d\lambda_o} = -\frac{2Ln}{\lambda_o^2} + \frac{2L}{\lambda_o}\frac{dn}{d\lambda_o}$$

Since m is an integer:

$$\frac{\Delta m}{\Delta\lambda_o} = -\frac{2Ln}{\lambda_o^2} + \frac{2L}{\lambda_o}\frac{dn}{d\lambda_o} \text{ and } -\Delta\lambda_o = \frac{\lambda_o^2}{2Ln}\left(1 - \frac{\lambda_o}{n}\frac{dn}{d\lambda_o}\right)^{-1}\Delta m$$

Setting $\Delta m = -1$ will allow calculation of the mode spacing

Ideal Metal-Semiconductor Junctions

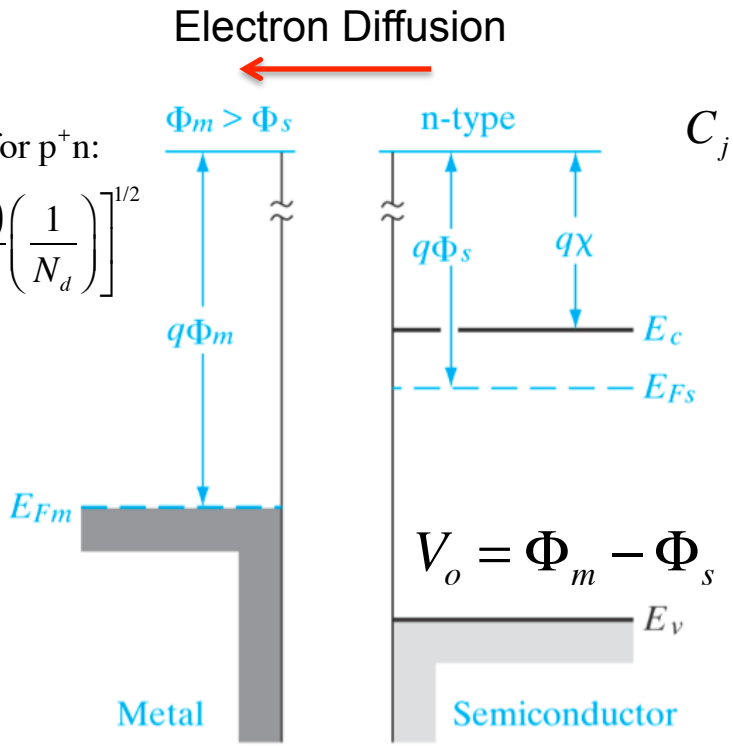
Rectifying Contacts

Case I: $q\Phi_m > q\Phi_s$, n-type Semiconductor

- Charge transfer occurs until Fermi levels align
- A depletion region is formed near the metal-semiconductor junction
- The positive charge due to uncompensated donors matches the negative charge on the metal
- Electric field and bending bands similar to p-n junctions (p^+-n approximation)

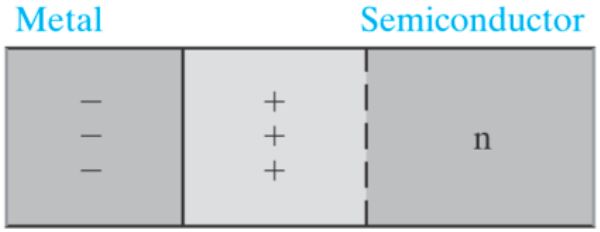
Approximation for p^+n:

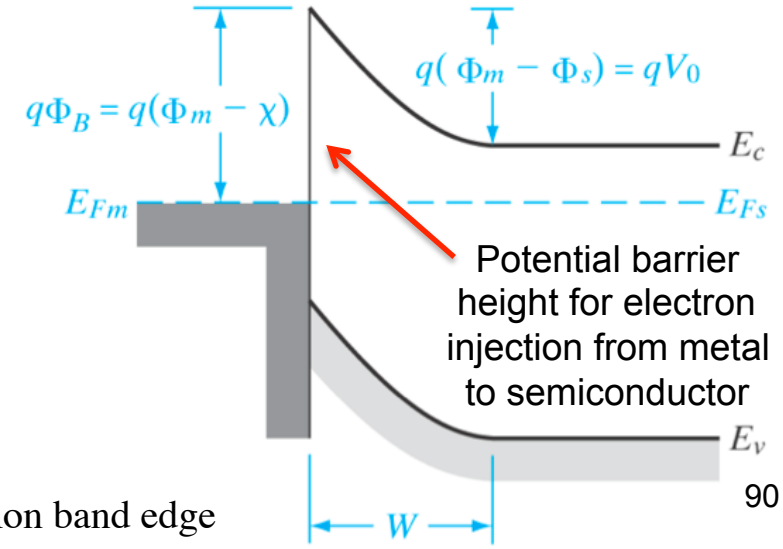
$$W \sim \left[\frac{2\varepsilon(V_0 - V)}{q}\left(\frac{1}{N_d}\right)\right]^{1/2}$$



$$V_o = \Phi_m - \Phi_s$$

$$C_j = \frac{\varepsilon_s A}{W}$$

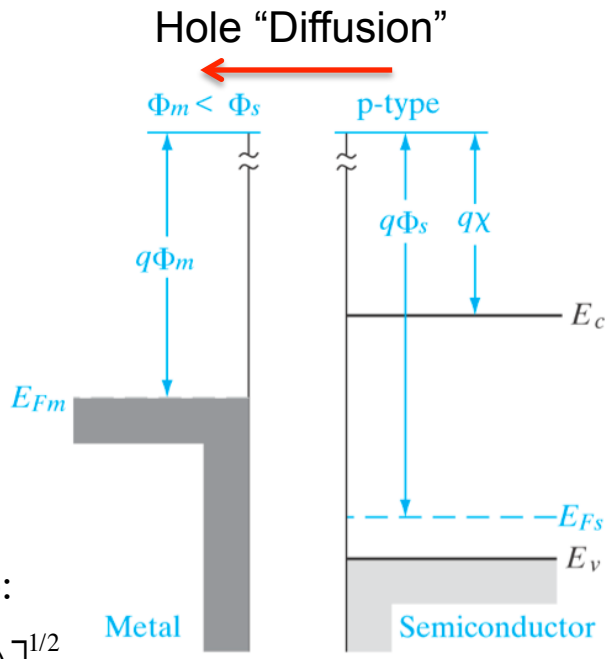


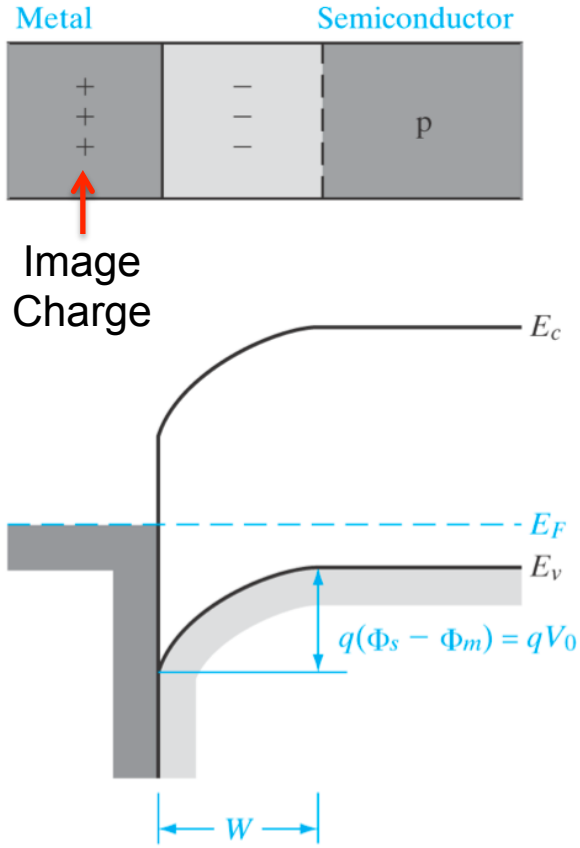


$q\chi \equiv$ electron affinity, measured from vacuum level to conduction band edge

Case II: $q\Phi_s > q\Phi_m$, p-type Semiconductor

- Charge transfer to align Fermi Levels results in a positive charge on the metal side and a negative charge on the semiconductor side
- Depletion region formed in semiconductor – ionized acceptors provide negative charge region
- Potential barrier V_o retards hole diffusion from semiconductor to metal
- Resembles n^+-p junction



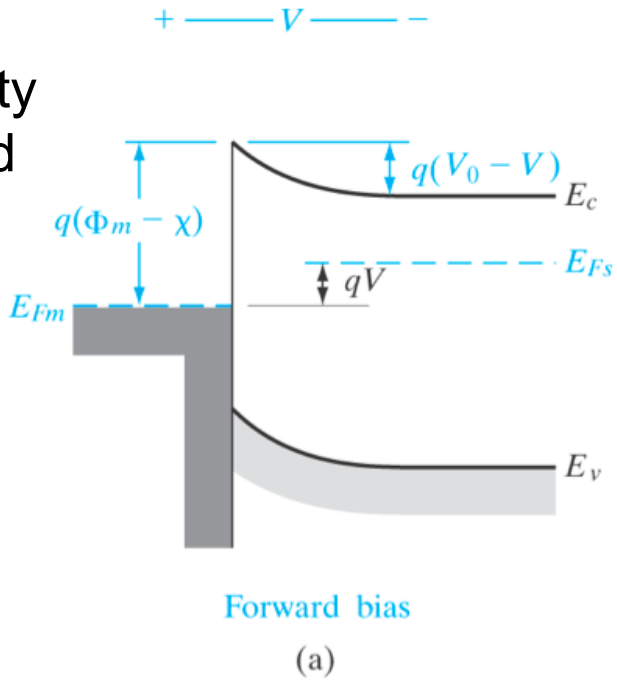


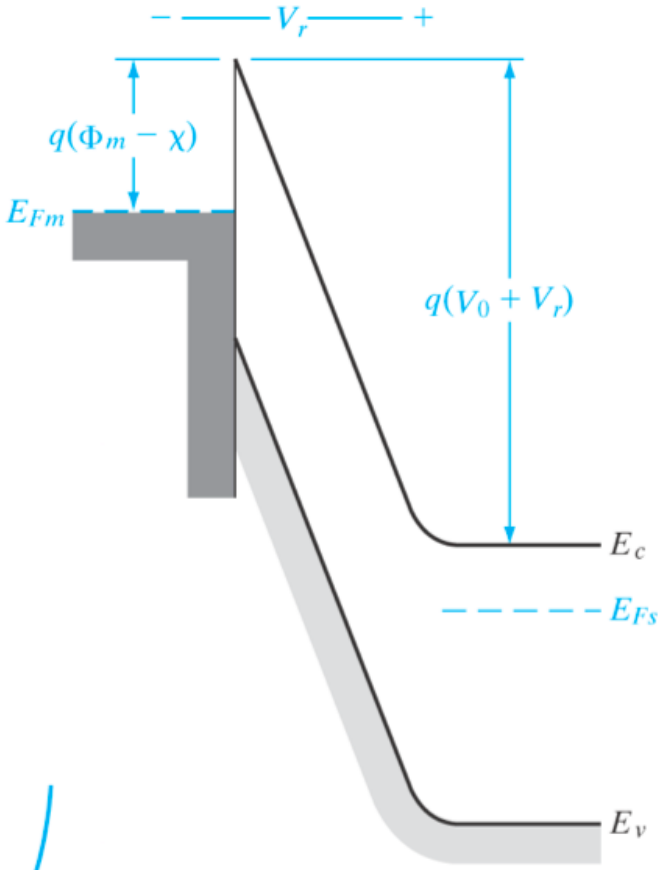
Approximation for n^+p:

$$W \sim \left[\frac{2\varepsilon(V_0 - V)}{q}\left(\frac{1}{N_a}\right)\right]^{1/2}$$

Rectifying Junctions

Reducing the barrier height causes majority carriers to be injected <u>into the metal from the semiconductor</u>





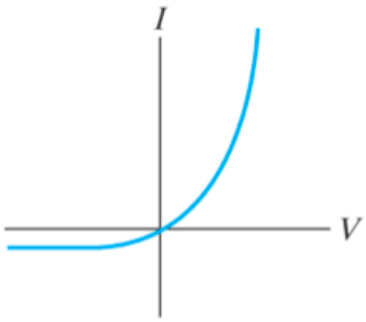
$I = I_o(e^{qV/kT} - 1)$

I_o is distinct from the p-n junction case:

$I_o \propto e^{-q\Phi_B/kT}$ (Boltzmann)

Φ_B is the barrier for electron injection

$\Phi_B = \Phi_m - \chi$ (Independent of applied voltage)



Forward Mode: Positive Bias to Metal

Comments:

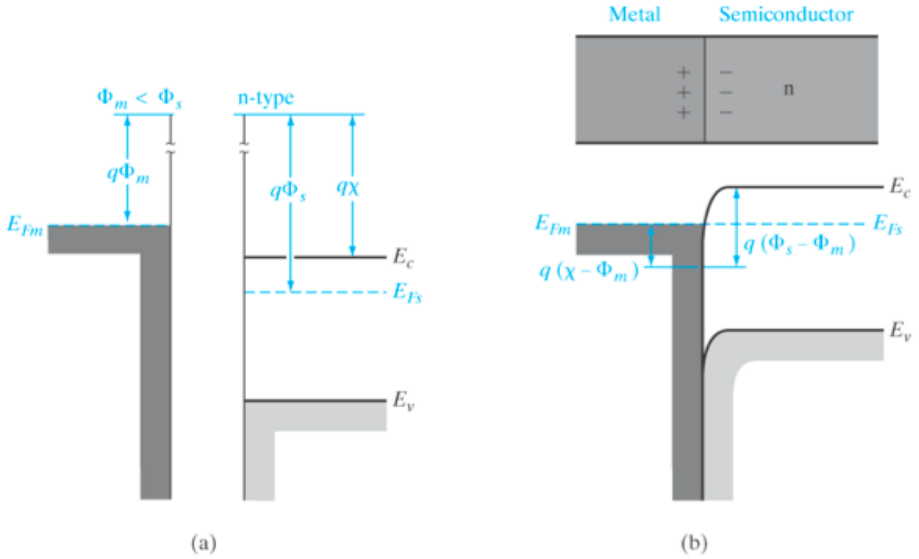
- The forward current is due to the injection of **majority carriers** from the semiconductor into the metal
- Because there is effectively no minority carrier charge storage, there is no charge storage delay time as the device bias conditions are changed (speed advantage)

Ideal Metal-Semiconductor Junctions

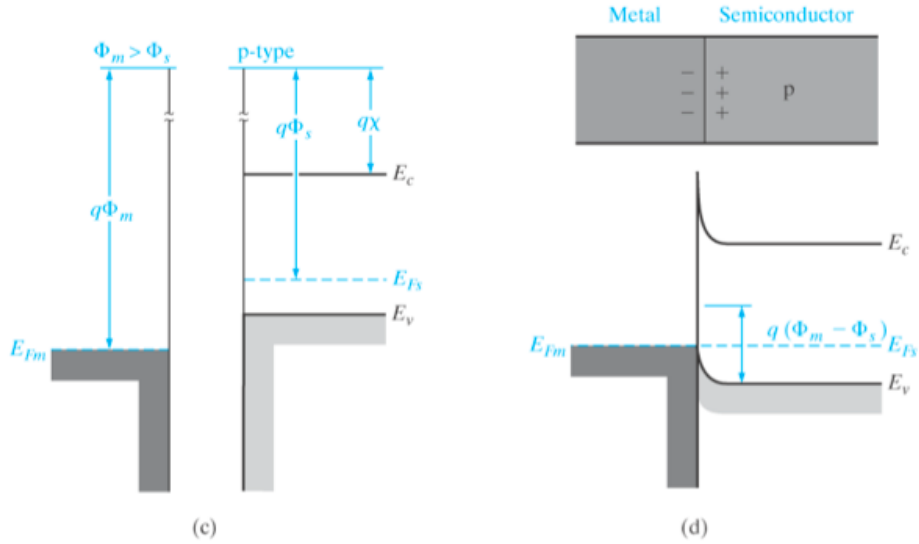
Ohmic Contacts

Ohmic Contacts

Case III:
$\Phi_m < \Phi_s$
n-type



Case IV:
$\Phi_m > \Phi_s$
p-type



- Charge transfer to align Fermi levels
- Induced charged in semiconductor provided by majority carriers as opposed to ionized impurities (no depletion region)
- Practical method of forming ohmic contacts involves heavy doping of semiconductor – small depletion width allows tunneling

Summary

Ideal Metal-Semiconductor Contacts

Contact Type: Doping and Work Function

	$\Phi_m > \Phi_s$	$\Phi_s > \Phi_m$
n-type	Rectifying	Ohmic
p-type	Ohmic	Rectifying

Summary: Metal-Semiconductor Contacts

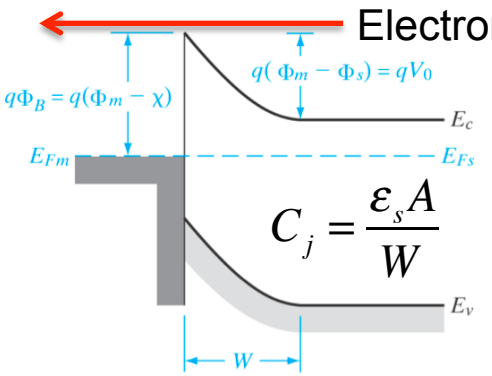
Forward Bias: Majority Carrier Injection Into Metal as Barrier is Reduced

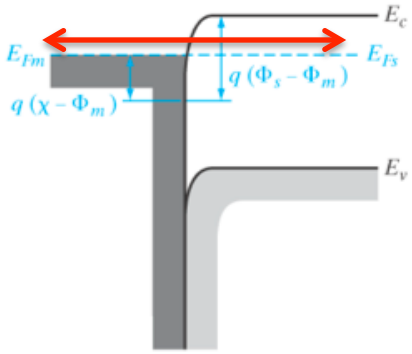
Reverse Bias: Thermionic Emission Over Barrier from Metal

$$I = I_o(e^{qV/kT} - 1)$$

$$I_o \propto e^{-q\Phi_B/kT}$$

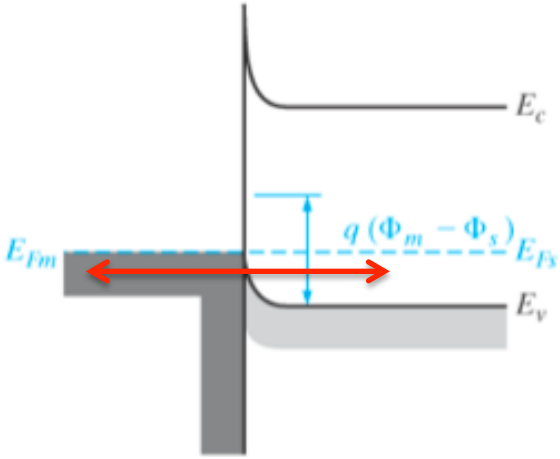
$$\Phi_B = \Phi_m - \chi$$

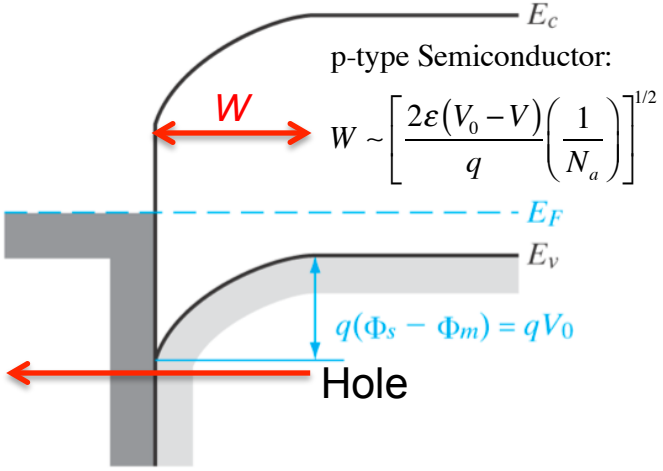




	$\Phi_m > \Phi_s$	$\Phi_s > \Phi_m$
n-type	Rectifying	Ohmic
p-type	Ohmic	Rectifying

Ohmic: Free Flow of Majority Carriers





Assignments

Assignments

- Homework assigned every Friday, due following Friday
- Reading from Streetman's book:
  - Mon 4/9: §'s 6.4.3, 6.4.4
  - Fri 4/13: § 6.4.5
- Chapters 5, 6, 7, 8, and 9 in Pierret cover similar material

Topics for Next Lecture

Outline, 4/13/18

- Continue MOS Capacitors

Thank You for Listening!

Instructional Objectives (1)

By the time of exam No. 1 (after 17 lectures), the students should be able to do the following:

1. Outline the classification of solids as metals, semiconductors, and insulators and distinguish direct and indirect semiconductors.
2. Determine relative magnitudes of the effective mass of electrons and holes from an E(k) diagram.
3. Calculate the carrier concentration in intrinsic semiconductors.
4. Apply the Fermi-Dirac distribution function to determine the occupation of electron and hole states in a semiconductor.
5. Calculate the electron and hole concentrations if the Fermi level is given; determine the Fermi level in a semiconductor if the carrier concentration is given.
6. Determine the variation of electron and hole mobility in a semiconductor with temperature, impurity concentration, and electrical field.
7. Apply the concept of compensation and space charge neutrality to calculate the electron and hole concentrations in compensated semiconductor samples.
8. Determine the current density and resistivity from given carrier densities and mobilities.
9. Calculate the recombination characteristics and excess carrier concentrations as a function of time for both low level and high level injection conditions in a semiconductor.
10. Use quasi-Fermi levels to calculate the non-equilibrium concentrations of electrons and holes in a semiconductor under uniform photoexcitation.
11. Calculate the drift and diffusion components of electron and hole currents.
12. Calculate the diffusion coefficients from given values of carrier mobility through the Einstein's relationship and determine the built-in field in a non-uniformly doped sample.

Instructional Objectives (3)

By the time of the Final Exam, after 44 class periods, the students should be able to do all of the items listed under A and B, plus the following:

26. Calculate the terminal parameters of a BJT in terms of the material properties and device structure.
27. Estimate the base transport factor “B” of a BJT and rank-order the internal currents which limit the gain of the transistor.
28. Determine the rank order of the electrical fields in the different regions of a BJT in forward active bias.
29. Calculate the threshold voltage of an ideal MOS capacitor.
30. Predict the C-V characteristics of an MOS capacitor.
31. Calculate the inversion charge in an MOS capacitor as a function of gate and drain bias voltage.
32. Estimate the drain current of an MOS transistor above threshold for low drain voltage.
33. Estimate the drain current of an MOS transistor at pinch-off.
34. Distinguish whether a MOSFET with a particular structure will operate as an enhancement or depletion mode device.
35. Determine the short-circuit current and open-circuit voltage for an illuminated p/n junction solar cell.

Tentative Schedule [1]

	JAN 17 Course overview	JAN 19 Intro to semiconductor electronics
JAN 22 Materials and crystal structures	JAN 24 Bonding forces and energy bands in solids	JAN 26 Metals, semiconductors, insulators, electrons, holes
JAN 29 Intrinsic and extrinsic material	JAN 31 Distribution functions and carrier concentrations	FEB 2 Distribution functions and carrier concentrations
FEB 5 Temperature dependence, compensation	FEB 7 Conductivity and mobility	FEB 9 Resistance, temperature, impurity concentration
FEB 12 Invariance of Fermi level at equilibrium	FEB 14 Optical absorption and luminescence	FEB 16 Generation and recombination

Subject to Change

Tentative Schedule [2]

FEB 19 Quasi-Fermi levels and photoconductive devices	FEB 21 Carrier diffusion	FEB 23 Built-in fields, diffusion and recombination
Feb 26 Review, discussion, problems (2/27 exam)	FEB 28 Steady state carrier injection, diffusion length	MAR 2 p-n junctions in equilibrium & contact potential
MAR 5 p-n junction Fermi levels and space charge	MAR 7 Continue p-n junction space charge	MAR 9 NO CLASS (EOH)
MAR 12 p-n junction current flow	MAR 14 Carrier injection and the diode equation	MAR 16 Minority and majority carrier currents
3/19-3/23 Spring Break MAR 26 Reverse-bias breakdown	MAR 28 Stored charge, diffusion and junction capacitance	MAR 30 Photodiodes, I-V under illumination

Subject to Change

Schedule & Policies

Important Information

- Course Website:
  - http://courses.engr.illinois.edu/ece340/
- Download and Review Syllabus / Course Information from Website!
- Course Coordinator: Prof. John Dallesasse
  - jdallesa@illinois.edu
  - Coordinates schedule, policies, absence issues, homework, quizzes, exams, etc.
- Contact Information and Office Hours for All ECE340 Professors & TAs in Syllabus
- Lecture Slides: Click on "(Sec. X)" next to my name in instructor list
- DRES Students: Contact Prof. Dallesasse ASAP
- Textbook:
  - "Solid State Electronic Devices," Streetman & Banerjee, 7th Edition
  - Supplemental: "Semiconductor Device Fundamentals," Pierret
  - Additional reference texts listed in syllabus

Key Points

- Attend Class!
  - 3 unannounced quizzes, each worth 5% of your grade
  - **You must take the quiz in your section**
  - Excused absences must be **pre-arranged** with the course director
  - Absences for illness, etc. need a note from the Dean
    - See policy on absences in the syllabus
- No Late Homework
  - Homework due on the date of an excused absence must be turned in ahead of time
  - You must turn in homework in your section
  - No excused absences for homework assignments
  - Top 10 of 11 homework assignments used in calculation of course grade
    - Do all of them to best prepare for the exams!
- No Cheating
  - Penalties are severe and will be enforced
- Turn Off Your Phone
  - No video recording, audio recording, or photography

Homework

- Assigned Friday, Due Following Friday
  - Due dates shown in syllabus
- Due at Start of Class
- Follow Guidelines in Syllabus
- Peer Discussions Related to Homework are Acceptable and Encouraged
- Directly Copying Someone Else's Homework is Not Acceptable
  - Graders have been instructed to watch for evidence of plagiarism
  - Both parties will receive a "0" on the problem or assignment

Absences

- The absence policy in the syllabus will be strictly enforced
- To receive an excused absence (quiz), you must:
  - Pre-arrange the absence with the course director (valid reason and proof required)
  - Complete an Excused Absence Form at the Undergraduate College Office, Room 207 Engineering Hall (333-0050)
    - The form must be signed by a physician, medical official, or the Emergency Dean (Office of the Dean of Students)
    - The Dean's Office has recently put a strict policy in place (3 documented days of illness)
  - Excused quiz score will be prorated based upon average of completed scores
  - No excused absences are given for homework, but only the best 10 of 11 are used to calculate your final grade
  - Excused absences are not given for exams, except in accordance with the UIUC Student Code
  - Unexcused work will receive a "0"
- Failure to take the final will result in an "incomplete" grade (if excused) or a "0" (if unexcused)

Exams

- Exam I: Tuesday February 27th, 7:30-8:30 pm
- Exam II: Thursday April 12th, 7:30-8:30 pm
- Final Exam: Date/Time To Be Announced
  - Determined by University F&S

Grading

Grading Criterion

Homework	10 %
Quizzes	15 %
Hour Exam I	20 %
Hour Exam II	20 %
Final Exam	35 %
Total	100 %

Historical Grade Trends*

	Spring 2016	Fall 2016	Spring 2017
A's	27 %	28 %	27 %
B's	37 %	26 %	38 %
C's	27 %	25 %	27%
D's	6 %	16 %	4 %
F's	3 %	5 %	4 %

*Past performance is not necessarily indicative of future results

My Recommendations

- Read the syllabus and information posted on the course website
- **Attend class** & participate
- Attend office hours (TA and Professors)
- **Read the book**
- Re-read the book
- Look at and read selected portions of the supplemental texts
- Form study groups to review concepts and discuss high-level approaches for solving homework problems
  - Don't form study groups to copy homework solutions
- **Don't miss any homework, quizzes, or exams**
  - It's hard to overcome a zero
- Ask questions in class!

Instructional Objectives (1)

By the time of exam No. 1 (after 17 lectures), the students should be able to do the following:

✔1. Outline the classification of solids as metals, semiconductors, and insulators and distinguish direct and indirect semiconductors.

✔2. Determine relative magnitudes of the effective mass of electrons and holes from an E(k) diagram.

✔3. Calculate the carrier concentration in intrinsic semiconductors.

✔4. Apply the Fermi-Dirac distribution function to determine the occupation of electron and hole states in a semiconductor.

✔5. Calculate the electron and hole concentrations if the Fermi level is given; determine the Fermi level in a semiconductor if the carrier concentration is given.

✔6. Determine the variation of electron and hole mobility in a semiconductor with temperature, impurity concentration, and electrical field.

✔7. Apply the concept of compensation and space charge neutrality to calculate the electron and hole concentrations in compensated semiconductor samples.

✔8. Determine the current density and resistivity from given carrier densities and mobilities.

✔9. Calculate the recombination characteristics and excess carrier concentrations as a function of time for both low level and high level injection conditions in a semiconductor.

✔10. Use quasi-Fermi levels to calculate the non-equilibrium concentrations of electrons and holes in a semiconductor under uniform photoexcitation.

✔11. Calculate the drift and diffusion components of electron and hole currents.

✔12. Calculate the diffusion coefficients from given values of carrier mobility through the Einstein's relationship and determine the built-in field in a non-uniformly doped sample.

Plus continuity equation, steady-state carrier injection, and diffusion length

https://my.ece.illinois.edu/courses/description.asp?ECE340

Instructional Objectives (2)

By the time of Exam No.2 (after 32 lectures), the students should be able to do all of the items listed under A, plus the following:

13. Calculate the contact potential of a p-n junction.
14. Estimate the actual carrier concentration in the depletion region of a p-n junction in equilibrium.
15. Calculate the maximum electrical field in a p-n junction in equilibrium.
16. Distinguish between the current conduction mechanisms in forward and reverse biased diodes.
17. Calculate the minority and majority carrier currents in a forward or reverse biased p-n junction diode.
18. Predict the breakdown voltage of a p+-n junction and distinguish whether it is due to avalanche breakdown or Zener tunneling.
19. Calculate the charge storage delay time in switching p-n junction diodes.
20. Calculate the capacitance of a reverse biased p-n junction diode.
21. Calculate the capacitance of a forward biased p-n junction diode.
22. Predict whether a metal-semiconductor contact will be a rectifying contact or an ohmic contact based on the metal work function and the semiconductor electron affinity and doping.
23. Calculate the electrical field and potential drop across the neutral regions of wide base, forward biased p+-n junction diode.
24. Calculate the voltage drop across the quasi-neutral base of a forward biased narrow base p+-n junction diode.
25. Calculate the excess carrier concentrations at the boundaries between the space-charge region and the neutral n- and p-type regions of a p-n junction for either forward or reverse bias.

Quiz 1 Statistics

- Average: 8.65
- Standard Deviation: 1.49

Quiz 2 Statistics

- Average: 4.59
- Standard Deviation: 1.86

Exam I Statistics

- Average (All Sections): 71.88
- Standard Deviation (All Sections): 15.44

Streetman Errata (6th Edition)

- Equation 4-30: “ΔxA” not δxA
- Equation 4-33b: “τ_p” not “τ_n”

Streetman Errata

- Equation 6-35 in Streetman's "Solid State Electronic Devices" 6th edition has a typo:

Equation in Streetman:

$$C_i = \frac{\varepsilon_s}{W}$$

Corrected Version:

$$C_d = \frac{\varepsilon_s}{W}$$

Final Exam Schedule

Course	Section	CRN	Date	Day	Start Time	End Time	Room	Exam Type
ECE 340	ALL	ALL	05/04/2018	F	1:30 PM	4:30 PM	1002 Electrical & Computer Eng Bldg	Combined
ECE 340	ALL	ALL	05/04/2018	F	7:00 PM	10:00 PM	2015 Electrical & Computer Eng Bldg	Conflict