ECE 340
Solid State Electronic Devices

M,W,F 12:00-12:50 (X), 2015 ECEB

Professor John Dallesasse
Department of Electrical and Computer Engineering
2114 Micro and Nanotechnology Laboratory
Tel: (217) 333-8416
E-mail: jdallesa@illinois.edu

Office Hours: Wednesday 13:00 – 14:00

ECE Illinois
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

Final Exam Schedule

Course	Section	CRN	Date	Day	Start Time	End Time	Room	Exam Type
ECE 340	ALL	ALL	05/04/2018	F	1:30 PM	4:30 PM	1002 Electrical & Computer Eng Bldg	Combined
ECE 340	ALL	ALL	05/04/2018	F	7:00 PM	10:00 PM	2015 Electrical & Computer Eng Bldg	Conflict

Tentative Schedule [3]

APR 2 LEDs and Diode Lasers	APR 4 Metal-semiconductor junctions	APR 6 MIS-FETs: Basic operation, ideal MOS capacitor
APR 9 MOS capacitors: flatband & threshold voltage	APR 11 Review, discussion, problems (4/12 exam)	APR 13 MOS capacitors: C-V analysis
APR 16 MOSFETs: Output & transfer characteristics	APR 18 MOSFETs: small signal analysis, amps, inverters	APR 20 Narrow-base diode
APR 23 BJT fundamentals	APR 25 BJT specifics	APR 27 BJT normal mode operation
APR 30 BJT common emitter amplifier and current gain	MAY 2 (LAST LECTURE) Review, discussion, problem solving	FINAL EXAM **Date & time to be announced**

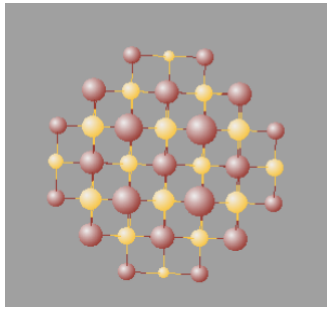
Subject to Change

Final Exam

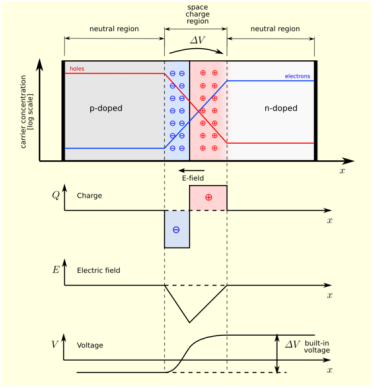
- **Main Areas:**
  - Semiconductor Fundamentals, pn Junctions, and Optoelectronic Devices
  - Bipolar Junction Transistors and the Narrow-Base Diode
  - MOS Capacitors & MOSFETs
- **Comments:**
  - The exam is inclusive − all assigned material (per syllabus)
  - No calculators or notes − writing utensils only

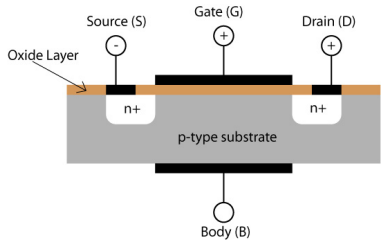
Course Purpose & Objectives

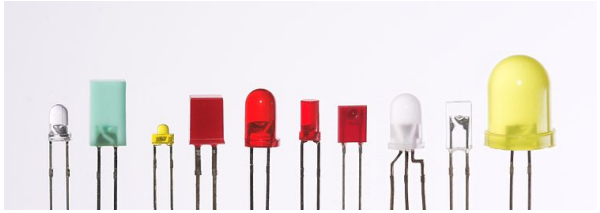
- Introduce key concepts in semiconductor materials
- Provide a basic understanding of p-n junctions
- Provide a basic understanding of light-emitting diodes and photodetectors
- Provide a basic understanding of field effect transistors
- Provide a basic understanding of bipolar junction transistors

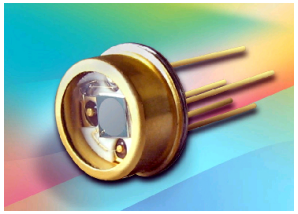


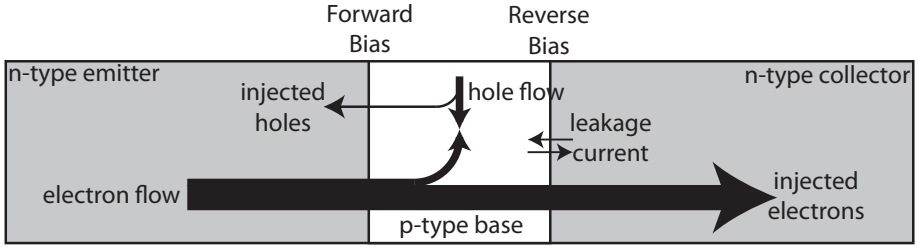












Instructional Objectives (1)

By the time of exam No. 1 (after 17 lectures), the students should be able to do the following:

1. Outline the classification of solids as metals, semiconductors, and insulators and distinguish direct and indirect semiconductors.
2. Determine relative magnitudes of the effective mass of electrons and holes from an E(k) diagram.
3. Calculate the carrier concentration in intrinsic semiconductors.
4. Apply the Fermi-Dirac distribution function to determine the occupation of electron and hole states in a semiconductor.
5. Calculate the electron and hole concentrations if the Fermi level is given; determine the Fermi level in a semiconductor if the carrier concentration is given.
6. Determine the variation of electron and hole mobility in a semiconductor with temperature, impurity concentration, and electrical field.
7. Apply the concept of compensation and space charge neutrality to calculate the electron and hole concentrations in compensated semiconductor samples.
8. Determine the current density and resistivity from given carrier densities and mobilities.
9. Calculate the recombination characteristics and excess carrier concentrations as a function of time for both low level and high level injection conditions in a semiconductor.
10. Use quasi-Fermi levels to calculate the non-equilibrium concentrations of electrons and holes in a semiconductor under uniform photoexcitation.
11. Calculate the drift and diffusion components of electron and hole currents.
12. Calculate the diffusion coefficients from given values of carrier mobility through the Einstein's relationship and determine the built-in field in a non-uniformly doped sample.

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Instructional Objectives (2)

By the time of Exam No.2 (after 32 lectures), the students should be able to do all of the items listed under A, plus the following:

13. Calculate the contact potential of a p-n junction.
14. Estimate the actual carrier concentration in the depletion region of a p-n junction in equilibrium.
15. Calculate the maximum electrical field in a p-n junction in equilibrium.
16. Distinguish between the current conduction mechanisms in forward and reverse biased diodes.
17. Calculate the minority and majority carrier currents in a forward or reverse biased p-n junction diode.
18. Predict the breakdown voltage of a p+-n junction and distinguish whether it is due to avalanche breakdown or Zener tunneling.
19. Calculate the charge storage delay time in switching p-n junction diodes.
20. Calculate the capacitance of a reverse biased p-n junction diode.
21. Calculate the capacitance of a forward biased p-n junction diode.
22. Predict whether a metal-semiconductor contact will be a rectifying contact or an ohmic contact based on the metal work function and the semiconductor electron affinity and doping.
23. Calculate the electrical field and potential drop across the neutral regions of wide base, forward biased p+-n junction diode.
24. Calculate the voltage drop across the quasi-neutral base of a forward biased narrow base p+-n junction diode.
25. Calculate the excess carrier concentrations at the boundaries between the space-charge region and the neutral n- and p-type regions of a p-n junction for either forward or reverse bias.

Instructional Objectives (3)

By the time of the Final Exam, after 44 class periods, the students should be able to do all of the items listed under A and B, plus the following:

26. Calculate the terminal parameters of a BJT in terms of the material properties and device structure.
27. Estimate the base transport factor “B” of a BJT and rank-order the internal currents which limit the gain of the transistor.
28. Determine the rank order of the electrical fields in the different regions of a BJT in forward active bias.
29. Calculate the threshold voltage of an ideal MOS capacitor.
30. Predict the C-V characteristics of an MOS capacitor.
31. Calculate the inversion charge in an MOS capacitor as a function of gate and drain bias voltage.
32. Estimate the drain current of an MOS transistor above threshold for low drain voltage.
33. Estimate the drain current of an MOS transistor at pinch-off.
34. Distinguish whether a MOSFET with a particular structure will operate as an enhancement or depletion mode device.
35. Determine the short-circuit current and open-circuit voltage for an illuminated p/n junction solar cell.

Current-Density Equations

Drift Diffusion

$$J_n = q\mu_n nE + qD_n \nabla n$$

$$J_p = q\mu_p pE - qD_p \nabla p$$

$$J_{cond} = J_n + J_p$$

- In semiconductor, in addition to an electron current density there is a hole current density
- Each current consist of the drift component cause by field and the diffusion component caused by the carrier concentration gradient

Continuity Equations

$$\frac{\partial n}{\partial t} = G_n - U_n + \frac{1}{q}\nabla \bullet J_n$$

$$\frac{\partial p}{\partial t} = G_p - U_p - \frac{1}{q}\nabla \bullet J_p$$

For a given volume of semiconductor, the rate change of carrier is the net effect of current flow into the volume and generation and recombination rates within the volume.

G_n *: electron generation rate*

G_p *: hole generation rate*

U_n *: electron recombination rate*

U_p *: hole recombination rate*

Maxwell Equations for Homogeneous and Isotropic Materials

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = -\frac{\partial D}{\partial t} + J_{cond} = J_{total}$$

$$\nabla \bullet D = \rho(x,y,z)$$

$$\nabla \bullet B = 0$$

$$B = \mu_o H$$

$$D(r,t) = \int_{-\infty}^{t} \varepsilon_s(t-t')E(r,t')dt'$$

$$= \varepsilon E + P$$

E: electric field

D: displacement vector

B: magnetic field

H: induction vector

ε_s: permittivity

μ_o: permeability

ρ: total electric charge density

J_{cond}: the conduction current density

$\times$: curl operator

$\bullet$: divergence operator

Key Points: Circuit Concepts

Capacitance :

Simple Form: $C = \left| \frac{Q}{V} \right|$

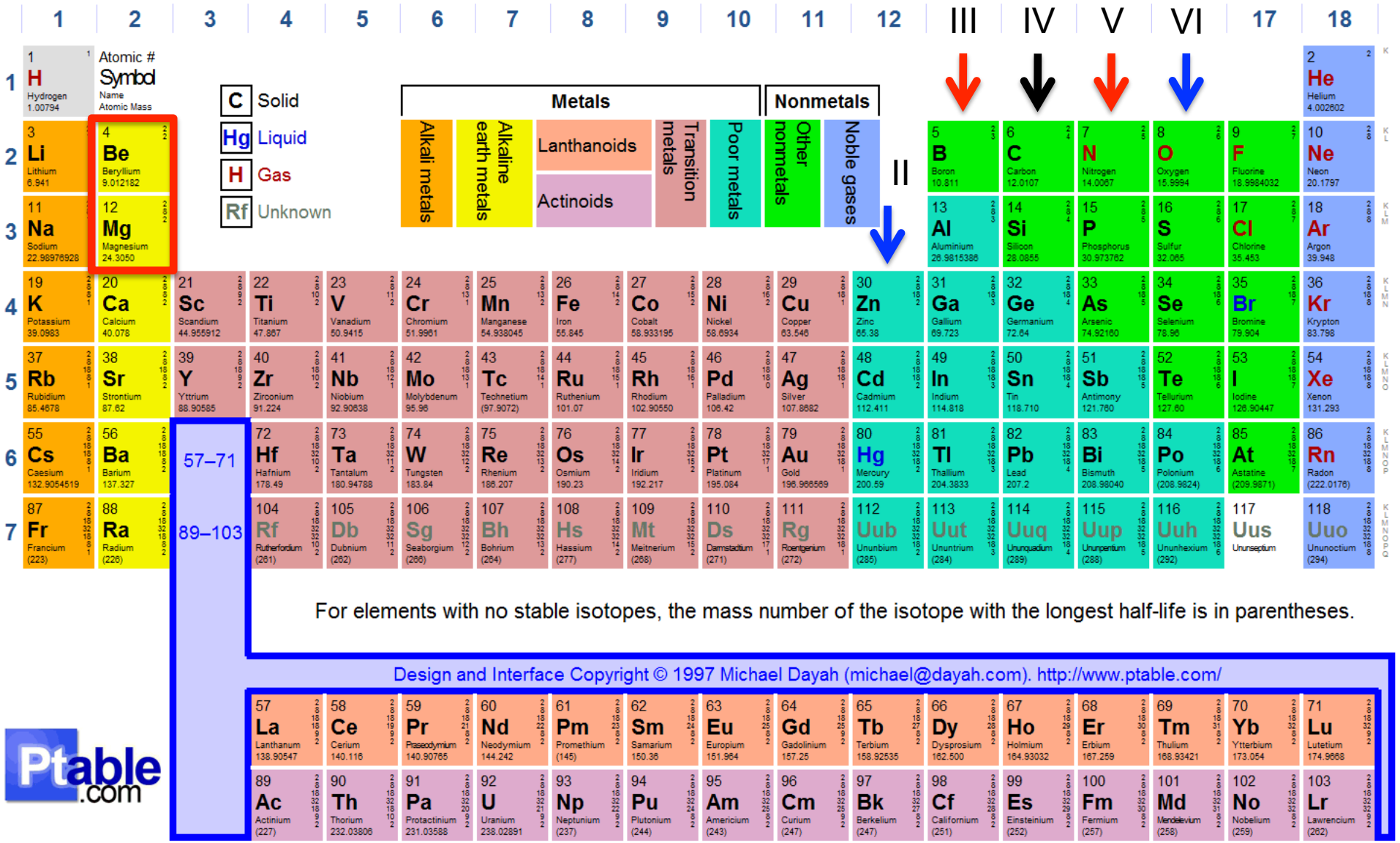
Differential Form: $C = \left| \frac{dQ}{dV} \right|$

Resistance / Conductance :

Resistance: $\rho = \frac{dV}{dI}$

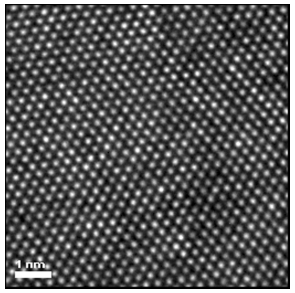
Conductance: $\sigma = \frac{dI}{dV}$

Periodic Table of the Elements



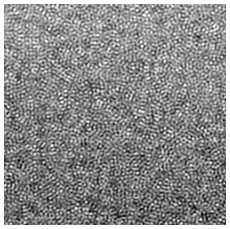
Types of Solids

HRTEM: [110] Si



http://spie.org/x31901.xml

TEM: Glassy Pd

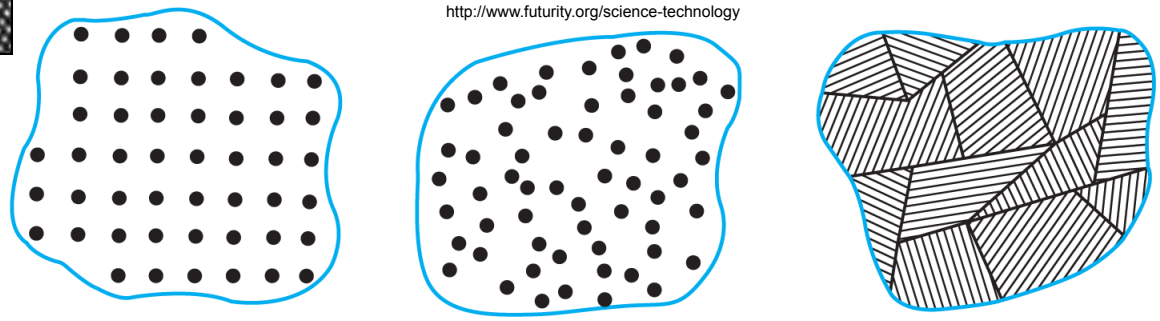


http://www.futurity.org/science-technology

SEM: Diamond on Si



http://isic.epfl.ch/page-53777-en.html



(a) Crystalline (b) Amorphous (c) Polycrystalline

Figure 1.1

Three types of solids, classified according to atomic arrangement: (a) crystalline and (b) amorphous materials are illustrated by microscopic views of the atoms, whereas (c) polycrystalline structure is illustrated by a more macroscopic view of adjacent single-crystalline regions, such as (a).

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Crystal Lattice

- **Lattice:** Periodic arrangement of a substance or "**basis**"
  - Atom, atomic pair, group of atoms, molecule, etc.
- **Primitive Cell:** The smallest unit cell that can be repeated in integral steps to produce the lattice
  - Contains a single lattice point
  - The Primitive Cell is a special form of the Unit Cell
- **Primitive Vectors: a, b, c**
  - (1 dimension) $r = p\,\mathbf{a}$
  - (2 dimension) $r = p\,\mathbf{a} + q\,\mathbf{b}$
  - (3 dimension) $r = p\,\mathbf{a} + q\,\mathbf{b} + r\,\mathbf{c}$
- **Unit Cell:** Lattice contains a volume which is representative of entire crystal and regularly repeated throughout the crystal
- **Basis Vectors**: Similar to primitive vectors, but used to replicate lattice through translation of unit cell

Simple Lattices and Unit Cells

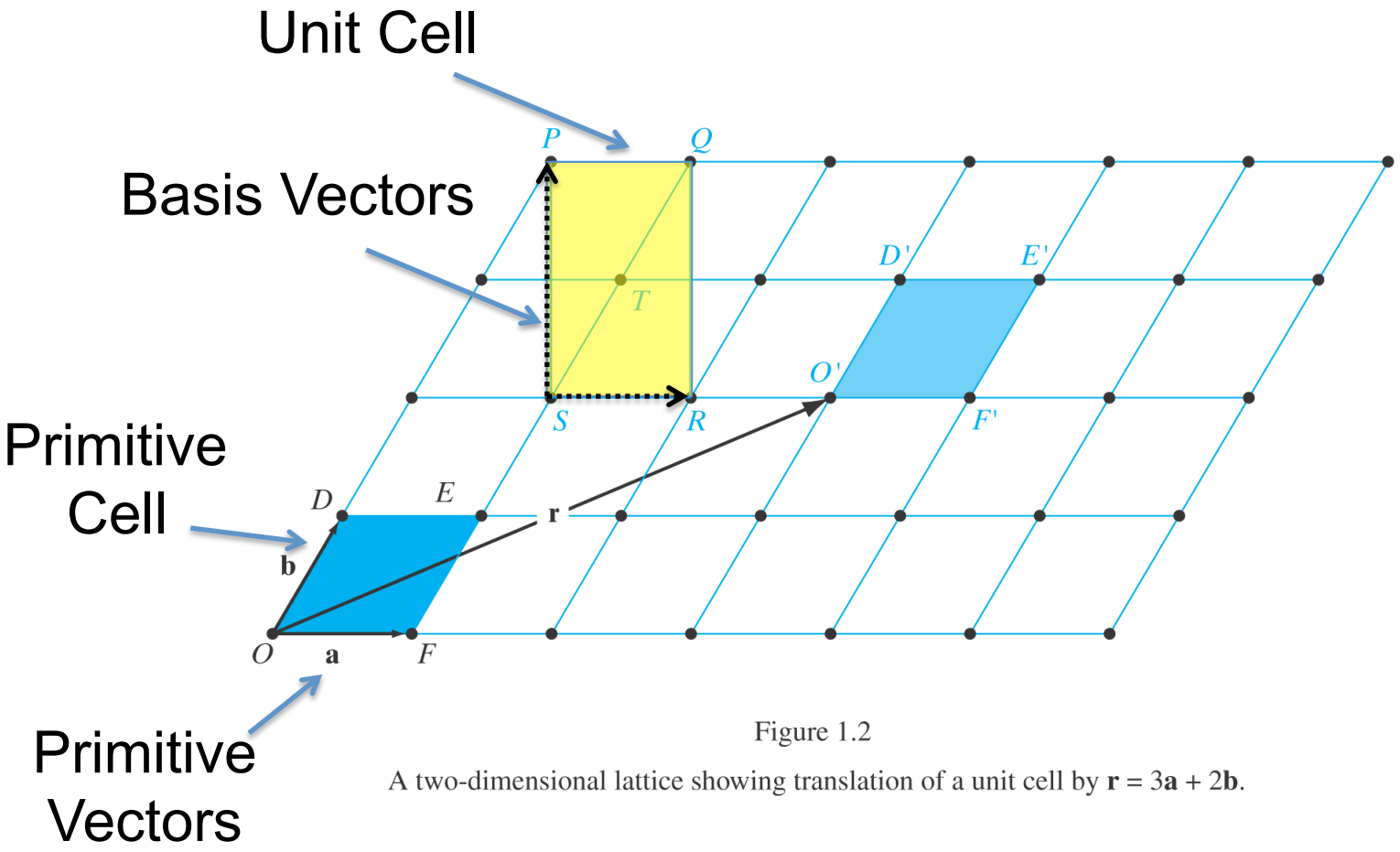

Figure 1.2

A two-dimensional lattice showing translation of a unit cell by $\mathbf{r} = 3\mathbf{a} + 2\mathbf{b}$.

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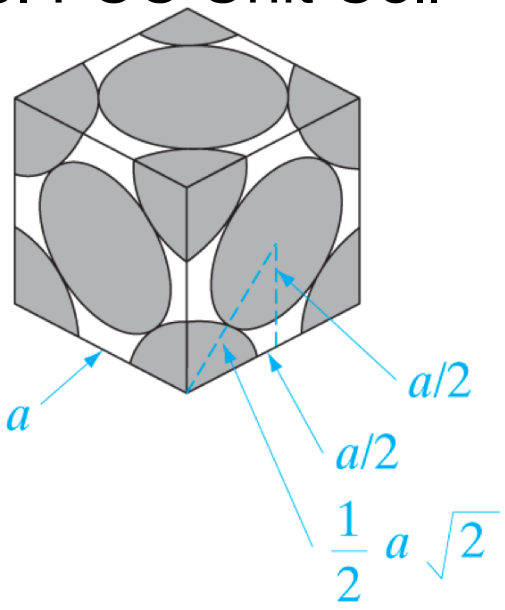
Other Important Definitions

- **<u>Lattice Constant:</u>** distance along the edge of a cubic unit cell ("a" in examples that follow)
  - More generally, the length of the basis vectors

NOTE: The lattice constant, in general, is **NOT** the distance between atoms (bond length).

i.e. FCC Unit Cell



Cubic Crystal Structures

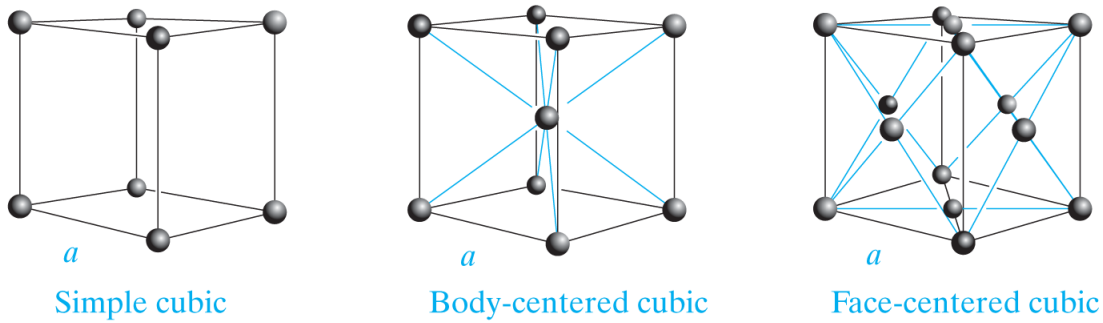


Figure 1.3

Unit cells for three types of cubic lattice structures.

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FCC Lattice

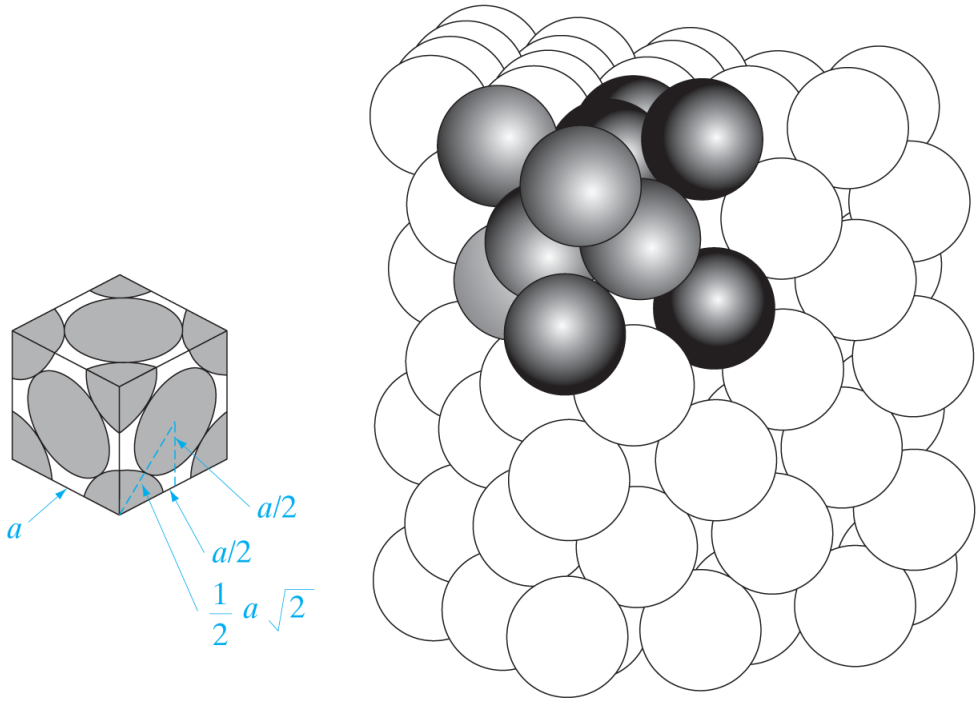

Figure 1.4

Packing of hard spheres in an fcc lattice.

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Diamond Structure

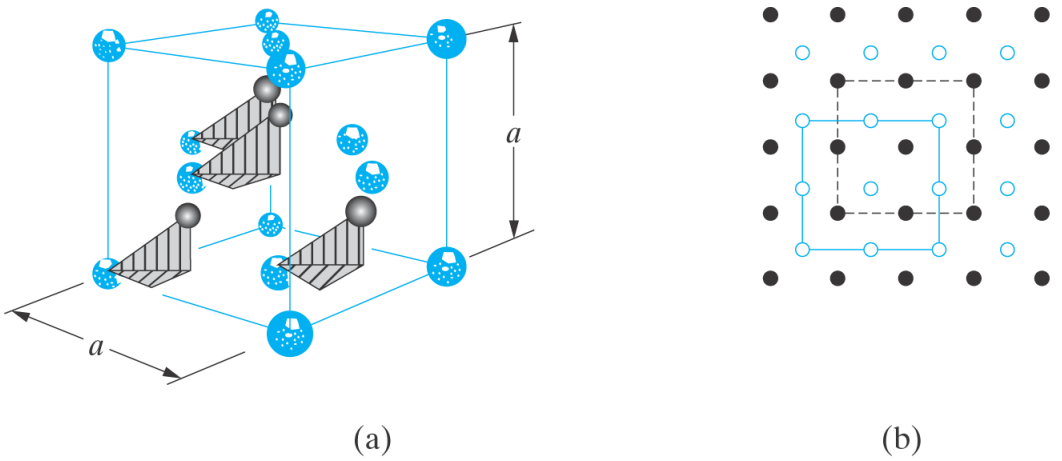

Figure 1.8

Diamond lattice structure: (a) a unit cell of the diamond lattice constructed by placing atoms $\frac{1}{4},\frac{1}{4},\frac{1}{4}$ from each atom in an fcc; (b) top view (along any ⟨100⟩ direction) of an extended diamond lattice. The colored circles indicate one fcc sublattice and the black circles indicate the interpenetrating fcc.

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Diamond Structure Bonding

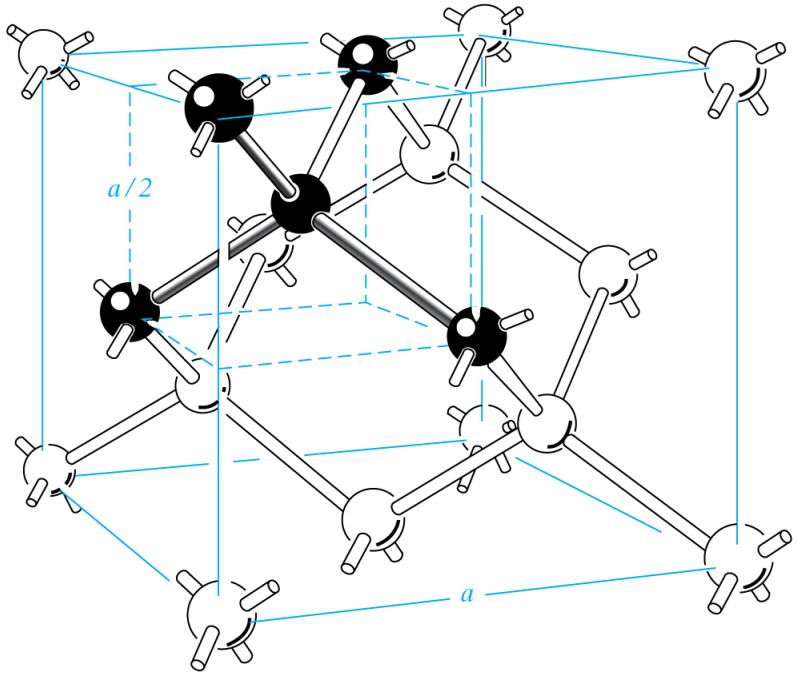

Figure 1.9

Diamond lattice unit cell, showing the four nearest neighbor structure.

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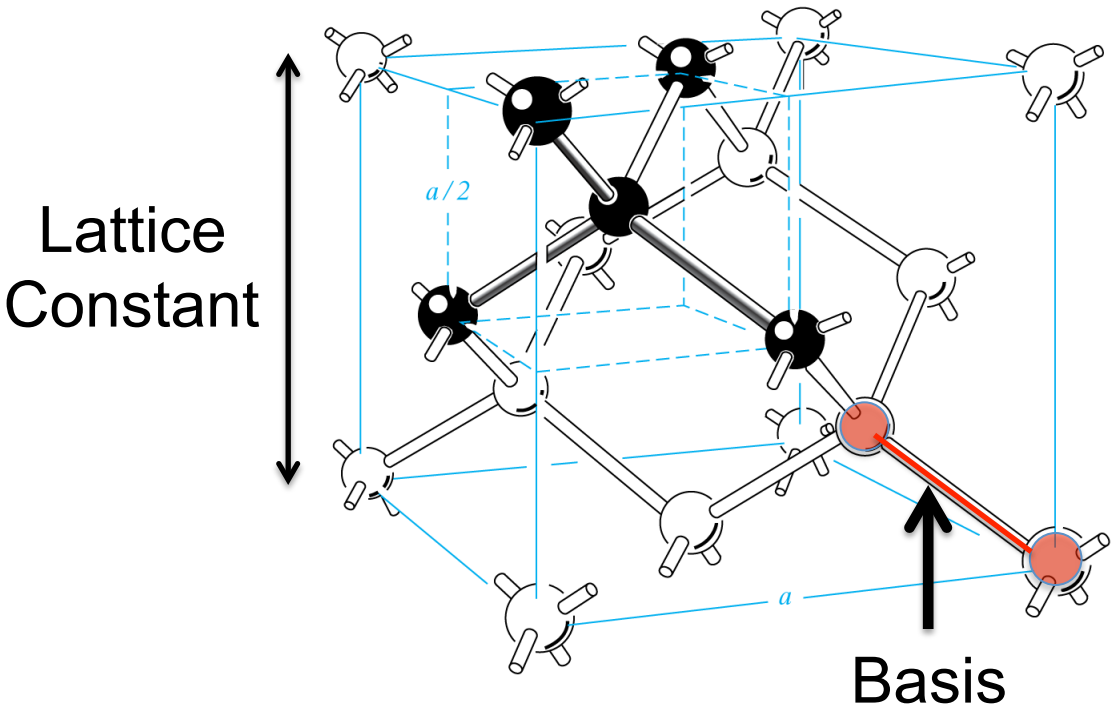
Diamond Structure Bonding

Diamond Unit Cell (not primitive cell)



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- The **Diamond Structure** is an **FCC Lattice** with a **Basis** of 2 atoms
- The **Unit Cell** has **4 Lattice Points**, and **8 Atoms**

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Determining Miller Indices

Crystallographic Notation
(hkl): Plane
[hkl]: Vector Normal to Plane
{hkl}: Family of Planes
<hkl>: Family of Vectors

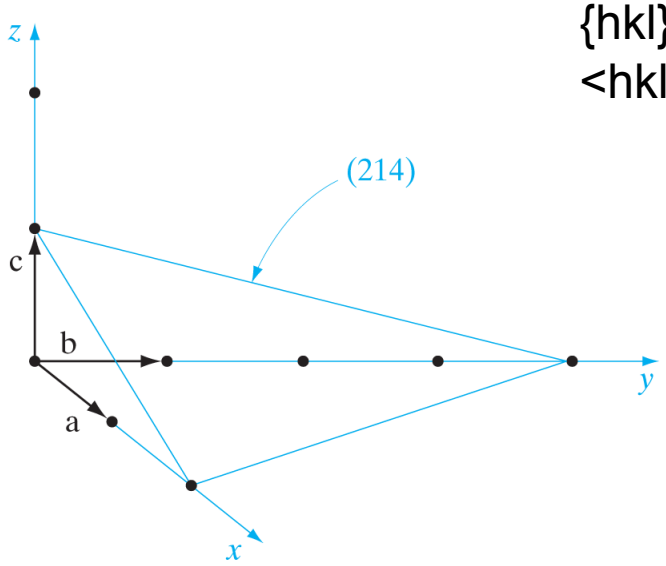

Figure 1.5
A (214) crystal plane.

Method
1. Determine intercept points
2. Take reciprocal
3. Multiply by least common multiple (smallest positive integer divisible by intercepts)

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Energy Bands

- As atoms are brought together to form a solid, the forces of attraction and repulsion between atoms will find a balance at the proper inter-atomic spacing for the crystal
- The discrete atomic energy levels form bands as the atoms are brought closer together. These bands become the conduction and valence band, separated by the forbidden gap where no electron states exist.
- Silicon Atom: $1s^2$ $2s^2$ $2p^6$ $3s^2$ $3p^2$ (missing 4 electrons to fill outer shell)
  - Core: $1s^2$ $2s^2$ $2p^6$ Valence: $3s^2$ $3p^2$

n=3	Available States	# of Electron
3S	2	2
3P	6	2
Total	8	4

- Assuming we have N Si atoms to form a crystal, we have N x (3s2 3p2) valence electrons

n=3	Available States	# of Electron
Total	Nx8	Nx4
Conduction Band	Nx4	0 (at 0°K)
Valence Band	Nx4	Nx4 (at 0°K)

Formation of Energy Bands

What happens when a collection of Si atoms are brought closer together?

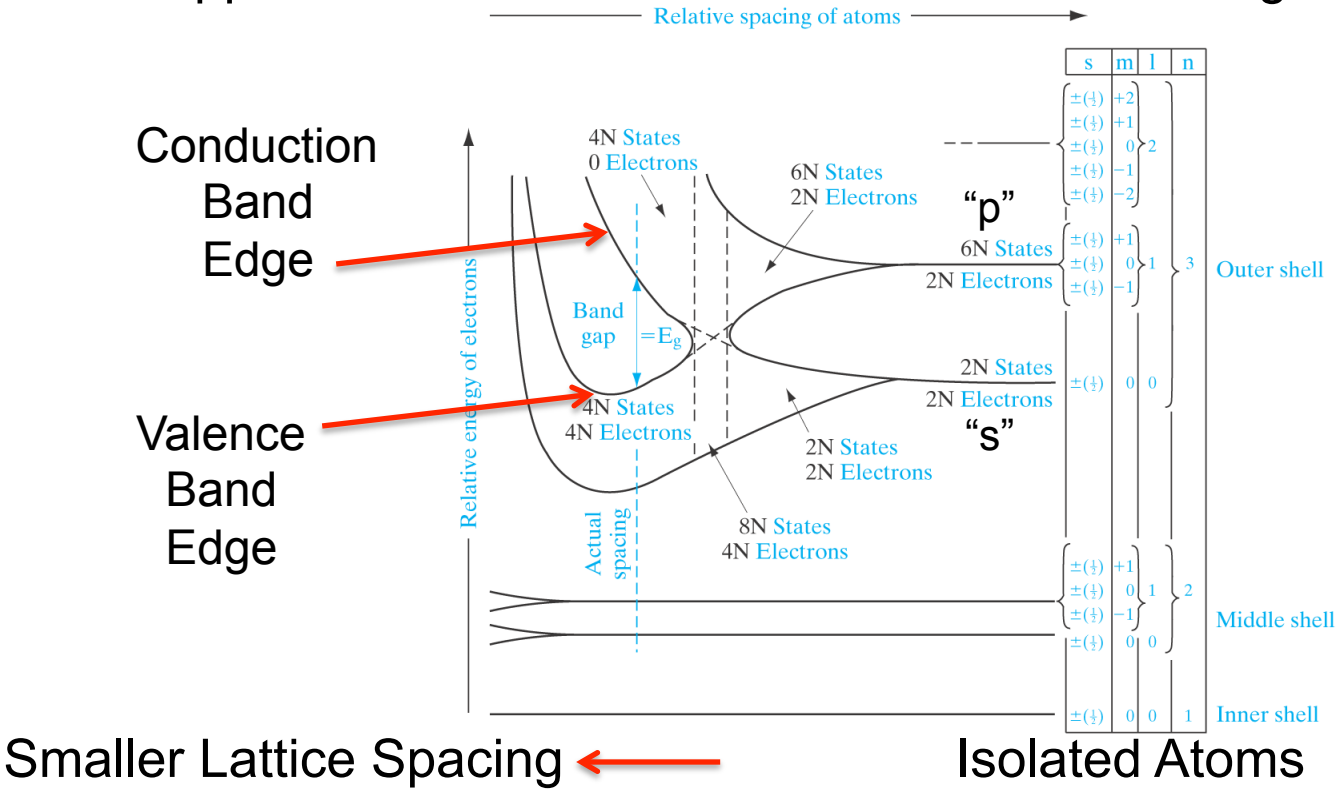

Figure 3.3

Energy levels in Si as a function of interatomic spacing. The core levels (**n** = 1,2) in Si are completely filled with electrons. At the actual atomic spacing of the crystal, the 2*N* electrons in the 3*s* subshell and the 2*N* electrons in the 3*p* subshell undergo sp^3 hybridization, and all end up in the lower 4*N* states (valence band), while the higher-lying 4*N* states (conduction band) are empty, separated by a band gap.

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Depiction of Bands in Real Space

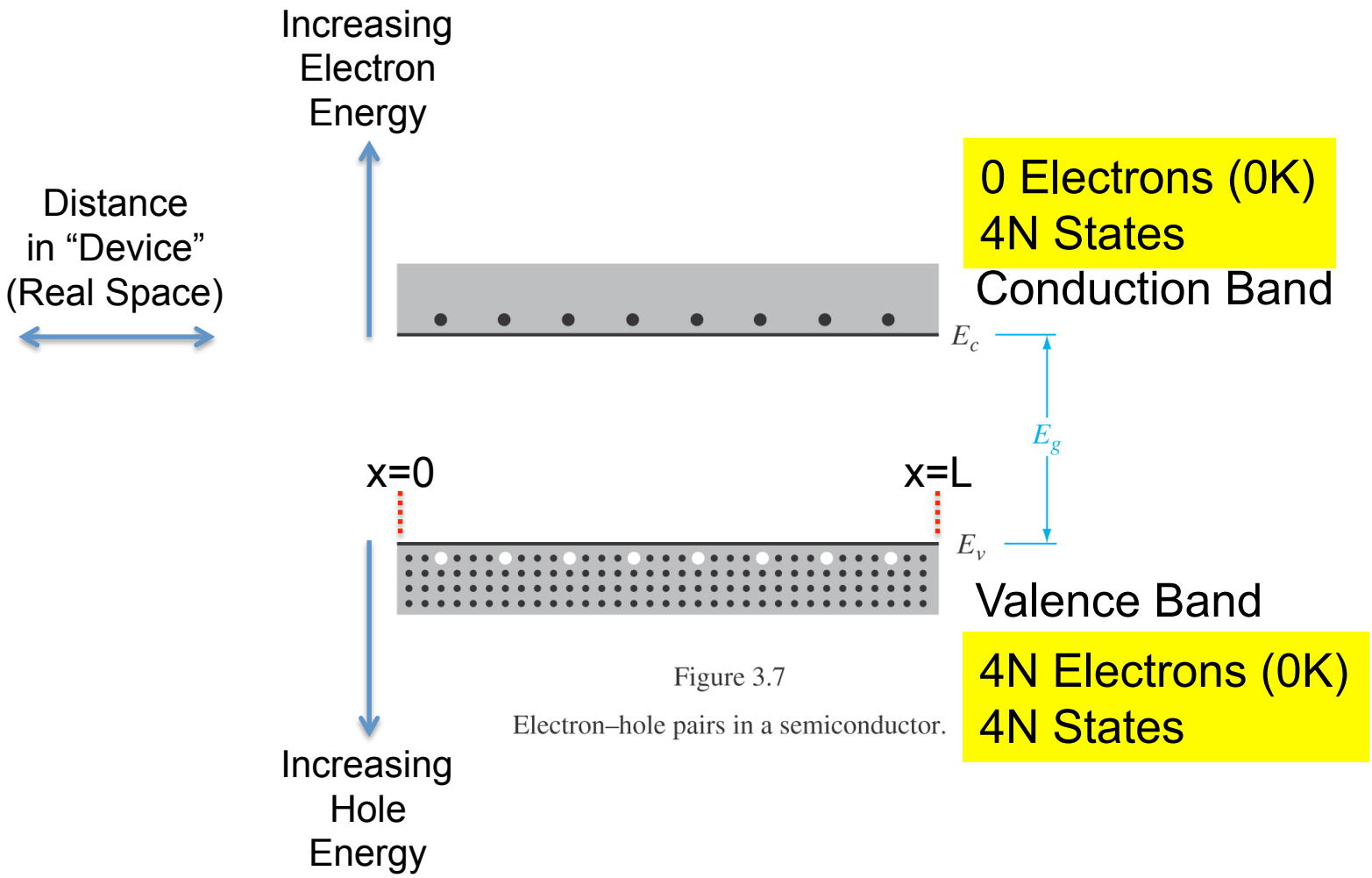

Figure 3.7

Electron–hole pairs in a semiconductor.

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Depictions in k-Space

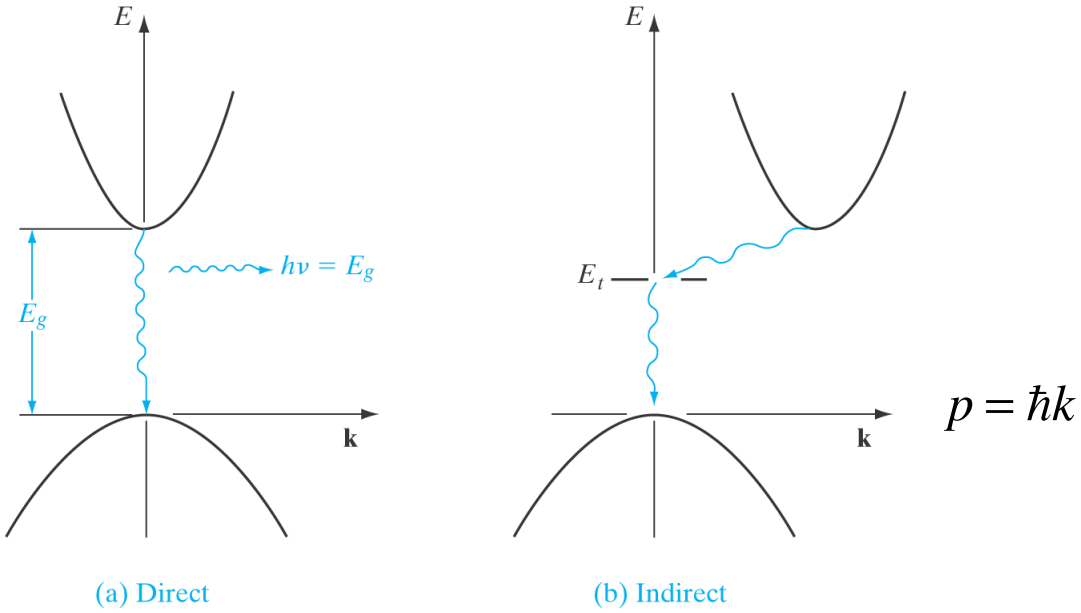

Figure 3.5

Direct and indirect electron transitions in semiconductors: (a) direct transition with accompanying photon emission; (b) indirect transition via a defect level.

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Insulators, Semiconductors, & Metals

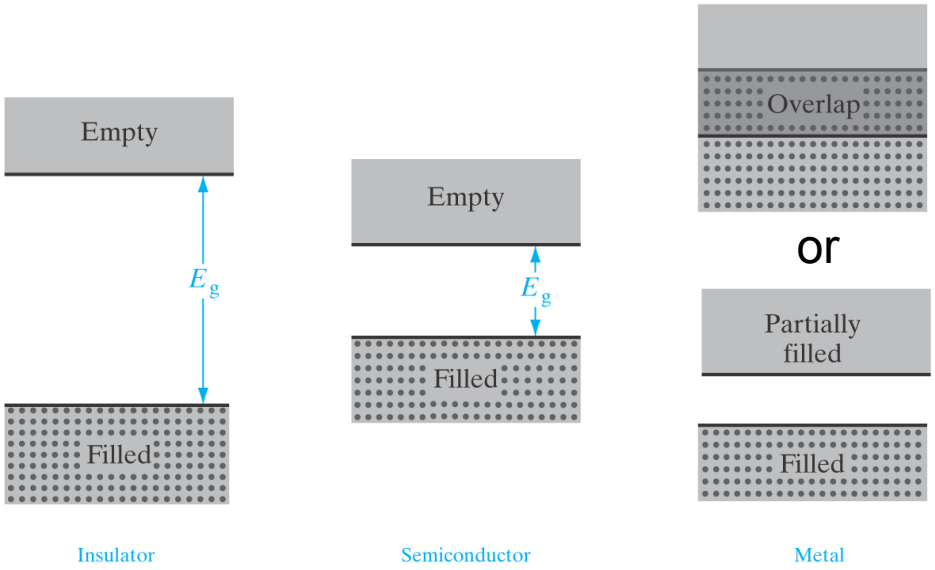

Figure 3.4

Typical band structures at 0 K.

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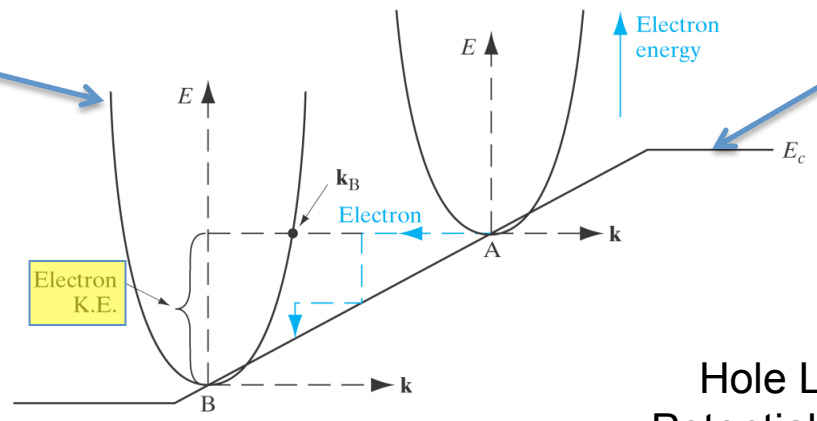
Inferring Electron Potential & Kinetic Energy From Band Diagrams

E ⟶

"k" Space

"Real" Space

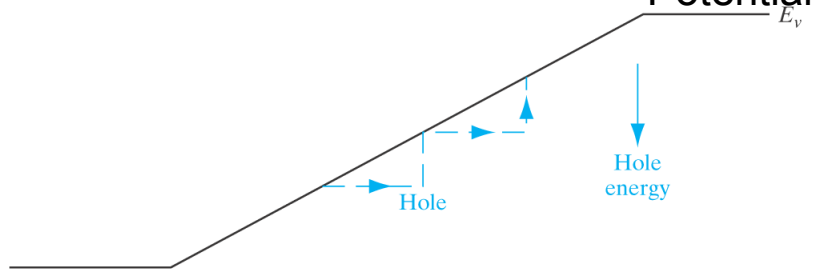
Electron Energy



Electron Higher Potential Energy

Electron Lower Potential Energy

Hole Lower Potential Energy



Hole Higher Potential Energy

Comments:

- The location of the band edge is an indication of the potential energy
- The location above (or below for holes) the band edge is an indication of kinetic energy

Hole Energy

Figure 3.9

Superimposition of the (E,**k**) band structure on the E-versus-position simplified band diagram for a semiconductor in an electric field. Electron energies increase going up, while hole energies increase going down. Similarly, electron and hole wave vectors point in opposite directions and these charge carriers move opposite to each other, as shown.

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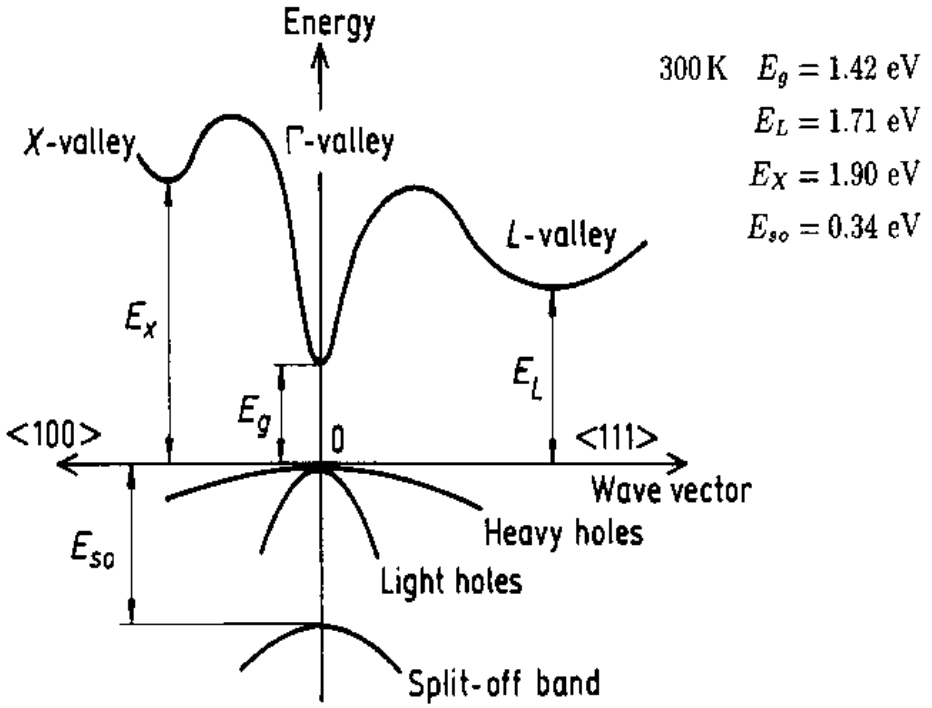
Electron and Hole Effective Mass

- In a semiconductor, the E(k) relationship looks parabolic near the conduction band minima and the valence band maxima
- The curvature of the parabola differs from the free space curvature, and depends upon which minima the electron or hole is in
- Near a minima, the E(k) relationship is:

$$E(k) = \frac{\hbar^2}{2m^* m_o} k^2$$

- This is similar to the E(k) relationship for a free electron, but with a mass $m^* m_o$ instead of m_o
- The parameter m* is the effective mass
- Tabulated effective masses are generally expressed as a multiple of the free-space electron mass m_o

GaAs Band Structure



http://www.ioffe.ru/SVA/NSM/Semicond/GaAs/bandstr.html

Extrinsic Material: Temperature Effects

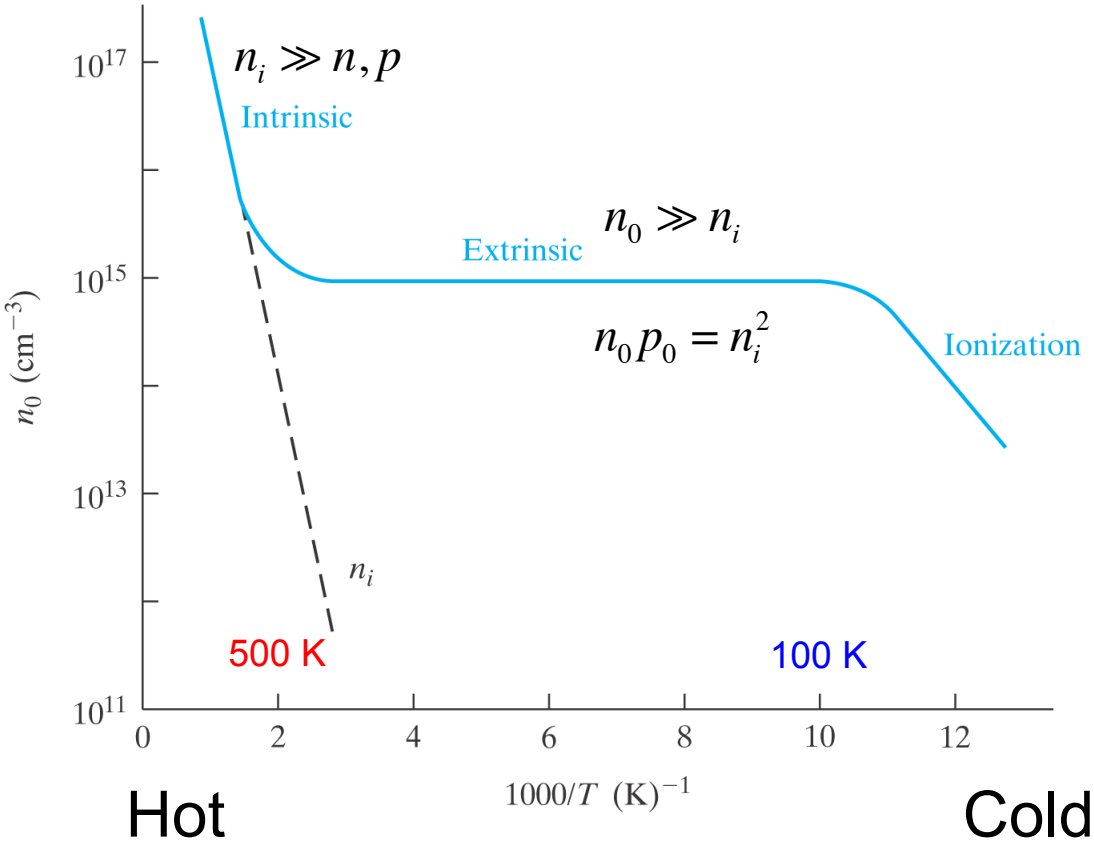


Figure 3.18

Carrier concentration vs. inverse temperature for Si doped with 10^{15} donors/cm^3.

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Fermi-Dirac Distribution

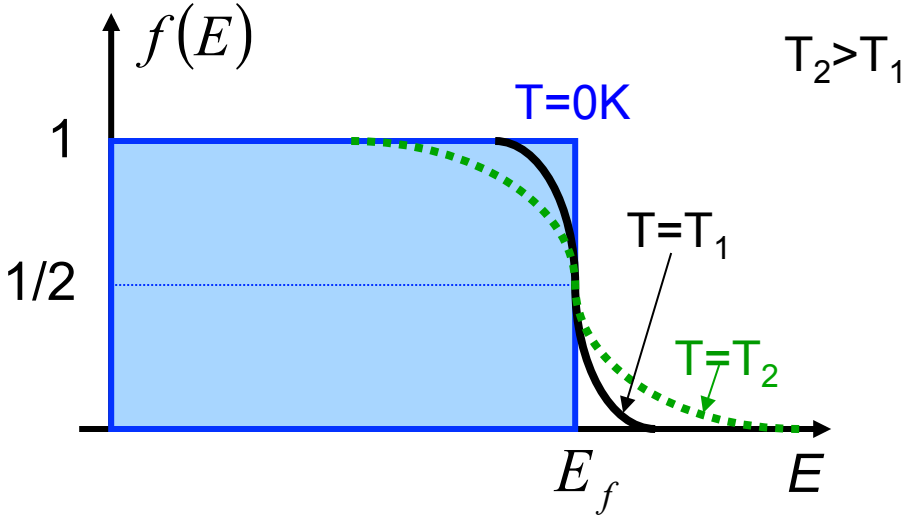
- The probability that an available energy state at E will be occupied by an electron at temperature T (temperature in Kelvin) is given by the **Fermi-Dirac Distribution Function**
- Fermi-Dirac distribution function:

$$f(E) = \frac{1}{1 + e^{(E - E_f)/kT}}$$

E_f : Fermi Level

k : Boltzmann Constant

$k = 8.62x10^{-5}\, eV / K$



(a) $E = E_f$ $\quad f(E = E_f) = \frac{1}{1 + e^0} = \frac{1}{2}$

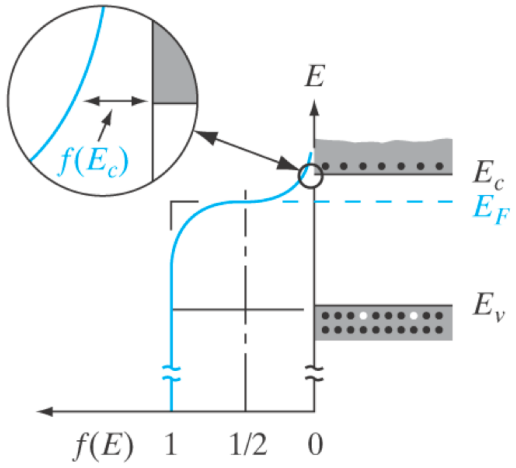
(b) T = 0K

$$f(E < E_f) = \frac{1}{1 + e^{-\infty}} = 1$$

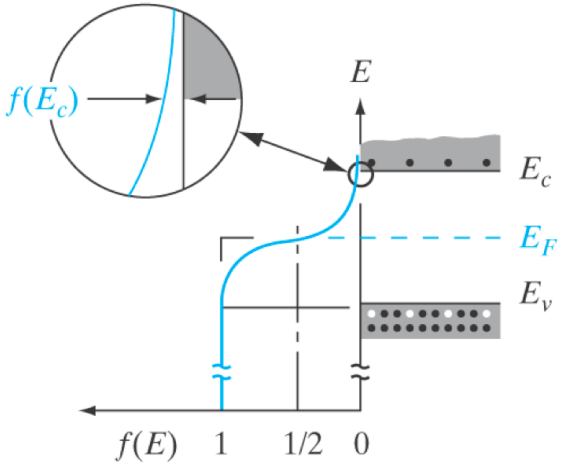
$$f(E > E_f) = \frac{1}{1 + e^{+\infty}} = 0$$

Fermi Level Versus Doping Type

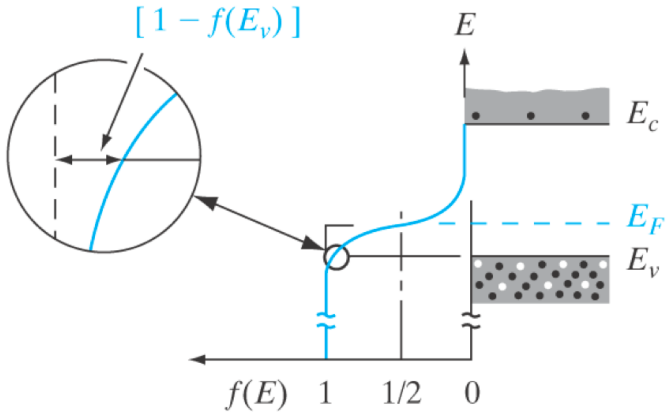
n-Type



Intrinsic

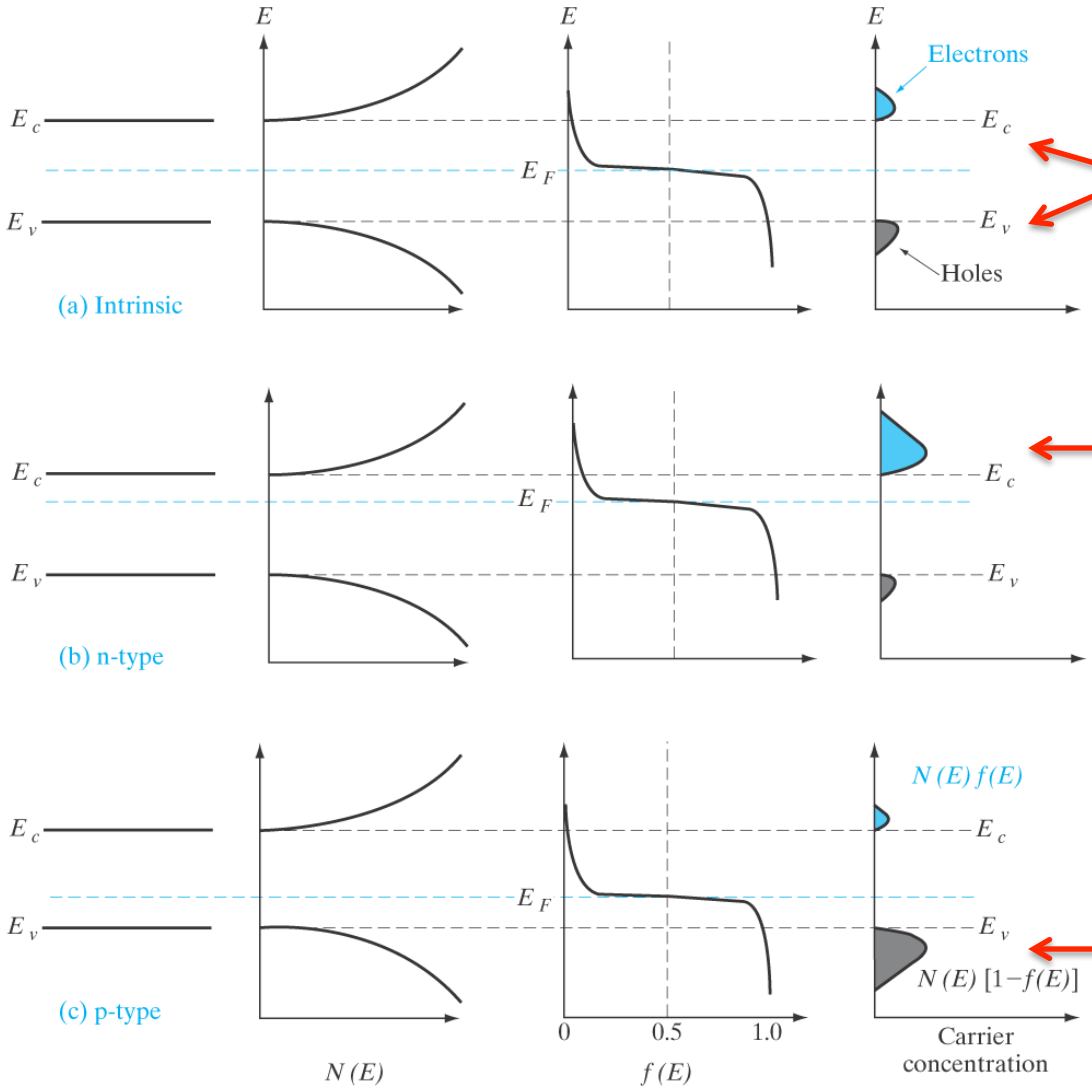


p-Type



Carrier Concentration at Thermal Equilibrium

The Fermi Function Multiplied by the Density of States



$$n_i = \int_{E_c}^{\infty} f(E) N_c(E) dE$$

$$= p_i = \int_{-\infty}^{E_V} [1 - f(E)] N_V(E) dE$$

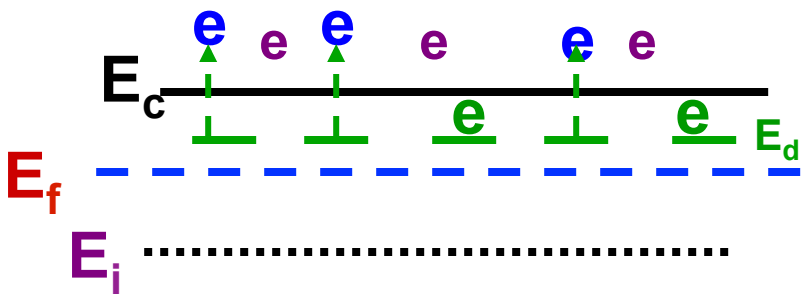
$$n_0 = \int_{E_c}^{\infty} f(E) N_c(E) dE$$

$$p_0 = \int_{-\infty}^{E_v} [1 - f(E)] N_v(E) dE$$

DOS: Appendix IV

Electron Concentration n_o in a n-type Semiconductor at Equilibrium

$n_o(K)=N_d$, majority carrier



$p_o(K)=n_i^2/n_o$, minority carrier

$$n_o = \int_{E_c}^{\infty} f(E) \bullet N(E)dE = \int_{E_c}^{\infty} \frac{1}{1+e^{(E-E_f)/kT}} \bullet \frac{\sqrt{2}}{\pi^2}\left(\frac{m^*}{\hbar^2}\right)^{3/2} E^{1/2}dE$$

$$\simeq \int_{E_c}^{\infty} e^{-(E-E_f)/kT} \bullet \frac{\sqrt{2}}{\pi^2}\left(\frac{m^*}{\hbar^2}\right)^{3/2} E^{1/2}dE = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} e^{(E_F-E_C)/kT}$$

$$= N_C(T)e^{(E_F-E_C)/kT} = N_C(T)e^{-(E_C-E_F)/kT}$$

$$n_o = \int_{E_c}^{\infty} f(E) \bullet N(E)dE$$

$$n_o = N_c(E_c) \bullet f(E_c)$$

$f(E) =$ Fermi Distribution Funtion

$N(E) =$ Density of States (cm^{-3})

$N_c(E_c) =$ Effective Density of States (located at conduction band edge)

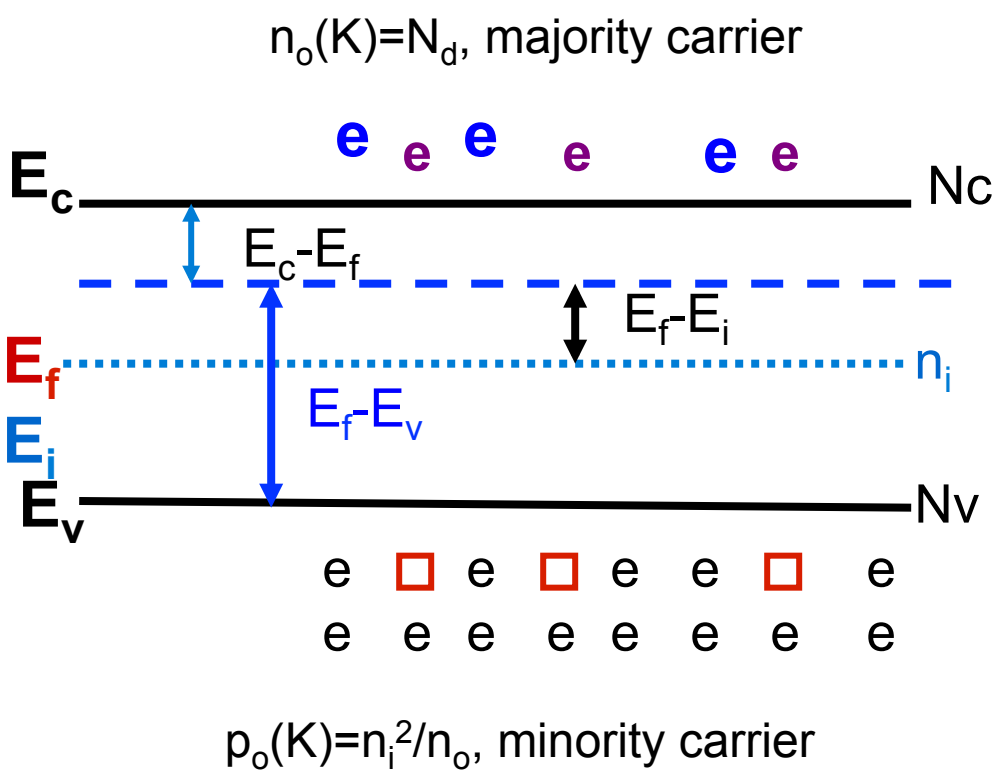
$$f(E_c) = \frac{1}{1+e^{(E_c-E_f)/kT}}$$

$$f(E_c) = e^{-(E_c-E_f)/kT} \quad if \quad e^{(E_c-E_f)/kT} >> 1$$

$$N_c(E_c) = 2\left(\frac{2\pi \bullet m_n^* kT}{h^2}\right)^{3/2}$$

$$n_o = N_c \bullet e^{-(E_c-E_f)/kT}$$

Referencing n_o and p_o to the Intrinsic Level & Intrinsic Carrier Concentration



$$n_i = N_c \bullet e^{-(E_c - E_i)/kT}$$

$$n_o = N_c \bullet e^{-(E_c - E_f)/kT}$$

$$n_o = n_i(T) \bullet e^{(E_f - E_i)/kT}$$

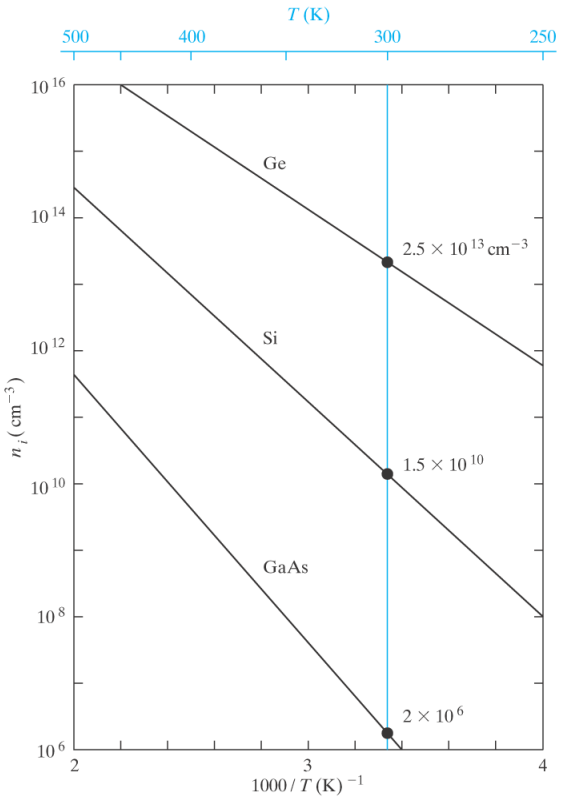
$$p_o = N_v \bullet e^{-(E_f - E_v)/kT}$$

$$p_o = n_i(T) \bullet e^{(E_i - E_f)/kT}$$

$$p_o \bullet n_o = n_i^2$$

Note: E_c-E_i=(E_c-E_f)+(E_f-E_i), so -(E_c-E_f)=(E_f-E_i)-(E_c-E_i)
Also: E_f-E_v=(E_f-E_i)+(E_i-E_v)

Intrinsic Concentration for Si, Ge, and GaAs



$$n_o p_o = N_c N_v e^{-(E_g)/kT} = n_i^2(T)$$

$$n_i(T) = \sqrt{N_c N_v}\, e^{-(E_g)/2kT}$$

$$n_i(T) = 2\left(\frac{2\pi kT}{h^2}\right)^2 \left(m_n^* m_p^*\right) e^{-(E_g)/2kT}$$

Intrinsic Carrier Concentration Depends Upon:

- Energy Gap E_g
- Temperature T
- Electron Effective Mass
- Hole Effective Mass

Figure 3.17

Intrinsic carrier concentration for Ge, Si, and GaAs as a function of inverse temperature. The room temperature values are marked for reference.

Note: Graph is neglecting $T^{3/2}$ term and $E_g(T)$

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Compensated Material

Charge Neutrality Equation :

$$p_o + N_d^+ = n_o + N_a^-$$

Equilibrium Concentration Product:

$$p_o n_o = n_i^2$$

Case 1: $N_D > N_A$ and $N_D - N_A \gg n_i$

Given GaAs at 300K with

$N_D = 1\times10^{17}\,cm^{-3}$

$N_A = 5\times10^{16}\,cm^{-3}$

Then:

$$n_o \simeq n_o - p_o = N_D^+ - N_A^- = 1\times10^{17} - 5\times10^{16}\,cm^{-3}$$

$$n_o \simeq 5\times10^{16}\,cm^{-3}$$

$$p_o = \frac{n_i^2}{n_o} \simeq \frac{\left(2\times10^6\right)^2}{5\times10^{16}} = 8\times10^{-5}\,cm^{-3}$$

Case 2: $N_A > N_D$ and $N_A - N_D \gg n_i$

Given Si at 300K with

$N_A = 1\times10^{17}\,cm^{-3}$

$N_D = 5\times10^{16}\,cm^{-3}$

Then:

$$p_o \simeq p_o - n_o = N_A^- - N_D^+ = 1\times10^{17} - 5\times10^{16}\,cm^{-3}$$

$$p_o \simeq 5\times10^{16}\,cm^{-3}$$

$$n_o = \frac{n_i^2}{p_o} \simeq \frac{\left(1.5\times10^{10}\right)^2}{5\times10^{16}} = 4.5\times10^{3}\,cm^{-3}$$

If the difference is close to the intrinsic concentration, need to solve 2 equations in 2 unknowns (quadratic formula)!

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Electron and Hole Current Density Relationship

Electrons: $(-q)nE_x = \left.\frac{dp_x}{dt}\right|_{field}$

Holes: $qnE_x = \left.\frac{dp_x}{dt}\right|_{field}$

$$\left.\frac{dp_x}{dt}\right|_{collision} = -\frac{p_x}{\bar{t}_n}$$

$$\left.\frac{dp_x}{dt}\right|_{collision} + \left.\frac{dp_x}{dt}\right|_{field} = 0$$

Electric field causes acceleration of the group of electrons (holes) in the crystal

Interaction of electrons (holes) with the crystal creates an opposing force to the applied electric field

The opposing force from the crystal balances the force due to the applied electric field at some mean value of velocity for the distribution of electrons (holes)

The mean velocity is the total momentum of all electrons (holes) divided by the total mass of all electrons $n \bullet m_n^*$

$$\langle v_x \rangle = \frac{p_x}{nm_n^*} = -\frac{q\bar{t}_n}{m_n^*}E_x$$

$$\langle v_x \rangle = \frac{p_x}{pm_p^*} = \frac{q\bar{t}_p}{m_p^*}E_x$$

The relationship between the value of the electric field and the mean value of the electron (hole) velocity at that field gives the mobility

$$\langle v_x \rangle = -\frac{q\bar{t}_n}{m_n^*}E_x = -\mu_n E_x$$

$$\langle v_x \rangle = \frac{q\bar{t}_p}{m_p^*}E_x = \mu_p E_x$$

The mean value of the electron (hole) velocity multiplied by the electron (hole) density multiplied by the charge carried and sign of the charge (-/+) gives the current density

$$J_{x,n} = -qn\langle v_x \rangle = qn\mu_n E_x$$

$$J_{x,p} = qp\langle v_x \rangle = qp\mu_p E_x$$

Electron and Hole Current in a Constant Field

Electron Current Relationship

$$J_{x,n} = -qn\langle v_x \rangle = qn\mu_n E_x = \sigma_n E_x$$

Electron Conductivity: $\sigma_n = qn\mu_n$

Electron Mobility: $\mu_n \equiv \dfrac{q\bar{t}_n}{m_n^*}$

Hole Current Relationship

$$J_{x,p} = qp\langle v_x \rangle = qp\mu_p E_x = \sigma_p E_x$$

Hole Conductivity: $\sigma_p = qp\mu_p$

Hole Mobility: $\mu_p \equiv \dfrac{q\bar{t}_p}{m_p^*}$

Combined Current

$$J_{x,total} = q(n\mu_n + p\mu_p)E_x = \sigma E_x$$

$$\sigma = q(n\mu_n + p\mu_p)$$

$V = IR$, or $I = \dfrac{1}{R}V$

Resistance of a Semiconductor Bar

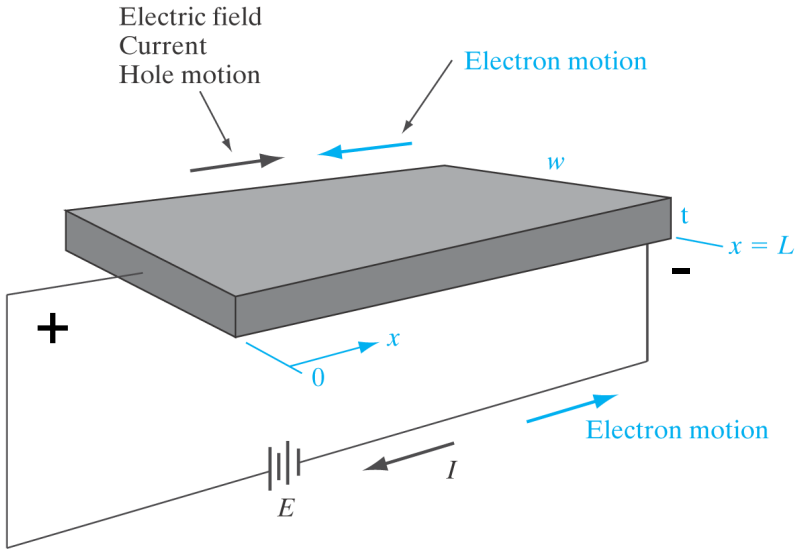

Figure 3.21

Drift of electrons and holes in a semiconductor bar.

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$$R = \frac{\rho L}{A} = \frac{\rho L}{wt} = \frac{L}{wt}\frac{1}{\sigma}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{q(n\mu_n + p\mu_p)}$$

- **Ohmic Contacts:** assume for now to be a "perfect" source of carriers (electrons or holes) – we will learn more later
- **Space Charge Neutrality and Dielectric Relaxation Time:** when a hole disappears through recombination at x=L, a new hole is created at x=0 in order to preserve space charge neutrality

σ Unit Analysis:

$$\frac{Coulombs}{carrier}\frac{carriers}{cm^3}\frac{cm^2}{V \bullet s} = \frac{Coulombs}{s}\frac{1}{V}\frac{1}{cm} = \frac{I}{V}\frac{1}{cm} = \frac{1}{\Omega}\frac{1}{cm} = \frac{S}{cm}$$

so $\rho = \frac{1}{\sigma} \Rightarrow$ units of $\Omega \bullet cm$

Mobility Impurity Dependence

More scattering centers translates to reduced mobility

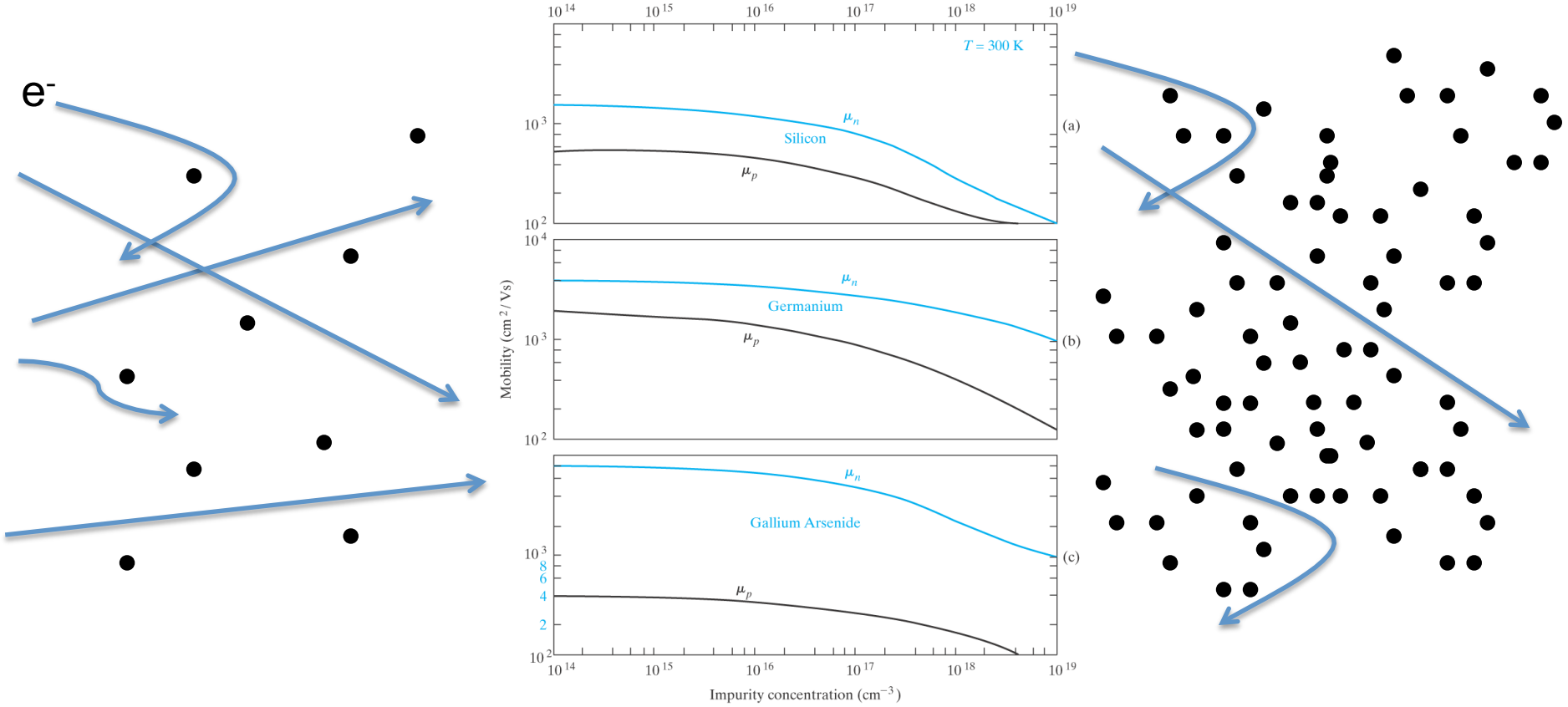

Figure 3.23

Variation of mobility with total doping impurity concentration ($N_a + N_d$) for Ge, Si, and GaAs at 300 K.

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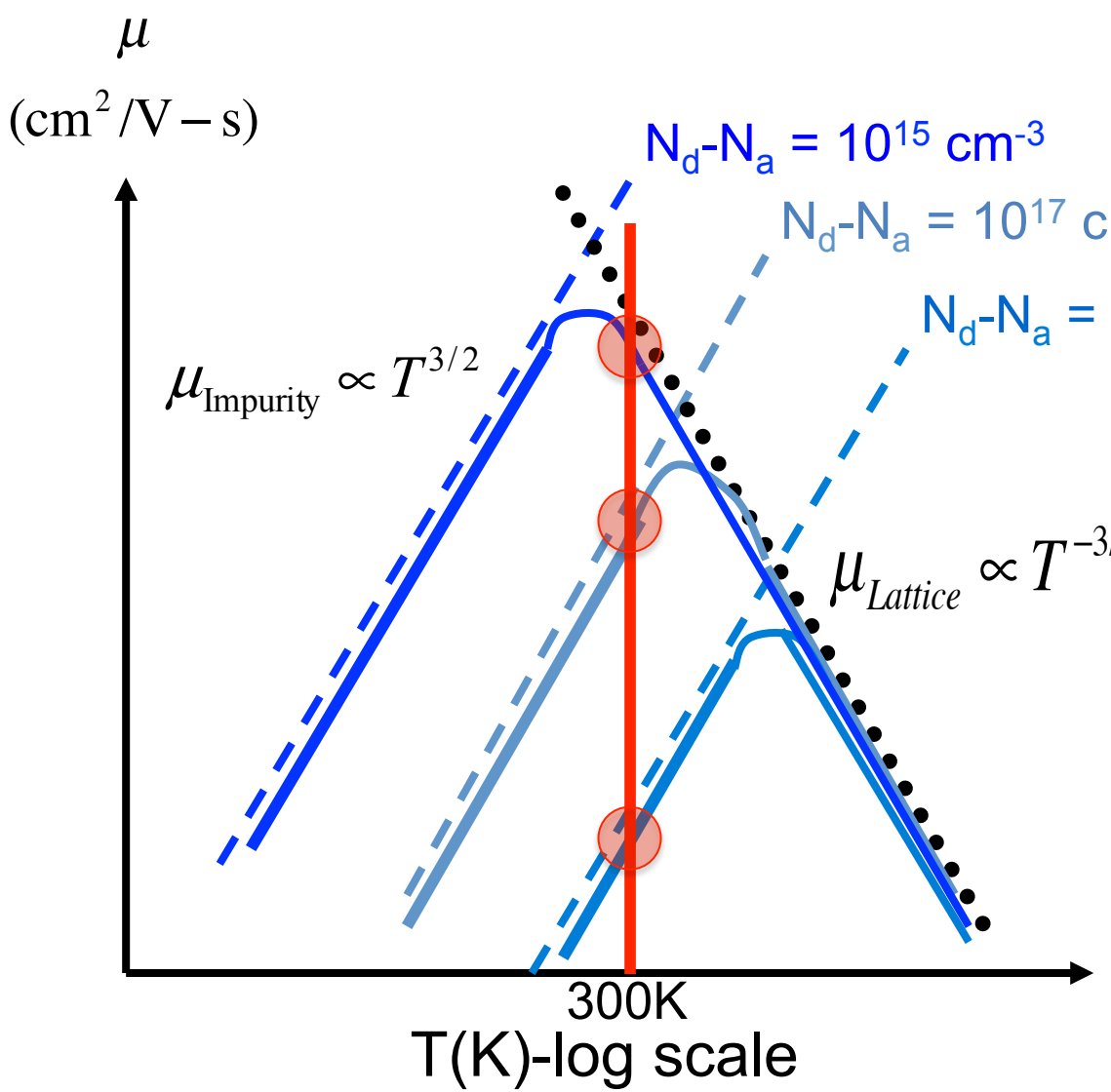
Effects of Both Temperature and Impurity Concentration on Mobility

$$\frac{1}{\mu} = \frac{1}{\mu_{Impurity}} + \frac{1}{\mu_{Lattice}} + \frac{1}{\mu_{Defect}} + \ldots$$

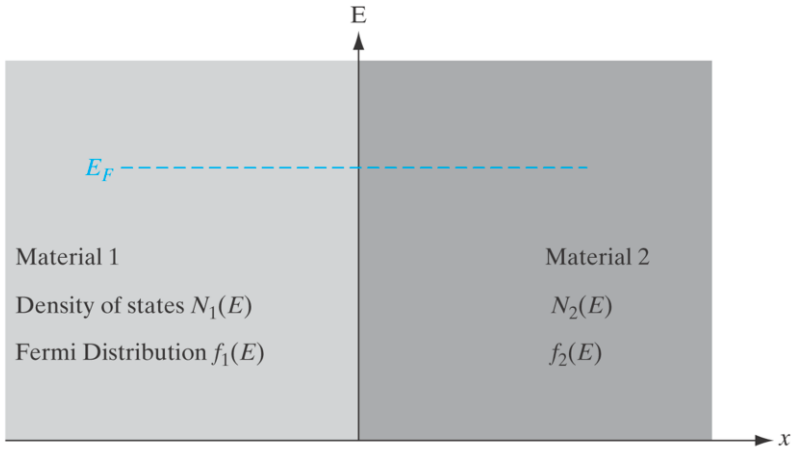


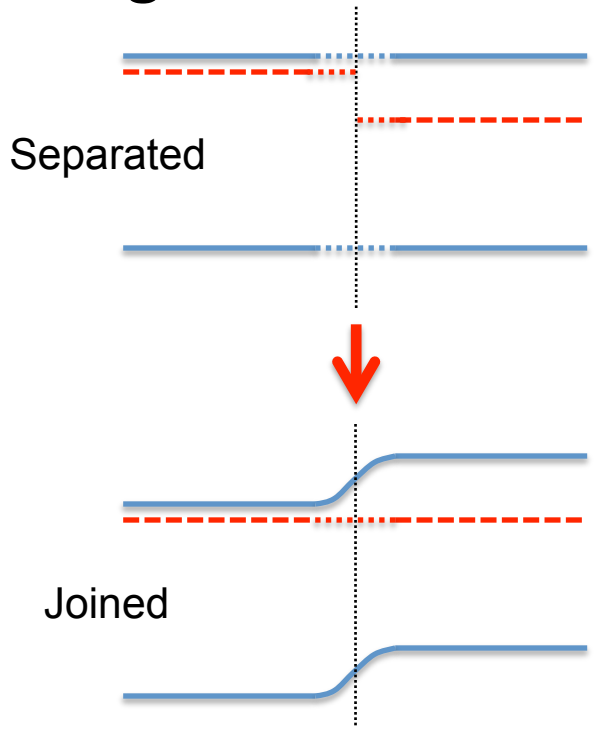
At T=300K

- For N_d-N_a=10^{19}cm^{-3}, the mobility is dominated by impurity scattering
- For N_d-N_a=10^{17}cm^{-3}, the mobility is dominated by impurity scattering
- For N_d-N_a=10^{15}cm^{-3}, the mobility is dominated by lattice scattering

Key Concept: Equilibrium Fermi Level

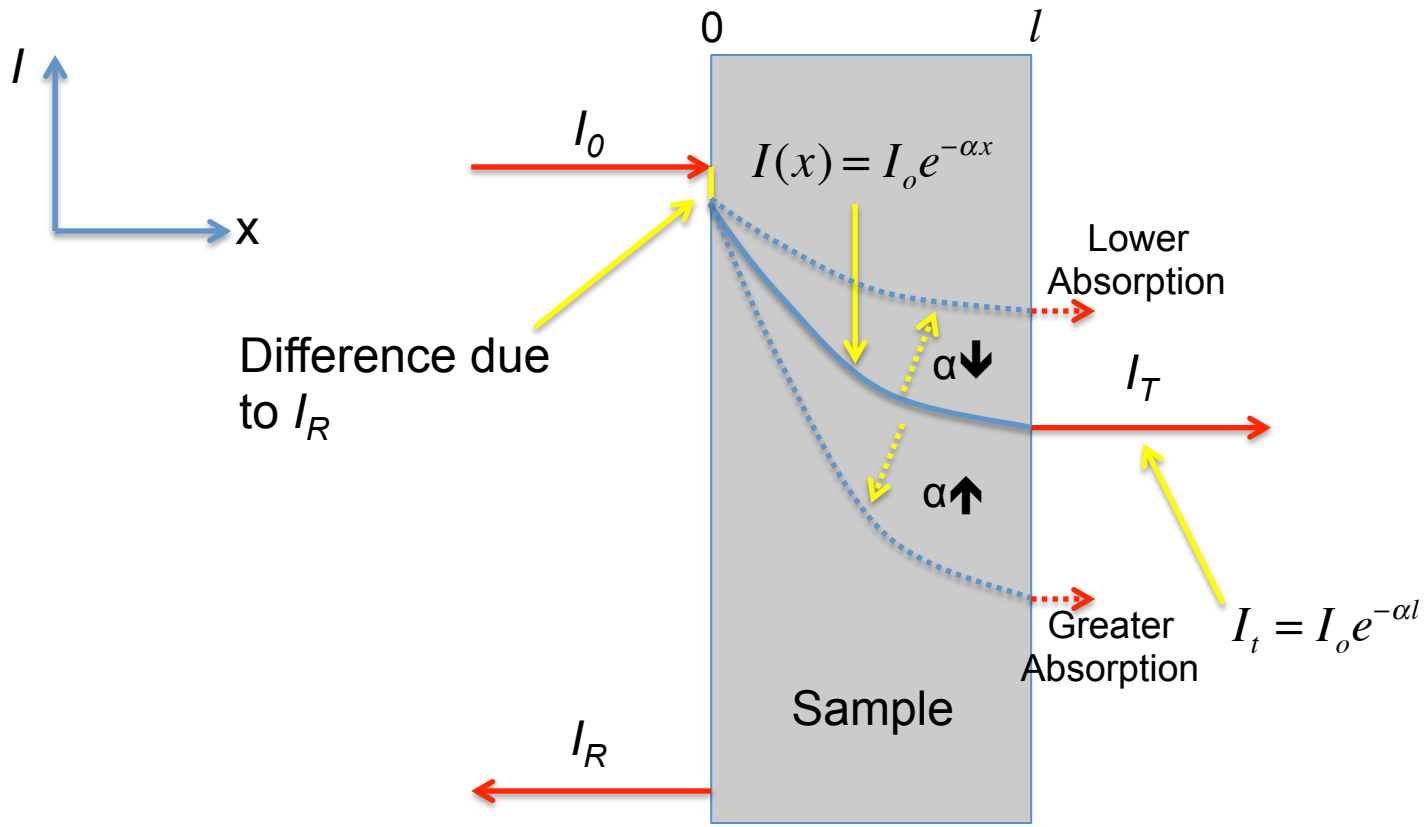
- No Discontinuity or Gradient Can Be Present in the **Equilibrium** Fermi Level E_F
  - In a drawing of energy versus position, E_F is a horizontal line that other energies are referenced against





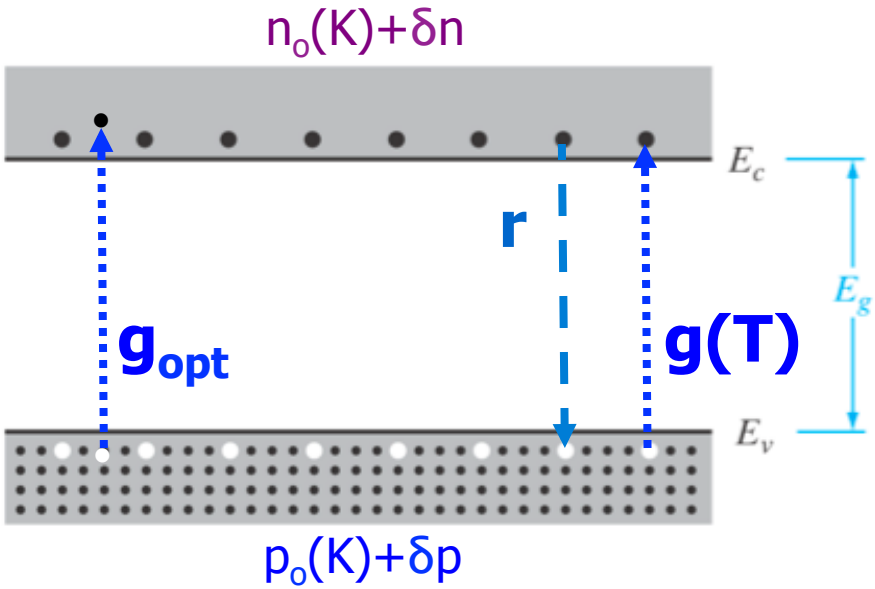
Absorption and Optical Intensity

- The photon flux drops exponentially in the sample
- The relationship for the generated carriers is more complicated due to diffusion



Steady State Generation

$$g = g(T) + g_{opt} = r = \alpha_r np$$
$$= \alpha_r (n_0 + \delta n)(p_0 + \delta p)$$
$$= \alpha_r \left[n_0 p_0 + n_0 \delta p + p_0 \delta n + \delta n \delta p \right]$$
$$= \alpha_r \left[n_0 p_0 + (n_0 + p_0)\delta n + \delta n^2 \right]$$
$$= \alpha_r n_0 p_0 + \alpha_r \left[(n_0 + p_0)\delta n + \delta n^2 \right]$$



Recall: $g(T) = \alpha_r n_i^2 = \alpha_r n_0 p_0$

so

$$g_{op} = \alpha_r \left[(n_0 + p_0)\delta n + \delta n^2 \right] \simeq \alpha_r (n_0 + p_0)\delta n = \frac{\delta n}{\tau_n}$$

$$\tau_n = \frac{1}{\alpha_n (n_0 + p_0)}$$

Assume δn small compared to majority carrier concentration

and

$$\delta n = \delta p = g_{op} \tau_n$$

Minority Carrier Lifetime

p-type Material

1) Assume low-level injection:

neglect $\delta n^2(t)$

2) p-type material so $p_0 \gg n_0$

$$\frac{d\delta n(t)}{dt} = -\alpha_r\left[\left(n_0 + p_0\right)\delta n(t) + \delta n^2(t)\right]$$

$$\simeq -\alpha_r\left(n_0 + p_0\right)\delta n(t) \simeq -\alpha_r p_0 \delta n(t)$$

The solution is:

$$\delta n(t) = \Delta n e^{-\alpha_r p_0 t} = \Delta n e^{-t/\tau_n}$$

where

$$\Delta n = \delta n(t=0)$$

$$\tau_n = \frac{1}{\alpha_r p_0} \quad \text{or}$$

$$\tau_n = \frac{1}{\alpha_r\left(n_0 + p_0\right)}$$

n-type Material

1) Assume low-level injection:

neglect $\delta p^2(t)$

2) n-type material so $n_0 \gg p_0$

$$\frac{d\delta p(t)}{dt} = -\alpha_r\left[\left(n_0 + p_0\right)\delta p(t) + \delta p^2(t)\right]$$

$$\simeq -\alpha_r\left(n_0 + p_0\right)\delta p(t) \simeq -\alpha_r n_0 \delta p(t)$$

The solution is:

$$\delta p(t) = \Delta p e^{-\alpha_r n_0 t} = \Delta n e^{-t/\tau_p}$$

where

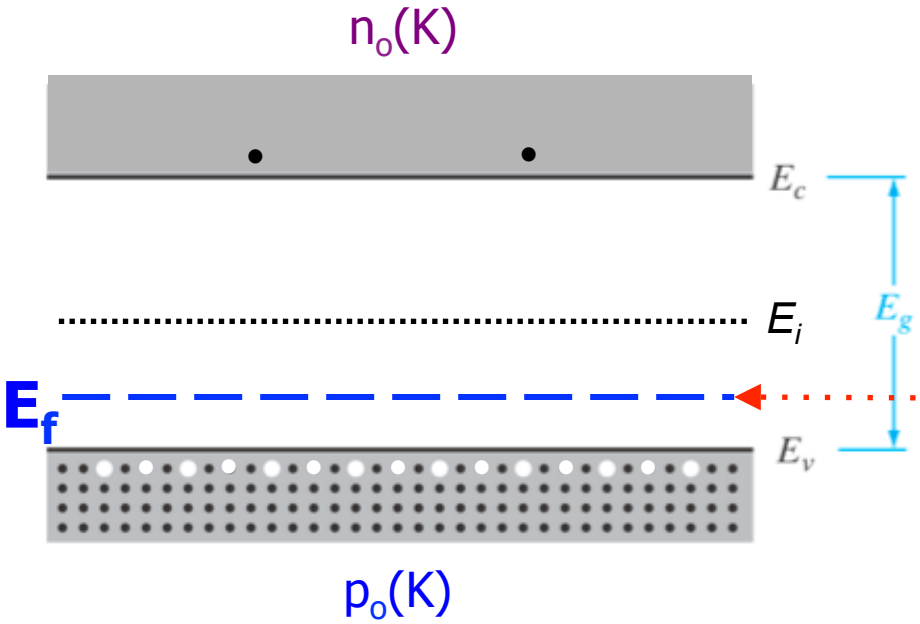
$$\Delta p = \delta p(t=0)$$

$$\tau_p = \frac{1}{\alpha_r n_0} \quad \text{or}$$

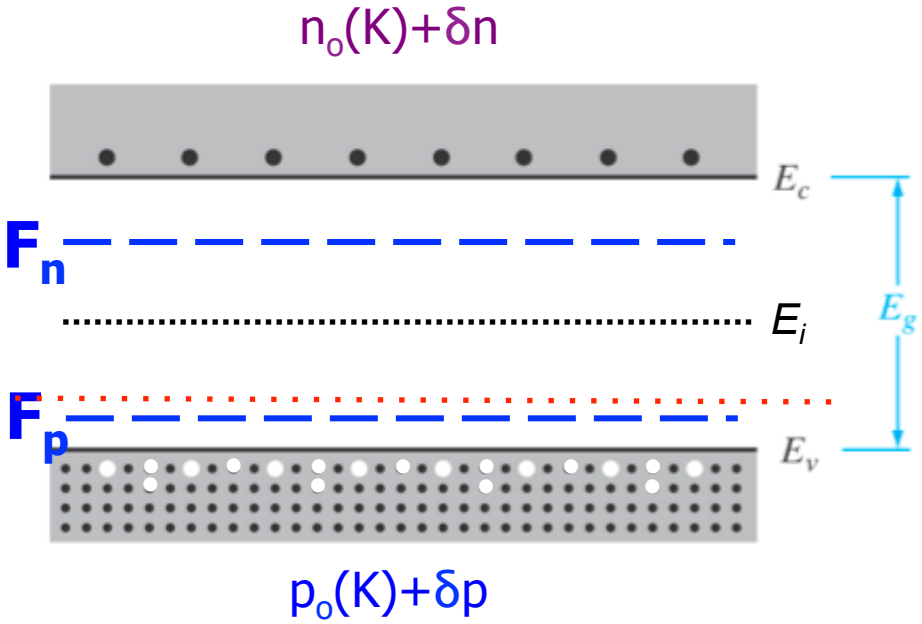
$$\tau_p = \frac{1}{\alpha_r\left(n_0 + p_0\right)}$$

Quasi-Fermi Levels in p-type Material

Equilibrium



With Excess Carriers

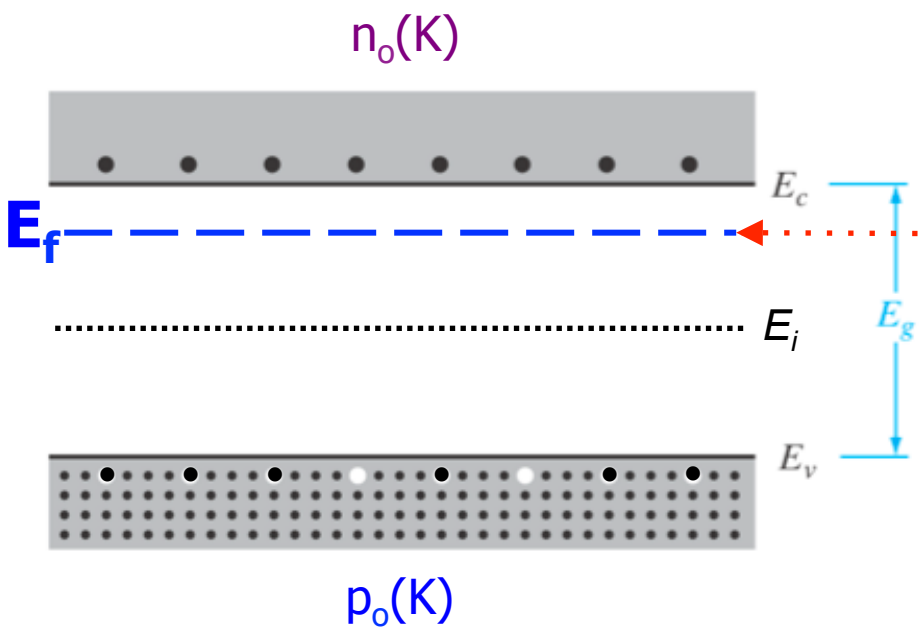


$$n = n_i e^{(F_n - E_i)/kT}$$

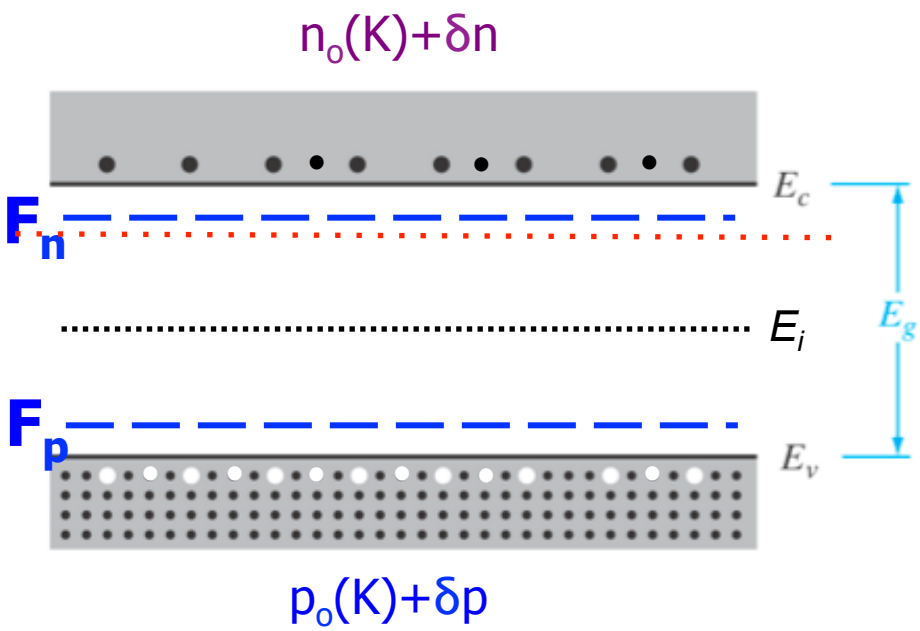
$$F_n - E_i = kT \ln\left(\frac{n}{n_i}\right)$$

Quasi-Fermi Levels in n-type Material

Equilibrium



With Excess Carriers



$$p = n_i e^{(E_i - F_p)/kT}$$

$$E_i - F_p = kT \ln\left(\frac{p}{n_i}\right)$$

Photoconductivity

Conductivity:

$$\sigma = q(n\mu_n + p\mu_p) = q(n_o + \delta n)\mu_n + q(p_o + \delta p)\mu_p$$

$$= q(n_o\mu_n + p_o\mu_p) + q(\delta n\mu_n + \delta p\mu_p)$$

$$= \sigma_o + \Delta\sigma$$

Photoconductivity Change:

$$\Delta\sigma = q(\delta n\mu_n + \delta p\mu_p) = qg_{opt}(\tau_n\mu_n + \tau_p\mu_p)$$

Since $\delta n = g_{opt}\tau_n$ and $\delta p = g_{opt}\tau_p$

- Maximizing Δσ:
  - Large mobility
  - Long carrier lifetime
- Other comments:
  - Photon energy higher than bandgap, but not by too much

Particle Flux and Current Density

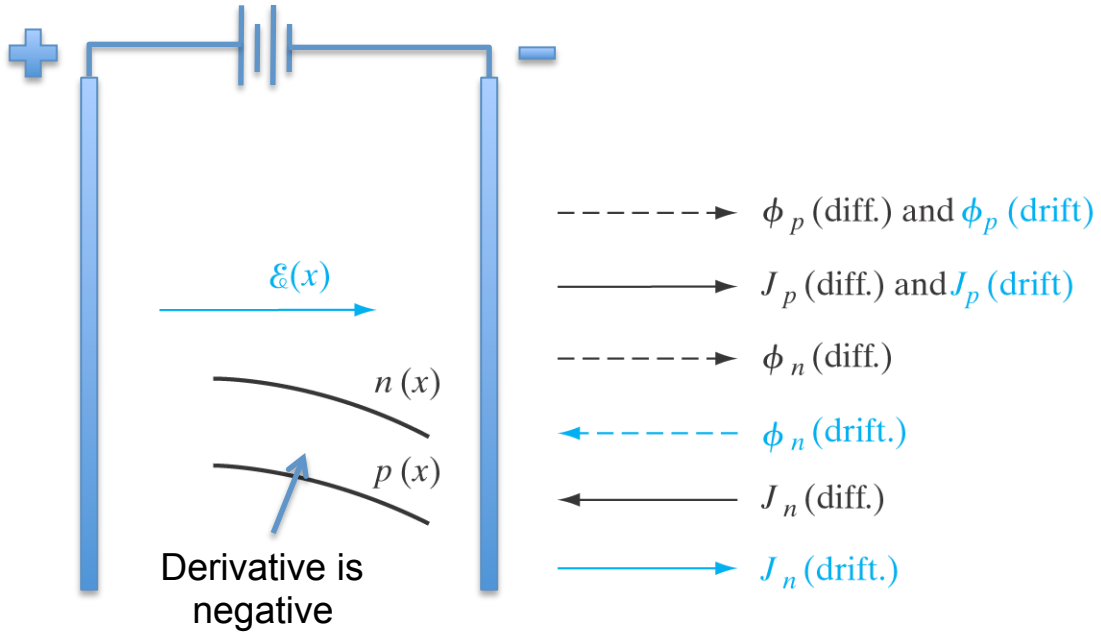


Figure 4.14

Drift and diffusion directions for electrons and holes in a carrier gradient and an electric field. Particle flow directions are indicated by dashed arrows, and the resulting currents are indicated by solid arrows.

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Total Current: Drift and Diffusion

$$J_n(x) = J_n(drift) + J_n(diff)$$

$$\rightarrow J_n(x) = qn(x)\mu_n\mathcal{E}(x) + qD_n\frac{dn(x)}{dx}$$

$$J_p(x) = J_p(drift) + J_p(diff)$$

$$\rightarrow J_p(x) = qp(x)\mu_p\mathcal{E}(x) - qD_p\frac{dp(x)}{dx}$$

$$\rightarrow J(x) = J_n(x) + J_p(x)$$

$$\boxed{J(x) = qn(x)\mu_n\mathcal{E}(x) + qD_n\frac{dn(x)}{dx} + qp(x)\mu_p\mathcal{E}(x) - qD_p\frac{dp(x)}{dx}}$$

Electron and Hole Continuity Equations

Hole Continuity Equation

$$\left.\frac{dp(x)}{dt}\right|_{x\to x+\Delta x} = \frac{1}{(+q)}\frac{J_p(x)-J_p(x+\Delta x)}{\Delta x} - \frac{\delta p}{\tau_p}$$

as $\Delta x \to 0$

$$\frac{\partial p(x,t)}{\partial t} = \frac{\partial(p_0+\delta p)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}$$

Electron Continuity Equation

$$\left.\frac{dn(x)}{dt}\right|_{x\to x+\Delta x} = \frac{1}{(-q)}\frac{J_n(x)-J_n(x+\Delta x)}{\Delta x} - \frac{\delta n}{\tau_n}$$

as $\Delta x \to 0$

$$\frac{\partial n(x,t)}{\partial t} = \frac{\partial(n_0+\delta n)}{\partial t} = \frac{\partial \delta n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$$

Note, as $\Delta x \to 0$:

$$\frac{J_p(x+\Delta x)-J_p(x)}{\Delta x} = \frac{\partial J_p(x)}{\partial x}$$

Assuming No Generation

Diffusion Equation: Where Drift is Negligible

Hole Diffusion Equation

$$\frac{\partial p(x,t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}$$

$$\text{and } J_p(x) = -qD_p\frac{dp(x)}{dx} = -qD_p\frac{\partial \delta p}{\partial x} \text{ (diffusion)}$$

$$\text{so } \frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial}{\partial x}\left(-qD_p\frac{\partial \delta p}{\partial x}\right) - \frac{\delta p}{\tau_p}$$

$$\therefore \boxed{\frac{\partial \delta p}{\partial t} = D_p\frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_p}}$$

Electron Diffusion Equation

$$\frac{\partial n(x,t)}{\partial t} = \frac{\partial \delta n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$$

$$\text{and } J_n(x) = qD_n\frac{dn(x)}{dx} = qD_n\frac{\partial \delta n}{\partial x} \text{ (diffusion)}$$

$$\text{so } \frac{\partial \delta n}{\partial t} = \frac{1}{q}\frac{\partial}{\partial x}\left(qD_n\frac{\partial \delta n}{\partial x}\right) - \frac{\delta n}{\tau_n}$$

$$\therefore \boxed{\frac{\partial \delta n}{\partial t} = D_n\frac{\partial^2 \delta n}{\partial x^2} - \frac{\delta n}{\tau_n}}$$

Diffusion Equations

Derivation of Einstein Relationship

- In equilibrium, there is no net current, and therefore no net motion of charge
- Also in equilibrium, the Fermi level is invariant

$$J_p(x) = qp(x)\mu_p\mathcal{E}(x) - qD_p\frac{dp(x)}{dx} = 0$$

$$\text{so } \mathcal{E}(x) = \frac{D_p}{\mu_p}\frac{1}{p(x)}\frac{dp(x)}{dx}$$

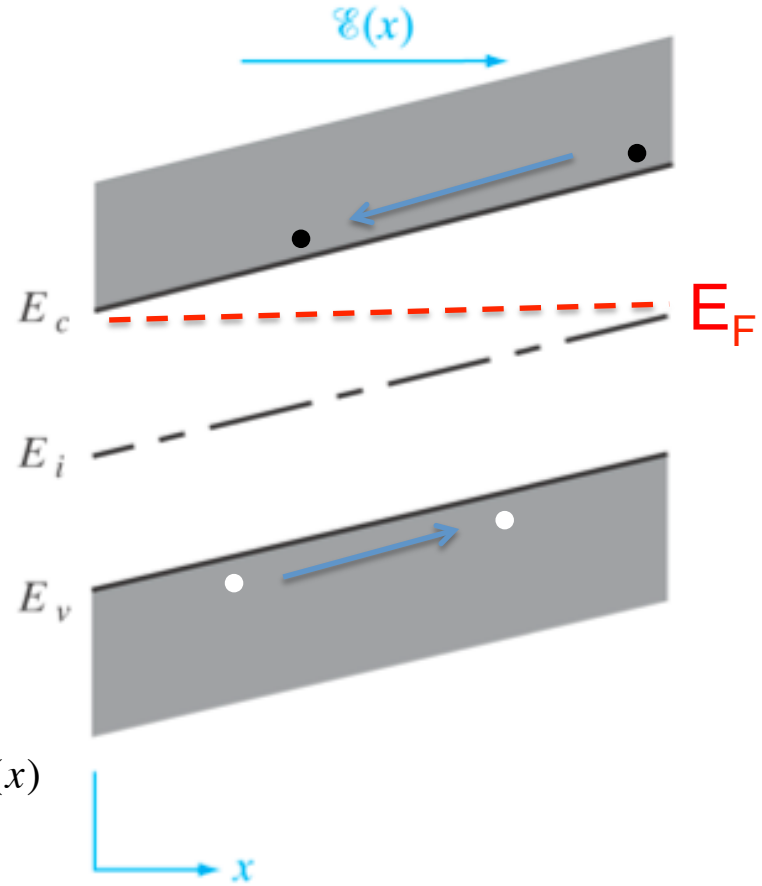
$$\text{since } p_o = n_i e^{(E_i - E_F)/kT}$$

$$\mathcal{E}(x) = \frac{D_p}{\mu_p}\frac{1}{n_i e^{(E_i-E_F)/kT}} n_i e^{(E_i-E_F)/kT}\frac{d}{dx}\left[\frac{E_i - E_F}{kT}\right]$$

$$\mathcal{E}(x) = \frac{D_p}{\mu_p}\frac{1}{kT}\left(\frac{dE_i}{dx} - \frac{dE_F}{dx}\right) = \frac{D_p}{\mu_p}\frac{1}{kT}\left(q\mathcal{E}(x) - 0\right) \text{ since } \frac{1}{q}\frac{dE_i}{dx} = \mathcal{E}(x)$$

$$\text{so } 1 = \frac{D_p}{\mu_p}\frac{q}{kT} \text{ and } \boxed{\frac{D_p}{\mu_p} = \frac{kT}{q}}$$

Internally Generated Field:



We can also have tilted bands with no externally applied potential!

Diffusion Length: Steady State

Steady-State Diffusion Equations

$$\frac{\partial \delta p}{\partial t} = D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_p}$$

In steady state, $\frac{\partial \delta p}{\partial t} = 0$

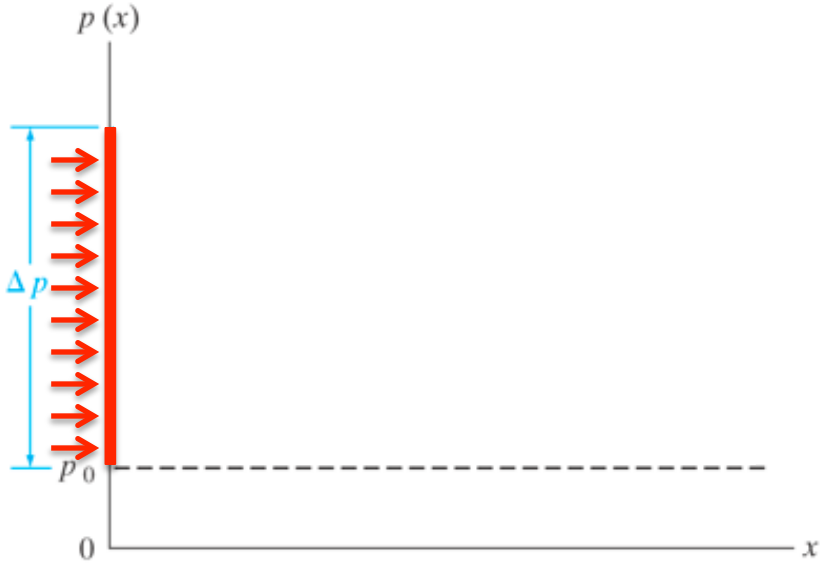
$$\therefore\ D_p \frac{\partial^2 \delta p}{\partial x^2} = \frac{\delta p}{\tau_p} \text{ or } \boxed{\frac{\partial^2 \delta p}{\partial x^2} = \frac{\delta p}{D_p \tau_p} \equiv \frac{\delta p}{L_p^2}}$$

$$\frac{\partial \delta n}{\partial t} = D_n \frac{\partial^2 \delta n}{\partial x^2} - \frac{\delta n}{\tau_n}$$

In steady state, $\frac{\partial \delta n}{\partial t} = 0$

$$\therefore\ D_n \frac{\partial^2 \delta n}{\partial x^2} = \frac{\delta n}{\tau_n} \text{ or } \frac{\partial^2 \delta n}{\partial x^2} = \boxed{\frac{\delta n}{D_n \tau_n} \equiv \frac{\delta n}{L_n^2}}$$

Hole Injection at x=0



(The injection mechanism is not specified.)

L_p is the average distance a hole travels before recombining.
L_n is the average distance an electron travels before recombining.

$$\boxed{L_n \equiv \sqrt{D_n \tau_n} \text{ and } L_p \equiv \sqrt{D_p \tau_p}}$$

Excess Hole Distribution and Current

Solution to Hole Diffusion Equation

$$\frac{d^2\delta p}{dx^2} = \frac{\delta p}{L_p^2} \qquad \frac{\partial \delta p}{\partial t} = 0$$

Solution:

$$\delta p(x) = C_1 e^{x/L_p} + C_2 e^{-x/L_p}$$

Boundary Condition:

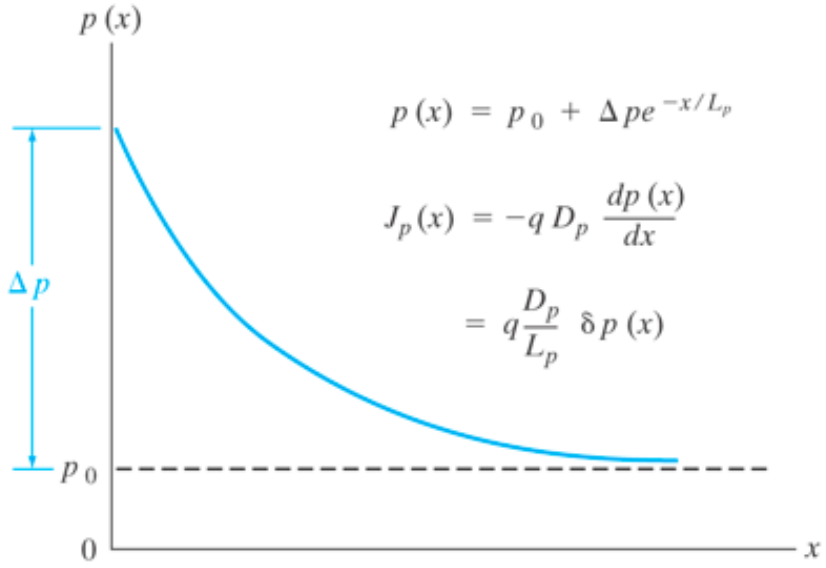
$$\delta p(x=0) = \Delta p = C_1 + C_2$$

$$\delta p(x=\infty) = 0 = C_1 e^{x/L_p}$$

$$C_1 = 0, \quad C_2 = \Delta p$$

$$\delta p(x) = \Delta p \bullet e^{-x/L_p}$$

Hole Injection at x=0



$$J_p(x) = -qD_p \frac{dp}{dx} = -qD_p \frac{d(p_o + \delta p)}{dx} = -qD_p \frac{d\delta p}{dx}$$

$$= q\frac{D_p}{L_p}\Delta p e^{-x/L_p} = q\frac{D_p}{L_p}\delta p(x)$$

Excess Electron Distribution and Current

Solution to Electron Diffusion Equation

$$\frac{d^2\delta n}{dx^2} = \frac{\delta n}{L_n^2} \qquad \frac{\partial \delta n}{\partial t} = 0$$

Solution:

$$\delta n(x) = C_1 e^{x/L_n} + C_2 e^{-x/L_n}$$

Boundary Condition:

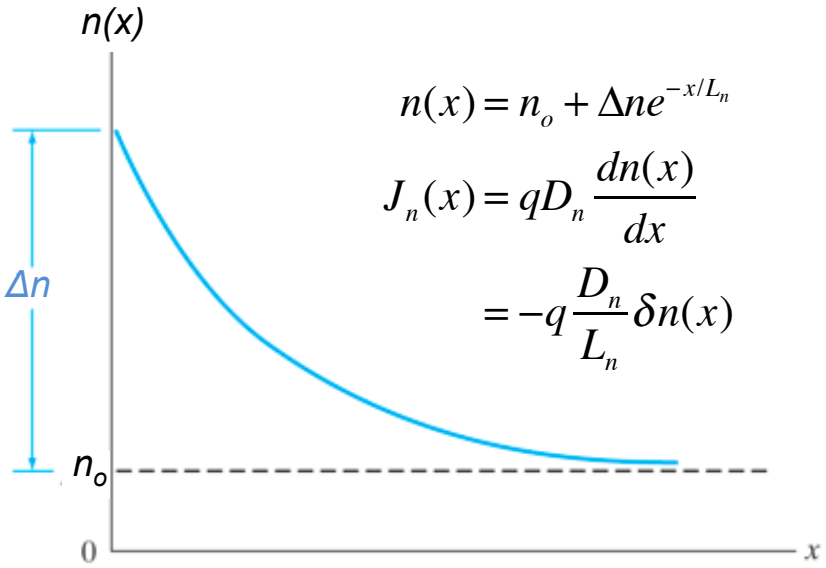
$$\delta n(x=0) = \Delta n = C_1 + C_2$$

$$\delta n(x=\infty) = 0 = C_1 e^{x/L_n}$$

$$C_1 = 0, \quad C_2 = \Delta n$$

$$\delta n(x) = \Delta n \bullet e^{-x/L_n}$$

Electron Injection at x=0



$$n(x) = n_o + \Delta n e^{-x/L_n}$$

$$J_n(x) = qD_n \frac{dn(x)}{dx}$$

$$= -q\frac{D_n}{L_n}\delta n(x)$$

$$J_n(x) = qD_n \frac{dn}{dx} = qD_n \frac{d(n_o + \delta n)}{dx} = qD_n \frac{d\delta n}{dx}$$

$$= -q\frac{D_n}{L_n}\Delta n e^{-x/L_n} = -q\frac{D_n}{L_n}\delta n(x)$$

Steady-State Carrier Injection

Diffusion Length

Diffusion Equation Derivation (Recap)

Continuity Equation

$$\frac{\partial p(x,t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}$$

Current Density Equation

$$J_p(x) = -qD_p\frac{dp(x)}{dx} = -qD_p\frac{\partial \delta p}{\partial x} \text{ (no drift)}$$

Diffusion Equation

$$\frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial}{\partial x}\left(-qD_p\frac{\partial \delta p}{\partial x}\right) - \frac{\delta p}{\tau_p} = D_p\frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_p}$$

$$\frac{\partial \delta p}{\partial t} = D_p\frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_p}$$

Excess Hole Distribution and Current

Solution to Hole Diffusion Equation

$$\frac{d^2\delta p}{dx^2} = \frac{\delta p}{L_p^2} \qquad \frac{\partial \delta p}{\partial t} = 0$$

Solution:

$$\delta p(x) = C_1 e^{x/L_p} + C_2 e^{-x/L_p}$$

Boundary Condition:

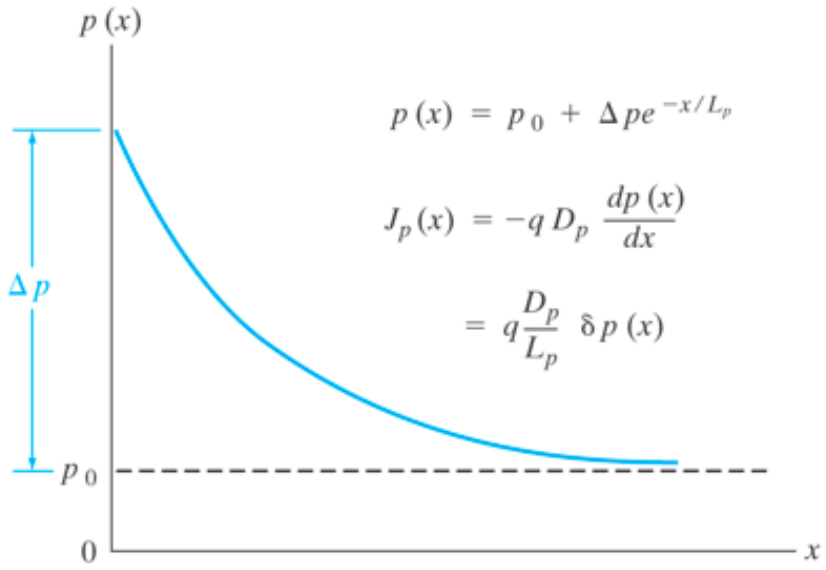
$$\delta p(x=0) = \Delta p = C_1 + C_2$$

$$\delta p(x=\infty) = 0 = C_1 e^{x/L_p}$$

$$C_1 = 0, \quad C_2 = \Delta p$$

$$\delta p(x) = \Delta p \bullet e^{-x/L_p}$$

Hole Injection at x=0



$$J_p(x) = -qD_p \frac{dp}{dx} = -qD_p \frac{d(p_o + \delta p)}{dx} = -qD_p \frac{d\delta p}{dx}$$

$$= q\frac{D_p}{L_p}\Delta p e^{-x/L_p} = q\frac{D_p}{L_p}\delta p(x)$$

$$L_n \equiv \sqrt{D_n \tau_n} \text{ and } L_p \equiv \sqrt{D_p \tau_p}$$

Excess Electron Distribution and Current

Solution to Electron Diffusion Equation

$$\frac{d^2\delta n}{dx^2} = \frac{\delta n}{L_n^2} \qquad \frac{\partial \delta n}{\partial t} = 0$$

Solution:

$$\delta n(x) = C_1 e^{x/L_n} + C_2 e^{-x/L_n}$$

Boundary Condition:

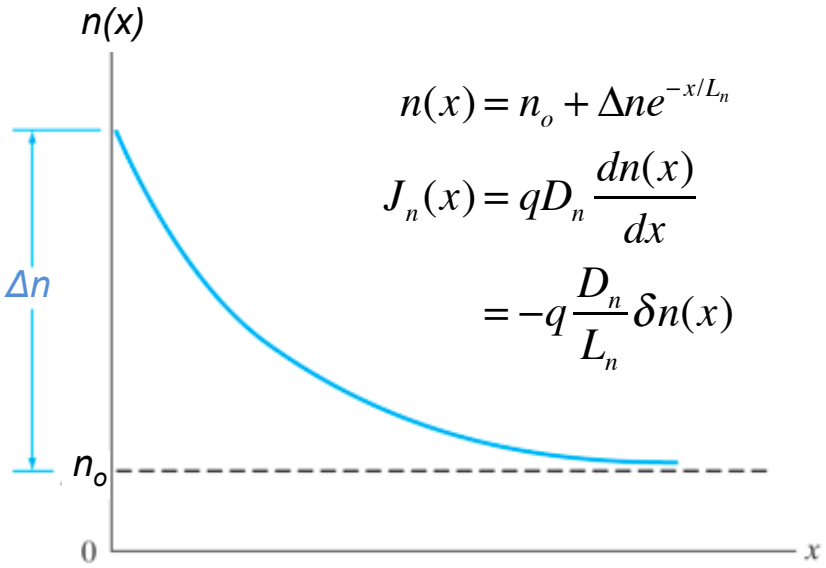
$$\delta n(x=0) = \Delta n = C_1 + C_2$$

$$\delta n(x=\infty) = 0 = C_1 e^{x/L_n}$$

$$C_1 = 0, \quad C_2 = \Delta n$$

$$\delta n(x) = \Delta n \bullet e^{-x/L_n}$$

Electron Injection at x=0



$$n(x) = n_o + \Delta n e^{-x/L_n}$$

$$J_n(x) = qD_n \frac{dn(x)}{dx}$$

$$= -q\frac{D_n}{L_n}\delta n(x)$$

$$J_n(x) = qD_n \frac{dn}{dx} = qD_n \frac{d(n_o + \delta n)}{dx} = qD_n \frac{d\delta n}{dx}$$

$$= -q\frac{D_n}{L_n}\Delta n e^{-x/L_n} = -q\frac{D_n}{L_n}\delta n(x)$$

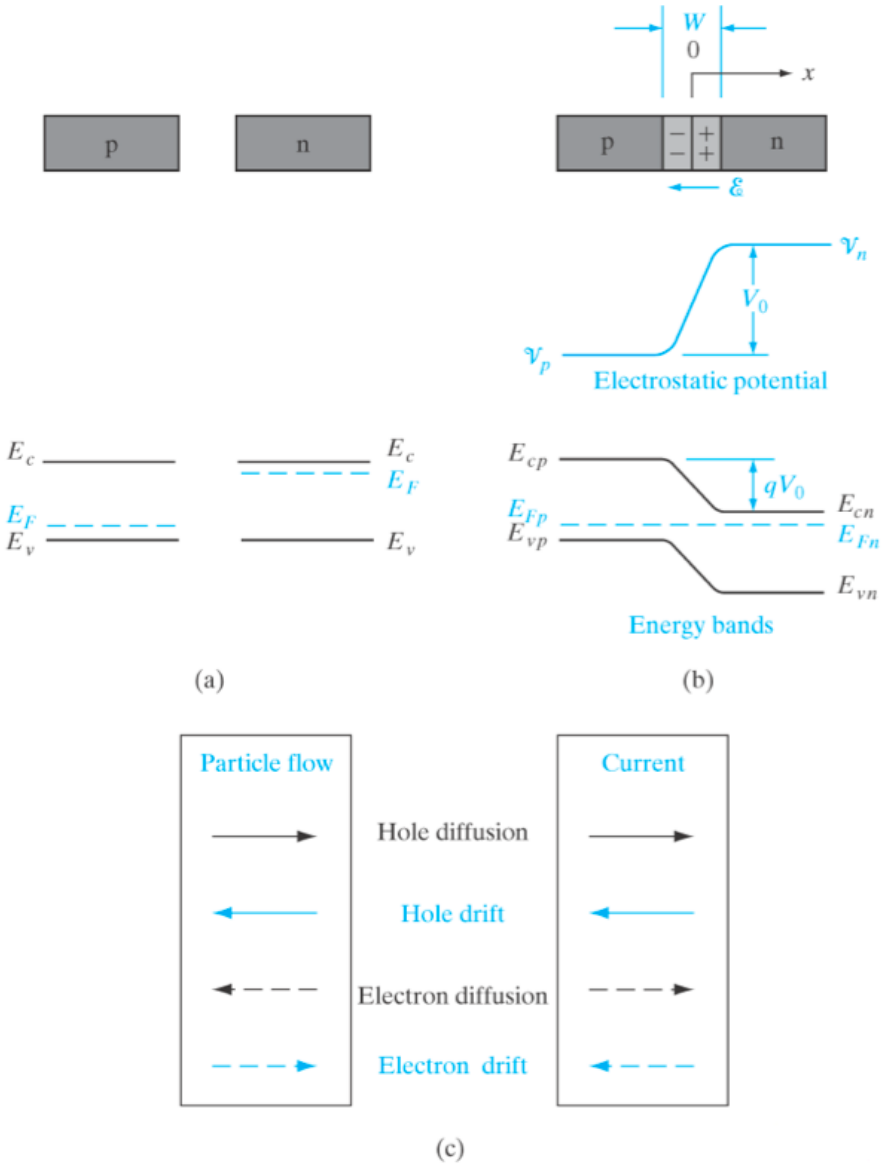
$$L_n \equiv \sqrt{D_n \tau_n} \text{ and } L_p \equiv \sqrt{D_p \tau_p}$$

Equilibrium Condition

pn Junctions

Equilibrium pn Junction

- In equilibrium, there is no net current flow across the junction, so:
  - J_p(Drift)+J_p(Diffusion)=0
  - J_n(Drift)+J_n(Diffusion)=0
- The electric field balances the diffusion current
- The potential difference $V_o=V_n-V_p$ develops in the direction opposite to the electric field $\mathcal{E}$
- V_o is the "***contact potential***" which is a built-in potential barrier
  - Contact potential cannot be measured
- W is the "***transition region***"
  - Also called "depletion region" and "space charge region"
- We can calculate V_o from the separation in the Fermi levels (this will be shown later)



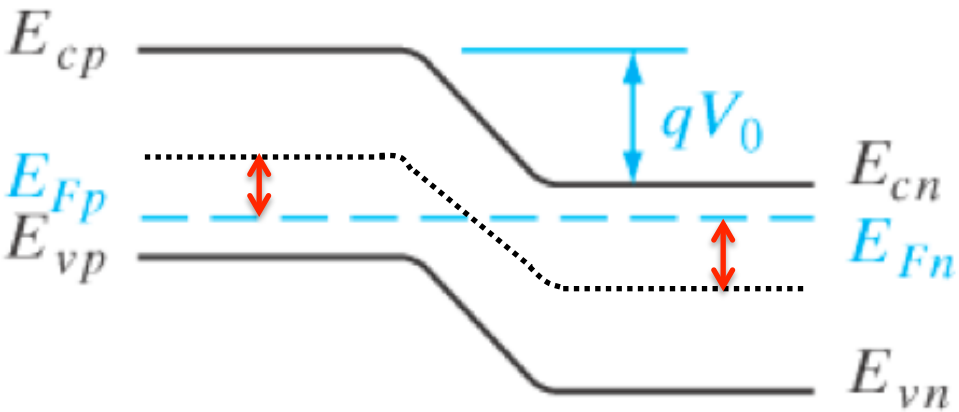
Contact Potential

Calculation of Contact Potential

Method 1:

- Use the relationship we derived last time

$$V_0 = \frac{kT}{q}\ln\frac{p_p}{p_n} = \frac{kT}{q}\ln\frac{N_a N_d}{n_i^2}$$



Method 2:

- Calculate the separation between the Fermi level and the intrinsic level on both sides of the junction, and sum the results

$$E_{ip} - E_F = kT\ln\frac{p_p}{n_i}$$

$$E_F - E_{in} = kT\ln\frac{n_n}{n_i}$$

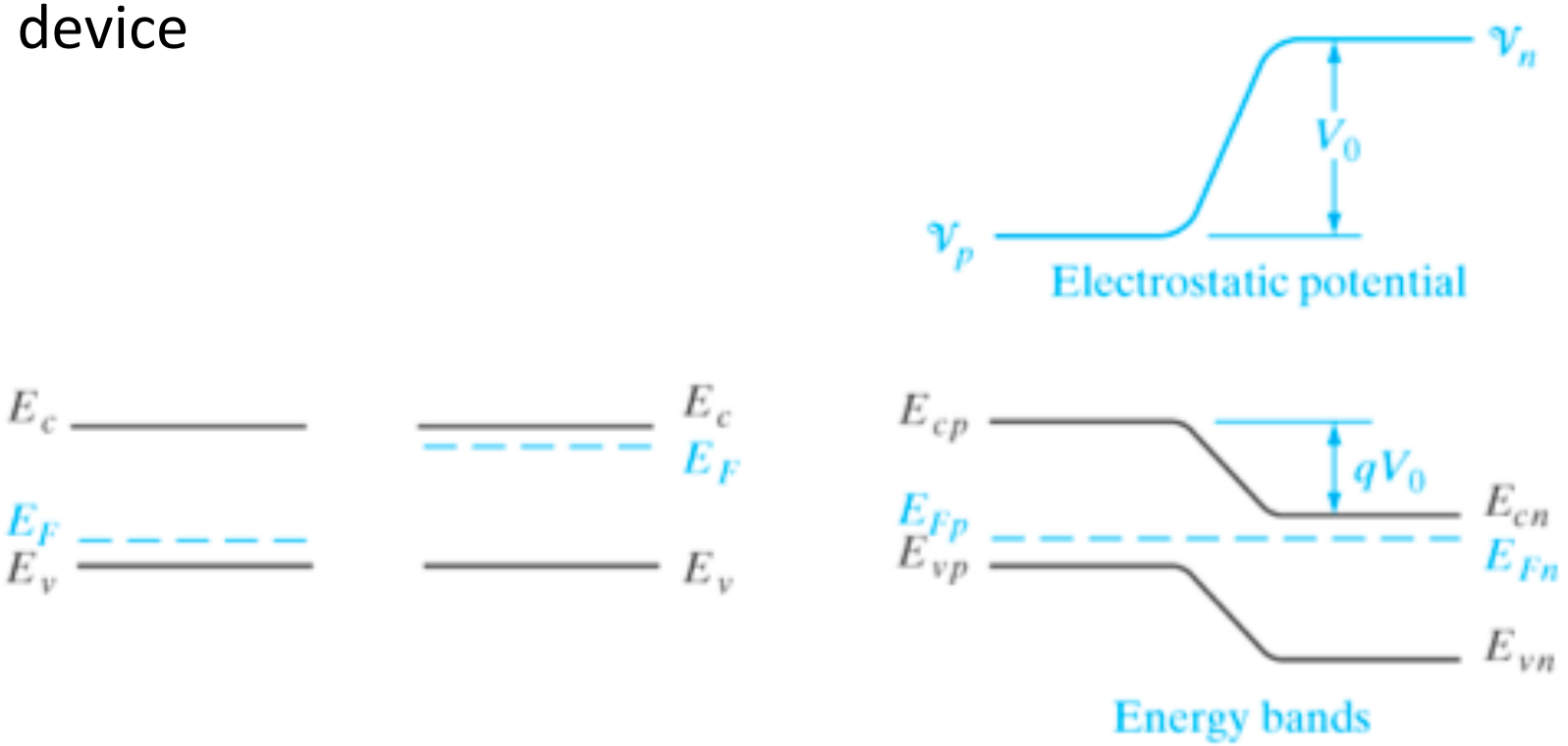
$$V_0 = \left(E_{ip} - E_F\right) + \left(E_F - E_{in}\right)$$

$$= kT\ln\frac{p_p}{n_i} + kT\ln\frac{n_n}{n_i}$$

Equilibrium Fermi Levels

Comments

- In equilibrium, the Fermi level is flat
- The equilibrium energy band separation is “q” times the contact potential
- The contact potential is a built-in potential barrier – don’t think of this as an external voltage that appears across the device



Space Charge at the Junction

Depletion Approximation

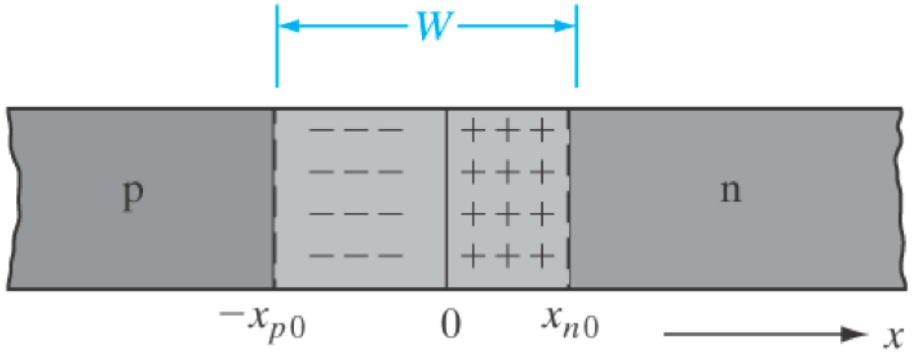
Depletion Approximation:

$$Q^+ = \left|Q^-\right|$$

$$qN_d^+ Ax_{no} = qN_a^- Ax_{po}$$

$$N_d^+ x_{no} = N_a^- x_{po}$$

$$x_{no} + x_{po} = W$$



- The depletion approximation states that carriers are depleted within the depletion width “W” and that charge neutrality applies outside of the depletion region

Calculation of Electric Field: n-Side

n-side of pn junction $(0 < x < x_{no})$:

$$p \approx 0,\ n \approx 0,\ N_a^- \approx 0,\ \frac{d\mathcal{E}(x)}{dx} = \frac{q}{\varepsilon}\left[N_d^+\right]$$

$$\int_{\mathcal{E}_o}^{0} d\mathcal{E}(x) = \frac{q}{\varepsilon}\left[N_d^+\right] \int_{0}^{x_{no}} dx$$

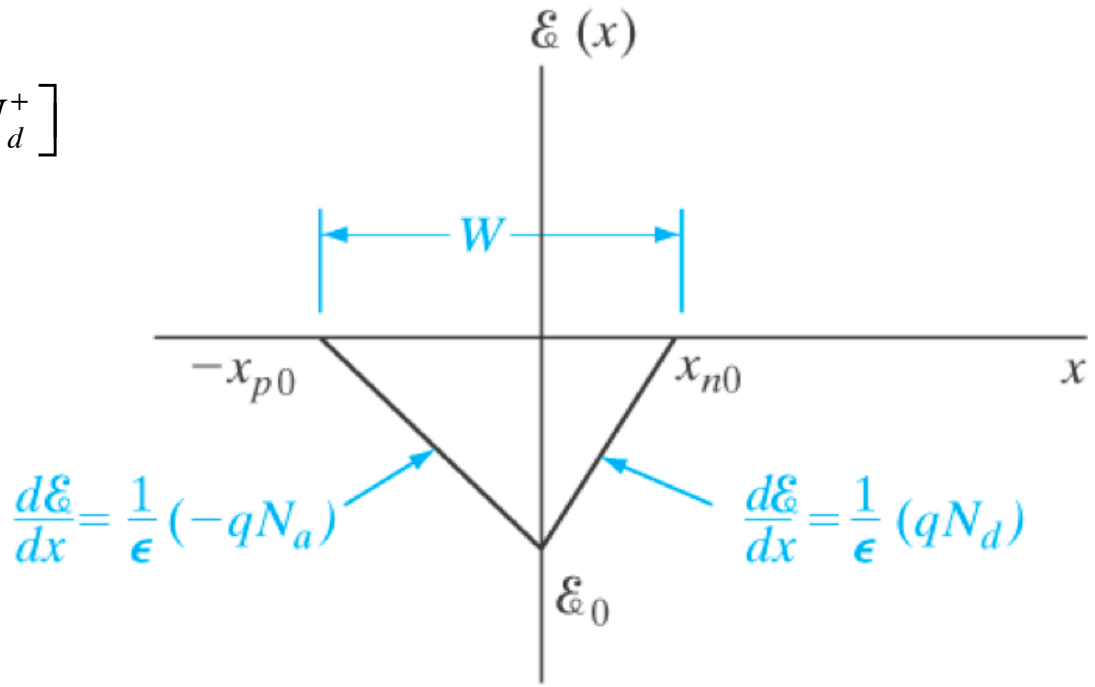
$$\left[0 - \mathcal{E}_o\right] = \frac{q}{\varepsilon}\left[N_d^+\right]\left[x_{no} - 0\right]$$

$$\mathcal{E}_o = -\frac{q}{\varepsilon}\left[N_d^+\right] x_{no}$$

To determine the value $\mathcal{E}(x)$:

$$\int_{\mathcal{E}(x)}^{0} d\mathcal{E}(x) = \frac{q}{\varepsilon}\left[N_d^+\right] \int_{x}^{x_{no}} dx$$

$$\mathcal{E}(x) = -\frac{q}{\varepsilon}\left[N_d^+\right] (x_{no} - x)$$



Maximum Value of Electric Field:

$$E_o = -\frac{q}{\varepsilon} N_d x_{n0} = -\frac{q}{\varepsilon} N_a x_{p0}$$

Electric Field and Contact Potential

Maximum Value of Electric Field:

$$\mathcal{E}_o = -\frac{q}{\varepsilon} N_d x_{n0} = -\frac{q}{\varepsilon} N_a x_{p0}$$

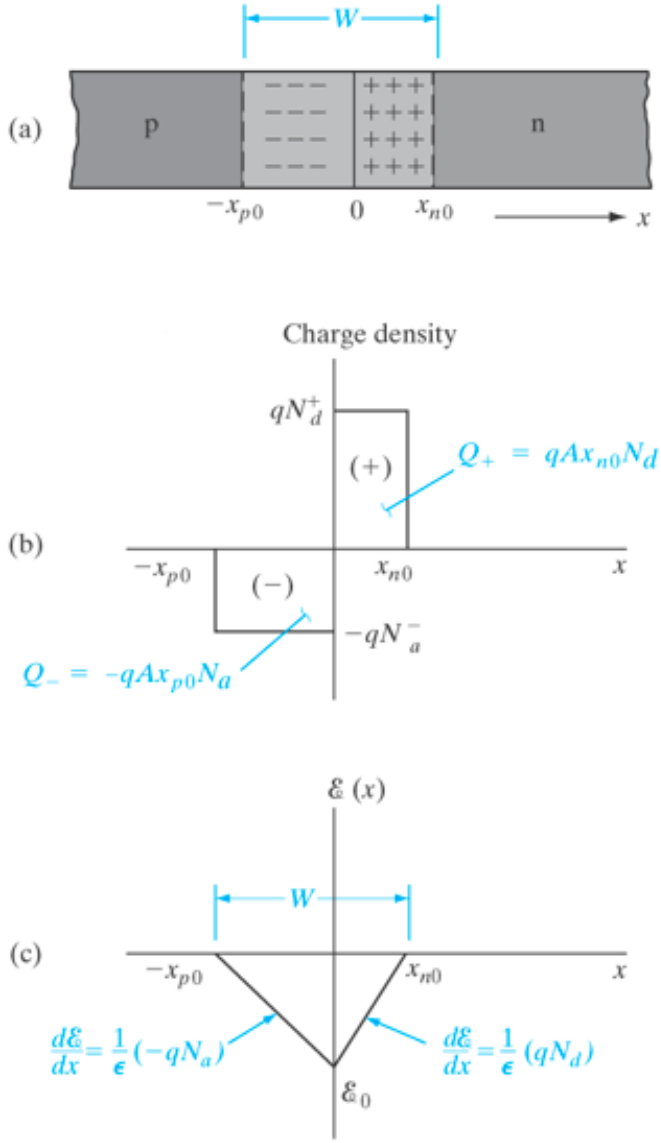
Relationship of Electric Field to Contact Potential:

$$\mathcal{E}(x) = -\frac{d\mathcal{V}(x)}{dx} \text{ or } -V_o = \int_{-x_{po}}^{x_{n0}} \mathcal{E}(x)\,dx$$

Since the electric field is the negative of the potential gradient at any point.

Recall that:

$$V_o = \frac{kT}{q} \ln \frac{p_p}{p_n} = \frac{kT}{q} \ln \frac{n_n}{n_p}$$



Calculation of Depletion Width

From the previous slide:

$$V_o = -\int_{-x_{po}}^{x_{n0}} \mathcal{E}(x)\,dx$$

So, the negative of the contact potential is the area under the $\mathcal{E}$(x) versus x triangle:

$$V_o = -\frac{1}{2}\mathcal{E}_o \cdot W = \frac{1}{2}\frac{q}{\varepsilon}N_d x_{no} W = \frac{1}{2}\frac{q}{\varepsilon}N_a x_{po} W$$

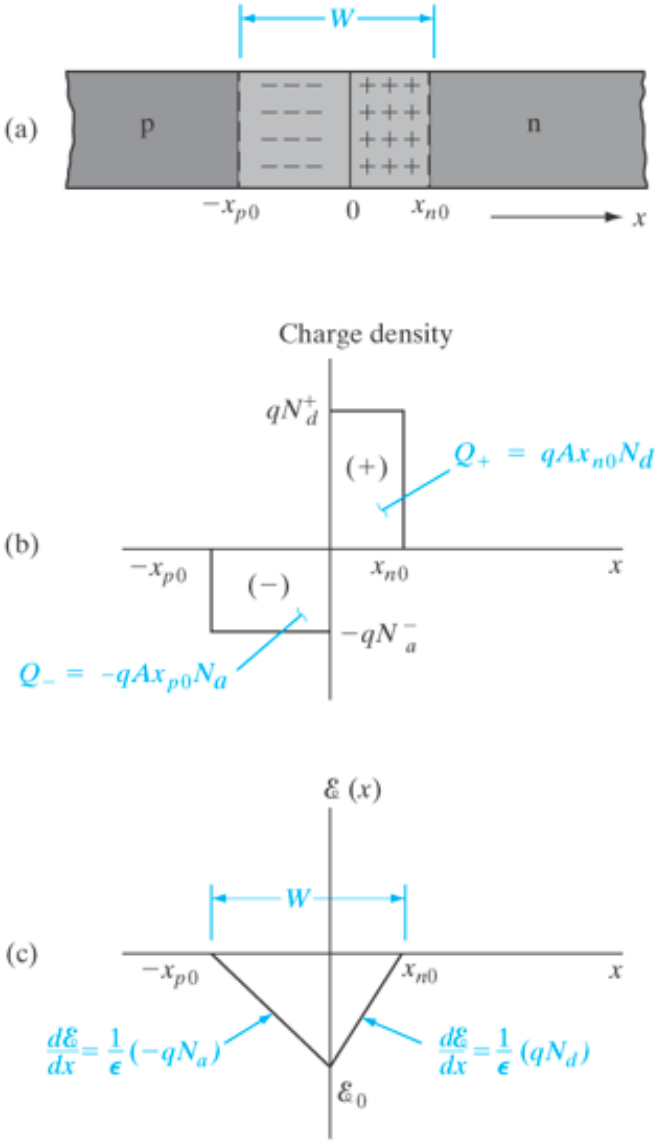
Balance of Charge Requirement: $Q_+ = |Q_-| \Leftrightarrow N_d x_{no} = N_a x_{po}$

$$x_{po} = x_{no}\frac{N_d}{N_a} \text{ and } W = x_{no} + x_{po} = x_{no}\left(1 + \frac{N_d}{N_a}\right),$$

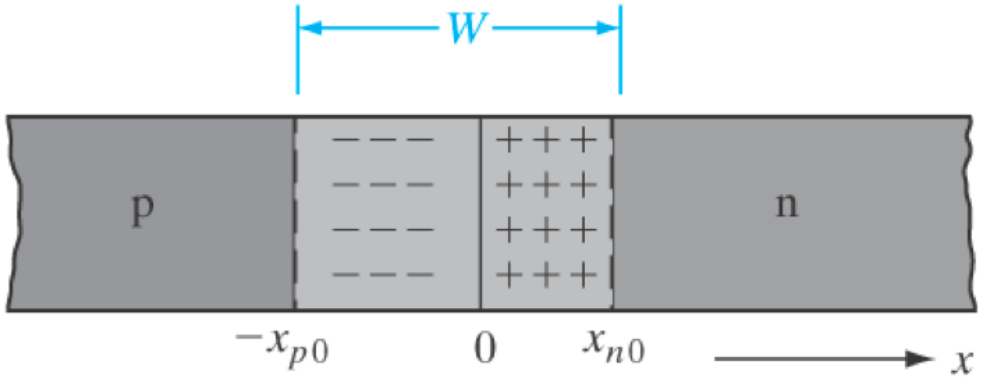
$$\text{so } x_{no} = W\left(\frac{N_a}{N_a + N_d}\right)$$

$$\text{and } V_o = \frac{1}{2}\mathcal{E}_o W = \frac{1}{2}\frac{q}{\varepsilon}\frac{N_a N_d}{N_a + N_d}W^2$$

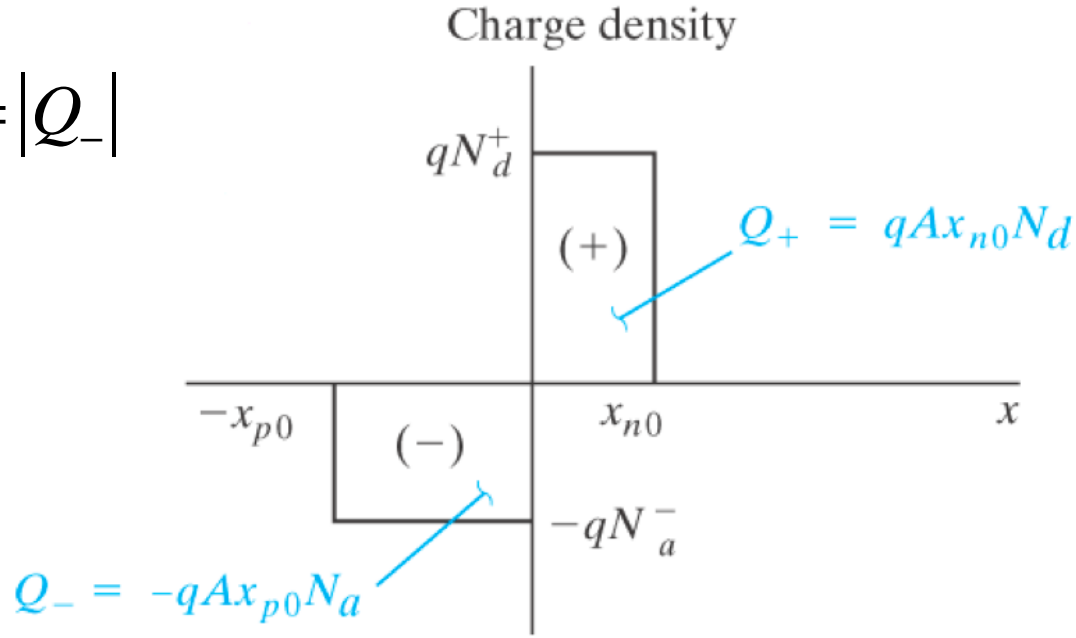
$$W = \left[\frac{2\varepsilon V_o}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2} = \left[\frac{2\varepsilon V_o}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}$$



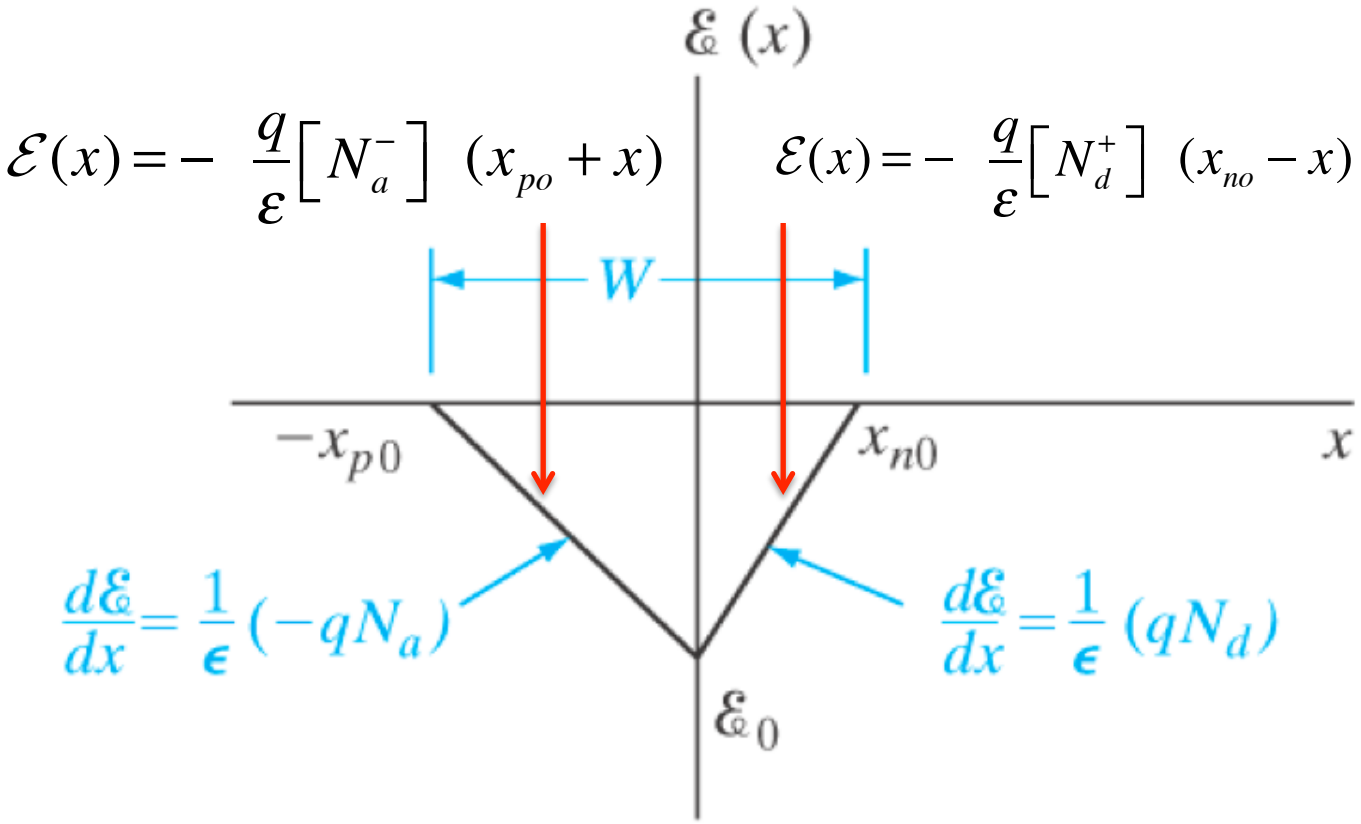
Space Charge



$$|Q_+| = |Q_-|$$



Junction Electric Field



$$\mathcal{E}_o = -\frac{q}{\varepsilon} N_d x_{n0} = -\frac{q}{\varepsilon} N_a x_{p0}$$

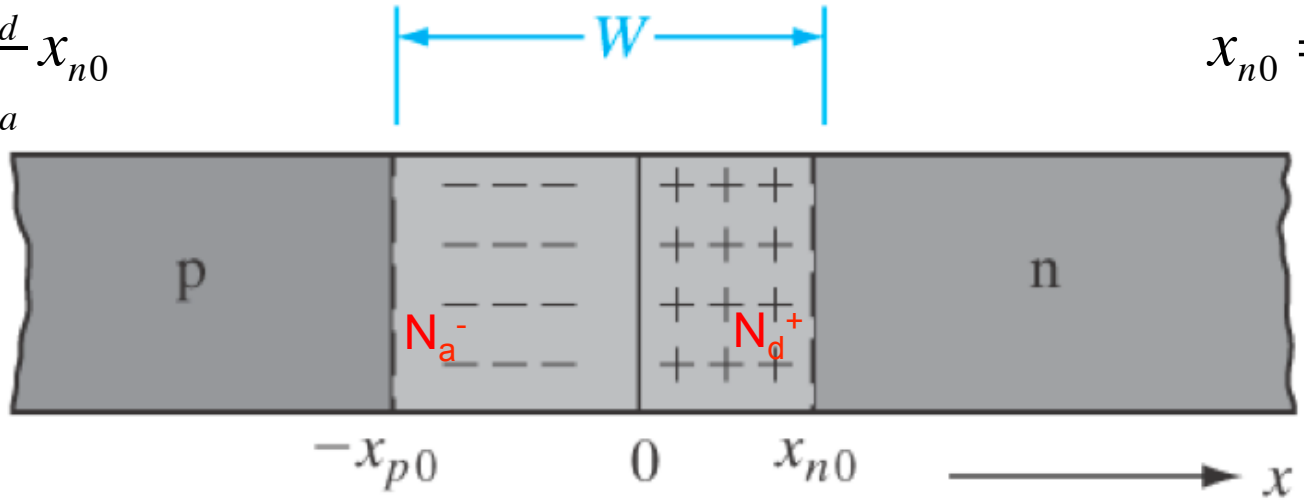
From Integrating Poisson's Equation

Depletion Width and Contact Potential

$$W = \left[\frac{2\varepsilon V_0}{q} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2}$$

$$x_{p0} = \frac{N_d}{N_a} x_{n0}$$

$$x_{n0} = \frac{N_a}{N_d} x_{p0}$$



$$x_{p0} = \left\{ \frac{2\varepsilon V_0}{q} \left[\frac{N_d}{N_a (N_a + N_d)} \right] \right\}^{1/2}$$

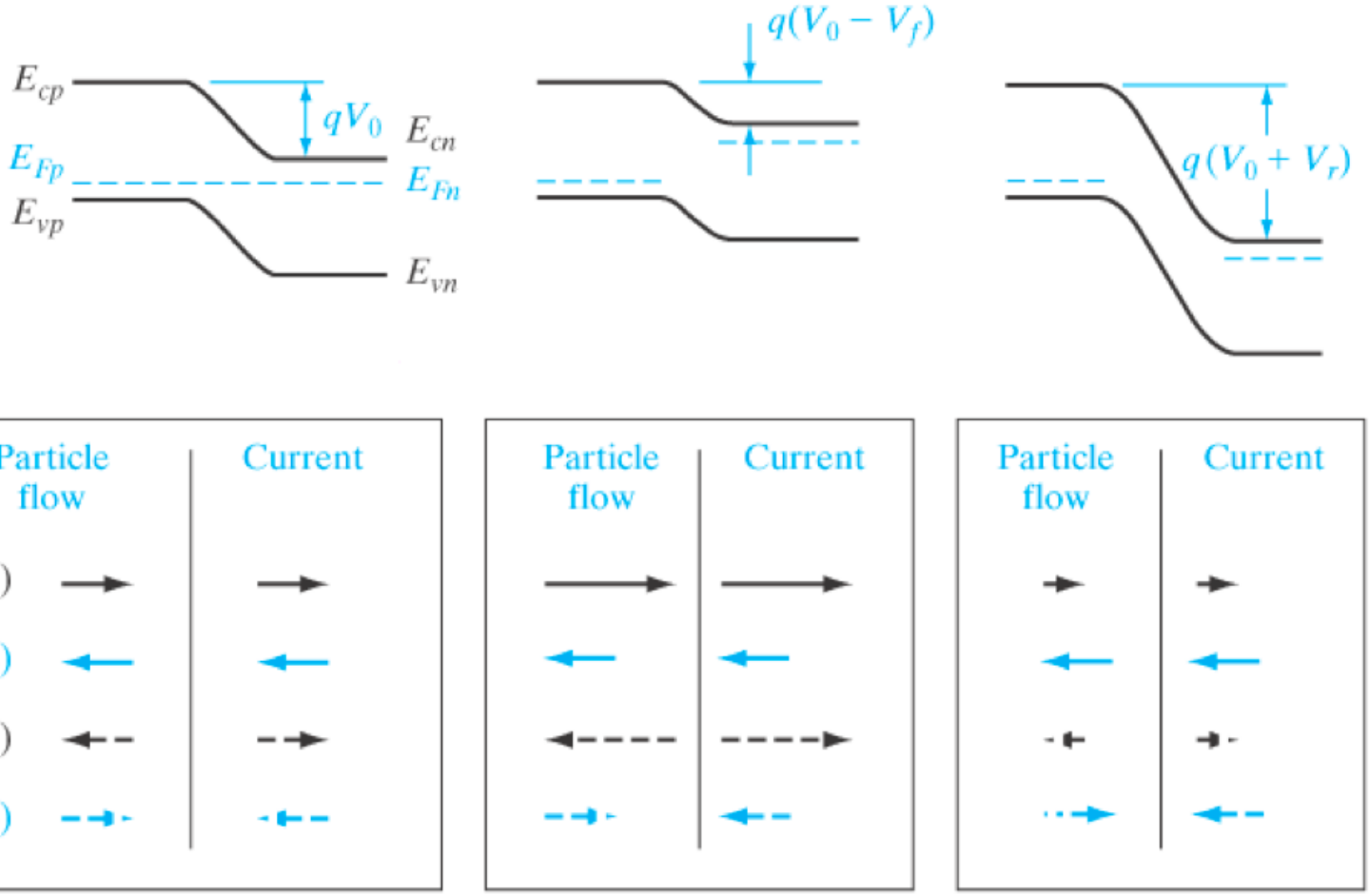
$$x_{n0} = \left\{ \frac{2\varepsilon V_0}{q} \left[\frac{N_a}{N_d (N_a + N_d)} \right] \right\}^{1/2}$$

Comments:

- The forward current is due to the injection of **majority carriers** from the semiconductor into the metal
- Because there is effectively no minority carrier charge storage, there is no charge storage delay time as the device bias conditions are changed (speed advantage)

pn Junctions Under Bias

Electron & Hole Motion



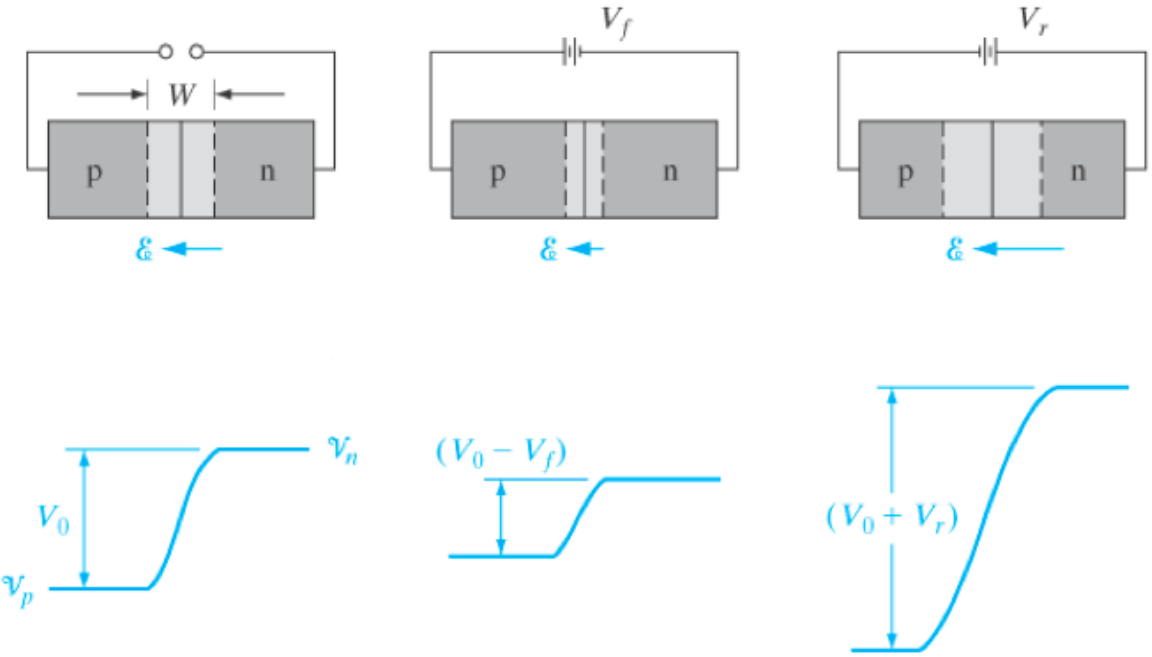
(1) Hole diffusion
(2) Hole drift
(3) Electron diffusion
(4) Electron drift

Bias and Field

- An applied bias reduces or increases the electrostatic potential barrier at the junction
- The magnitude of the maximum value of the electric field is lower in a junction under forward bias
- The magnitude maximum value of the electric is higher in a junction under reverse bias

$$\mathcal{E}(x) = -\frac{d\mathcal{V}(x)}{dx}$$

$$V_o - V = -\int_{-x_p}^{x_n} \mathcal{E}(x)\,dx$$

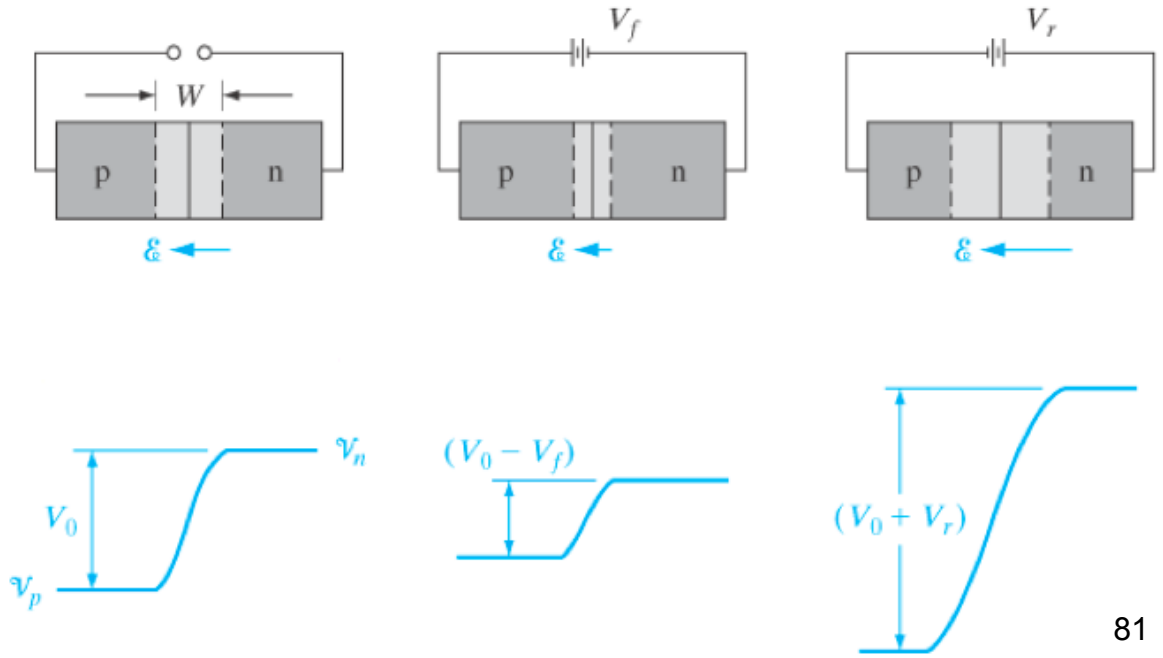


Bias and Depletion Width

- Field is produced by charge, a smaller field implies less charge so the depletion width under forward bias (lower field) is smaller
- The depletion width under reverse bias (higher field) is larger

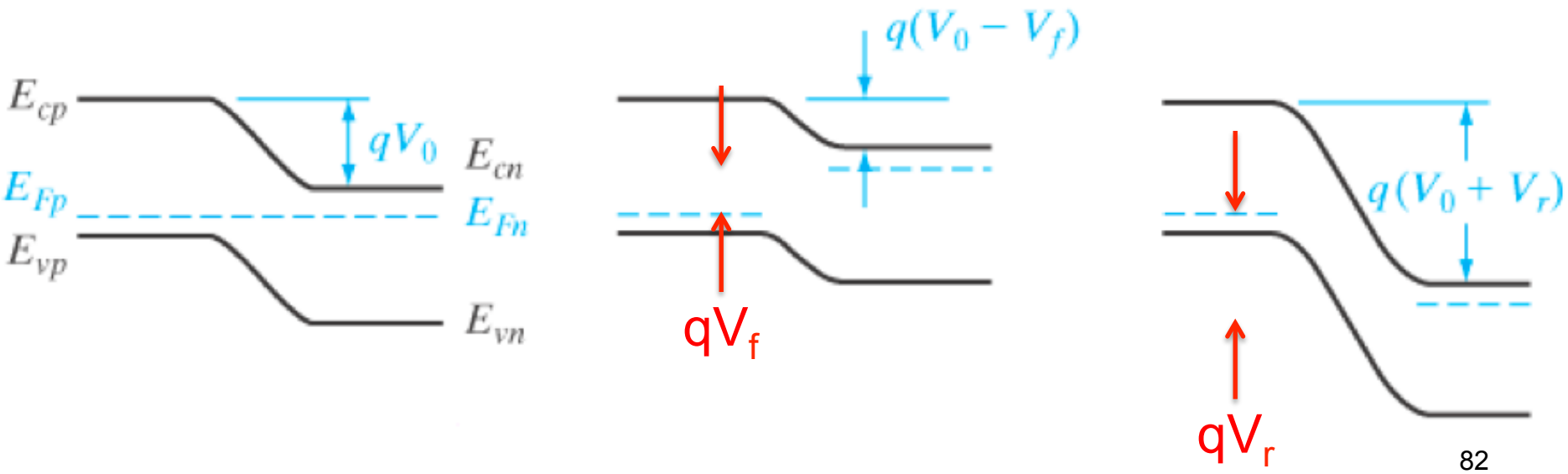
$$W = \left[\frac{2\varepsilon(V_o - V)}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2\varepsilon(V_o - V)}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}$$



Bias and Fermi Level Separation

- When we apply voltage we are no longer in equilibrium
- The application of a voltage creates a difference in the electrochemical potential across the junction boundary
- The Fermi levels become separated by an energy of “q” times the applied voltage
- The band separation is $q(V_0-V)$



Reverse Bias Current: Thermal Generation

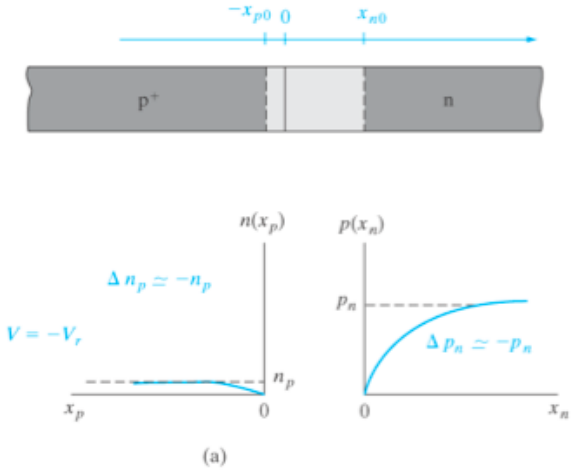
$$\Delta p_n = p_n\left(e^{q(-V_r)/kT} - 1\right) \simeq -p_n \quad \text{for} \quad V_r \gg kT/q$$

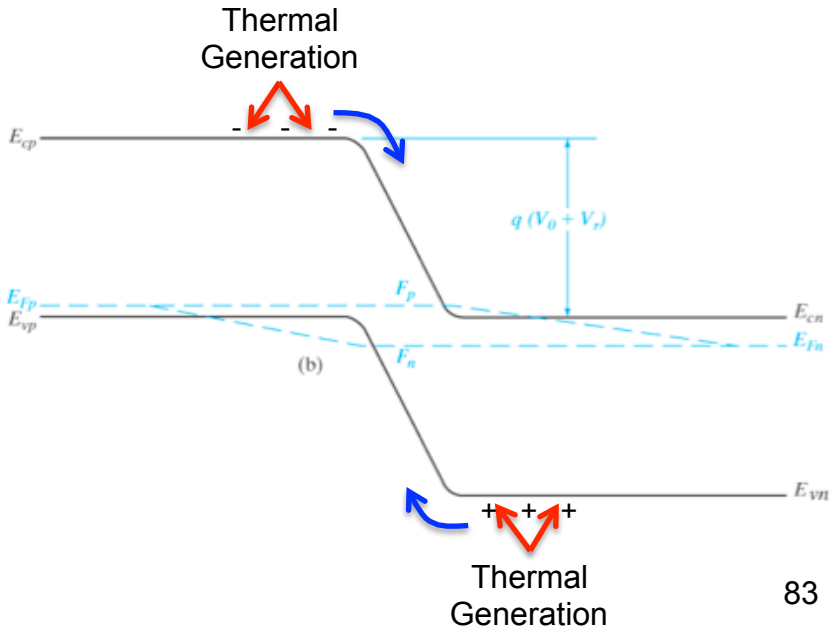
Similarly: $\Delta n_p \simeq -n_p$

- Minority carrier concentration approaches zero at the edge of the depletion region
- This effect is termed “minority carrier extraction”
- The reverse saturation current is a drift current, but the minority carriers arrive at the depletion region by way of diffusion

Reverse Bias:

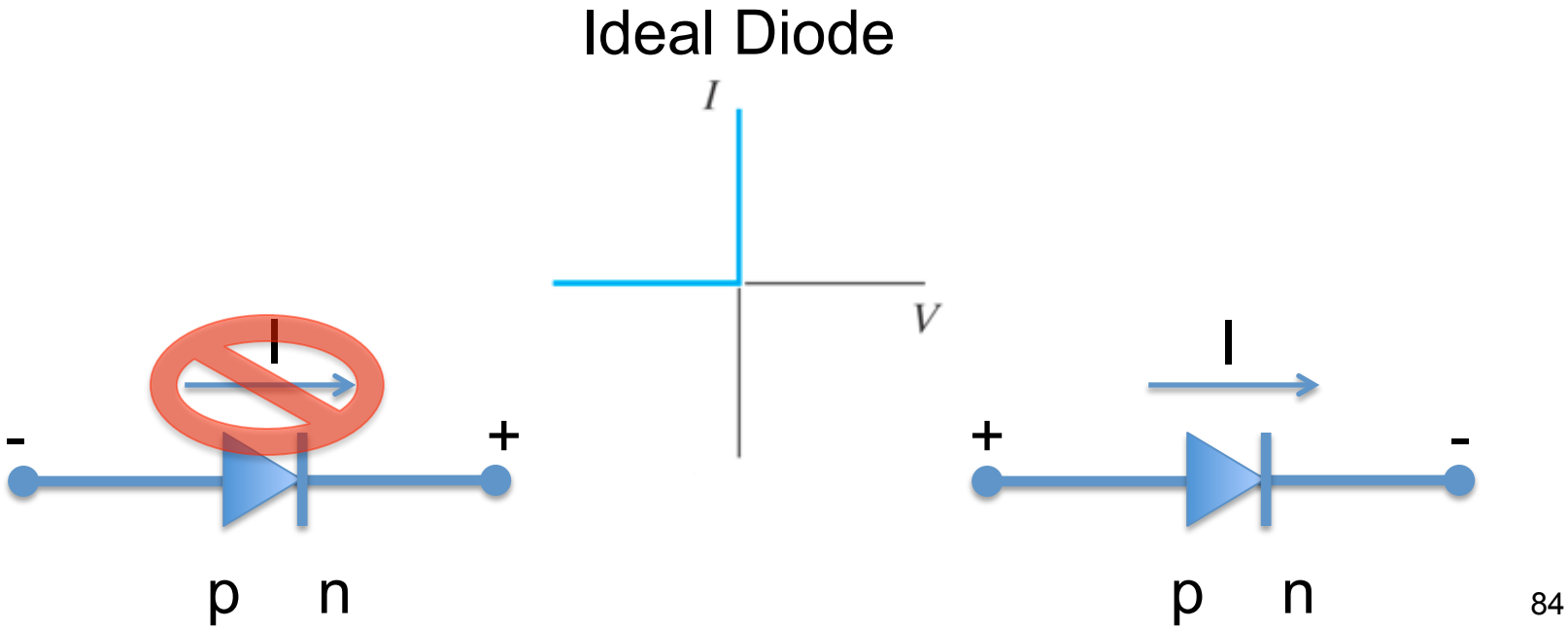
$$pn = n_i^2 e^{(F_n - F_p)/kT} \approx 0$$





Terminology

- ***Rectifier:*** A device that passes current in one bias direction (positive to negative) and blocks the flow of current in the opposite bias direction (negative to positive)



Carrier Injection

Minority Carrier Injection: Forward Bias

$$\frac{p(x_{no})}{p_n} = e^{qV/kT} \quad \text{so:}$$

$$p(x_{no}) = p_n e^{qV/kT}$$

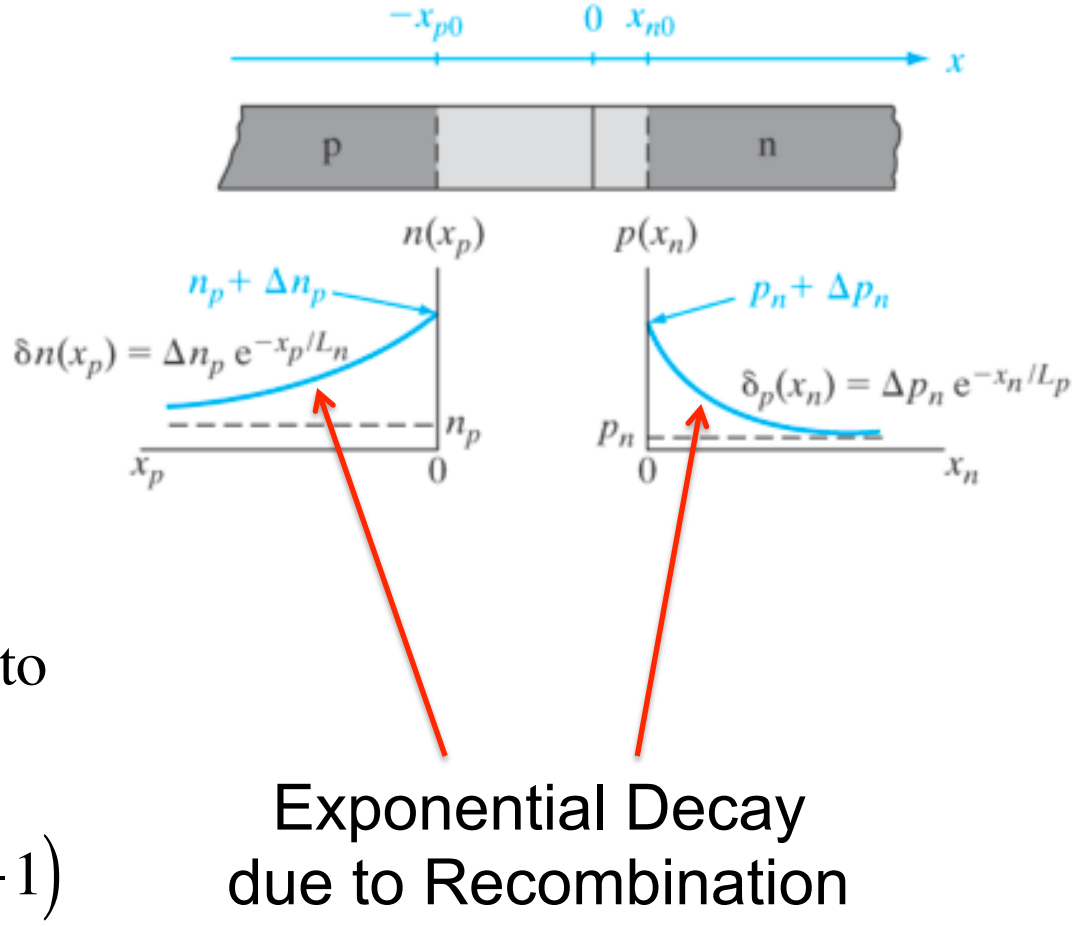
Therefore:

$$\Delta p_n = p(x_{no}) - p_n = p_n e^{qV/kT} - p_n$$

$$= p_n\left(e^{qV/kT} - 1\right)$$

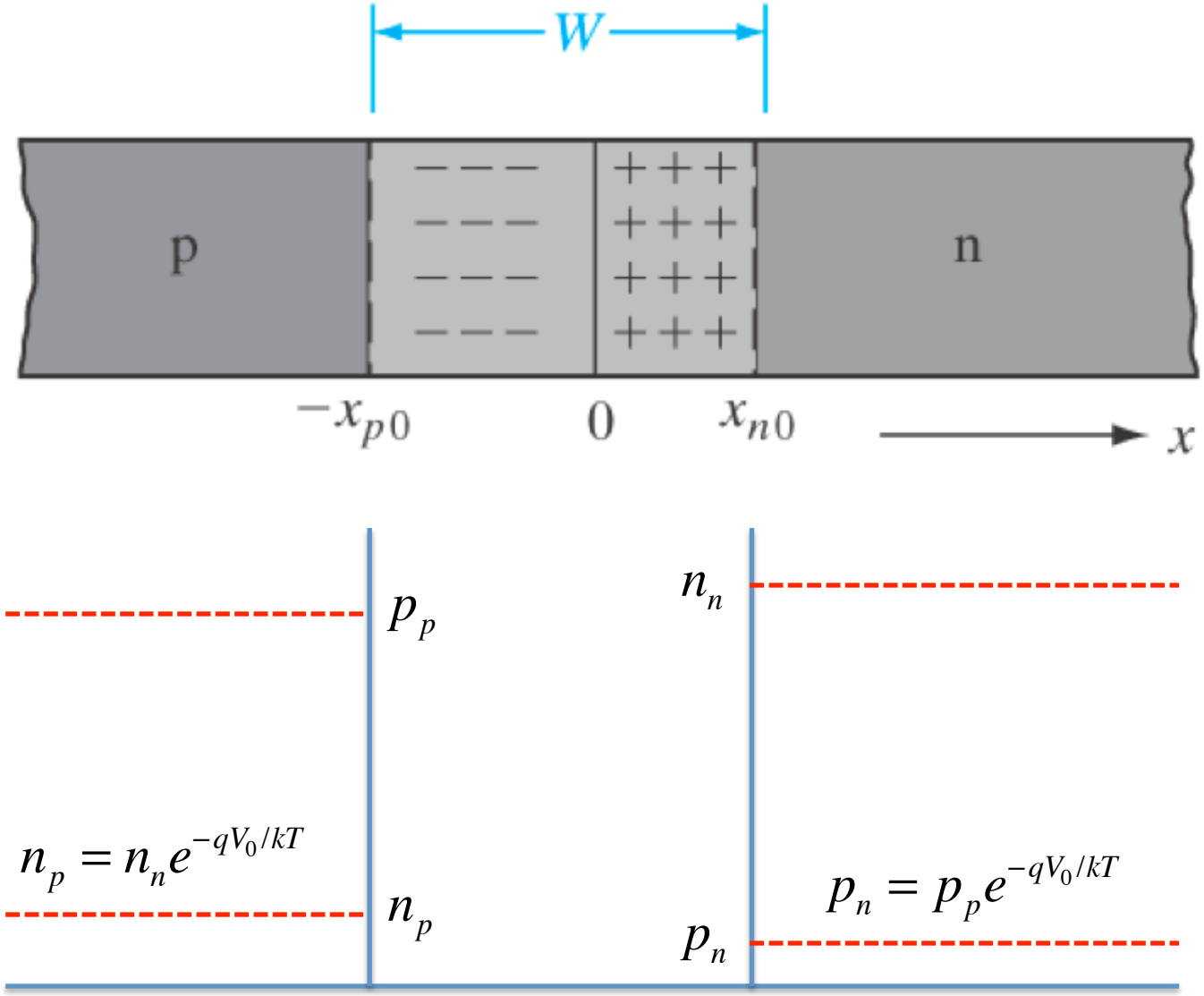
A similar approach can be used to derive the expression for Δn_p :

$$\Delta n_p = n(-x_{po}) - n_p = n_p\left(e^{qV/kT} - 1\right)$$

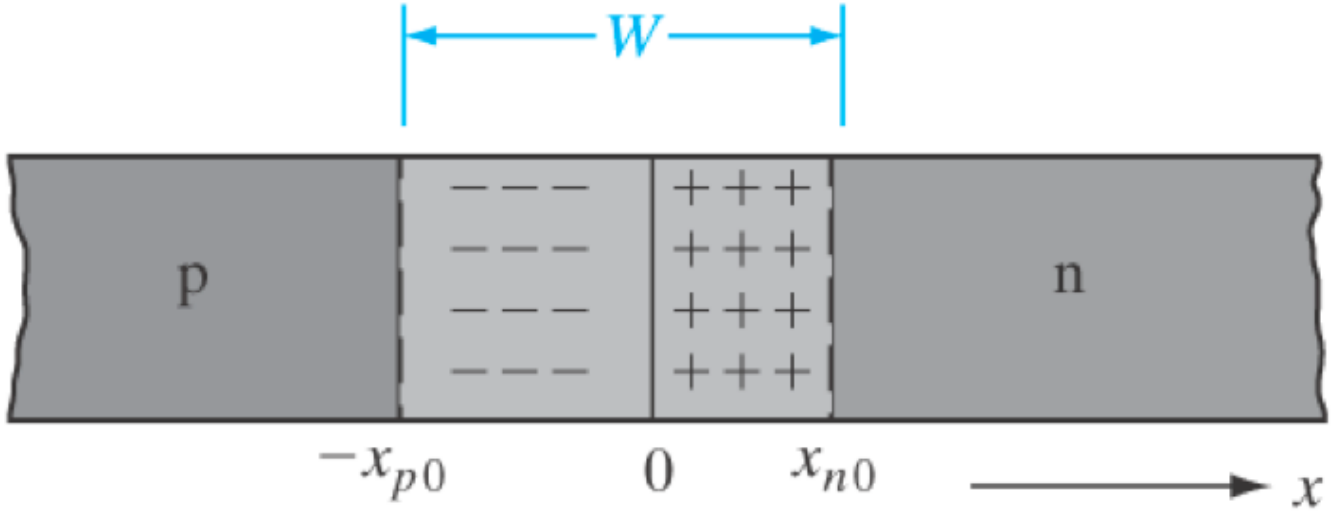


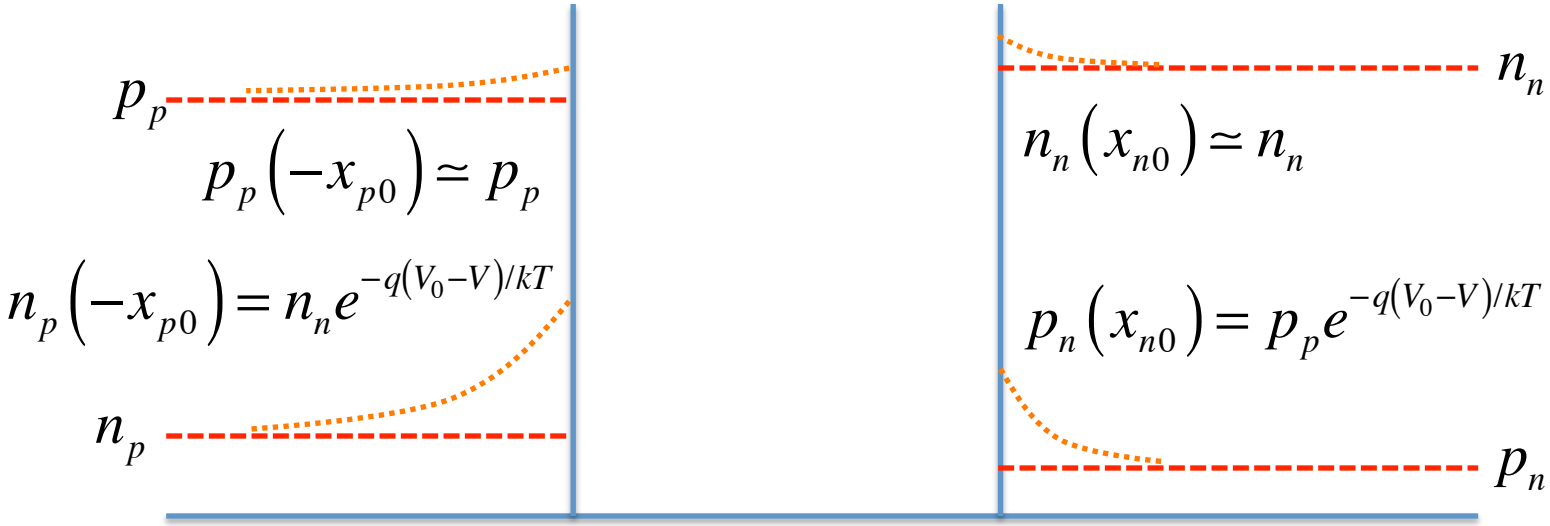
Exponential Decay due to Recombination

Equilibrium Carrier Concentrations

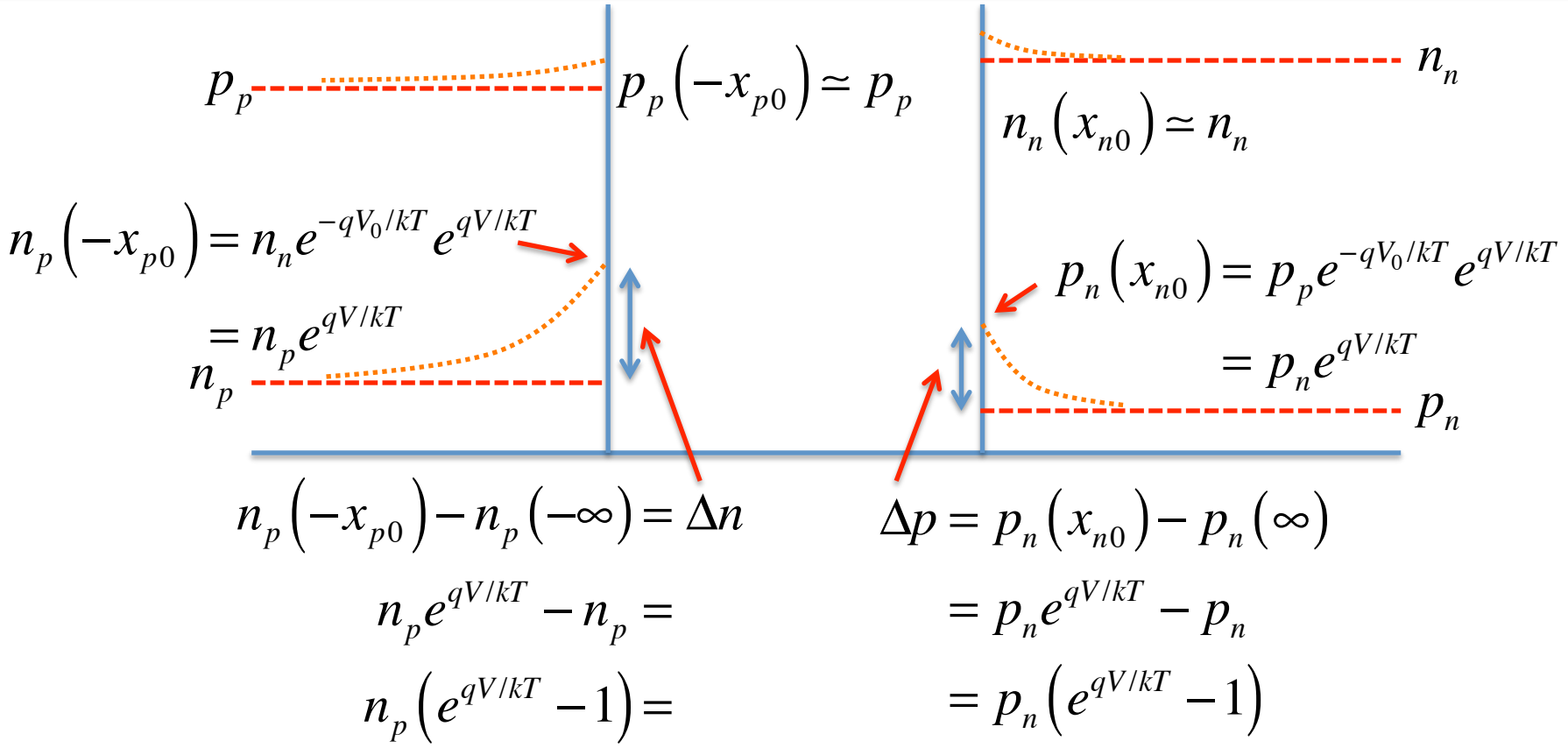


Forward Bias





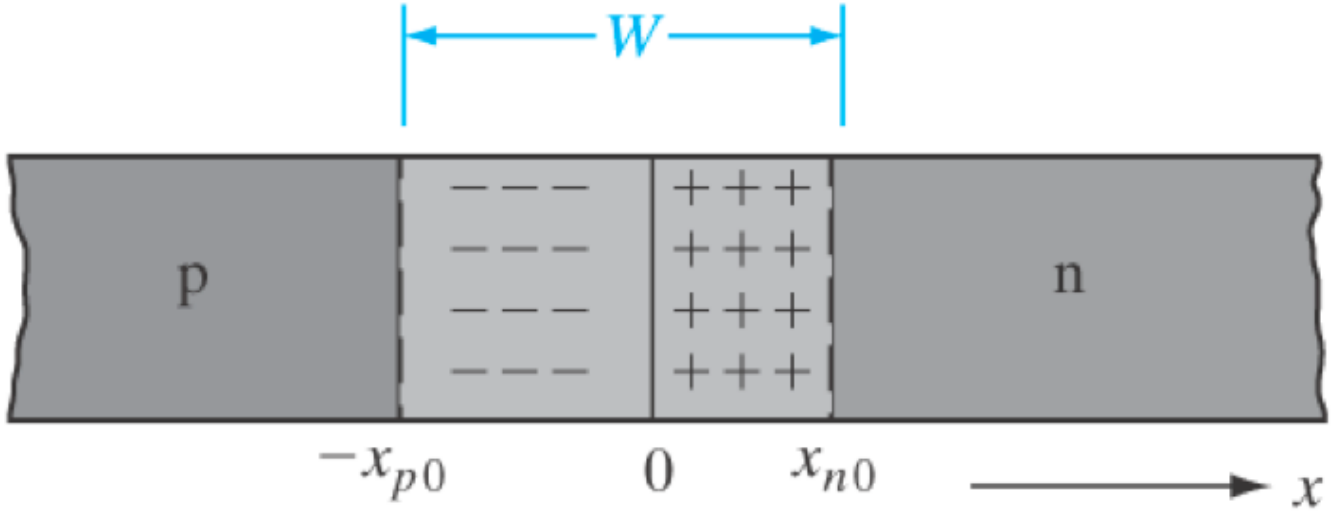
Forward Bias

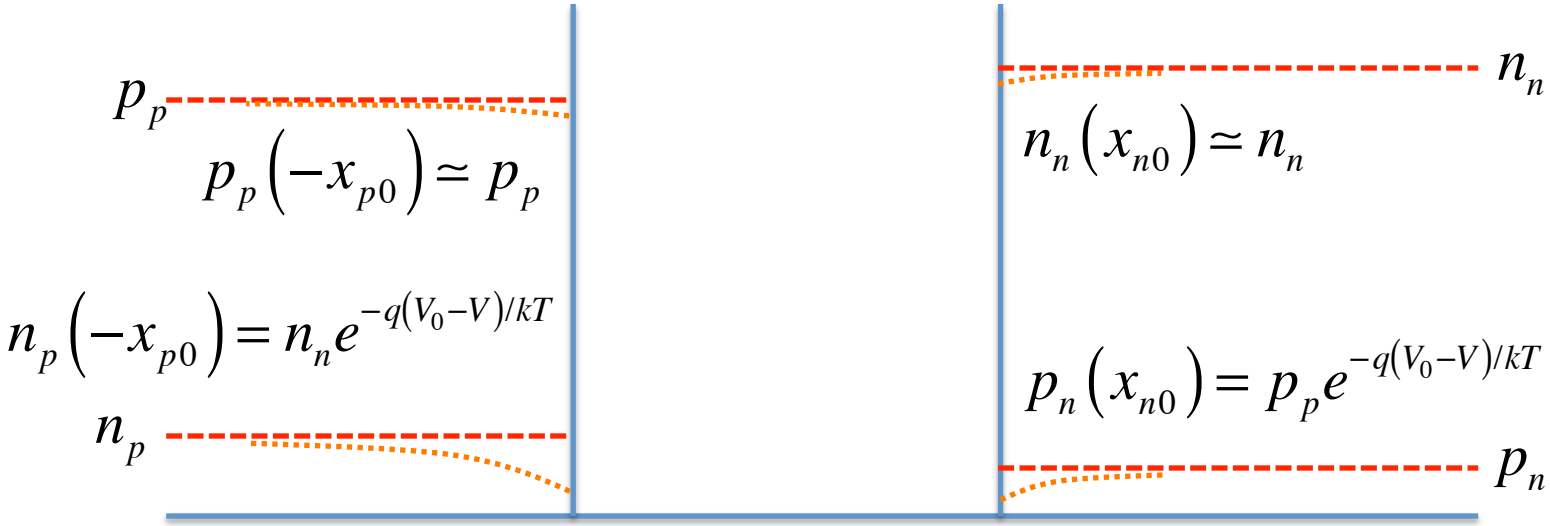


Space Charge Neutrality, Low - Level Injection :

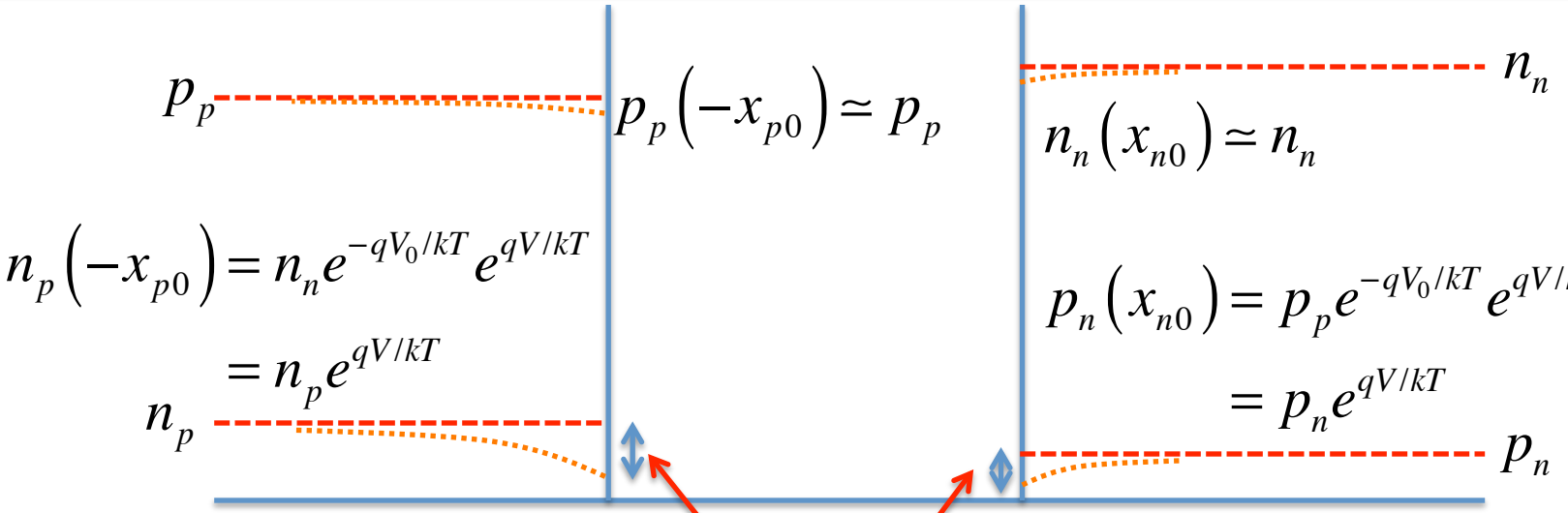
$$p_p(-x_{p0}) = p_p(-\infty) + \Delta n = p_p + n_p(e^{qV/kT} - 1) \simeq p_p$$

Reverse Bias





Reverse Bias



$$n_p(-x_{p0}) - n_p(-\infty) = \Delta n$$

$$n_p e^{qV/kT} - n_p =$$

$$n_p\left(e^{qV/kT} - 1\right) =$$

$$-n_p \simeq$$

$$\Delta p = p_n(x_{n0}) - p_n(\infty)$$

$$= p_n e^{qV/kT} - p_n$$

$$= p_n\left(e^{qV/kT} - 1\right)$$

$$\simeq -p_n$$

Injected Carrier Distribution: Holes

Solution to Hole Diffusion Equation

$$\frac{d^2\delta p}{dx^2} = \frac{\delta p}{L_p^2} \qquad \frac{\partial \delta p}{\partial t} = 0$$

Solution:

$$\delta p(x) = C_1 e^{x/L_p} + C_2 e^{-x/L_p}$$

Boundary Condition:

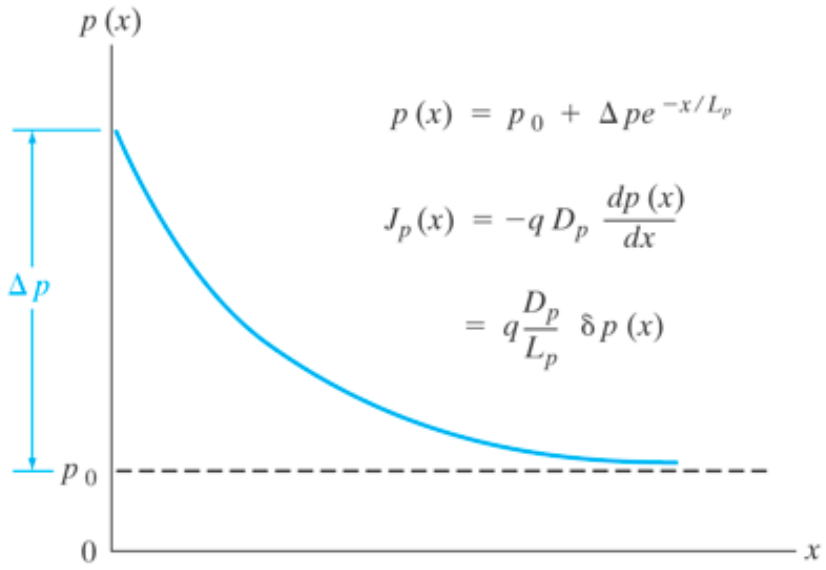
$$\delta p(x=0) = \Delta p = C_1 + C_2$$

$$\delta p(x=\infty) = 0 = C_1 e^{x/L_p}$$

$$C_1 = 0, \quad C_2 = \Delta p$$

$$\delta p(x) = \Delta p \cdot e^{-x/L_p}$$

Hole Injection at x=0



$$J_p(x) = -qD_p \frac{dp}{dx} = -qD_p \frac{d(p_o + \delta p)}{dx} = -qD_p \frac{d\delta p}{dx}$$

$$= q\frac{D_p}{L_p}\Delta p e^{-x/L_p} = q\frac{D_p}{L_p}\delta p(x)$$

Electron Diffusion in –x Direction

Solution to Electron Diffusion Equation

$$\frac{d^2\delta n}{dx^2} = \frac{\delta n}{L_n^2} \qquad \frac{\partial \delta n}{\partial t} = 0$$

Solution:

$$\delta n(x) = C_1 e^{x/L_n} + C_2 e^{-x/L_n}$$

Boundary Condition:

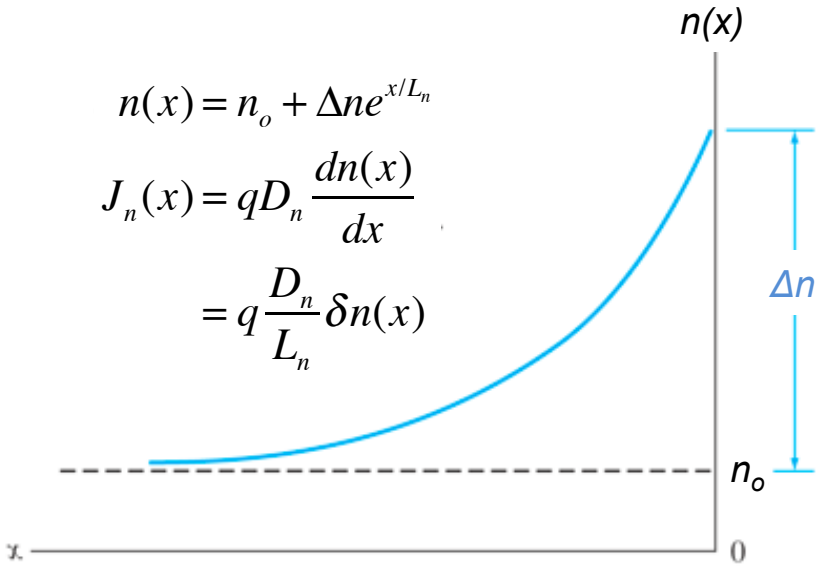
$$\delta n(x=0) = \Delta n = C_1 + C_2$$

$$\delta n(x=-\infty) = 0 = C_2 e^{-x/L_n}$$

$$C_2 = 0, \quad C_1 = \Delta n$$

$$\delta n(x) = \Delta n \cdot e^{x/L_n}$$

Electron Injection at x=0



$$J_n(x) = qD_n \frac{dn}{dx} = qD_n \frac{d(n_o + \delta n)}{dx} = qD_n \frac{d\delta n}{dx}$$

$$= q\frac{D_n}{L_n}\Delta n e^{x/L_n} = q\frac{D_n}{L_n}\delta n(x)$$

The Ideal Diode Equation

Hole Current

Recall that:

$$J_p(x) = q\frac{D_p}{L_p}\delta p(x)$$

Therefore:

$$I_p(x_n = 0) = \frac{qAD_p}{L_p} p_n\left(e^{qV/kT} - 1\right)$$

Electron Current

Recall that:

$$J_n(x) = -q\frac{D_n}{L_n}\delta n(x)$$

Therefore:

$$I_n(x_p = 0) = -\frac{qAD_p}{L_p} n_p\left(e^{qV/kT} - 1\right)$$

Relative to the transformed coordinate system

Total Current

Neglecting recombination in the transition region:

$$\left|I_p(x_n = 0)\right| = \left|I_p(x_p = 0)\right| \text{ and } \left|I_n(x_p = 0)\right| = \left|I_n(x_n = 0)\right|$$

Since x_p is defined in the -x direction:

$$I_n(x_n = 0)_{x_n axis} = -I_n(x_p = 0)_{x_p axis}$$

The total current is the sum of the electron and hole currents:

$$I = I_p(x_n = 0) + I_n(x_n = 0)_{x_n axis} = I_p(x_n = 0) - I_n(x_p = 0)_{x_p axis} \text{ so:}$$

$$I = \frac{qAD_p}{L_p}\Delta p_n + \frac{qAD_n}{L_n}\Delta n_p = qA\left(\frac{D_p}{L_p} p_n + \frac{D_n}{L_n} n_p\right)\left(e^{qV/kT} - 1\right)$$

A Few Points

$$I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)\left(e^{qV/kT} - 1\right) = I_o\left(e^{qV/kT} - 1\right)$$

n-side current p-side current

- The dominant current contribution comes from injection from the more heavily doped side into the more lightly doped side
- Reducing the doping level on either side of the junction increases the minority carrier concentrations p_n and n_p which would tend to increase the current for a given voltage
- Increasing the doping level will cause the diffusion length to decrease, which would tend to cause current to increase for a given voltage
- An asymmetrical junction (p^+n or pn^+) will both increase the minority carrier concentration and reduce the diffusion length

Quasi Fermi Levels in the Forward Biased Junction

Quasi Fermi Levels in the Forward-Biased Junction

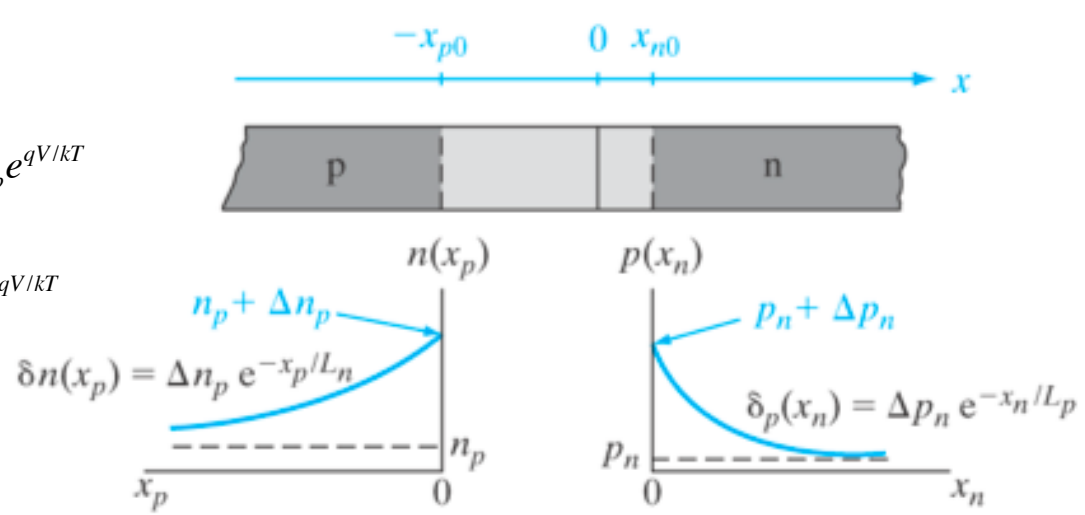
$$p(x_p = 0) \sim p_p(x_p = \infty) = p_p$$

$$n(x_p = 0) = n_p(x_p = \infty) + \Delta n$$

$$= n_p\left(e^{qV/kT} - 1\right) \sim n_p e^{qV/kT}$$

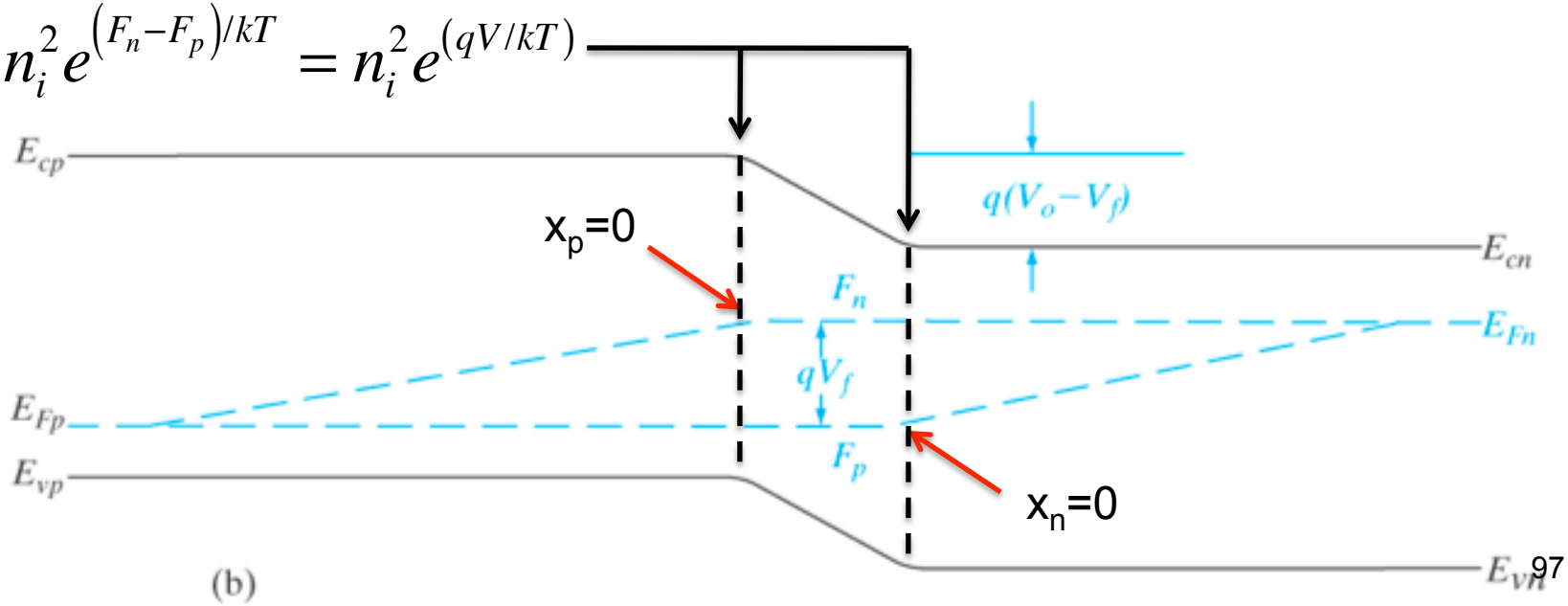
so:

$$pn(x_p = 0) = p_p n_p e^{qV/kT} = n_i^2 e^{qV/kT}$$



(a)

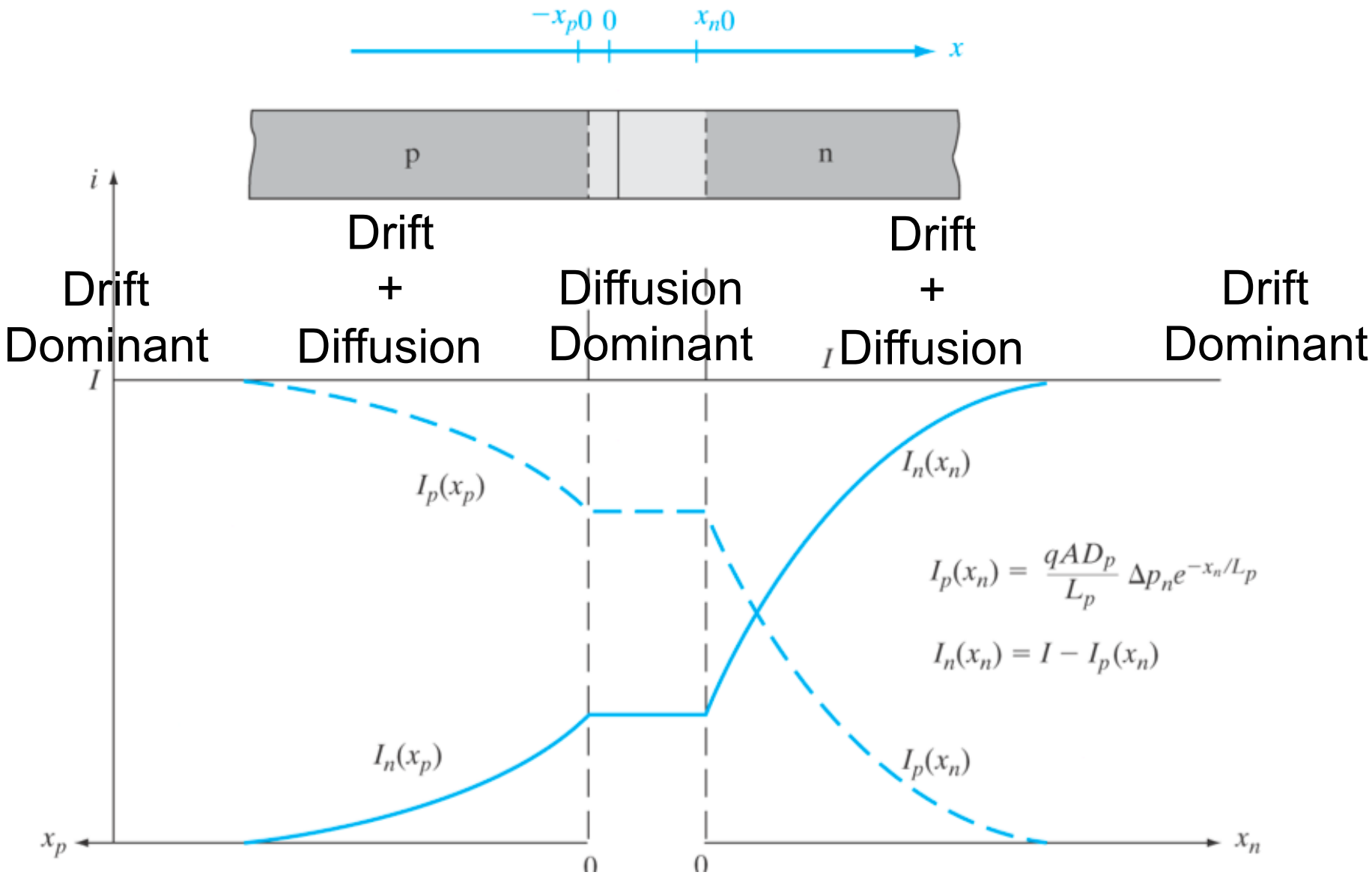
$$pn = n_i^2 e^{(F_n - F_p)/kT} = n_i^2 e^{(qV/kT)}$$



(b)

Majority and Minority Currents

Drift & Diffusion in Forward Bias

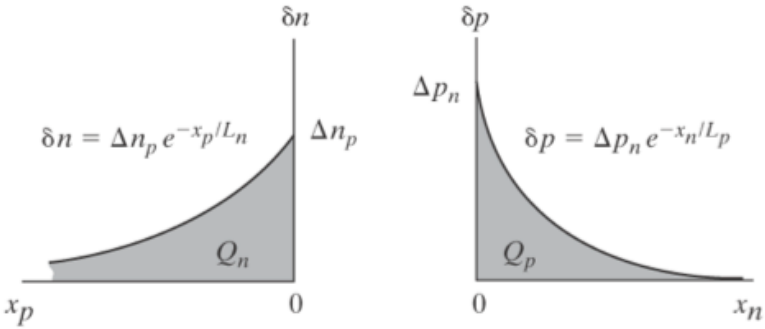


Charge Control Model

Another Derivation of the Ideal Diode Equation

Charge Control Approximation

- In steady state, the rate at which we replenish charge across the x_n=0 or x_p=0 plane must equal the amount of minority carrier charge lost through recombination in an interval of time
- The rate at which charge is lost is equal to the total amount of charge divided by the charge lifetime



$$Q_n = -qA\int_0^\infty \delta n(x_p)\,dx_p \qquad Q_p = qA\int_0^\infty \delta p(x_n)\,dx_n$$

$$I_n(x_p=0) = \frac{Q_n}{\tau_n} = \frac{-qAL_n}{\tau_n}\,\Delta n_p \qquad I_p(x_n=0) = \frac{Q_p}{\tau_p} = \frac{qAL_p}{\tau_p}\,\Delta p_n$$

$$Q_p = qA\int_0^\infty \delta p(x_n)\,dx_n = qA\Delta p_n\int_0^\infty e^{-x_n/L_p}\,dx_n$$
$$= qA\Delta p_n L_p$$

$$Q_n = qA\int_0^\infty \delta n(x_p)\,dx_p = qA\Delta n_p\int_0^\infty e^{-x_p/L_n}\,dx_p$$
$$= qA\Delta n_p L_n$$

Note: x_p and x_n coordinate systems

$$I = I_p(x_n=0) - I_n(x_p=0) = qA\left(\frac{D_p}{L_p}\,\Delta p_n + \frac{D_n}{L_n}\,\Delta n_p\right)$$
$$= qA\left(\frac{D_p p_n}{L_p} + \frac{D_n n_p}{L_n}\right)(e^{qV/kT} - 1)$$

Recall that: $L_n \equiv \sqrt{D_n\tau_n}$ and $L_p \equiv \sqrt{D_p\tau_p}$

so: $\tau_n = \dfrac{L_n^2}{D_n}$ and $\tau_p = \dfrac{L_p^2}{D_p}$

Summary of Models

Summary of Models

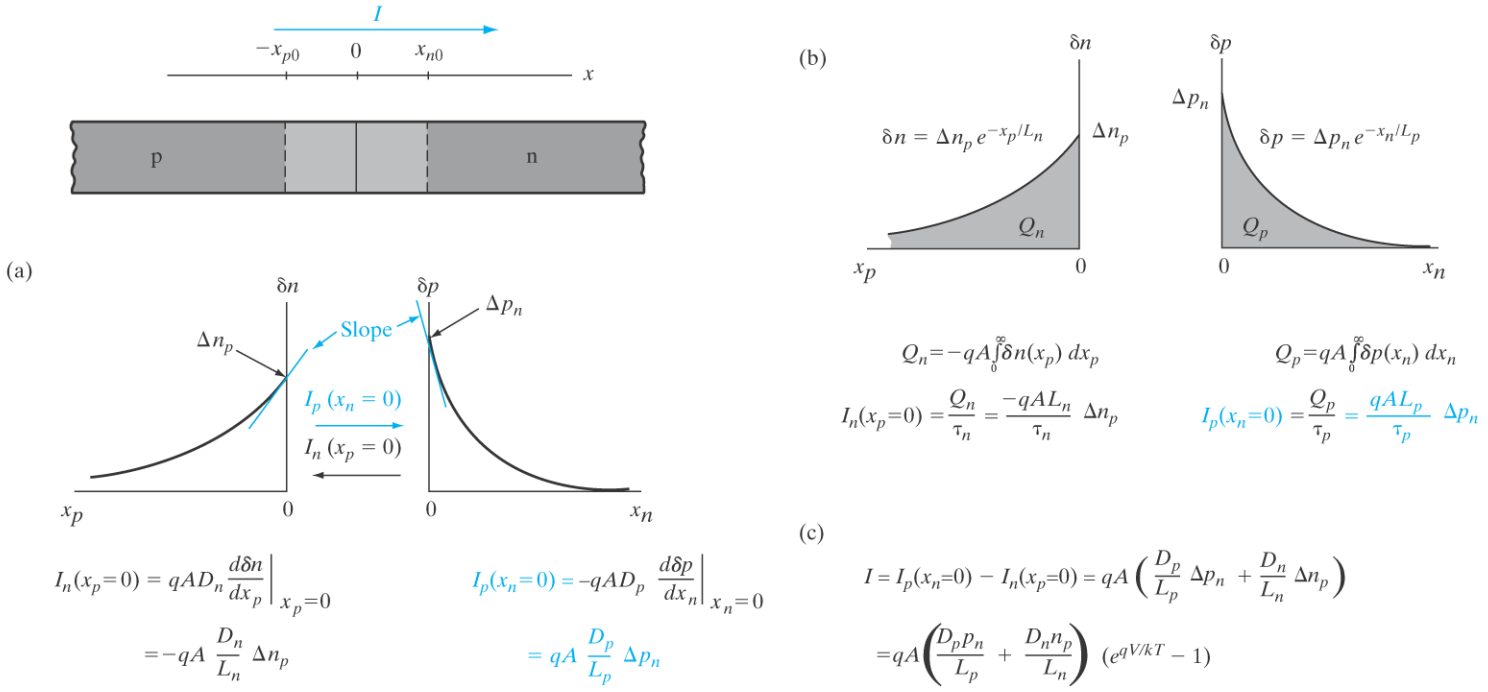


Figure 5.16

Two methods for calculating junction current from the excess minority carrier distributions: (a) diffusion currents at the edges of the transition region; (b) charge in the distributions divided by the minority carrier lifetimes; (c) the diode equation.

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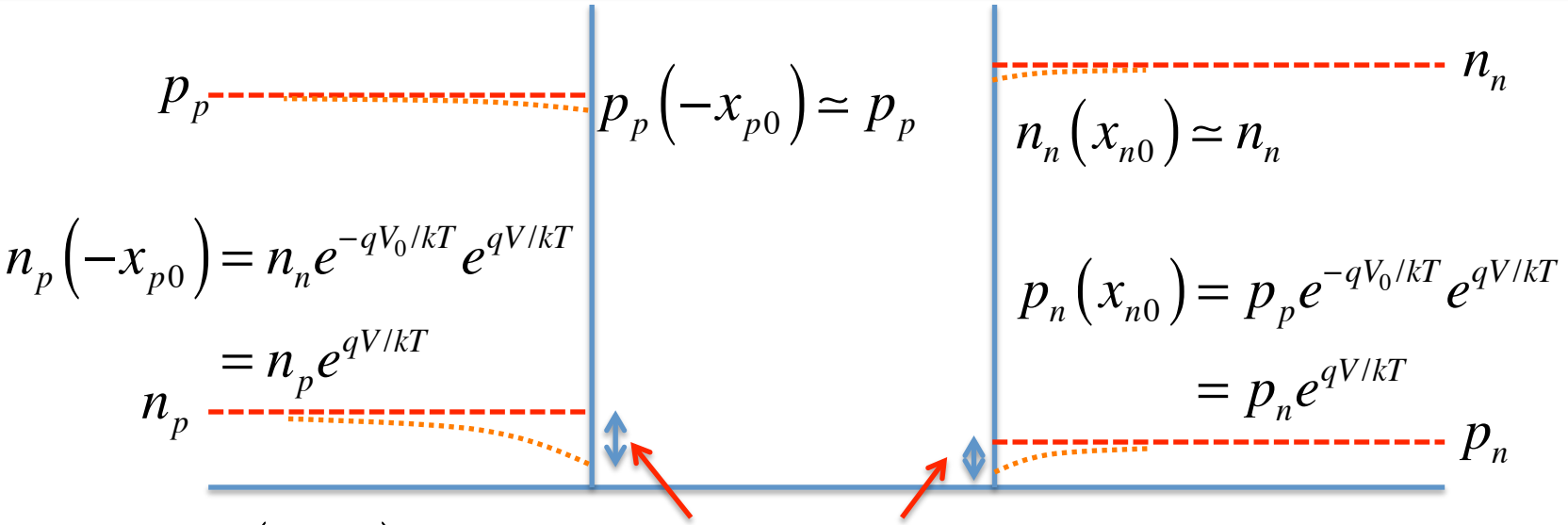
Reverse Bias

Reverse Bias Charge Distribution



$$n_p(-x_{p0}) - n_p(-\infty) = \Delta n$$

$$n_p e^{qV/kT} - n_p =$$

$$n_p\left(e^{qV/kT} - 1\right) =$$

$$-n_p \simeq$$

$$\Delta p = p_n(x_{n0}) - p_n(\infty)$$

$$= p_n e^{qV/kT} - p_n$$

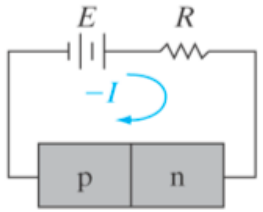
$$= p_n\left(e^{qV/kT} - 1\right)$$

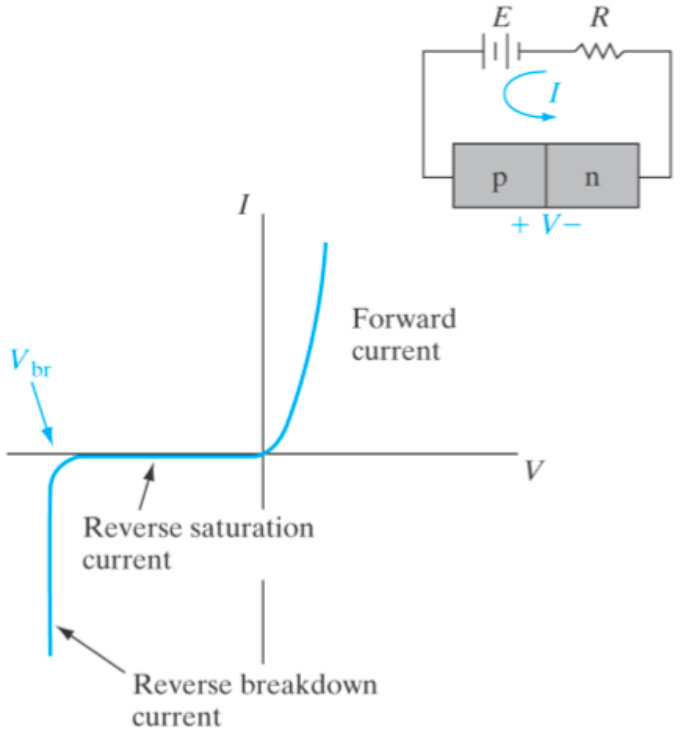
$$\simeq -p_n$$

Reverse Breakdown

Reverse Breakdown

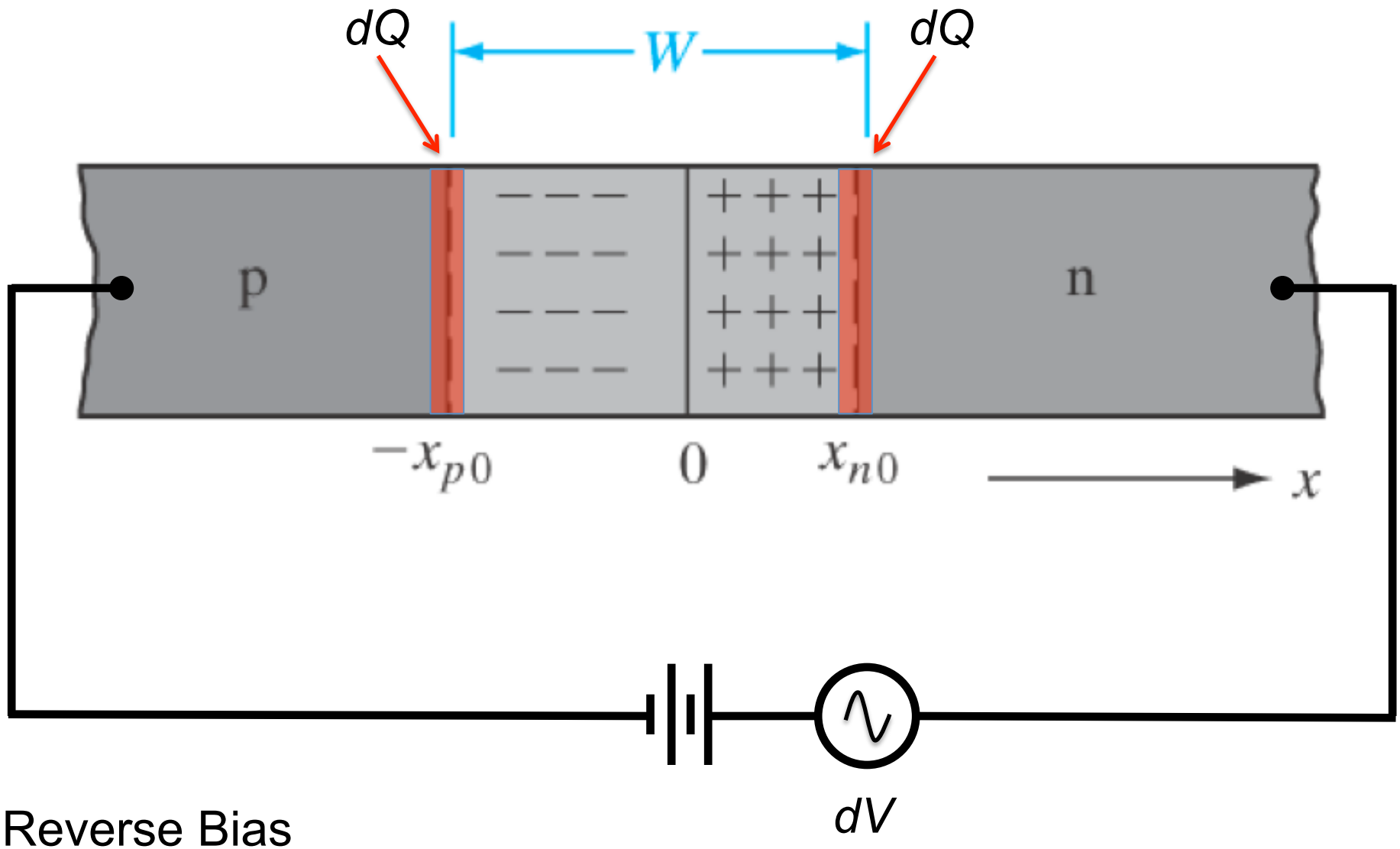
- Reverse “breakdown” is not a destructive process (despite the name) – as long as current is limited externally
- The two types of breakdown discussed here are Zener effect breakdown and avalanche breakdown
- Zener breakdown occurs at a few volts
- Avalanche breakdown occurs at higher voltages





Capacitance of p-n Junctions

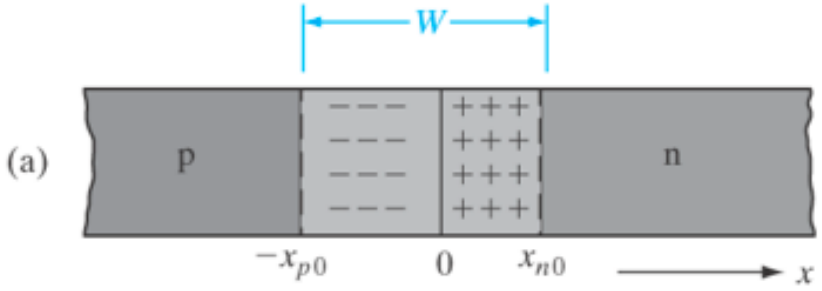
Depletion Capacitance

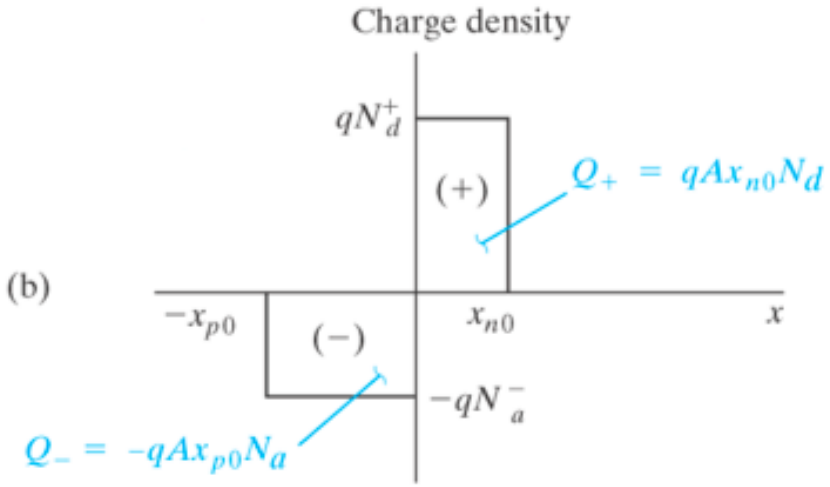


Reverse Bias

Junction Capacitance

- Junction Capacitance: due to the dipole in the transition region
  - Dominant in reverse bias
- Charge Storage Capacitance: due to lagging of voltage as current changes due to charge storage effects
  - Dominant in forward bias
- More complex than C=Q/V





$$C = \left|\frac{dQ}{dV}\right|$$

Under bias: $$W = \left[\frac{2\varepsilon(V_o - V)}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2}$$

$$|Q| = qAx_{no}N_d = qAx_{po}N_a$$

$$x_{no} = \frac{N_a}{N_a + N_d}W \text{ and } x_{po} = \frac{N_d}{N_a + N_d}W \text{ so:}$$

$$|Q| = qA\frac{N_d N_a}{N_d + N_a}W = A\left[2q\varepsilon(V_o - V)\left(\frac{N_d N_a}{N_d + N_a}\right)\right]^{1/2}$$

Junction Capacitance [2]

$$|Q| = qA\frac{N_d N_a}{N_d + N_a}W = A\left[2q\varepsilon\left(V_o - V\right)\left(\frac{N_d N_a}{N_d + N_a}\right)\right]^{1/2} \quad \text{define } V' = V_0 - V$$

$$\frac{dV'}{dV} = \frac{d\left(V_0 - V\right)}{dV} = -1 \text{ so } dV' = -dV$$

$$C_j = \left|\frac{dQ}{dV}\right| = \left|\frac{dQ}{-dV'}\right| = \left|\frac{dQ}{-d\left(V_o - V\right)}\right| = \frac{A}{2}\left[\frac{2q\varepsilon}{\left(V_o - V\right)}\left(\frac{N_d N_a}{N_d + N_a}\right)\right]^{1/2}$$

Key Point: A p-n junction has a voltage-variable capacitance

Varactor: device that makes use of this property

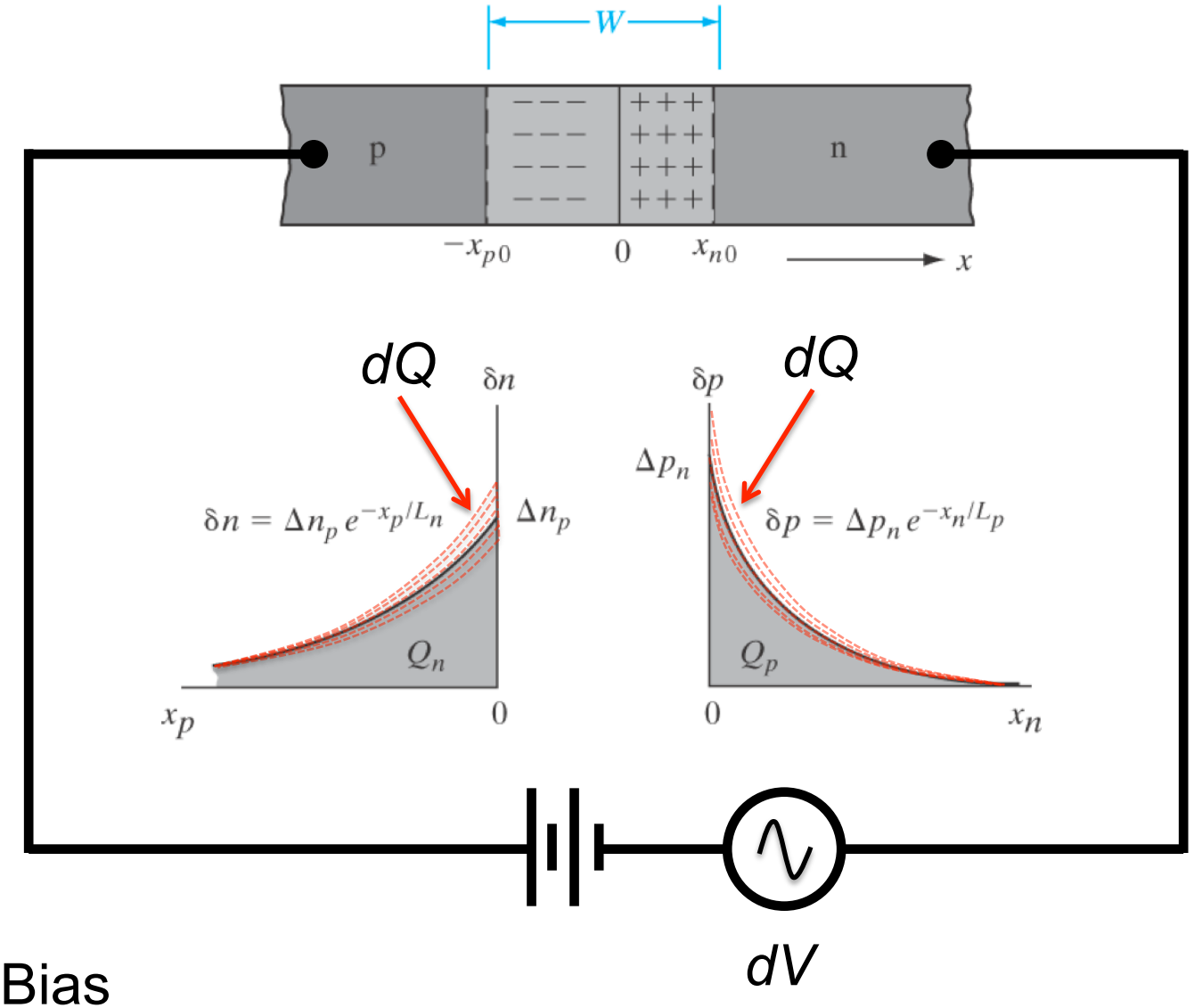
$$\text{Note: } C = \frac{A}{2}\left(\frac{4\varepsilon^2}{4\varepsilon^2}\right)^{1/2}\left[\frac{2q\varepsilon}{\left(V_o - V\right)}\left(\frac{N_d N_a}{N_d + N_a}\right)\right]^{1/2} = \varepsilon A\left[\frac{q}{2\varepsilon\left(V_o - V\right)}\left(\frac{N_d N_a}{N_d + N_a}\right)\right]^{1/2} = \frac{\varepsilon A}{W}$$

For a p^+n junction, $N_a \gg N_d$ and $x_{no} \simeq W$ while x_{po} is negligible:

$$C_j = \frac{A}{2}\left[\frac{2q\varepsilon}{\left(V_o - V\right)}N_d\right]^{1/2} \quad \leftarrow \quad p^+n$$

Capacitance can be used to measure doping density!

Diffusion Capacitance



Forward Bias

Charge Storage (Diffusion) Capacitance

$$C_{sp} = \frac{q}{kT} Q_p = \frac{q}{kT} I_p \tau_p$$

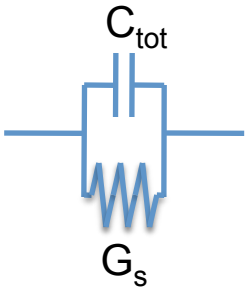
$$C_{sn} = \frac{q}{kT} Q_n = \frac{q}{kT} I_n \tau_n$$

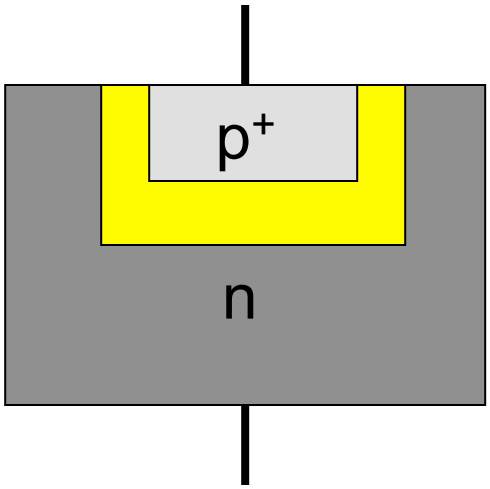
$$I = \frac{Q}{\tau} = I_p + I_n = \frac{Q_p}{\tau_p} + \frac{Q_n}{\tau_n}$$

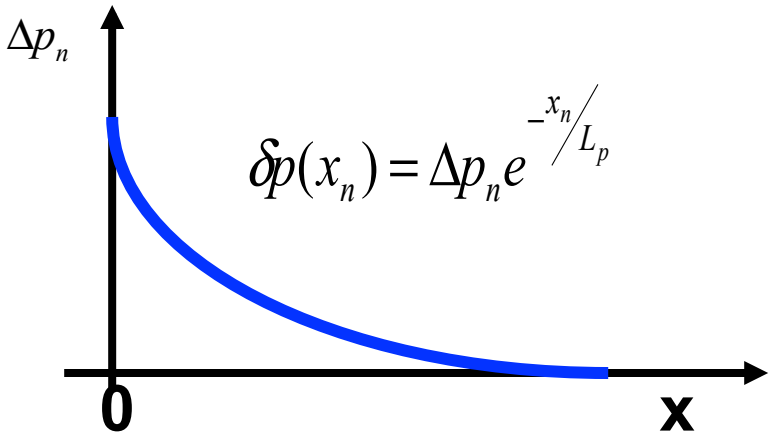
$C_s = C_{sp} + C_{sn}$ for a p-n junction

$C_s = C_{sp}$ for a p^+-n junction

$C_s = C_{sn}$ for a p-n^+ junction







$$C_s = \left| \frac{dQ_p}{dV} \right|$$

$$Q_p = qAL_p \Delta p_n = qAL_p p_n (e^{qV/kT} - 1)$$

$$C_s = \left| \frac{dQ_p}{dV} \right| = \frac{q^2}{kT} AL_p p_n e^{qV/kT} = \frac{q}{kT} Q_p$$

$$C_s = \left| \frac{dQ_p}{dV} \right| = \frac{q}{kT} Q_p = \frac{q}{kT} I \tau_p$$

$$I = \frac{Q_p}{\tau_p} = \frac{qAL_p}{\tau_p} p_n (e^{qV/kT} - 1)$$

But also: $G_s = \frac{dI}{dV} = \frac{q}{kT} I$

So we see: $C_s = G_s \tau_p$

$$i(t) = G_s v(t) + C_s \frac{dv(t)}{dt}$$

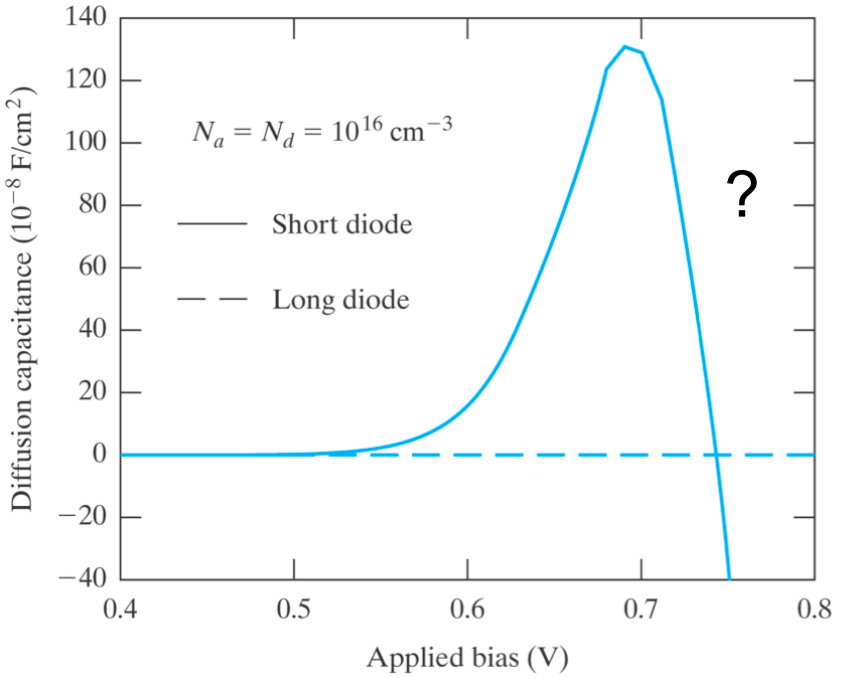
$$f_t = \frac{G_s}{2\pi C_s}$$

The charge storage capacitance can be a serious limitation for forward biased p-n junction in high frequency circuits, where f_t is cut off frequency.

Diffusion Capacitance

From Streetman:

- Under forward bias, the charge storage (diffusion capacitance) C_s can become dominant
- Long diodes – dimensions larger than diffusion lengths: no diffusion capacitance (typical case for laser diodes, direct-gap materials)
- Short diodes – reclaimable charge is 2/3 of the total stored charge (typical case for Si)



Note: There is debate regarding this material - possibly incorrect!

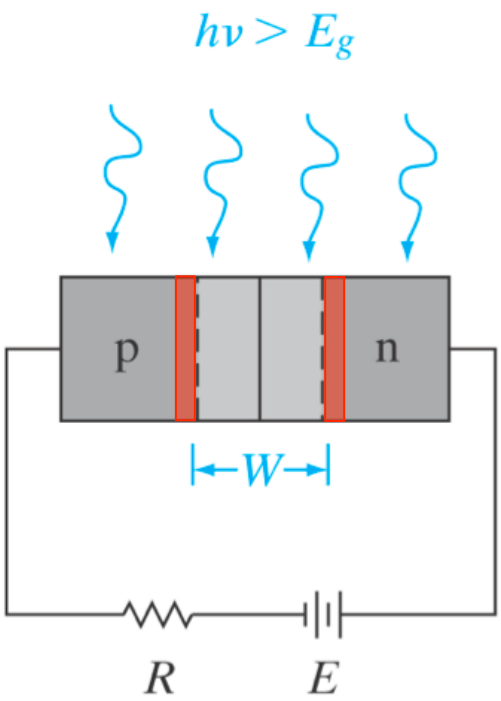
Short Diode Capacitance: $C_s = \frac{dQ_p}{dV} = \frac{1}{3}\frac{q^2}{kT} Acp_n e^{qV/kT}$

A-C Conductance: $G_s = \frac{dI}{dV} = \frac{qAL_p p_n}{\tau_p}\frac{d}{dV}\left(e^{qV/kT}\right) = \frac{q}{kT} I$

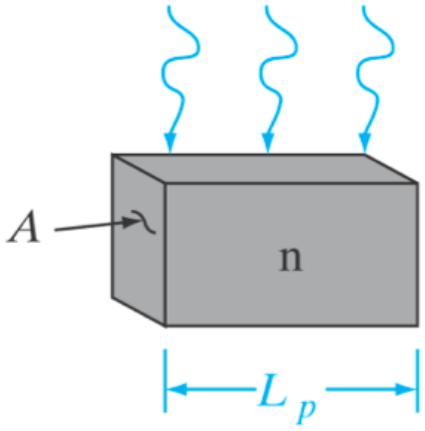
Current and Voltage in Illuminated Junctions

Optical Generation of Carriers

Absorption of Light



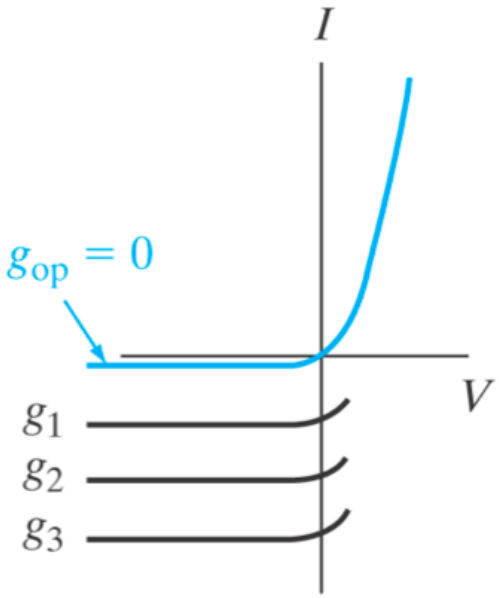
Current from EHP Generated Within a Diffusion Length



$$\delta p_{op} = g_{op}\tau_p$$

$$I_{op} = q\,A L_p g_{op}$$

I-V Characteristic of Illuminated Junction

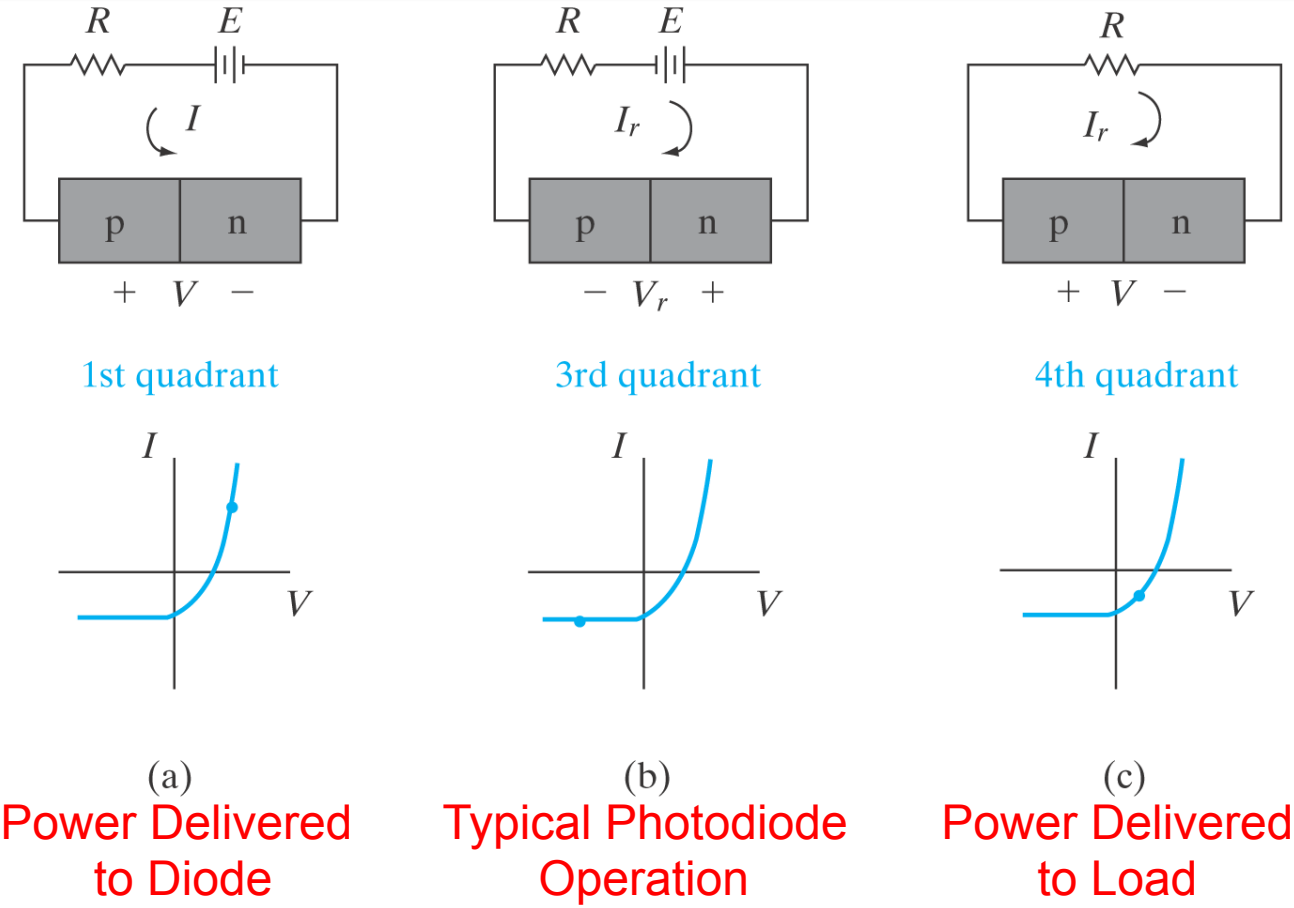


$g_3 > g_2 > g_1$

For both p-side and n-side contributions: $I_{op} = qAg_{op}\left(L_p + L_n + W\right)$

Optical excitation creates an additional term g_{op}

Photodiode Operating Regions



Operation of an illuminated junction in the various quadrants of its *I–V* characteristic; in (a) and (b), power is delivered to the device by the external circuit; in (c) the device delivers power to the load.

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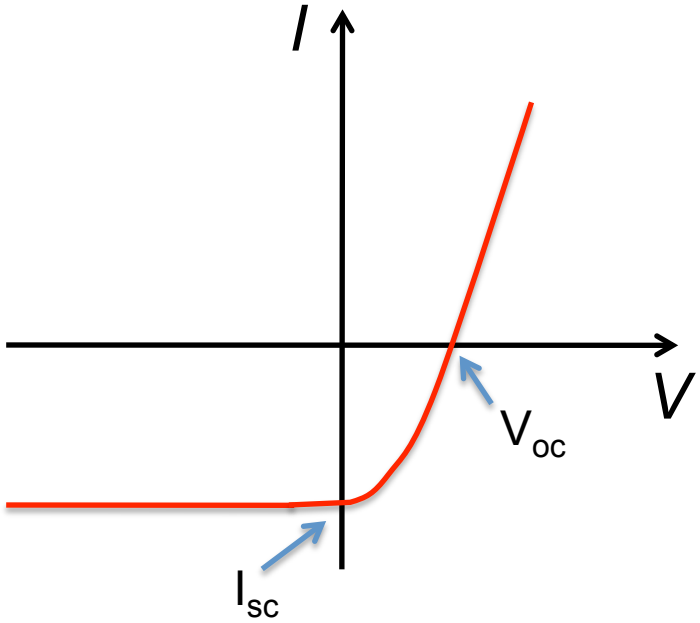
V_{oc} and I_{sc}

- I_{sc} is the short circuit current: if the device is shorted under illumination, a current flows from n to p
  - From the diode equation under illumination, $I=-I_{op}$
- Under open circuit conditions, there is a voltage across the device that can be measured V_{oc}
- As the minority carrier concentration increases, the carrier lifetime decreases
  - Since $g_{th}=p_n/\tau_n$ we see g_{th} increases
- V_{oc} cannot increase indefinitely – it is limited by the contact potential V_o
- ***Photovoltaic effect:*** appearance of a voltage across an illuminated junction

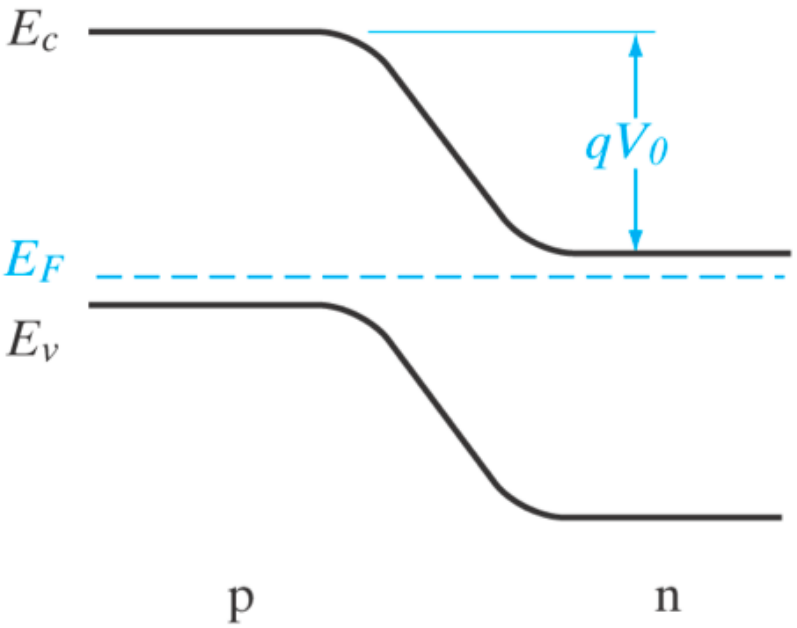


$$V_{oc} \simeq \frac{kT}{q} \ln \frac{g_{op}}{g_{th}}$$

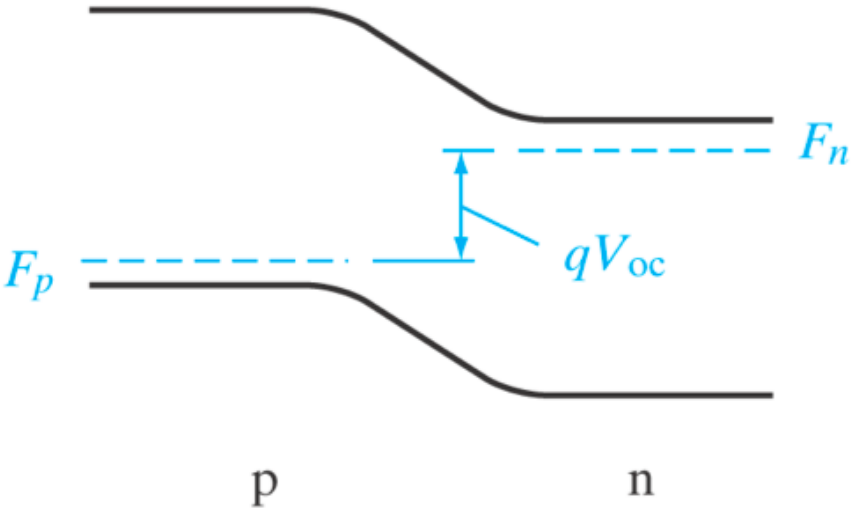
To Be Derived Shortly

V_{oc} Under Illumination

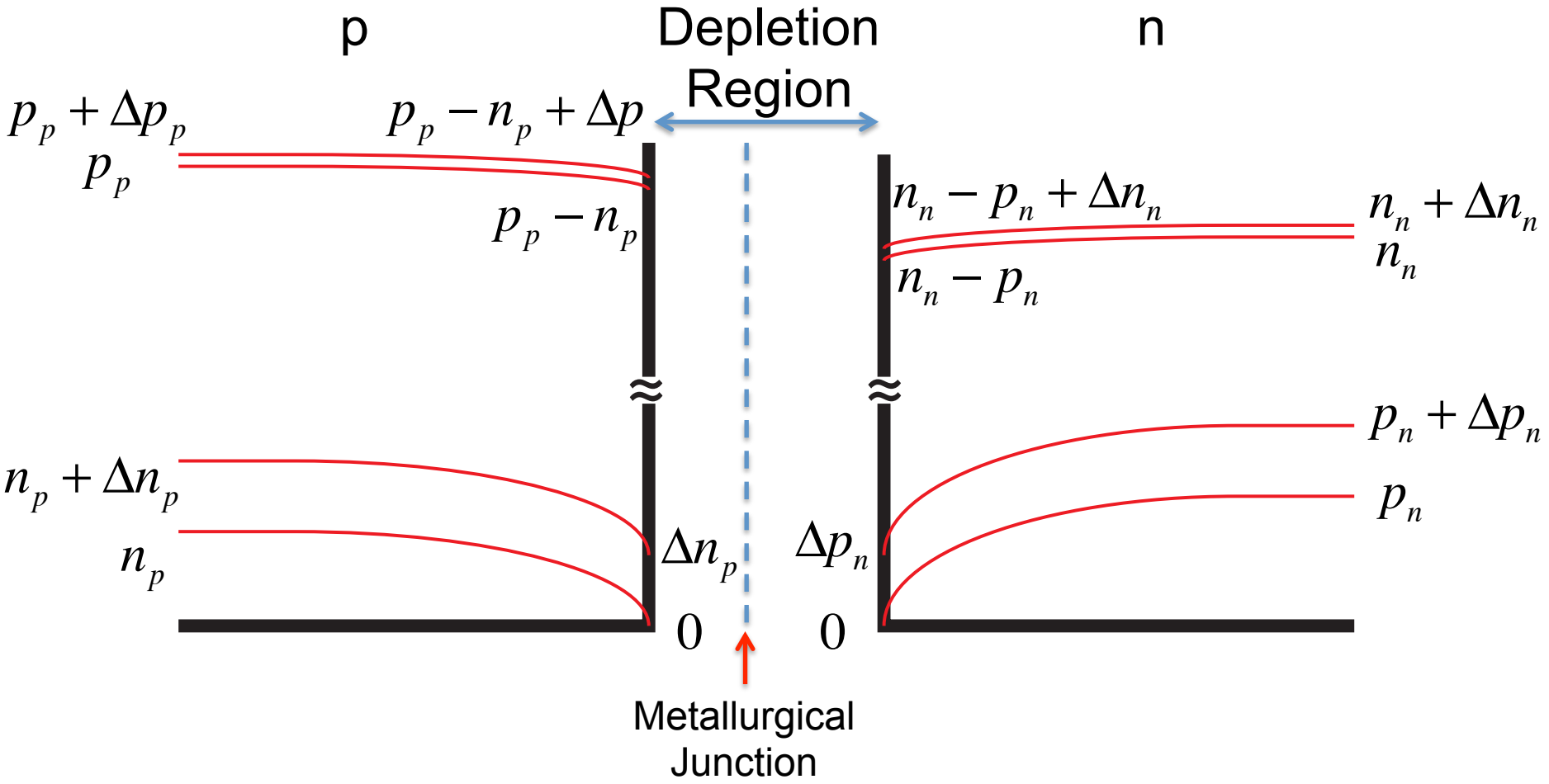
Equilibrium Junction



Junction Under Illumination



Uniformly Illuminated Junction



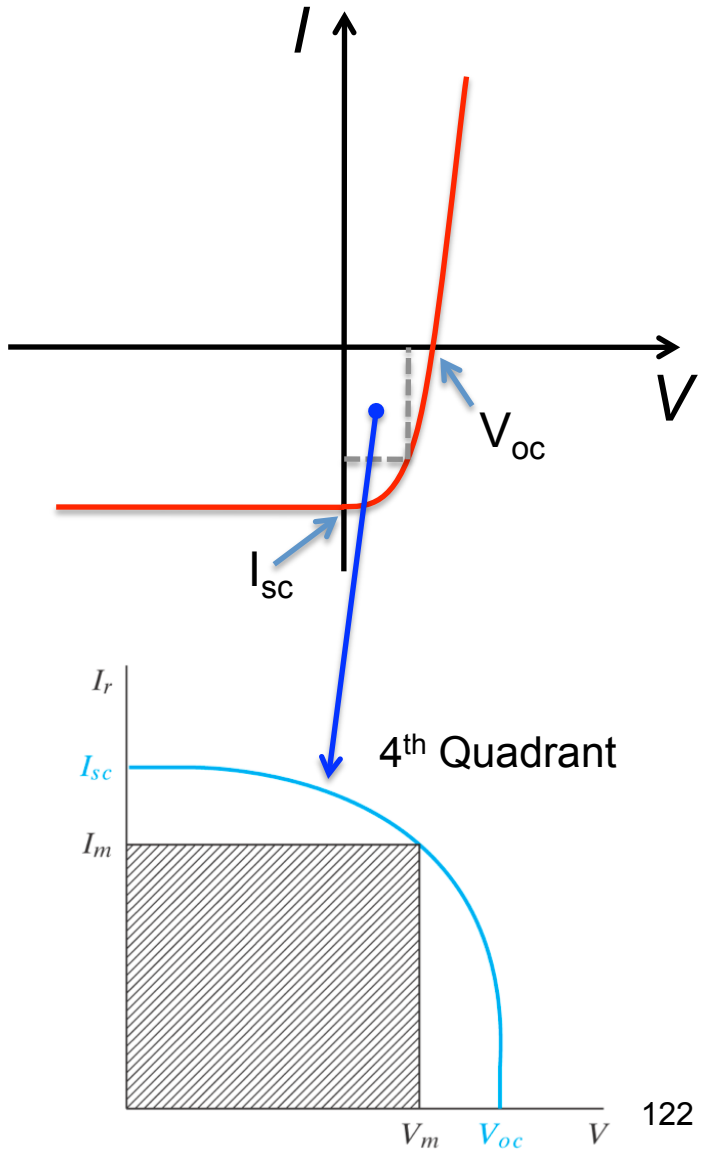
$$\Delta n_p = \Delta p_p = g_{op}\tau_n$$

$$\Delta n_n = \Delta p_n = g_{op}\tau_p$$

Solar Cells

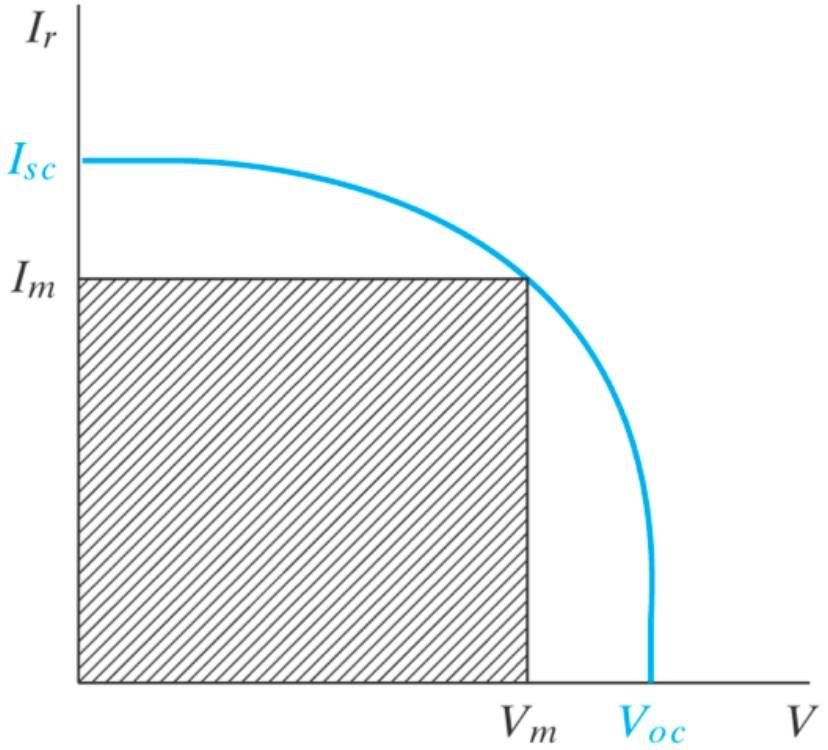
Solar Cell I-V

- Solar cells operate in the 4th quadrant
- The voltage output is limited to be less than the contact potential, which in turn is generally less than the bandgap voltage E_g/q
  - Si solar cell: V_{oc}<1V
  - Current depends upon illuminated area, but typically I_{op} ~10-100 mA for a 1 cm^2 cell
  - To increase voltage, cells are connected in series, to increase current, cells are connected in parallel
- An optimal solar cell has a large-area junction located near the surface
  - Junction depth less than L_p or L_n
  - Holes/electrons generated near surface can diffuse to the junction



Fill Factor

- V_{oc} and I_{sc} determined by solar cell characteristics, solar spectra, and solar intensity
- Maximum power is delivered when the product $V \bullet I_r$ is maximized
- A key solar cell figure of merit is the "fill factor"
- A second key figure of merit is efficiency
- For terrestrial use, the solar intensity is ~1000 W/m^2
- The power produced by the cell under peak conditions is ~$\eta \bullet$1000 W/m^2



Example:

For a Si solar cell with $I_{sc} = 100\text{mA}$ and $V_{oc} = 0.8\text{V}$, assuming a fill factor of 0.7 what is the max power?

$$P_{\max} = (f.f.) I_{sc} V_{oc} = (0.7)(100)(0.7) = 56 \text{ mW}$$

Fill Factor:

$$f.f. = \frac{I_m V_m}{I_{sc} V_{oc}}$$

V_{oc} and I_{sc} Revisited

$$I = I_{th}\left(e^{qV/kT} - 1\right) - I_{op}$$

For I_{sc}, V = 0:

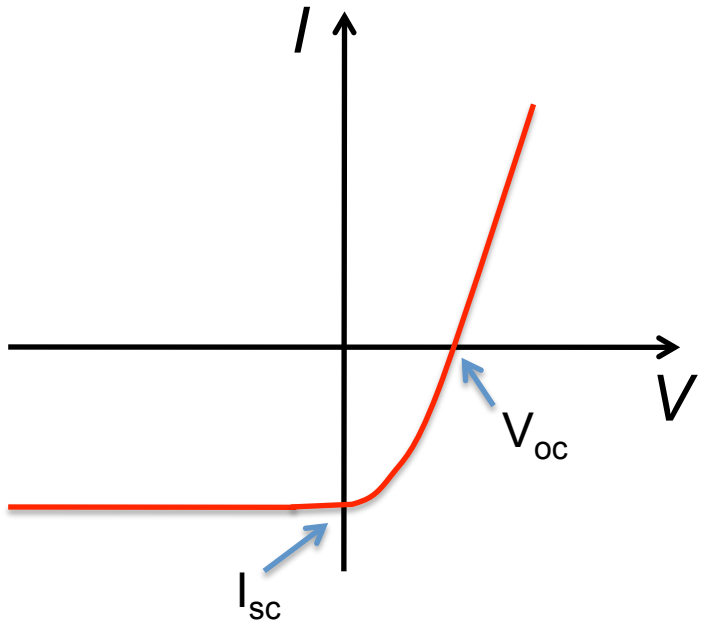
$$I_{sc} = -I_{op} = -qAg_{op}\left(L_p + L_n + W\right)$$

$$V_{oc} = \frac{kT}{q}\ln\left[\frac{I_{op}}{I_{th}} + 1\right] \quad \text{where} \quad I = 0$$

$$= \frac{kT}{q}\ln\left[\frac{L_p + L_n + W}{\left(\left(L_p/\tau_p\right)p_n + \left(L_n/\tau_n\right)n_p\right)}g_{op} + 1\right]$$

$$V_{oc} = \frac{kT}{q}\ln\left[\frac{g_{op}}{g_{th}}\right] \quad \text{for } g_{op} >> g_{th}$$

$$g_{th} = p_n/\tau_n \quad \text{and} \quad g_{op} = \delta p_n/\tau_n$$



- g_{th} is the equilibrium thermal generation-recombination term

$$I = qA\left(\frac{L_p}{\tau_p}p_n + \frac{L_n}{\tau_n}n_p\right)\left(e^{qV/kT} - 1\right) - qAg_{op}\left(L_p + L_n + W\right)$$

Solar Cell Figure of Merit Summary

- ***Open Circuit Voltage, V_{oc}:*** Voltage appearing at the terminals of a solar cell (or photodiode) under illumination with no load
- ***Short Circuit Current, I_{sc}:*** Current flowing between the terminals of a solar cell (or photodiode) under illumination with a low impedance connection (short) between the terminals
- ***Fill Factor, F.F.:*** Maximum power extraction ($P_m=V_m \times I_m$) divided by $V_{oc} \times I_{sc}$
- ***External Quantum Efficiency:*** Current per second (in electrons, not Coulombs) divided by the number of incident photons per second at a given wavelength

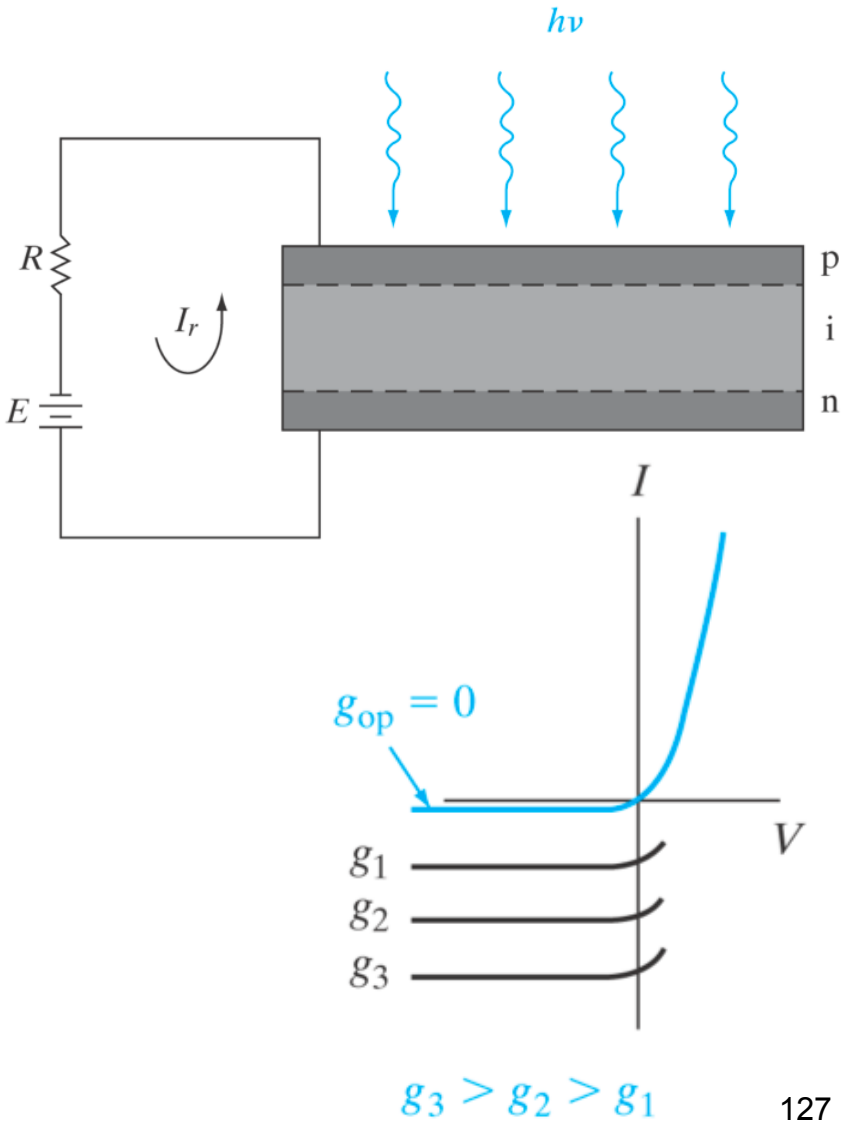
Photodetectors

PIN Illustration

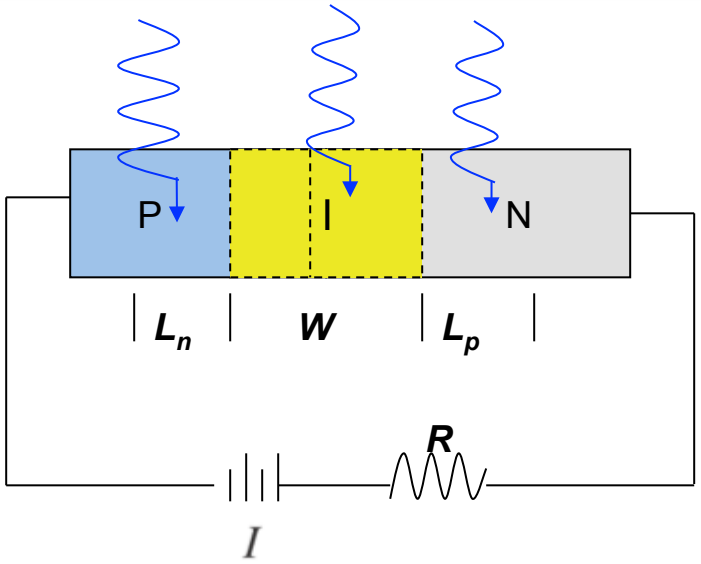
- In the 3rd quadrant, the current is independent of voltage and proportional to the optical generation rate
- In most applications, a key figure of merit is the photodiode bandwidth
- Carrier diffusion is slow, but drift is fast – absorption in the depletion region is preferred "depletion layer photodiode"
- There is also a design trade-off between sensitivity and speed – if the depletion width is too wide bandwidth is reduced

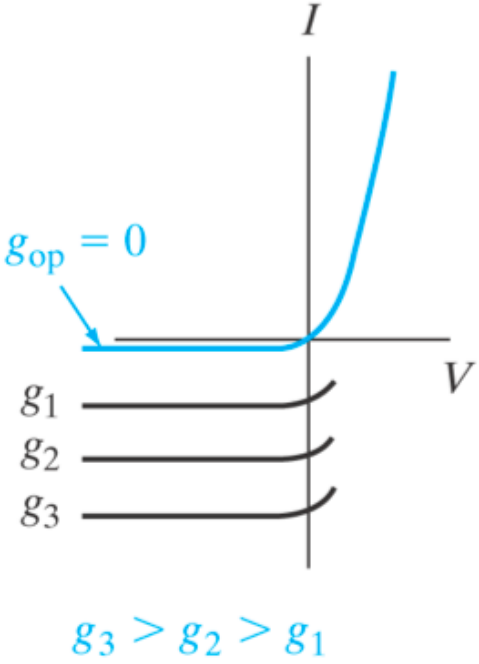
External Quantum Efficiency:

$$\eta_Q = \frac{\text{electrons out}}{\text{photons in}} = \frac{\left(J_{op}/q\right)}{\left(P_{op}/h\nu\right)}$$



Photodetectors: PIN Diodes





$$I = I_{th}\left(e^{qV/kT} - 1\right) - I_{op}$$

$$I_{op} = qAg_{op}\left(L_p + L_n + W\right)$$

To improve PIN bandwidth,

we need to reduce the photogeneration in

the diffuson layers L_n and L_p and optimize W.

$$I_{op} = qAg_{op}(W) = qA\frac{\delta n}{\tau}W = q\eta\left(\frac{P_{abs}}{h\nu}\right)$$

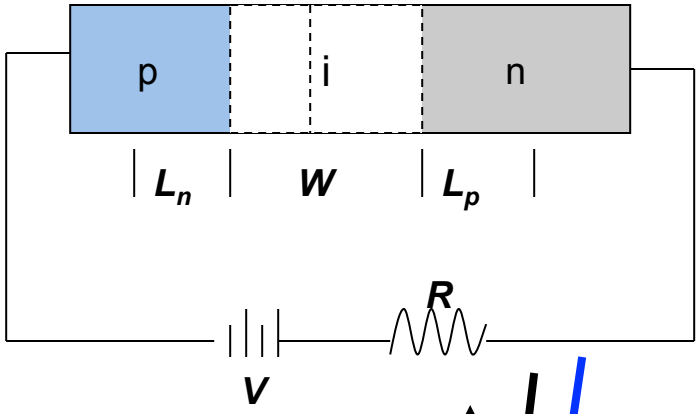
$$\eta = \frac{I_{op}/q}{P_{abs}/h\nu} = \frac{I_{op}/q}{P_{inc}(1-e^{-\alpha\ell})/h\nu}$$

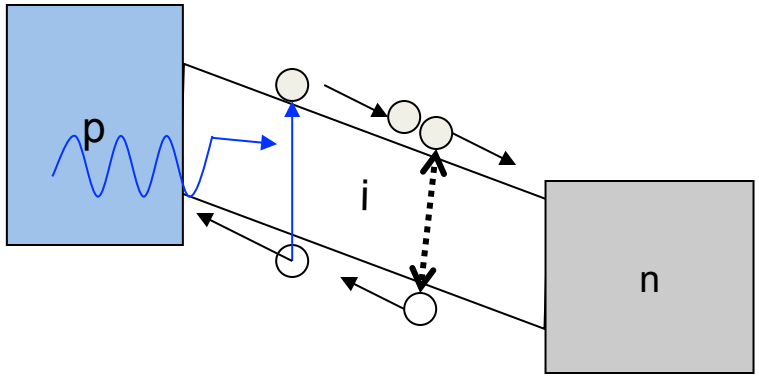
$$I_p = qA\delta n v_d = q\eta\left(\frac{P_{abs}}{h\nu}\right)\left(\frac{\tau v_d}{W}\right) = I_{op}\left(\frac{\tau}{t_r}\right)$$

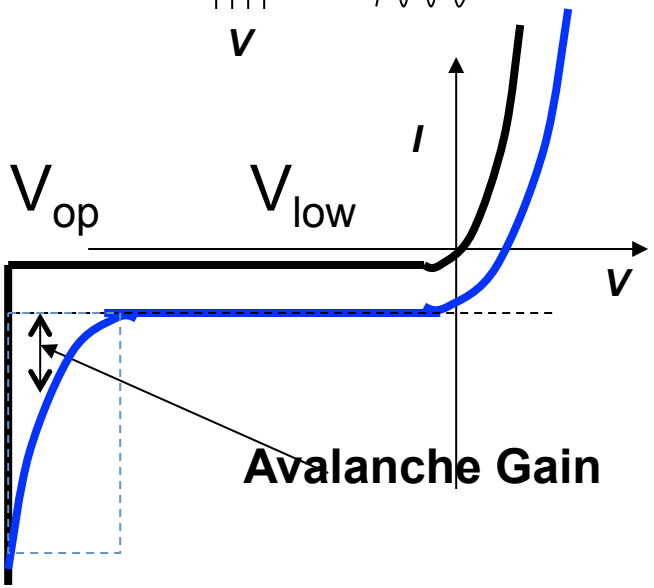
Drift is fast, diffusion is slow

Avalanche Photodiodes

An electron or a hole with high energy in "I" layer can break chemical or ionic bonds in crystal to create an EHP







- An avalanche photodiode (APD) is operated under a reverse-bias voltage that is sufficient to enable avalanche multiplication
- An electron or a hole with high energy in the "i" layer can break chemical or ionic bonds in crystal to create an EHP
- The multiplication results in internal current gain

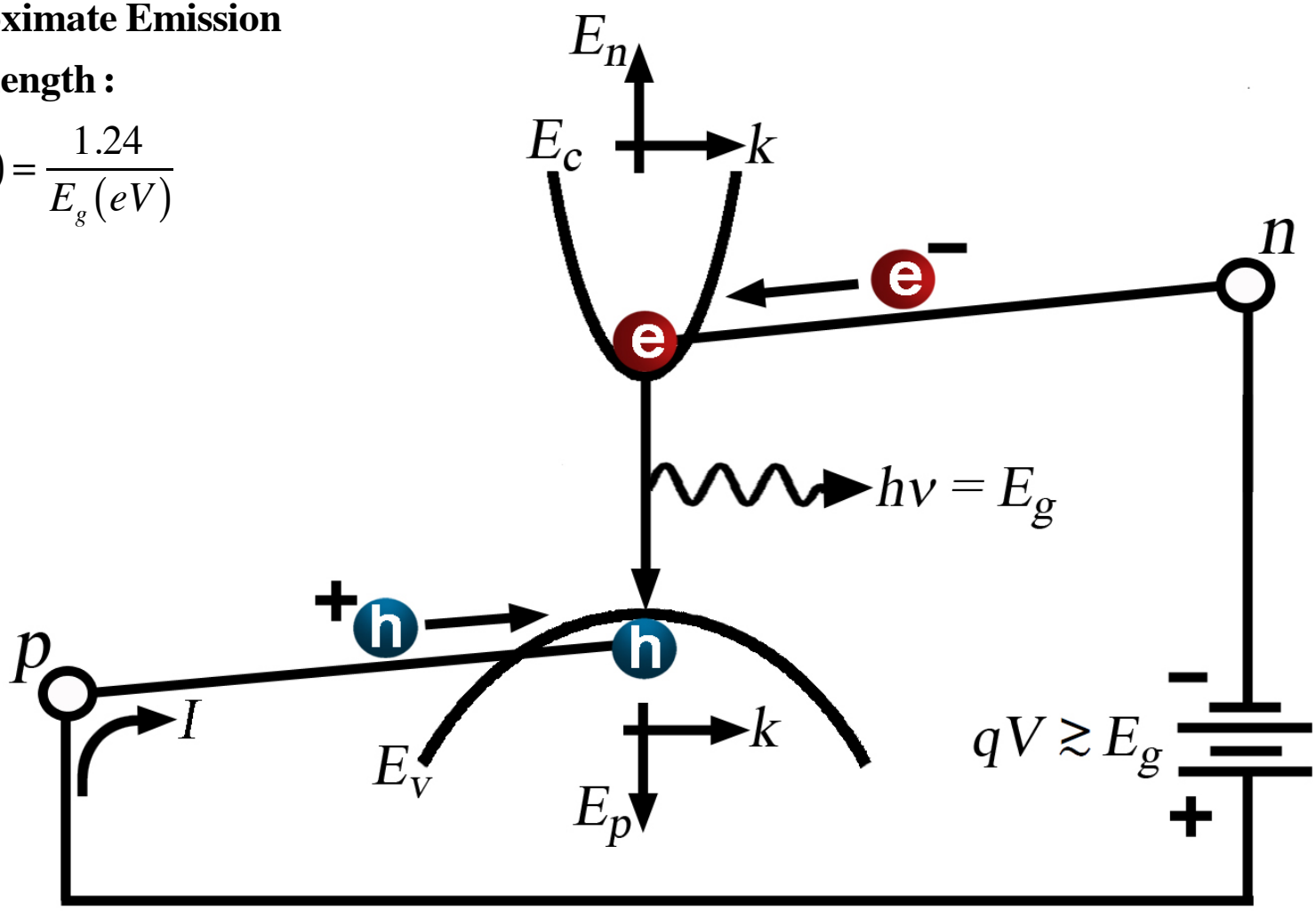
$$Gain \equiv \frac{\left(I_p - I_d\right)_{V_{op}}}{\left(I_p - I_d\right)_{V_{low}}}$$

Light Emitting Diodes

The Ultimate Lamp

Approximate Emission Wavelength :

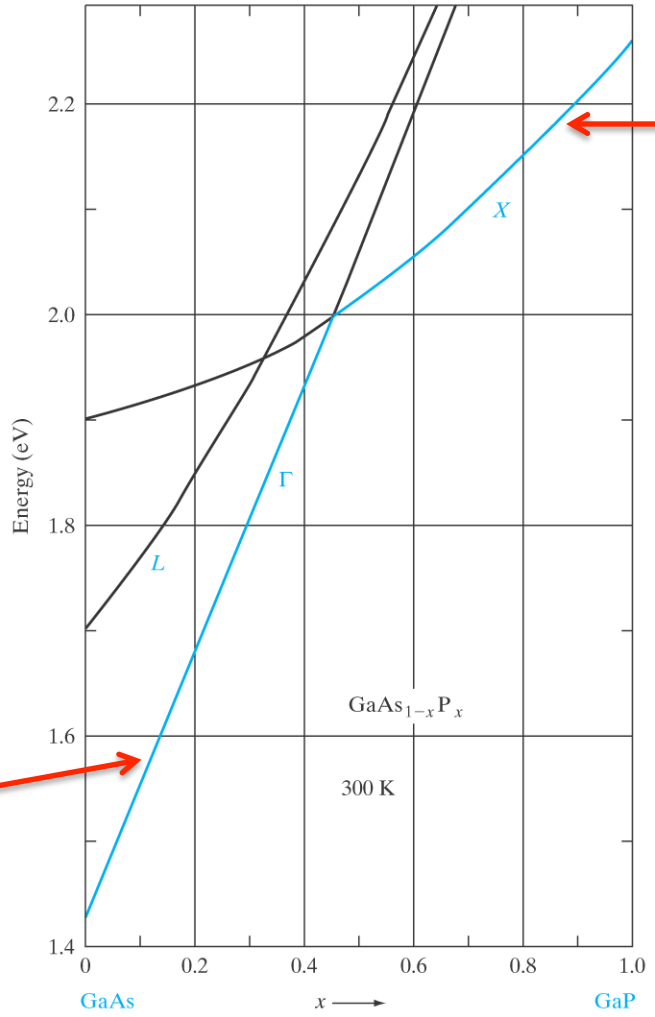
$$\lambda(\mu m) = \frac{1.24}{E_g(eV)}$$

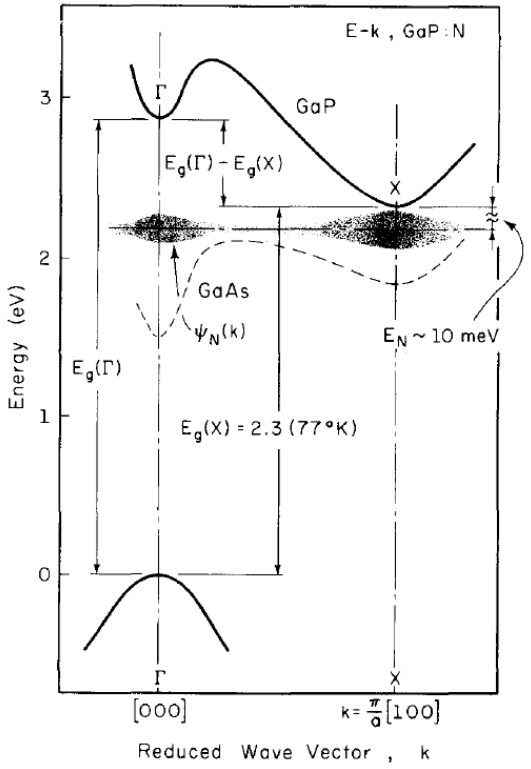


$$\eta_{ext} = (\text{Internal Radiative Efficiency}) \times (\text{Extraction Efficiency})$$

Indirect Gap:
N-Doping

Direct Gap





Holonyak, et al.,
J. Appl. Phys. **44**, 5517 (1973)

Figure 8.11

Conduction band energies as a function of alloy composition for $GaAs_{1-x}P_x$.

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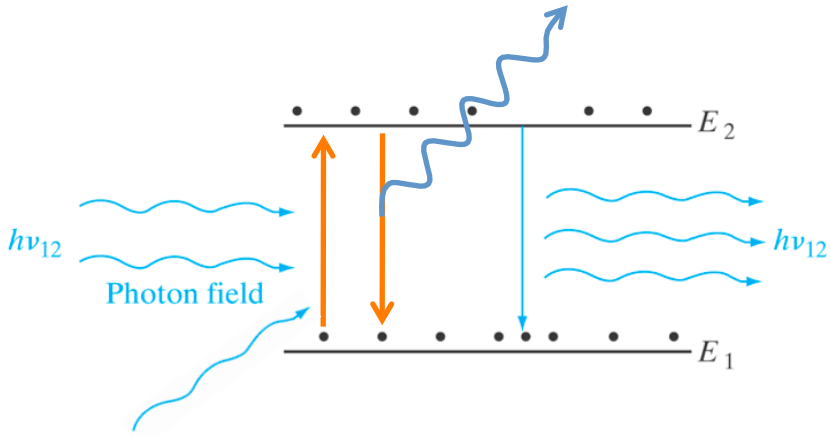
Lasers

Transitions in a 2-State System

LASER: Light Amplification through Stimulated Emission of Radiation



3 EHP-Photon Processes:

- ***Spontaneous Emission:*** Random transition from E_2 to E_1
- ***Absorption:*** Transition from E_1 to E_2 caused by interaction with and annihilation of a photon
- ***Stimulated Emission:*** Transition caused by presence of photon field
  - Stimulated photons have the same energy and phase as the stimulating photon field (monochromatic, coherent)

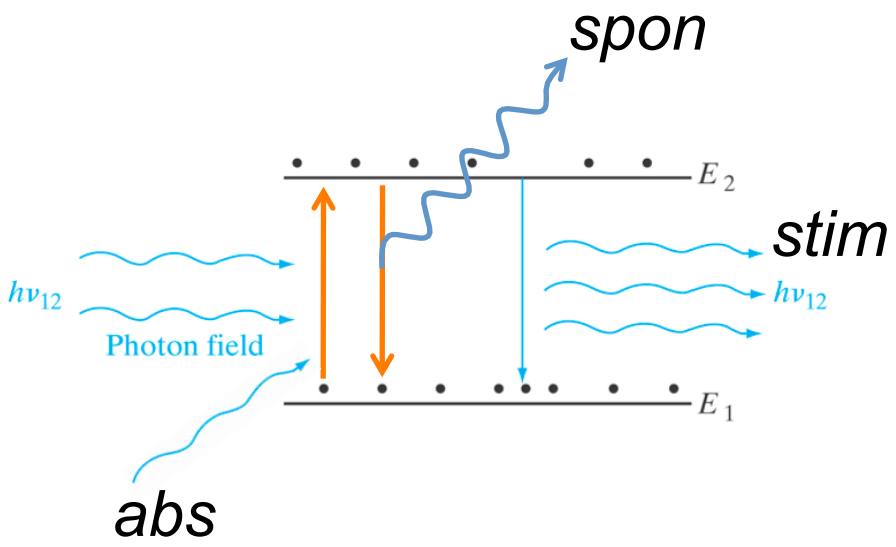
Steady State Condition

For :

$$R_{stim} = B_{21} n_2 \rho(\nu_{12})$$

$$R_{abs} = B_{12} n_1 \rho(\nu_{12})$$

$$R_{spon} = A_{21} n_2$$



In steady state, the emission rates must balance the absorption rate:

$$\left.\frac{dn_2}{dt}\right|_{abs} + \left.\frac{dn_2}{dt}\right|_{spon} + \left.\frac{dn_2}{dt}\right|_{stim} = 0 \quad \text{or}$$

$$B_{12} n_1 \rho(\nu_{12}) = A_{21} n_2 + B_{21} n_2 \rho(\nu_{12})$$

$$[\text{Absorption}] = [\text{Spontaneous Emission}] + [\text{Stimulated Emission}]$$

B_{12}, A_{21}, and B_{21} are the Einstein Coefficients

In steady state, the absorption is the sum of spontaneous and stimulated emission

Conditions for Lasing

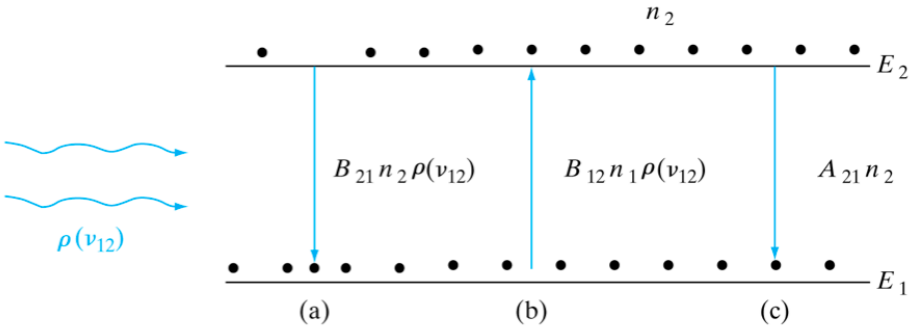
$$\frac{\text{Stimulated emission rate}}{\text{Spontaneous emission rate}} = \frac{B_{21} n_2 \rho(\nu_{12})}{A_{21} n_2} = \frac{B_{21}}{A_{21}} \rho(\nu_{12})$$

A large photon field energy density enhances the stimulated emission rate (optical resonant cavity).

$$\frac{\text{Stimulated emission rate}}{\text{Absorption rate}} = \frac{B_{21} n_2 \rho(\nu_{12})}{B_{12} n_1 \rho(\nu_{12})} = \frac{B_{21}}{B_{12}} \frac{n_2}{n_1}$$

For stimulated emission to exceed absorption, $n_2 > n_1$

Population Inversion needed for stimulated emission to exceed absorption ("***negative temperature***" concept).



$$\frac{n_2}{n_1} = e^{-(E_2 - E_1)/kT} = e^{-h\nu_{12}/kT}$$

if $n_2 > n_1$ then T is "negative"

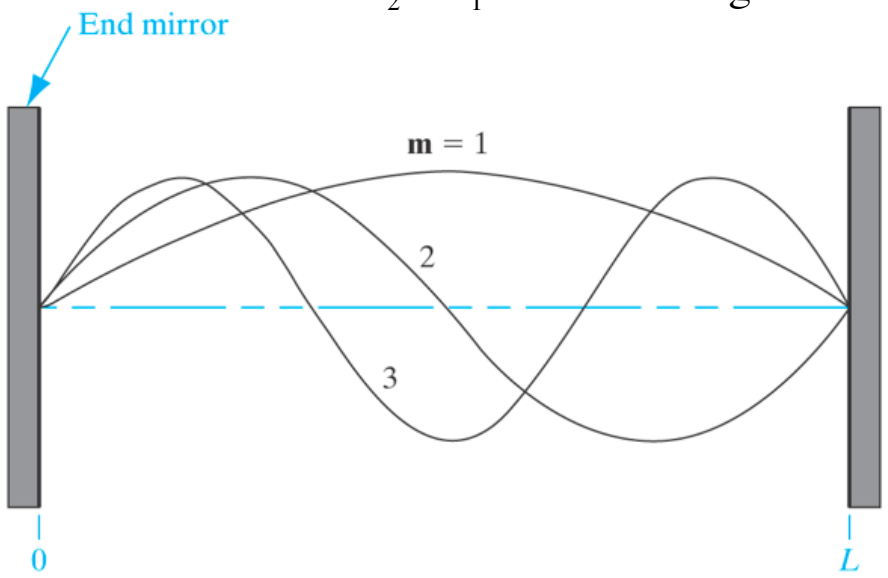
Optical Resonant Cavity: Fabry-Perot Modes

$$L = m\left(\frac{\lambda}{2}\right)$$

λ is the wavelength in the cavity. For a refractive index n:

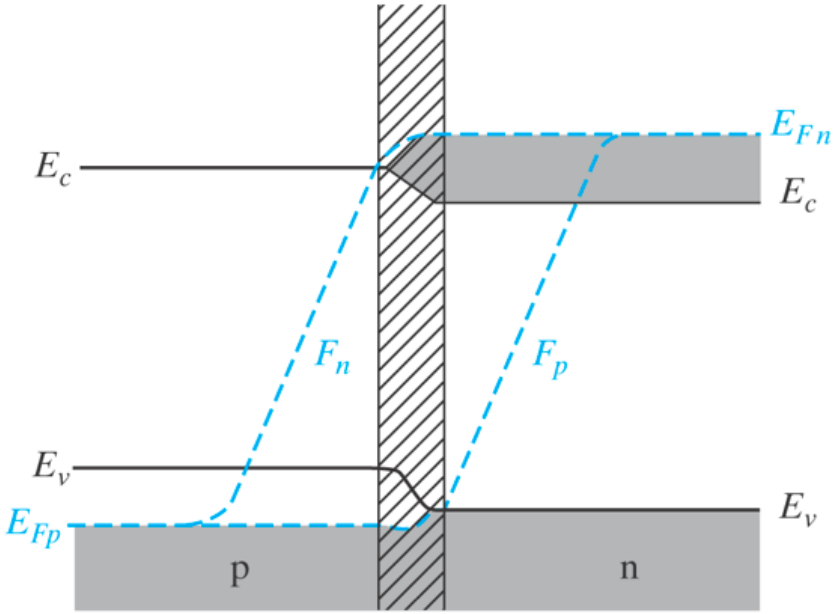
$$\lambda = \frac{\lambda_o}{n}$$

where λ_o is the wavelength in free space.



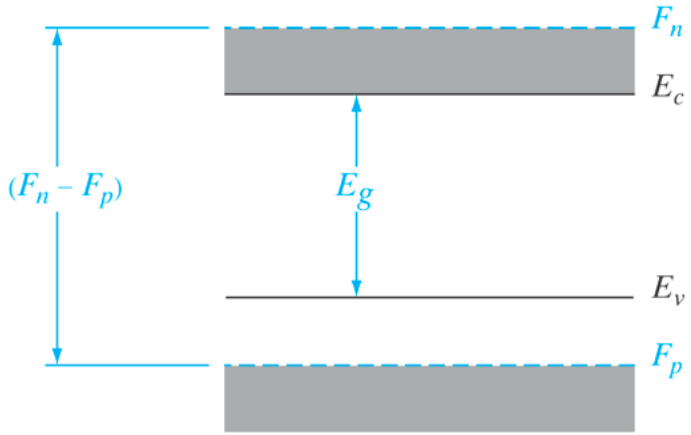
Junction Under Large Bias

- If the bias is such that electrons and holes are injected into and across the transition region in high concentration, an ***inversion region*** is created
- The population inversion is described using quasi-Fermi levels
- The separation of the quasi-Fermi levels is a measure of the population inversion
- Direct bandgap material required



$$n = N_c e^{-(E_c - F_n)/kT} = n_i e^{(F_n - E_i)/kT}$$

$$p = N_v e^{-(F_p - E_v)/kT} = n_i e^{(E_i - F_p)/kT}$$

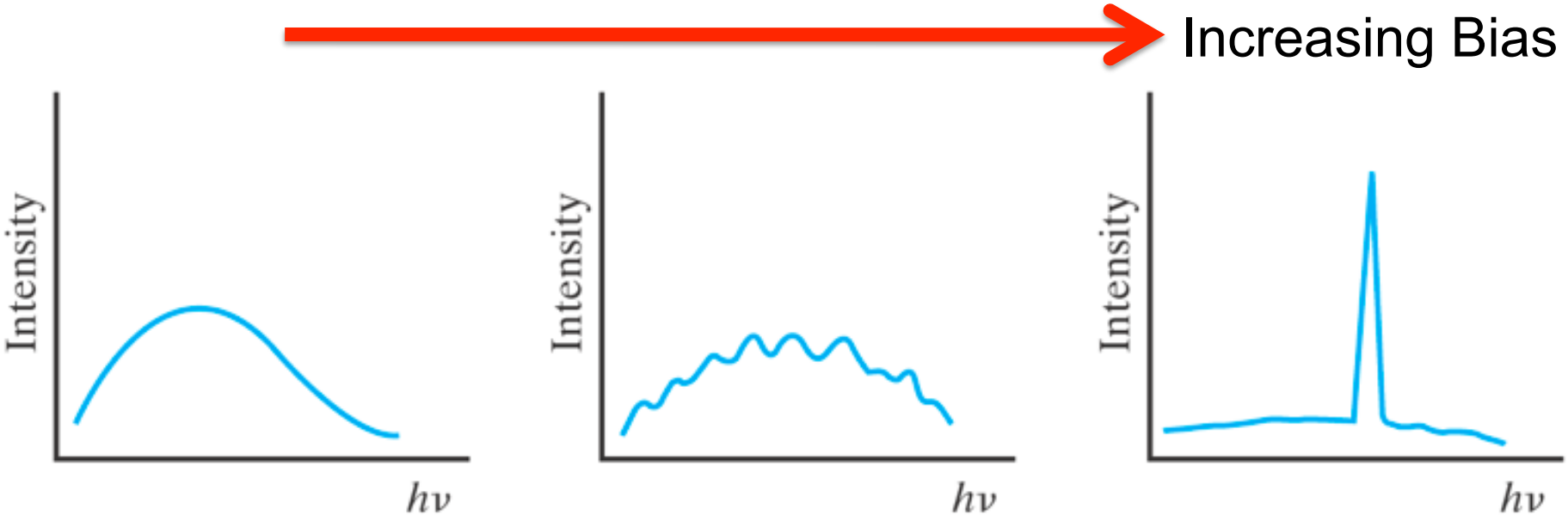


Lasing Condition (Population Inversion):

$(F_n - F_p) > h\nu$ or, for band-edge transitions

$(F_n - F_p) > E_g$

Laser Mode Spacing



$$L = m\left(\frac{\lambda_o}{2n}\right) \text{ so } m = \frac{2Ln}{\lambda_o} \text{ and } \frac{dm}{d\lambda_o} = -\frac{2Ln}{\lambda_o^2} + \frac{2L}{\lambda_o}\frac{dn}{d\lambda_o}$$

Since m is an integer:

$$\frac{\Delta m}{\Delta\lambda_o} = -\frac{2Ln}{\lambda_o^2} + \frac{2L}{\lambda_o}\frac{dn}{d\lambda_o} \text{ and } -\Delta\lambda_o = \frac{\lambda_o^2}{2Ln}\left(1 - \frac{\lambda_o}{n}\frac{dn}{d\lambda_o}\right)^{-1}\Delta m$$

Setting $\Delta m = -1$ will allow calculation of the mode spacing

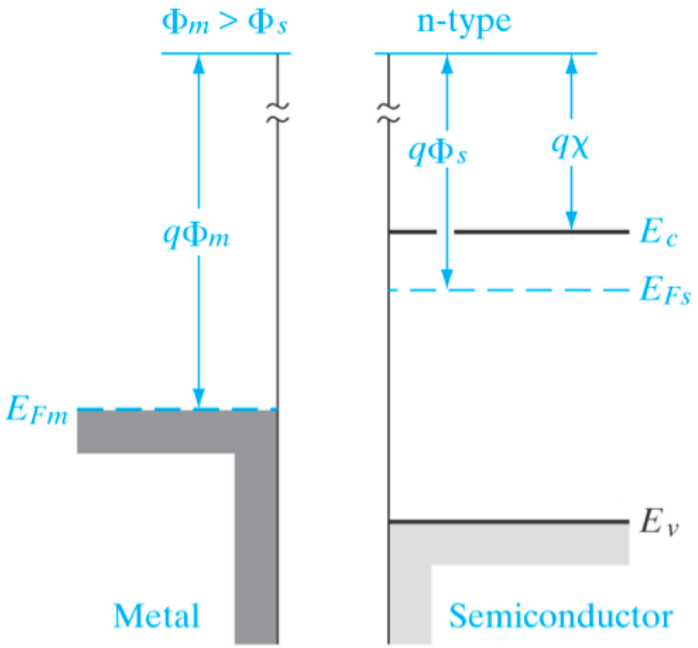
Ideal Metal-Semiconductor Junctions

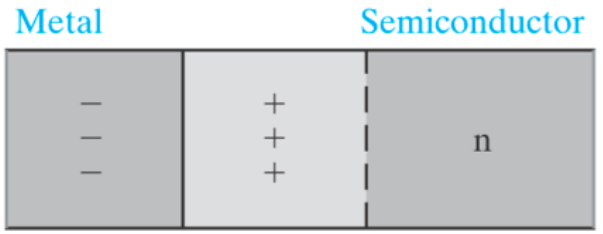
Schottky Barriers

Fermi Level Alignment

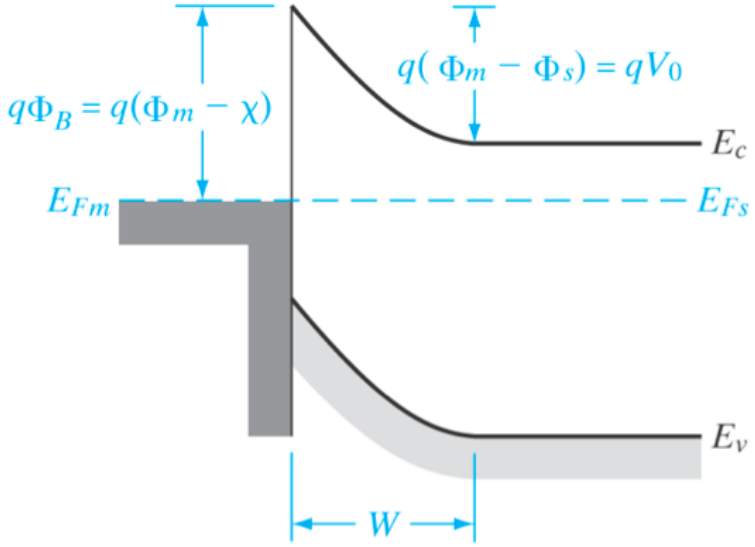
- Metal Work Function $q\Phi_m$: Energy needed to remove an electron at the Fermi level to the vacuum
- When a metal and a semiconductor are brought together, charge transfer occurs to align the Fermi levels ($q\Phi_m = q\Phi_s$)

Metal/Semiconductor Not in Contact



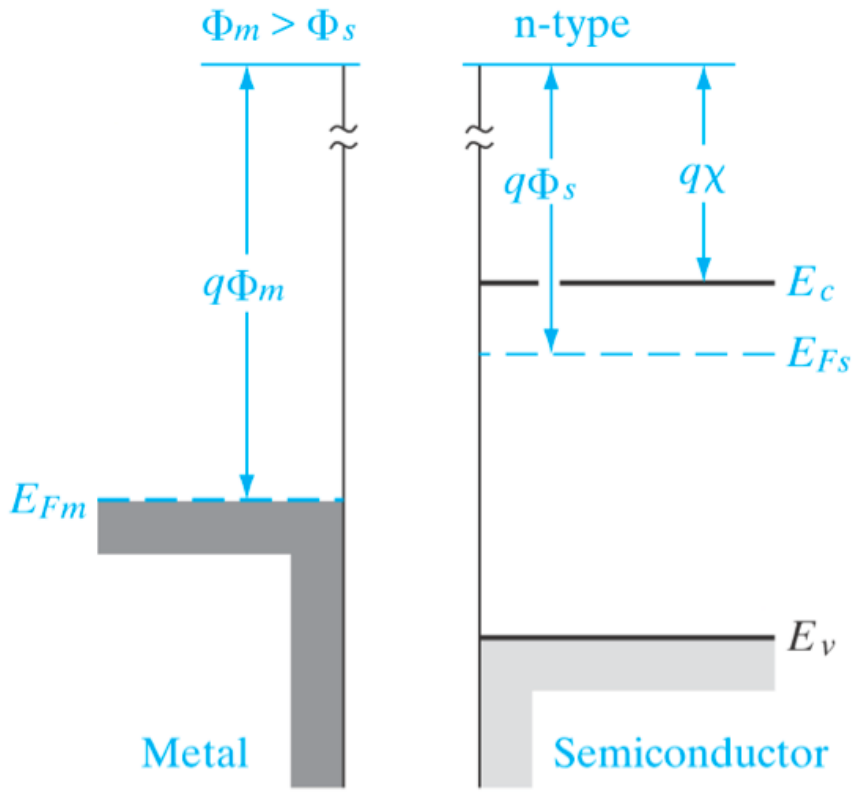


Metal/Semiconductor in Contact



Electron Affinity

- ***Electron Affinity:*** The energy from vacuum level to the conduction band edge



Ideal Metal-Semiconductor Junctions

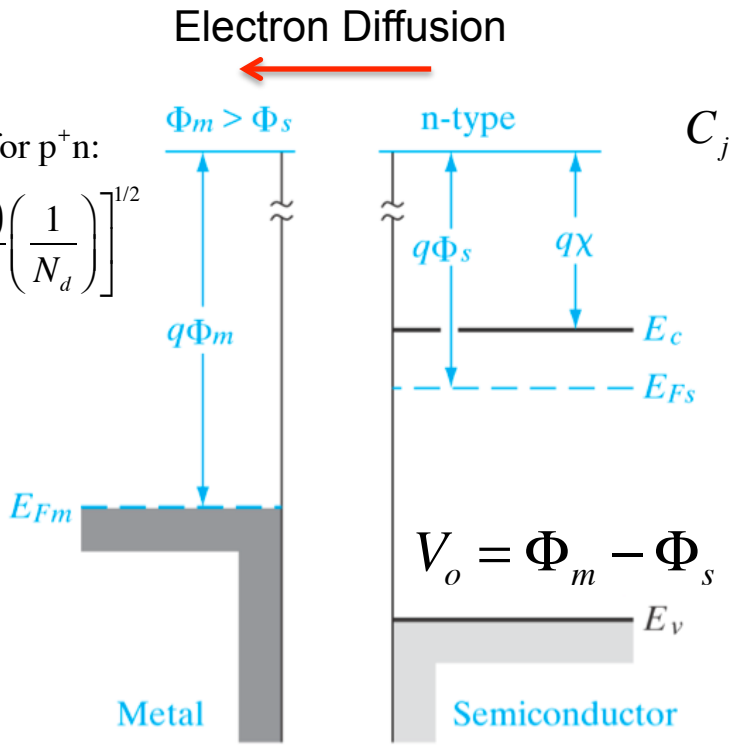
Rectifying Contacts

Case I: $q\Phi_m > q\Phi_s$, n-type Semiconductor

- Charge transfer occurs until Fermi levels align
- A depletion region is formed near the metal-semiconductor junction
- The positive charge due to uncompensated donors matches the negative charge on the metal
- Electric field and bending bands similar to p-n junctions (p^+-n approximation)

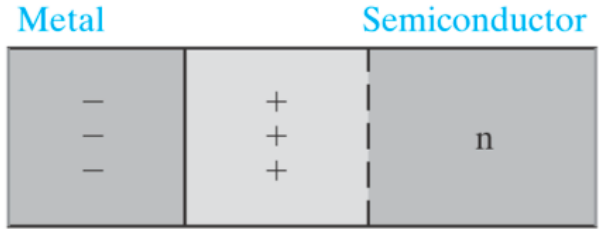
Approximation for p^+n:

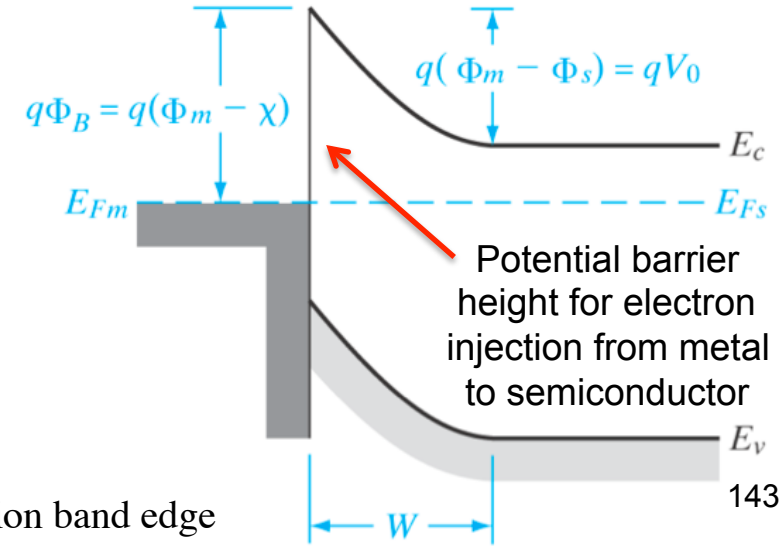
$$W \sim \left[\frac{2\varepsilon(V_0 - V)}{q}\left(\frac{1}{N_d}\right)\right]^{1/2}$$



$$V_o = \Phi_m - \Phi_s$$

$$C_j = \frac{\varepsilon_s A}{W}$$

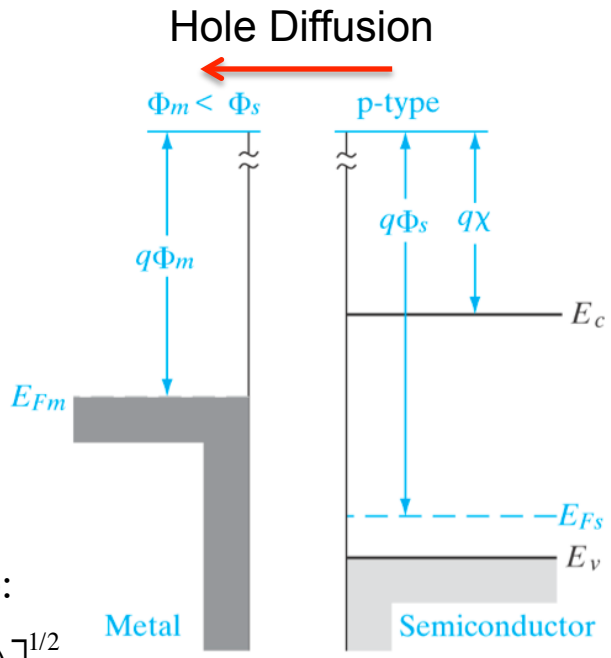


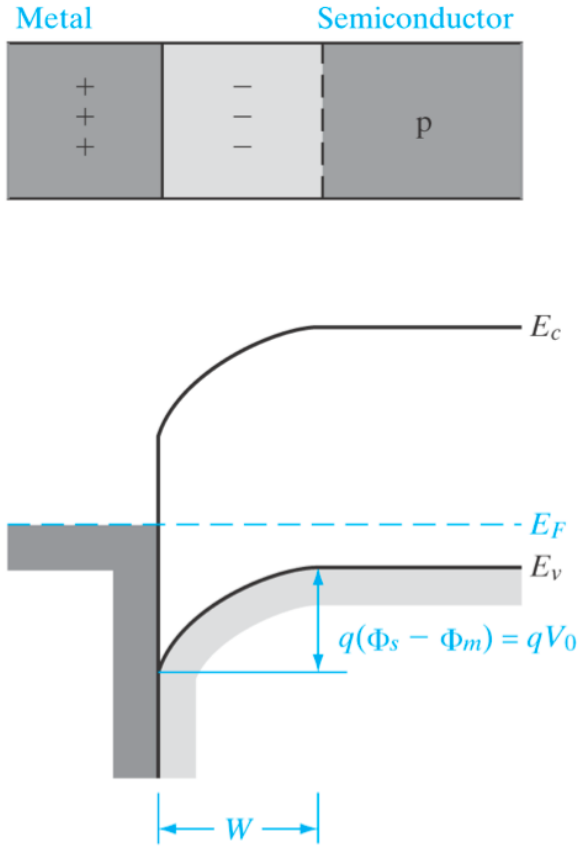


$q\chi \equiv$ electron affinity, measured from vacuum level to conduction band edge

Case II: $q\Phi_s > q\Phi_m$, p-type Semiconductor

- Charge transfer to align Fermi Levels results in a positive charge on the metal side and a negative charge on the semiconductor side
- Depletion region formed in semiconductor – ionized acceptors provide negative charge region
- Potential barrier V_o retards hole diffusion from semiconductor to metal
- Resembles n^+-p junction



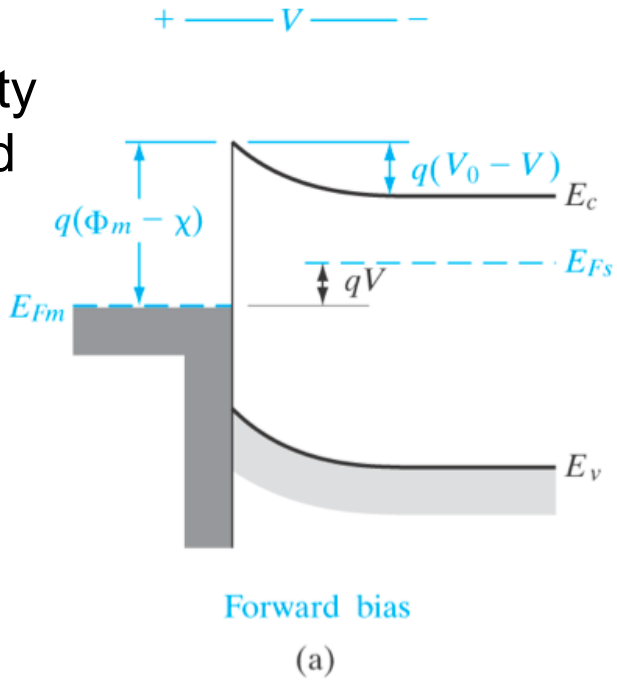


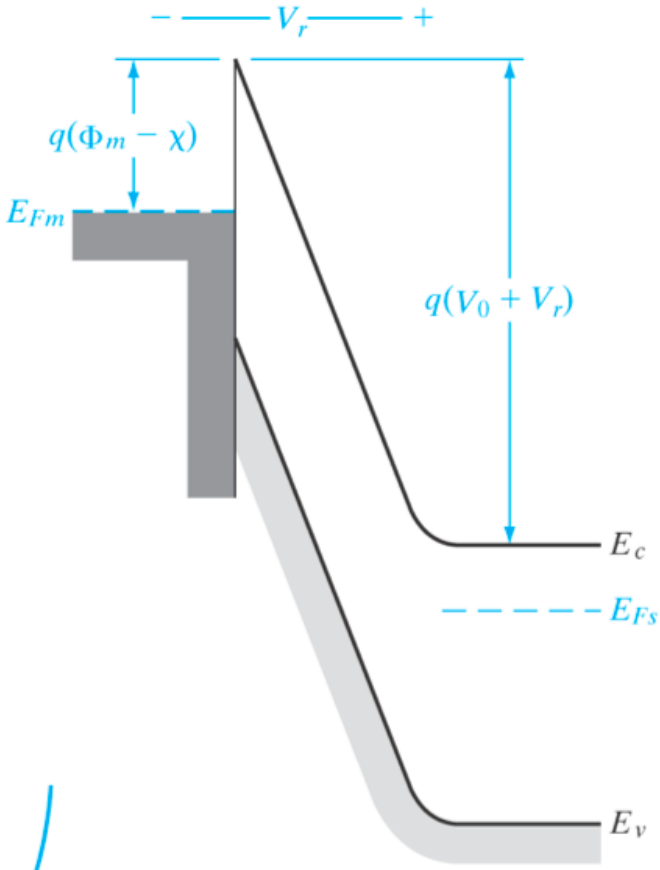
Approximation for n^+p:

$$W \sim \left[\frac{2\varepsilon(V_0 - V)}{q}\left(\frac{1}{N_a}\right)\right]^{1/2}$$

Rectifying Junctions

Reducing the barrier height causes majority carriers to be injected into the metal from the semiconductor





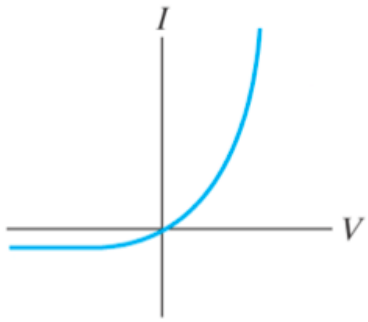
$I = I_o(e^{qV/kT} - 1)$

I_o is distinct from the p-n junction case:

$I_o \propto e^{-q\Phi_B/kT}$ (Boltzmann)

Φ_B is the barrier for electron injection

$\Phi_B = \Phi_m - \chi$ (Independent of applied voltage)



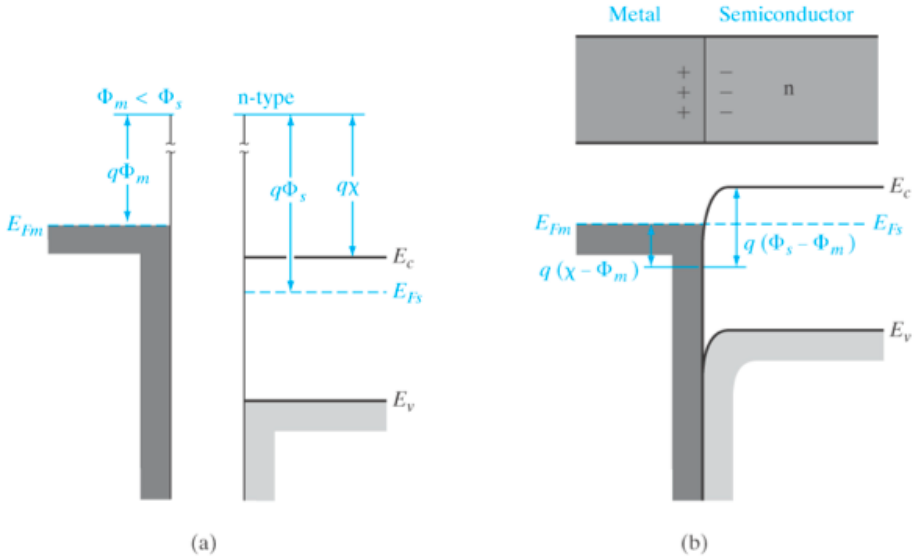
Forward Mode: Positive Bias to Metal

Ideal Metal-Semiconductor Junctions

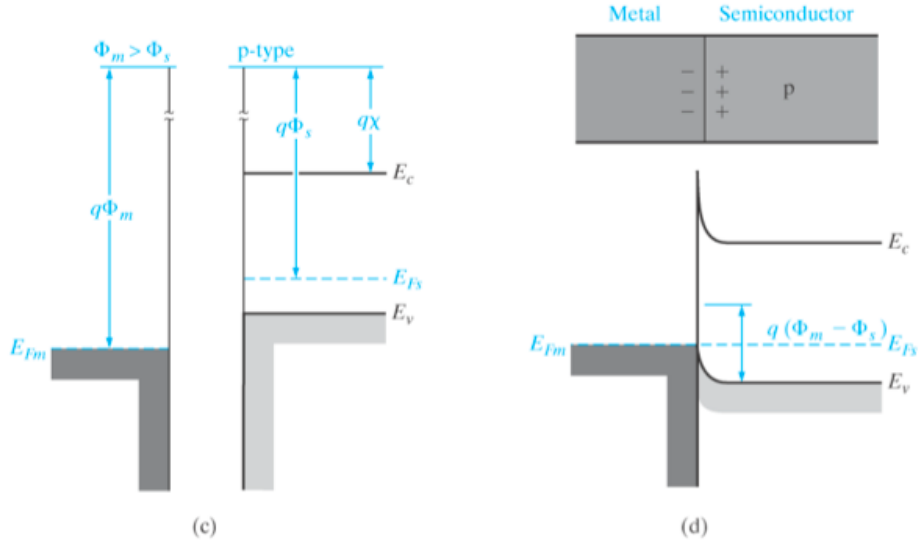
Ohmic Contacts

Ohmic Contacts

Case III:
$\Phi_m < \Phi_s$
n-type



Case IV:
$\Phi_m > \Phi_s$
p-type



- Charge transfer to align Fermi levels
- Induced charged in semiconductor provided by majority carriers as opposed to ionized impurities (no depletion region)
- Practical method of forming ohmic contacts involves heavy doping of semiconductor – small depletion width allows tunneling

Contact Type: Doping and Work Function

	$\Phi_m > \Phi_s$	$\Phi_s > \Phi_m$
n-type	Rectifying	Ohmic
p-type	Ohmic	Rectifying

Metal-Insulator-Semiconductor Field-Effect Transistor (MISFETs)

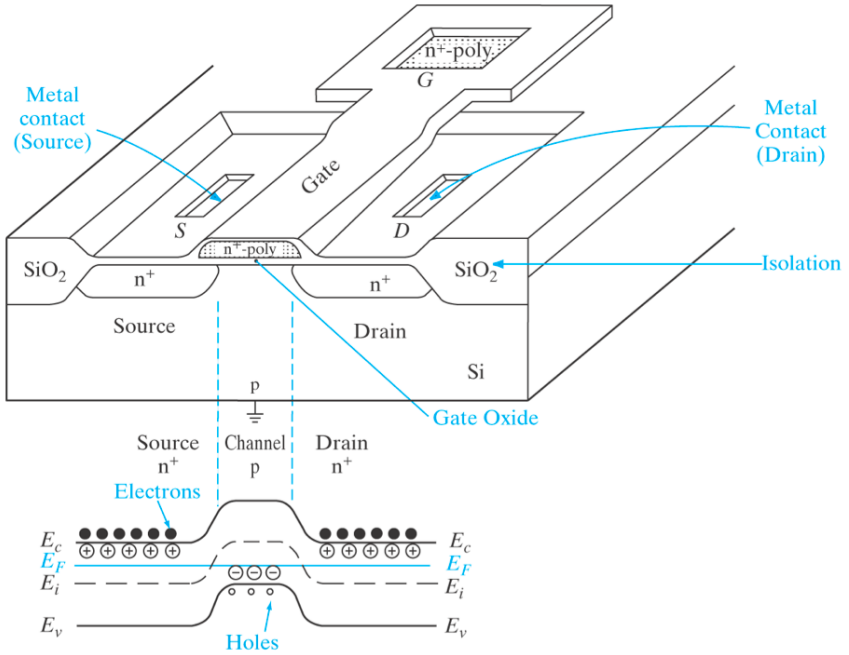
Some Definitions

- FET: Field Effect Transistor
  - JFET (Junction FET): The control (gate) voltage varies the depletion width of a reverse-biased p-n junction
  - MESFET (MEtal-Semiconductor FET): The p-n junction is replaced by a Schottky barrier diode in reverse bias
  - MISFET (Metal-Insulator-Semiconductor FET): The metal electrode is separated from the semiconductor by an insulator
  - MOSFET (Metal-Oxide-Semiconductor FET): The insulator is an oxide
  - FETs are majority carrier, **unipolar** devices
  - FETs are voltage-controlled devices
- BJT: Bipolar Junction Transistor
  - The action of both majority carriers and minority carriers is important
    - **Bipolar device**
  - BJTs are current-controlled devices
- Key Applications: amplification and switching

MISFET Cross Section

- Channel current is controlled by a voltage applied at a gate electrode
- A positive voltage applied to the gate induces positive charge in the metal of the gate
- In the semiconductor, negative charges are induced in response to the positive charge on the gate
- The induced electrons are mobile and form a conductive channel
- Current flows from drain to source
- The gate voltage varies the conductance of the channel – voltage controlled potential barrier or voltage-controlled resistor

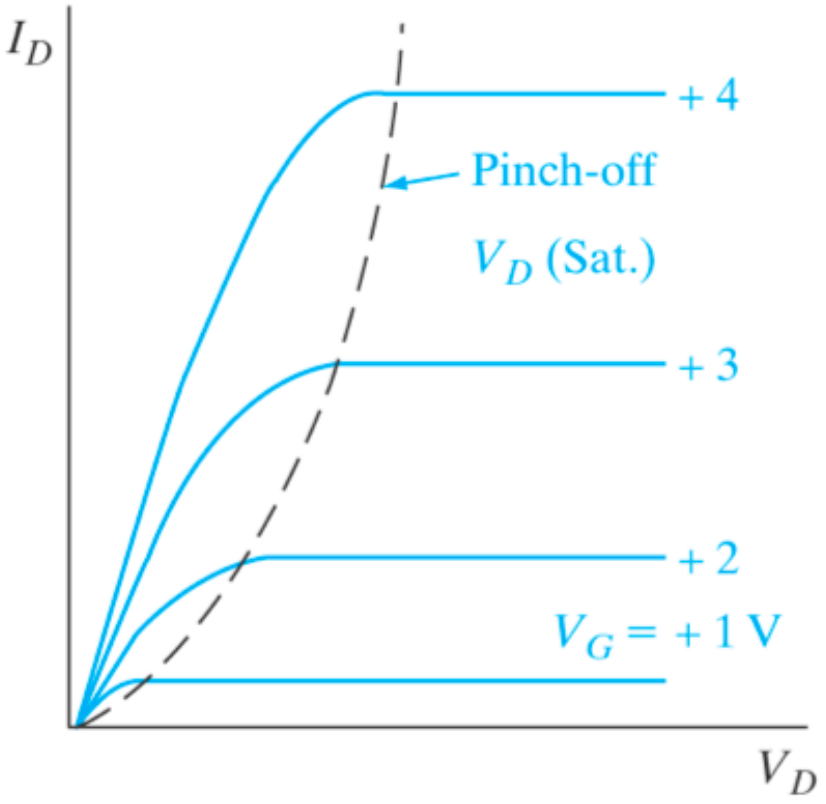
Enhancement Mode n-Channel MOSFET



MISFET I-V Characteristic

- Threshold Voltage: voltage required to induce a conductive channel ("normally off" device)
  - For an n-channel device V_T is positive
  - For a p-channel device V_T is negative
- "Normally On" devices are called "***depletion mode***" transistors, while "Normally Off" devices are called "***enhancement mode***" transistors
- Initially, current increases linearly with voltage ("linear regime")
- Current flow causes a voltage drop across the channel, and eventually as V_D is increased the channel is pinched off and the I-V enters the "saturation region" (V_G-V_D<V_T)

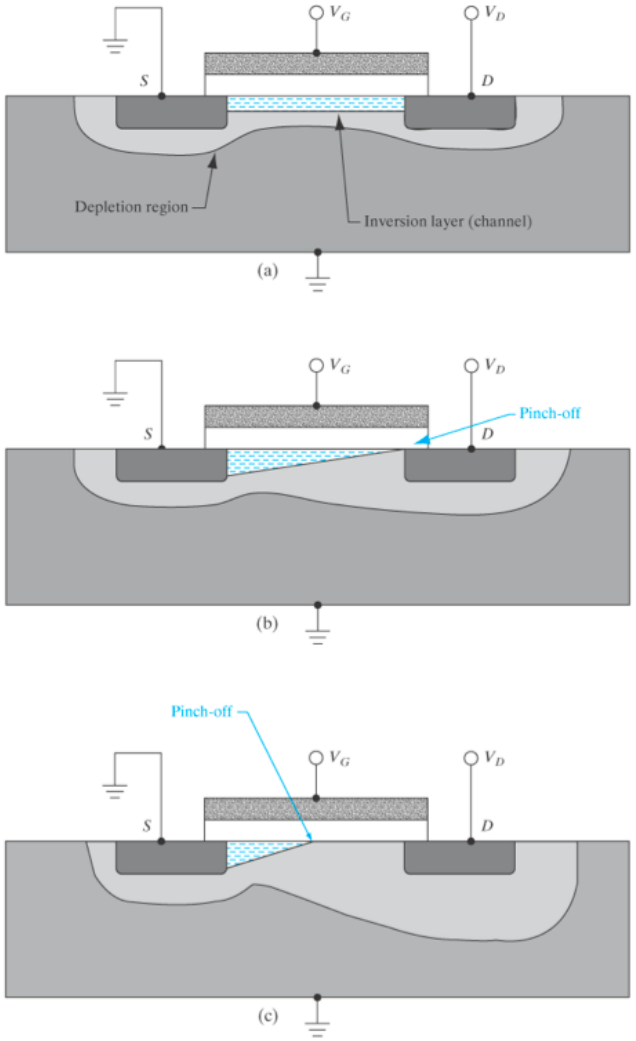
Current-Voltage Characteristic



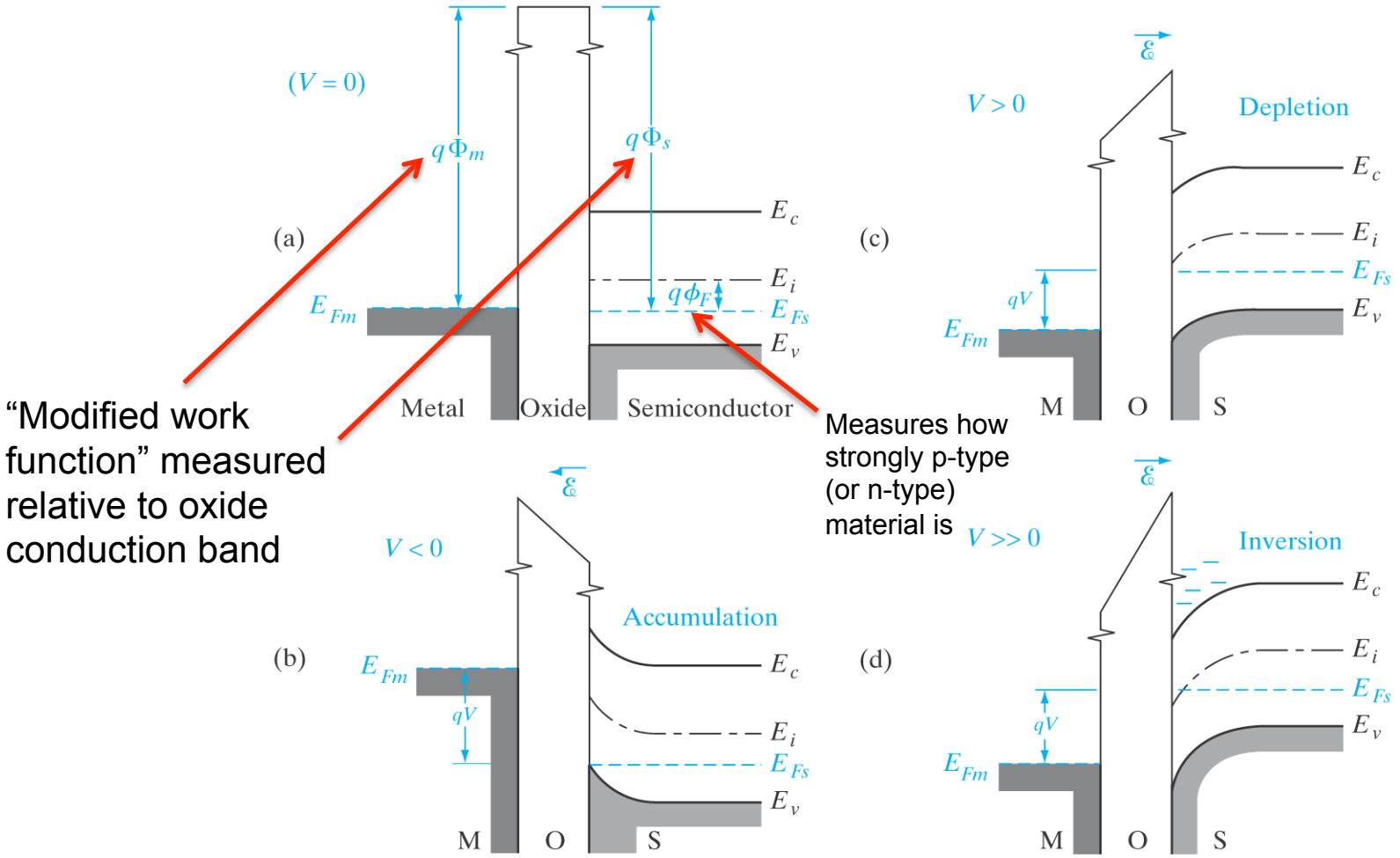
Visualizing the Channel

- In the saturation region, electrons are pulled from the channel and travel at the saturation drift velocity due to the high electric fields between the edge of the channel and the drain
- MOSFETs are useful in digital circuits by switching them from the “on” state to the “off” state
- The dc input impedance of the gate is large as it sits on an insulating oxide
- The ac input impedance is small (capacitive) – power is dissipated in switching
- The n-channel device is preferred in silicon: electron mobility is higher than hole mobility

Channel With Increasing V_D



The Ideal MOS Capacitor



"Modified work function" measured relative to oxide conduction band

Measures how strongly p-type (or n-type) material is

Assume $\Phi_m = \Phi_s$

Figure 6.12

Band diagram for the ideal MOS structure at: (a) equilibrium; (b) negative voltage causes hole accumulation in the p-type semiconductor; (c) positive voltage depletes holes from the semiconductor surface; (d) a larger positive voltage causes inversion–an "n-type" layer at the semiconductor surface.

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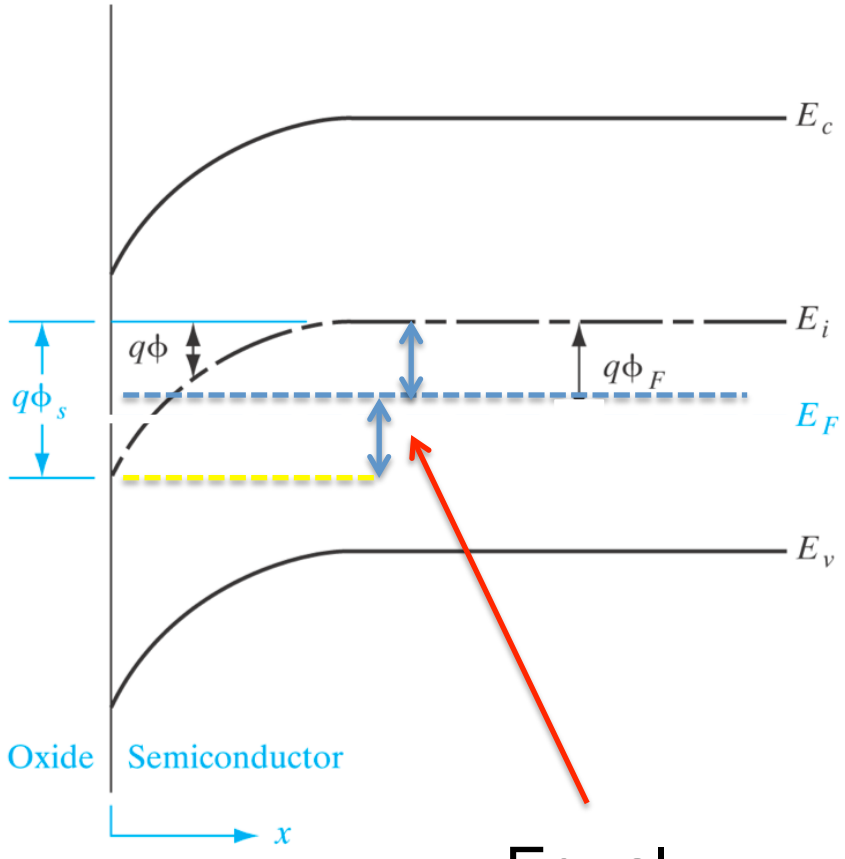
Strong Inversion

- In the case where Φ_s has moved past the intrinsic level by an amount Φ_F the material is as n-type as it was p-type and is in "strong inversion"



Strong Inversion:

$$\phi_s(inv.) = 2\phi_F = 2\frac{kT}{q}\ln\frac{N_a}{n_i}$$

for p-type material

Equal Distance from E_i

Carrier Concentration

In Equilibrium

For Electrons:

$$n_o = n_i e^{(E_F - E_i)/kT} = n_i e^{-q\phi_F/kT}$$

Using the term $\phi(x)$ shown in the figure at any point x:

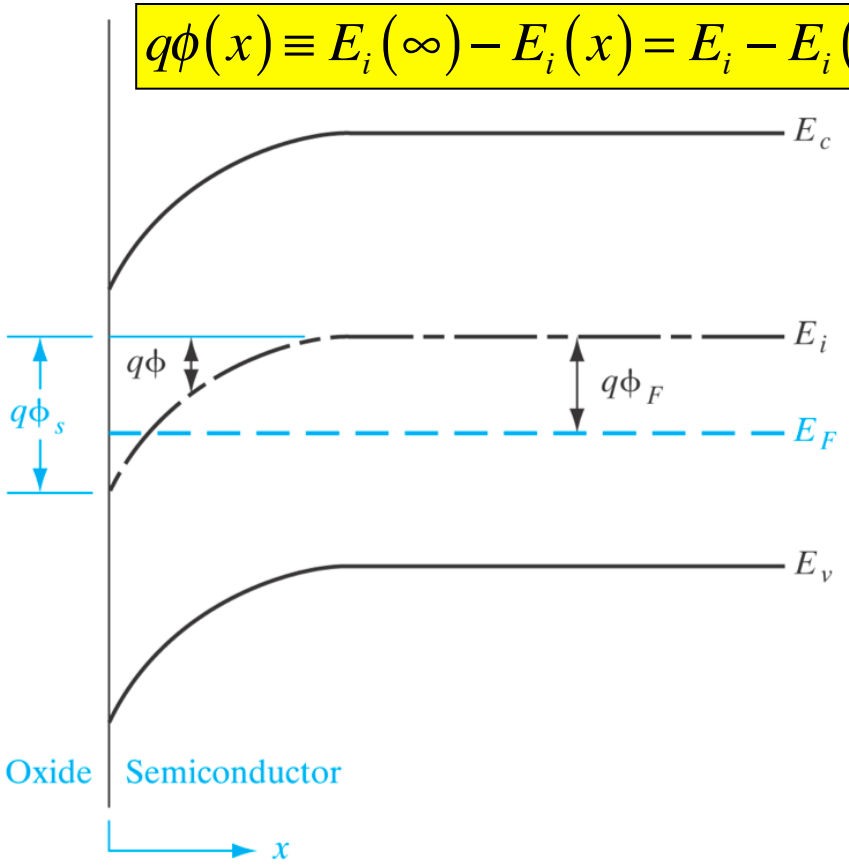
$$n(x) = n_i e^{-q(\phi_F - \phi(x))/kT} = n_o e^{q\phi(x)/kT}$$

For Holes:

$$p_o = n_i e^{q\phi_F/kT}$$

$$p(x) = p_o e^{-q\phi(x)/kT}$$

$$q\phi(x) \equiv E_i(\infty) - E_i(x) = E_i - E_i(x)$$



$$\phi(x=0) = \phi_s$$

$$\phi(x=\infty) = 0$$

Solving for $\mathcal{E}$(x) and Φ(x)

Poisson's Equation:

$$\frac{\partial^2\phi}{\partial x^2} = -\frac{\rho(x)}{\varepsilon_s} \quad \left[\text{from } \nabla^2\phi = -\frac{\rho}{\varepsilon}\right]$$

Charge Density Expression:

$$\rho(x) = q\left(N_d^+ - N_a^- + p(x) - n(x)\right)$$

Using these plus the expressions from the previous slide:

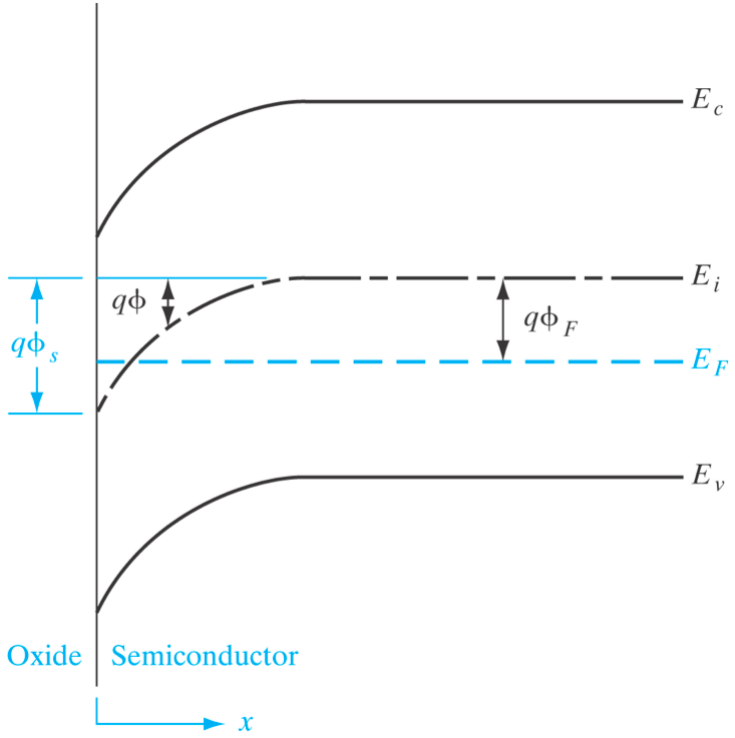
$$\frac{\partial^2\phi}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial x}\right) = -\frac{q}{\varepsilon_s}\left[n_o - p_o + p_o e^{-q\phi(x)/kT} - n_o e^{q\phi(x)/kT}\right]$$

$$= -\frac{q}{\varepsilon_s}\left[p_o\left(e^{-q\phi(x)/kT} - 1\right) - n_o\left(e^{q\phi(x)/kT} - 1\right)\right]$$

Also, $\mathcal{E}(x) = -\dfrac{\partial\phi}{\partial x}$

At equilibrium:

$$N_d^+ - N_a^- + p_0 - n_0 = 0 \quad \Rightarrow \quad N_d^+ - N_a^- = n_0 - p_0$$



Electron and Hole Concentration:

$$n_o = n_i e^{-q\phi_F/kT}$$

$$n(x) = n_i e^{-q\left(\phi_F - \phi(x)\right)/kT} = n_o e^{q\phi(x)/kT}$$

$$p_o = n_i e^{q\phi_F/kT}$$

$$p(x) = p_o e^{-q\phi(x)/kT}$$

Electric Field and Debye Length

$$\frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial x}\right) = -\frac{q}{\varepsilon_s}\left[p_o\left(e^{-q\phi(x)/kT} - 1\right) - n_o\left(e^{q\phi(x)/kT} - 1\right)\right] = -\frac{\partial \mathcal{E}}{\partial x}$$

$$\frac{\partial \phi}{\partial x} = -\mathcal{E}$$

but $$-\frac{\partial \mathcal{E}}{\partial x} = -\frac{\partial \phi}{\partial x}\frac{\partial \mathcal{E}}{\partial \phi} = -\frac{\partial \phi}{\partial x}\frac{\partial\left(-\frac{\partial \phi}{\partial x}\right)}{\partial \phi} = \frac{\partial \phi}{\partial x}\frac{\partial\left(\frac{\partial \phi}{\partial x}\right)}{\partial \phi} = \mathcal{E}\frac{\partial \mathcal{E}}{\partial \phi}$$

Derivative Chain Rule

Integrating from the bulk (flat band, $\mathcal{E} = 0$) to the surface:

$$\int_0^{\mathcal{E}} \frac{\partial \phi}{\partial x} d\left(\frac{\partial \phi}{\partial x}\right) = \int_0^{\mathcal{E}} \mathcal{E}\, d\mathcal{E} = -\frac{q}{\varepsilon_s}\int_0^{\phi}\left[p_o(e^{-q\phi/kT} - 1) - n_o(e^{q\phi/kT} - 1)\right] d\phi$$

$$\mathcal{E}^2 = \left(\frac{2kTp_o}{\varepsilon_s}\right)\left[\left(e^{-\frac{q\phi(x)}{kT}} + \frac{q\phi(x)}{kT} - 1\right) + \frac{n_o}{p_o}\left(e^{\frac{q\phi(x)}{kT}} - \frac{q\phi(x)}{kT} - 1\right)\right]$$

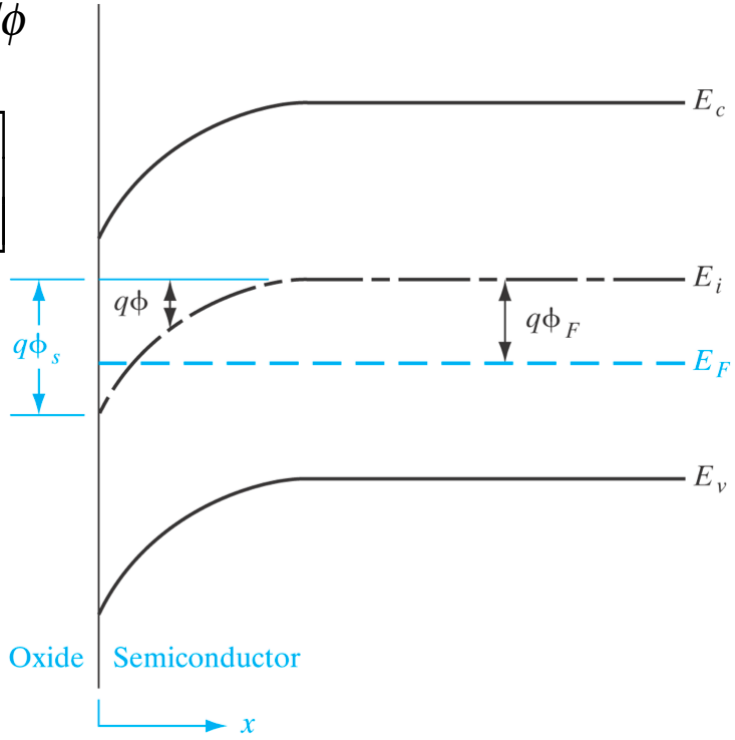
At the surface (x=0) the electric field is:

$$\mathcal{E}_s = \frac{\sqrt{2}kT}{qL_D}\left[\left(e^{-\frac{q\phi_s}{kT}} + \frac{q\phi_s}{kT} - 1\right) + \frac{n_o}{p_o}\left(e^{\frac{q\phi_s}{kT}} - \frac{q\phi_s}{kT} - 1\right)\right]^{\frac{1}{2}}$$

where L_D is the "Debye screening length"

$$L_D = \sqrt{\frac{\varepsilon_s kT}{q^2 p_o}}$$

The magnitude of the distance needed to screen out charge

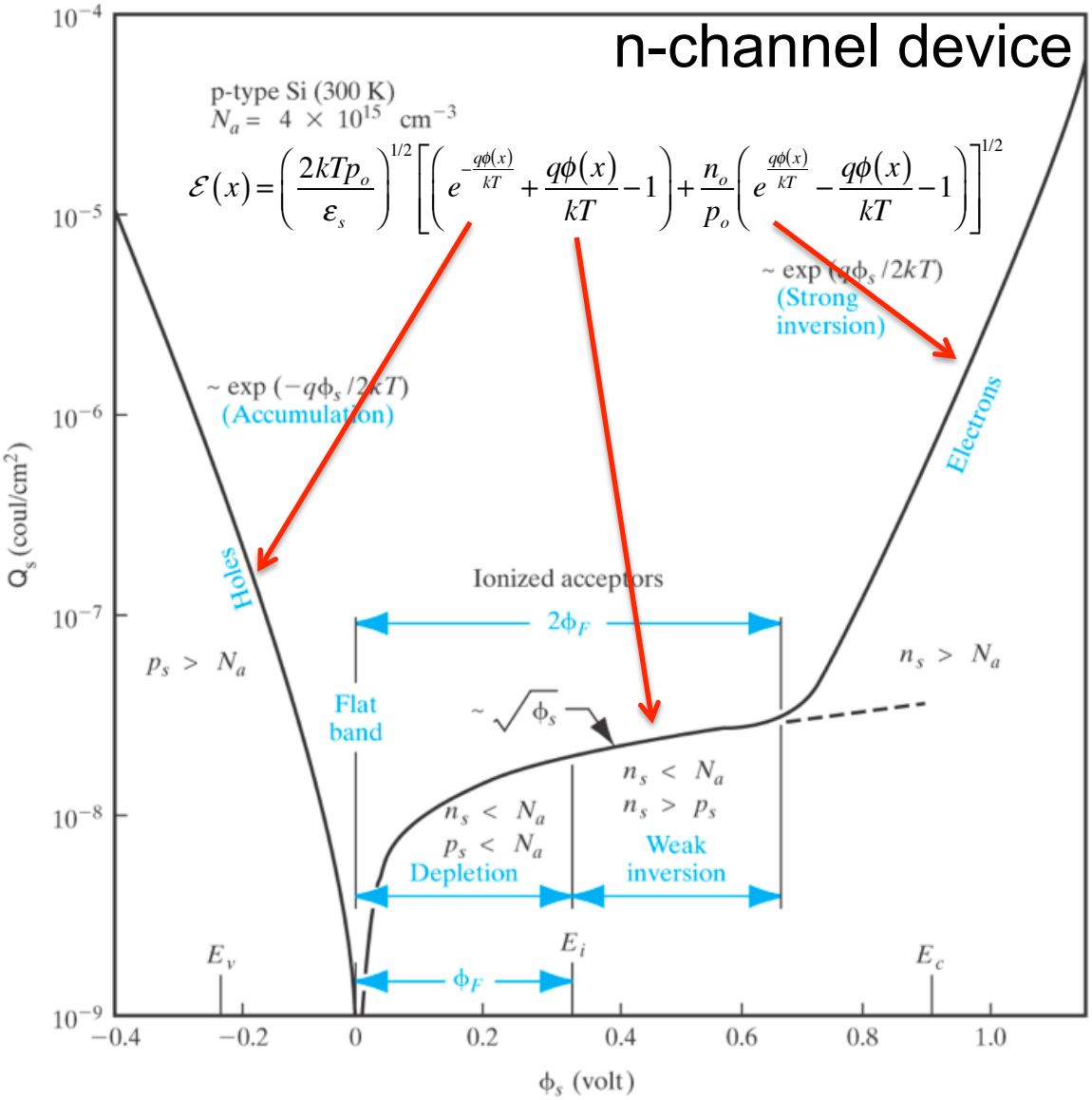


Space Charge Versus Surface Potential

Using Gauss' Law at the surface:

$$Q_s = -\varepsilon_s \mathcal{E}_s$$

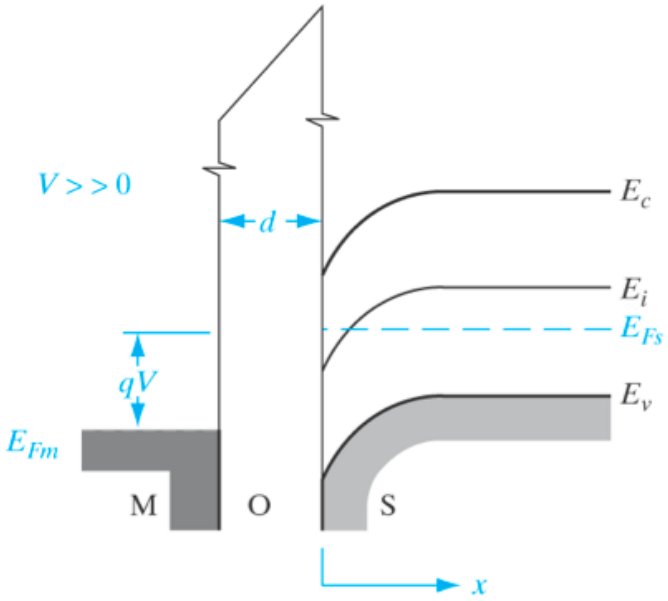
- When the surface potential is zero, the net space charge is zero
- When the surface potential is negative, majority carriers (holes) are attracted to the surface forming an accumulation layer (~20 nm thick)
- When the surface potential is positive, initially a depletion region forms (space charge), and then an inversion layer with mobile electrons (triangular potential well with a thickness of ~5 nm)



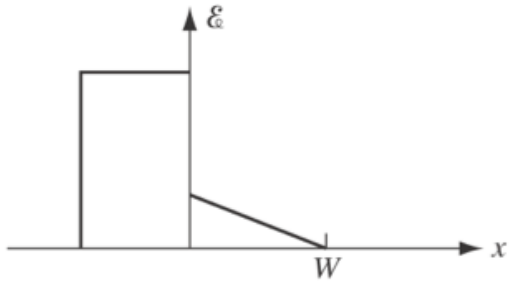
Charge Density, Field, and Potential

n-channel device

+V to Metal

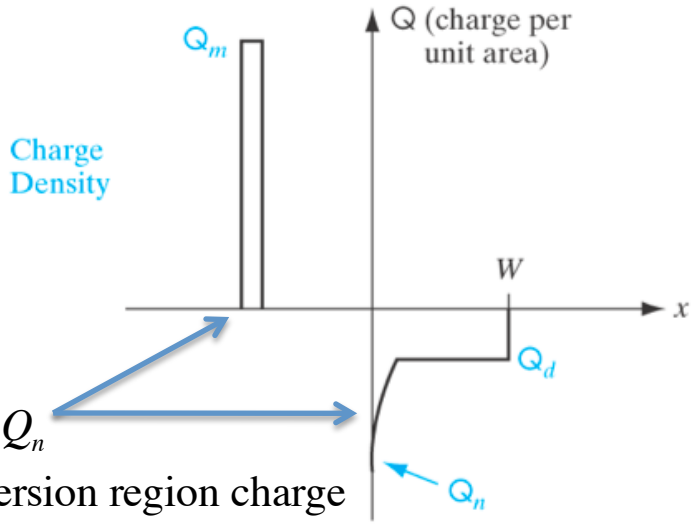


Electric Field



$V = V_i + \phi_s$

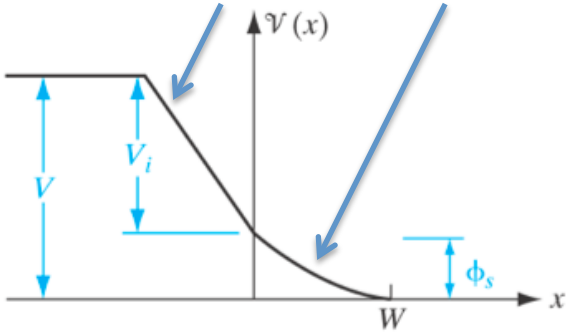
The applied voltage is distributed between the insulator and semiconductor



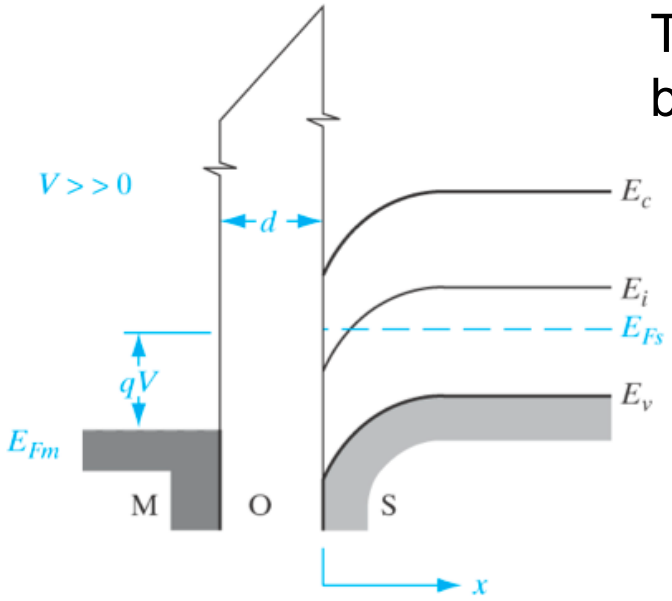
$Q_m = -Q_s = qN_aW - Q_n$

Q_n is the mobile inversion region charge

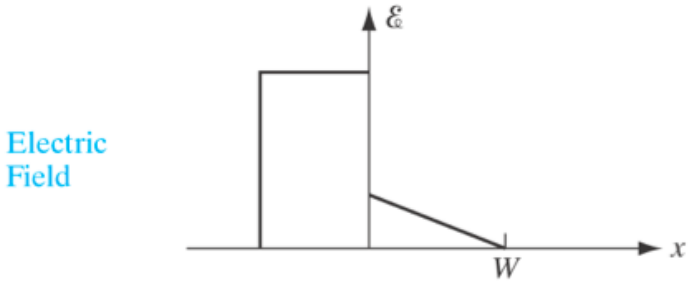
Electrostatic Potential



Insulator Voltage



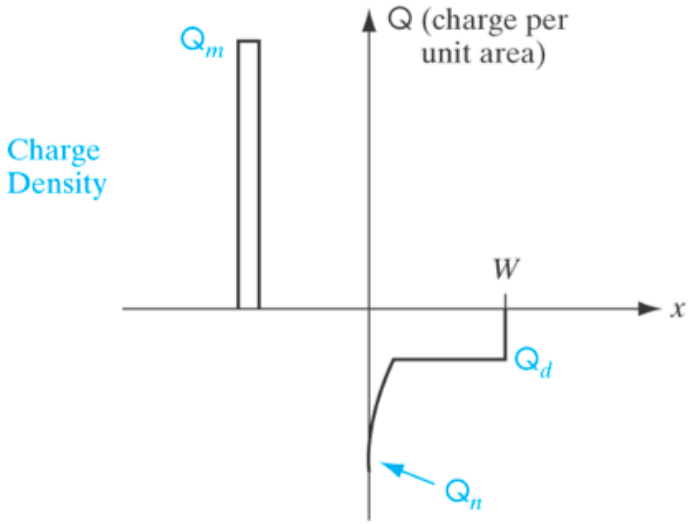
The voltage across the insulator is charge divided by the capacitance

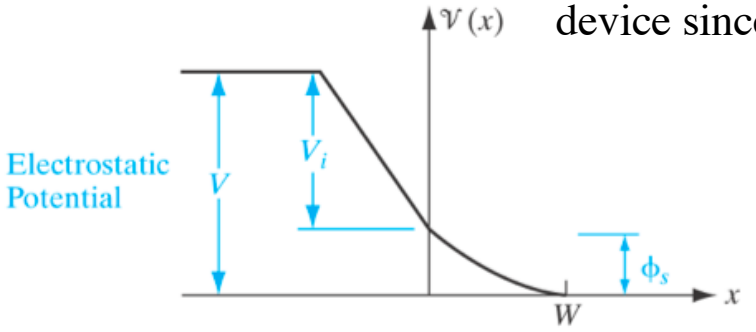


$$V_i = \frac{-Q_s d}{\varepsilon_i} = \frac{-Q_s}{C_i}$$

C_i is the insulator capacitance

V_i is positive for a n-channel device since Q is negative.





MOS C-V Characteristic

Depletion Width & Threshold Voltage

The depletion approximation is used to solve for the depletion width W as a function of ϕ_s

Recall, for n^+p:

$$W = \left[\frac{2\varepsilon(V_0 - V)}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2} \simeq \left[\frac{2\varepsilon(V_0 - V)}{qN_a}\right]^{1/2}$$

$$W = \left[\frac{2\varepsilon_s \phi_s}{qN_a}\right]^{1/2}$$ ← p-type material, "n-channel" device

The depletion width increases until strong inversion is reached. After that, increasing voltage produces stronger inversion as opposed to more depletion.

$$W_m = \left[\frac{2\varepsilon_s \phi_s(inv.)}{qN_a}\right]^{1/2} = 2\left[\frac{\varepsilon_s kT \ln(N_a / n_i)}{q^2 N_a}\right]^{1/2} \quad \text{since} \quad \phi_s(inv.) = 2\phi_F = 2\frac{kT}{q}\ln\frac{N_a}{n_i}$$

The charge per unit area at strong inversion is:

$$Q_d = -qN_a W_m = -2\left(\varepsilon_s qN_a \phi_F\right)$$

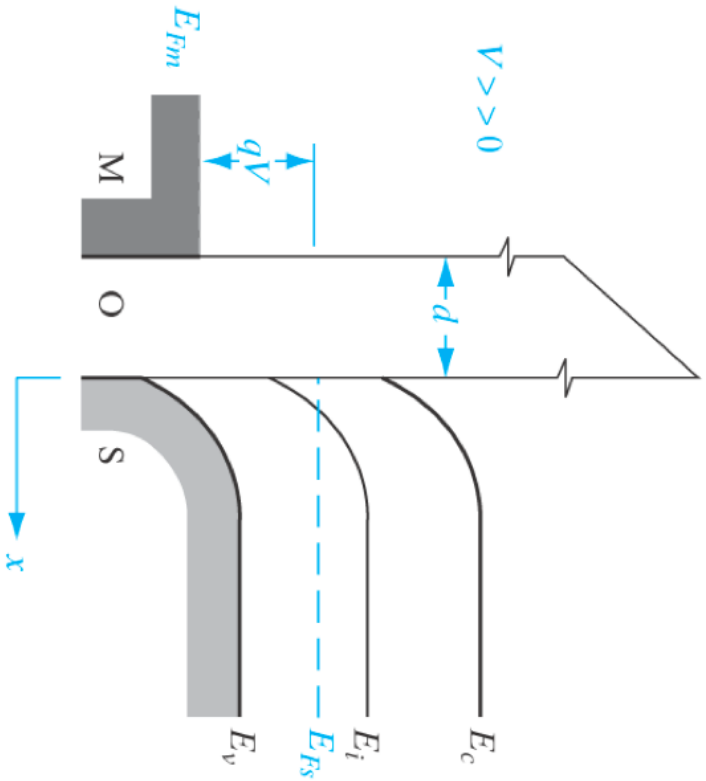
The applied voltage to achieve strong inversion must be large enough to create Q_d plus the surface potential $\phi_s(inv.)$:

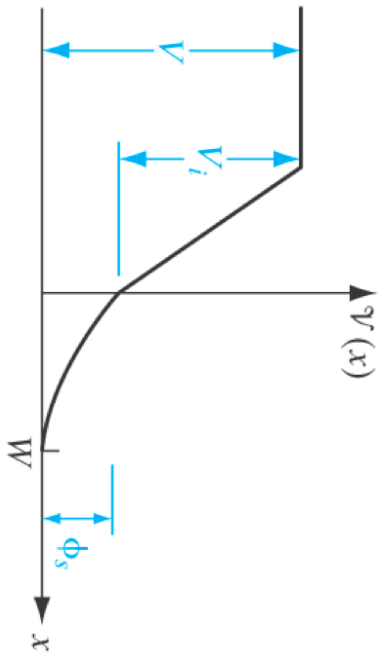
Threshold Voltage : $$V_T = -\frac{Q_d}{C_i} + 2\phi_F \text{ since } \phi_s(inv.) = 2\phi_F$$

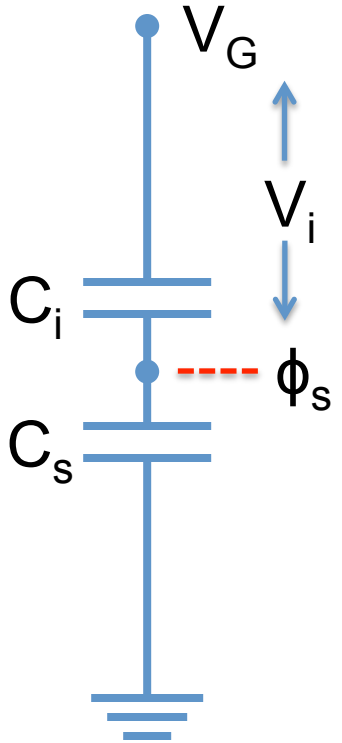
Capacitance and Voltage in Depletion

n-channel case

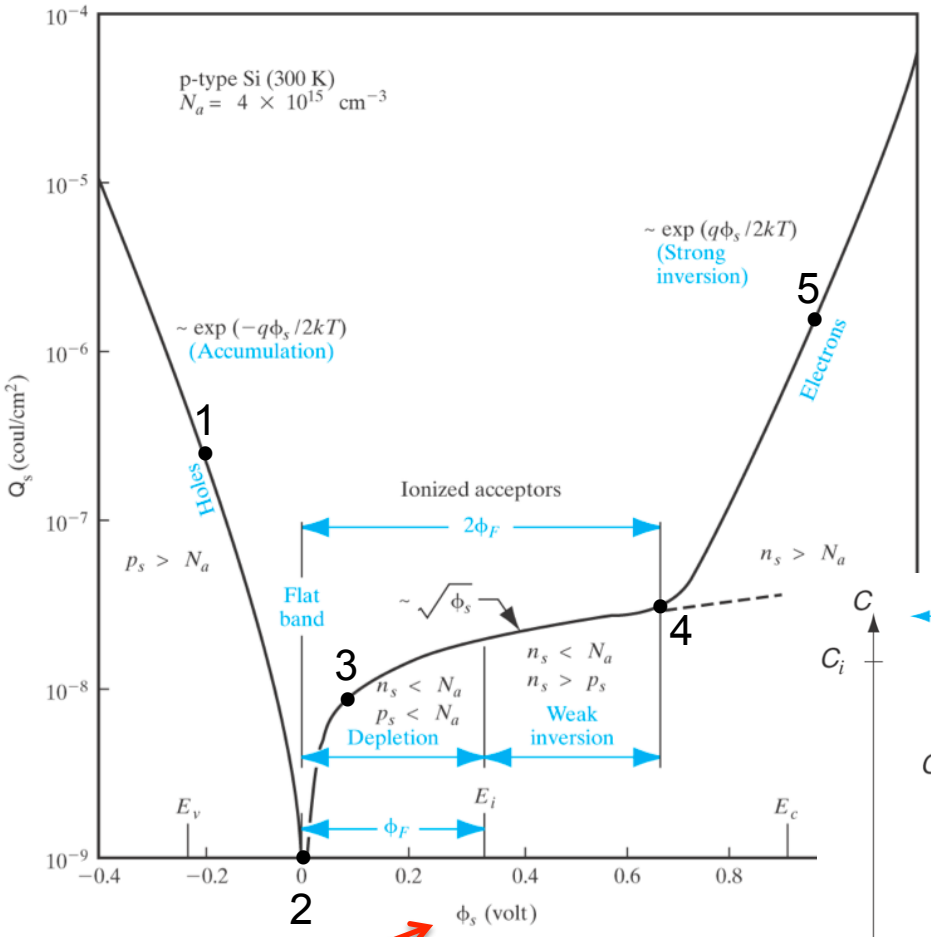
$$V_i = V_G - \phi_s = \frac{-Q_s}{C_i} = \frac{-Q_s d}{\varepsilon_i}$$



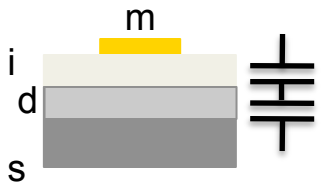




Capacitance Derivation

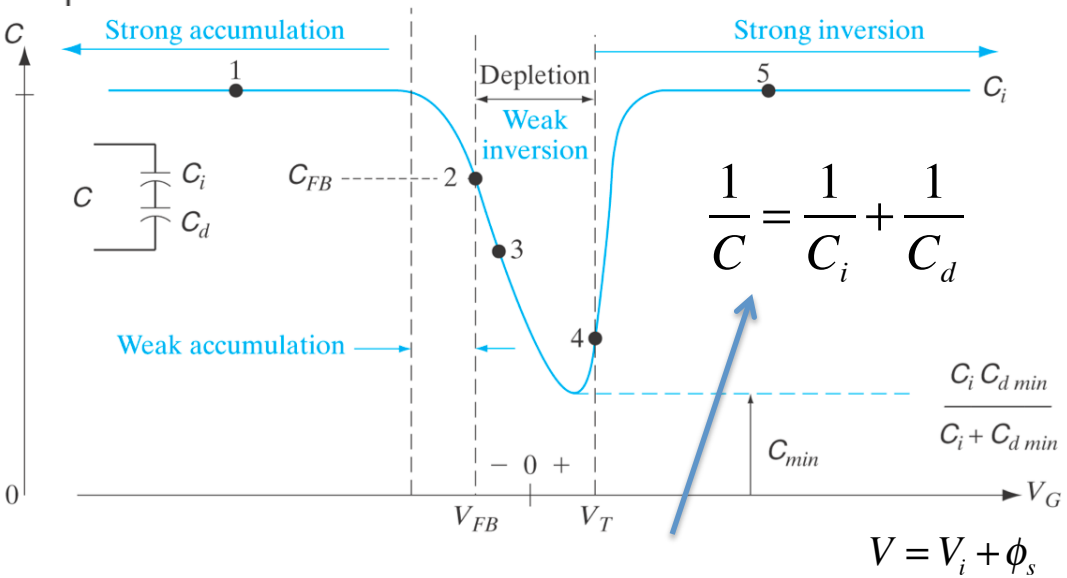


$$\phi_s = V_G - V_i$$



- The semiconductor capacitance C_s is the slope of the Q_s-ϕ_s plot
- The insulator capacitance C_i is fixed, and determined by oxide thickness and dielectric constant
- The total capacitance is calculated using the relationship for capacitors in series

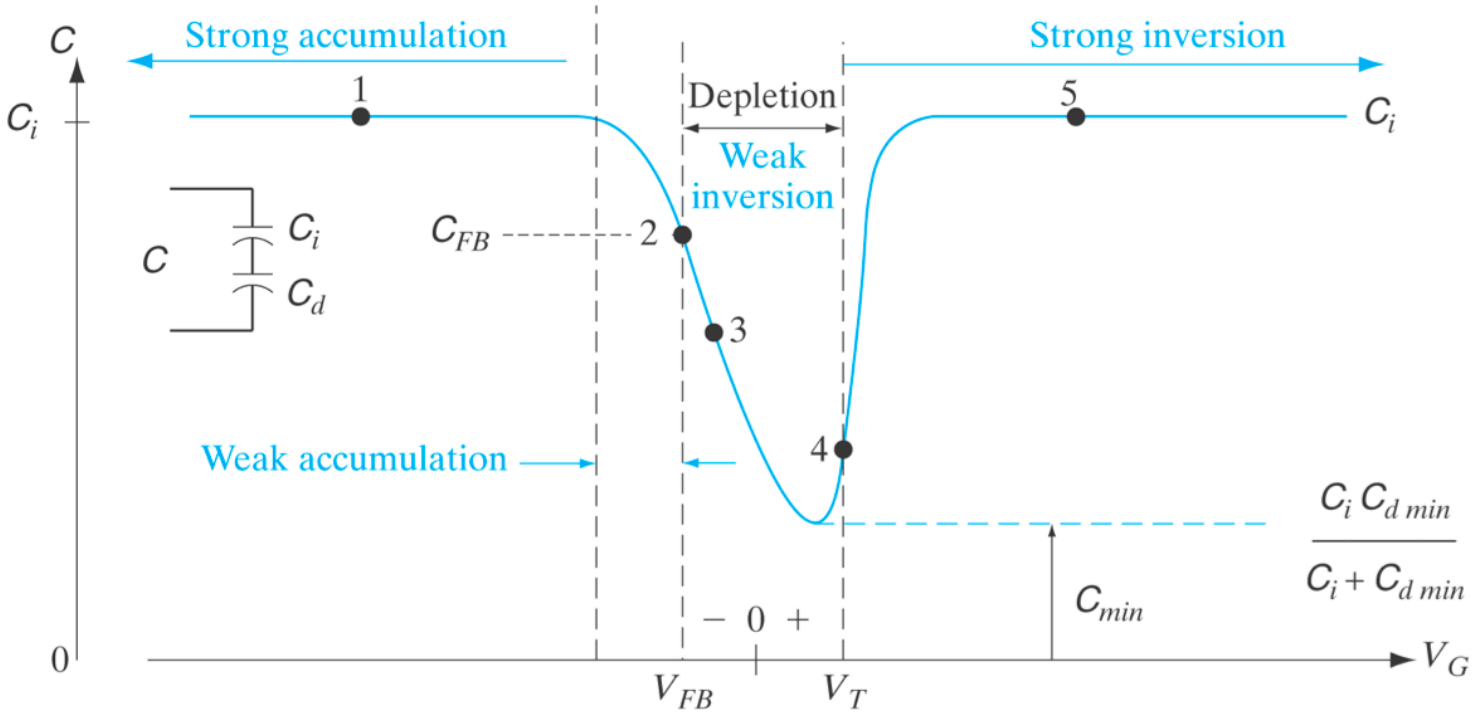
$$C_s = \frac{dQ}{dV} = \frac{dQ_s}{d\phi_s} \qquad C_d = \frac{\varepsilon_s}{W} \qquad C_i = \frac{\varepsilon_i}{d}$$



The smallest capacitance dominates

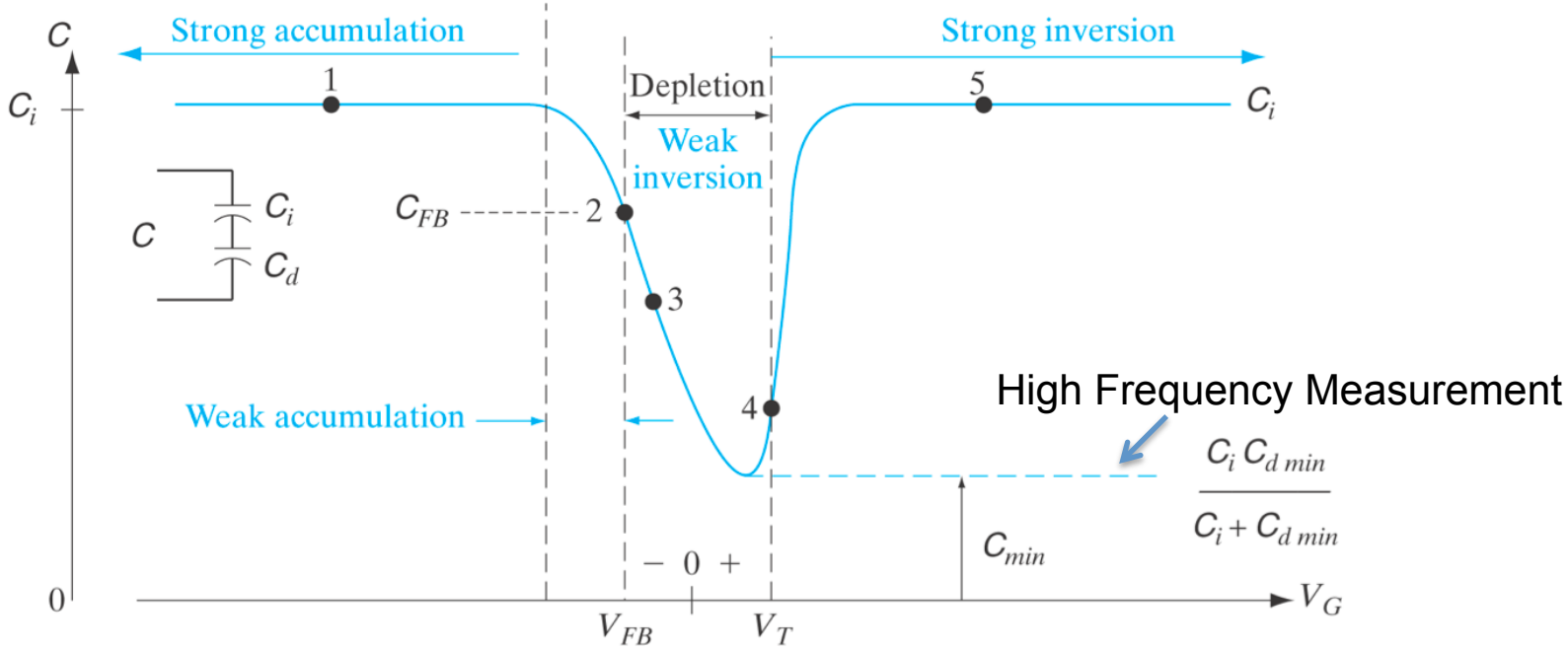
Capacitance: Commentary

- Point 1: The semiconductor capacitance is large – accumulation charge (majority carriers) changes substantially with surface potential, and the semiconductor is acting like the “metal” of the other electrode and the insulator capacitance is dominant
- Point 2: The semiconductor is becoming depleted, and a depletion capacitance in series with the insulator capacitance needs to be taken into account
- Point 3: The semiconductor depletion capacitance decreases as the depletion width expands



Capacitance: High Frequency Effects

- Point 4: Under weak inversion, charge density is beginning to increase at the semiconductor-oxide interface, dQ_s/dV begins to increase and C_i becomes more dominant
- Point 5: In strong inversion, inversion charge increases exponentially with surface potential and total capacitance is defined by the insulator capacitance
- High Frequency: If the gate voltage is varied rapidly, the charge in the inversion layer cannot change in response (generation-recombination, carrier lifetime)
- Comment: MOSFET versus MOS high frequency capacitance is different due to source and drain acting as sources for inversion carriers



Summary: Capacitance

In General:

$$C_i = \frac{\varepsilon_i}{d}, \qquad C_s = \frac{dQ}{dV}, \qquad C = \frac{C_i C_s}{C_i + C_s}$$

1) For large $\frac{dQ}{dV}$ such that $C_s \gg C_i$:

$C \sim C_i$

$\rightarrow$ Accumulation, Inversion, Low Frequency Signal

2) In Depletion or High Frequency Case:

$$C_i = \frac{\varepsilon_i}{d}$$

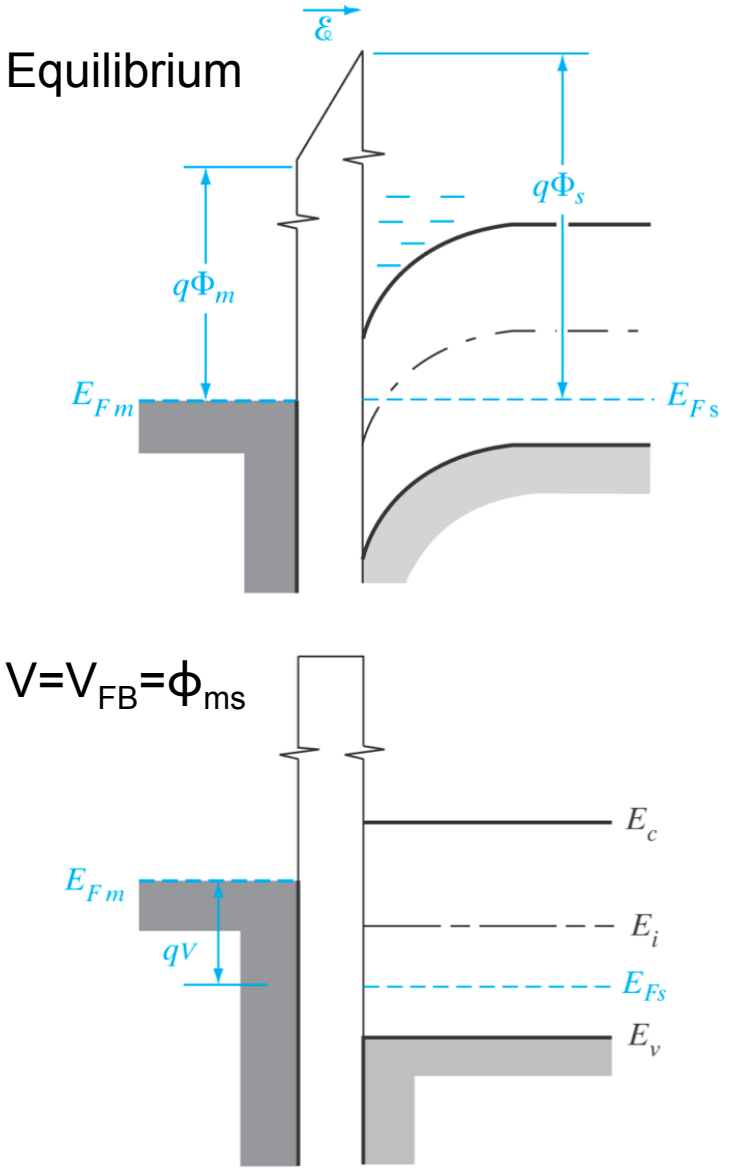
$C_s \sim C_d = \frac{\varepsilon_s}{W}$ $\leftarrow$ W is the depletion width

$$C = \frac{C_i C_d}{C_i + C_d}$$

Effects of Real Surfaces

Band Alignment: Differing Work Functions

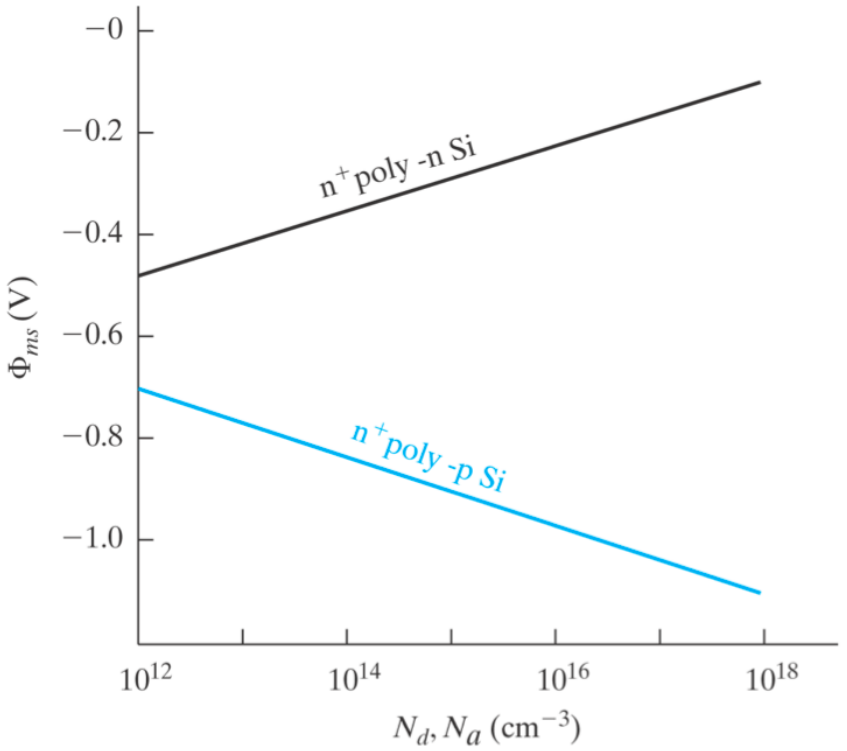
- Alignment of the metal and semiconductor work functions results in a potential difference between the metal and semiconductor
- The potential difference creates an electric field across the insulator, and the metal is positively charged relative to the semiconductor
- If ϕ_{ms} is sufficiently negative, an inversion layer forms with no applied voltage
- To obtain a flat band condition, a negative voltage must be applied to the metal



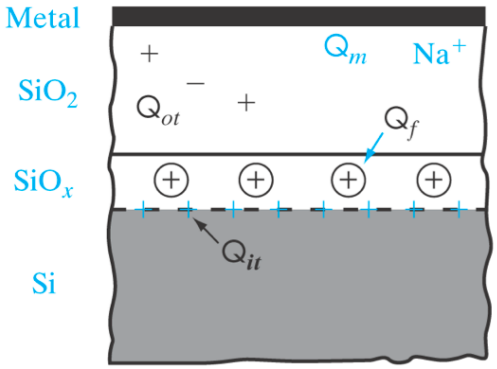
Work Function Difference

- Ideal case assumed that the metal and semiconductor work functions were the same
- Typical materials depart from the ideal case
- The semiconductor modified work function varies with semiconductor doping density (Fermi level changes with doping)
- Note that Φ_{ms} is always negative, and most negative for heavily p-doped material (think of this as the difference between n^+ crystalline silicon and n or p-type material)
- Fermi level alignment can produce an inversion layer with no voltage applied

Metal-Semiconductor Work Function Difference

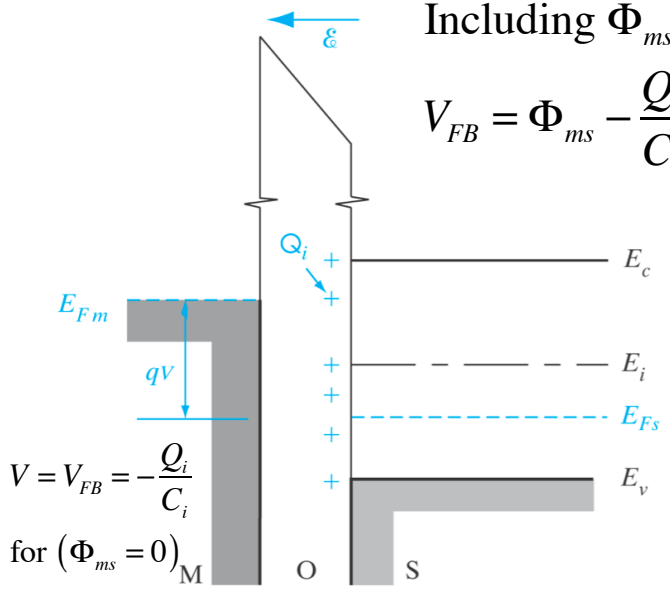


Interface Charge & Flat Band Voltage



Q_m Mobile ionic charge
Q_{ot} Oxide trapped charge
Q_f Oxide fixed charge
Q_{it} Interface trap charge

~10^{10} charges/cm^2 on {100} Si



Including Φ_{ms} :

$$V_{FB} = \Phi_{ms} - \frac{Q_i}{C_i}$$

$$V = V_{FB} = -\frac{Q_i}{C_i}$$

for $(\Phi_{ms} = 0)$

- Charge in the oxide and at the interface comes from a variety of sources
- Mobile ionic charge is typically from contaminants such as sodium
- Oxide trapped charge arises from imperfections in the SiO_2
- Interface states occur abrupt transition between the oxide and semiconductor
- Oxide fixed charge occurs in the transition region between fully oxidized silicon and unoxidized crystalline silicon
- The net positive charge in the oxide requires a negative voltage in the metal to achieve flat bands

Key Definition

- ***Flat Band Voltage:*** Voltage that must be applied to the gate to achieve unbent (flat) bands. This compensates for the work function difference and any interface charge.

$$V_{FB} = \Phi_{ms} - \frac{Q_i}{C_i}$$

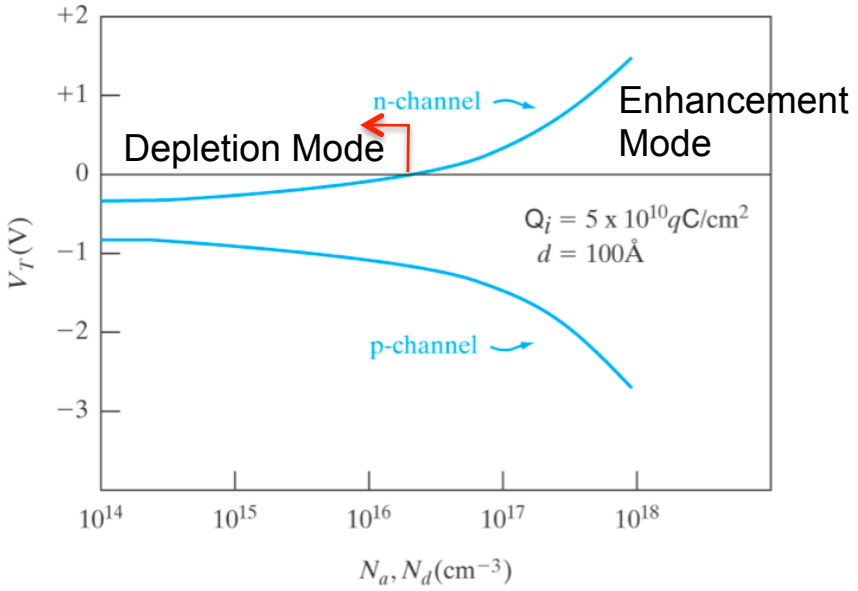
Threshold Voltage

Non-Ideal Case

Threshold Voltage: Non-Ideal Case

- The ideal expression for threshold voltage is modified by the work function difference and interface charge
- p-channel: the negative voltage applied to introduce a channel must be larger than V_T
- n-channel: if V_T is positive, a voltage larger than this must be applied to induce a channel (normally "OFF")
- n-channel: if V_T is negative, a channel exists at V=0 and a negative voltage must be applied to turn the device off (depletion mode, or normally "ON")

$V_T =$	Φ_{ms}	$-\frac{Q_i}{C_i}$	$-\frac{Q_d}{C_i}$	$+ 2\phi_F$
	(−)	(−)	(+) n channel (−) p channel	(+) n channel (−) p channel



$$V_T = \Phi_{ms} - \frac{Q_i}{C_i} - \frac{Q_d}{C_i} + 2\phi_F$$

Add V_{FB} to $V_{T,ideal}$

Example

Assuming a n^+ polysilicon gate with an n-channel MOS device (p-substrate):

Given $N_a = 5\times10^{15}\,cm^{-3}$, an oxide thickness of 100A, and an interface charge density $Q_i = 4\times10^{10}\,qC/cm^2$, determine C_i, C_{min}, W_m, V_{FB}, and V_T.

$$\phi_F = \frac{kT}{q}\ln\frac{N_a}{n_i} = 0.0259\ln\frac{5\times10^{15}}{1.5\times10^{10}} = 0.329eV$$

$$W_m = 2\left[\frac{\varepsilon_s\phi_F}{qN_a}\right]^{1/2} = 2\left[\frac{11.8\times8.85\times10^{-14}\times0.329}{1.6\times10^{-19}\times5\times10^{15}}\right]^{1/2} = 4.15\times10^{-5}\,cm = 0.415\,\mu m$$

From the figure, $\Phi_{ms} \approx -0.95V$

$$Q_i = 4\times10^{10}\times1.6\times10^{-19} = 6.4\times10^{-9}\,C/cm^2$$

$$C_i = \frac{\varepsilon_i}{d} = \frac{3.9\times8.85\times10^{-14}}{0.1\times10^{-5}} = 3.45\times10^{-7}\,F/cm^2$$

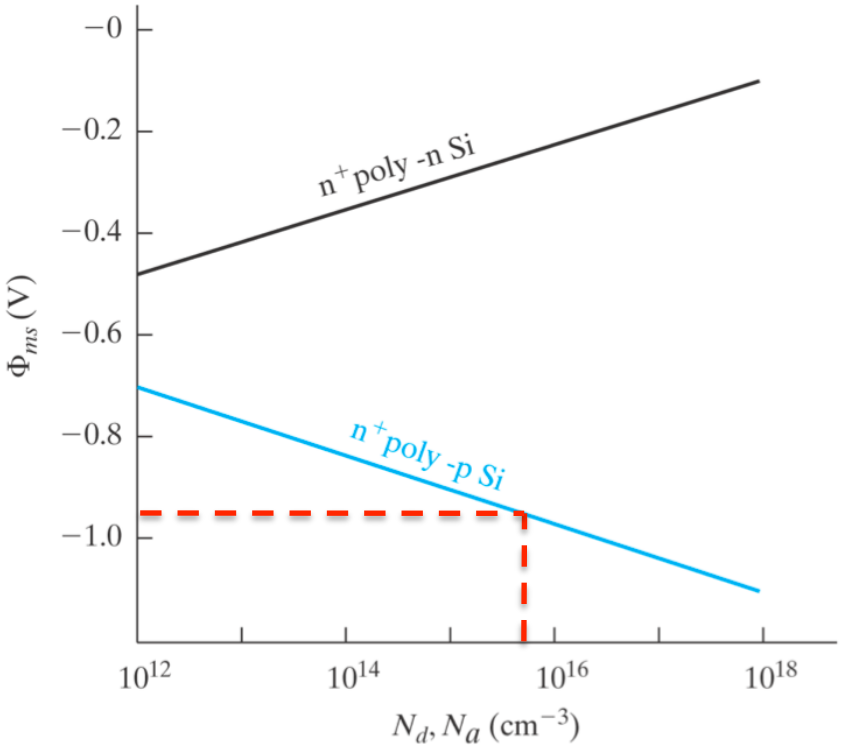
$$V_{FB} = \Phi_{ms} - \frac{Q_i}{C_i} = -0.95 - \frac{6.4\times10^{-9}}{3.45\times10^{-7}} = -0.969V$$

$$Q_d = -qN_aW_m = -1.6\times10^{-19}\times5\times10^{15}\times4.15\times10^{-5} = -3.32\times10^{-8}\,C/cm^2$$

$$V_T = V_{FB} - \frac{Q_d}{C_i} + 2\phi_F = -0.969 + \frac{3.32\times10^{-8}}{3.45\times10^{-7}} + 0.658 = -0.215V$$

$$C_d = \frac{\varepsilon_i}{W_m} = \frac{11.8\times8.85\times10^{-14}}{4.15\times10^{-5}} = 2.5\times10^{-8}\,F/cm^2$$

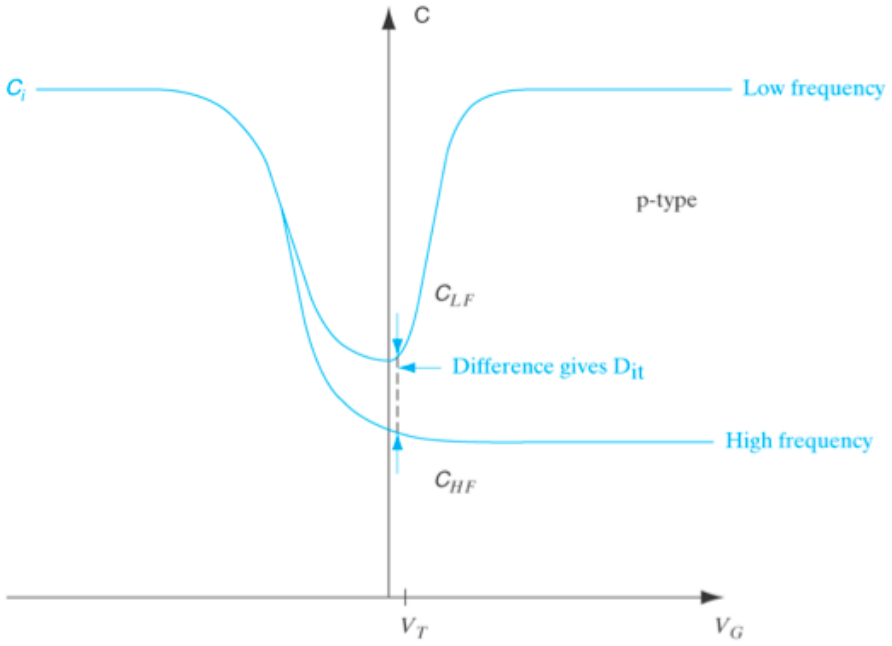
$$C_{min} = \frac{C_iC_d}{C_i + C_d} = \frac{3.45\times10^{-7}\times2.5\times10^{-8}}{3.45\times10^{-7} + 2.5\times10^{-8}} = 2.33\times10^{-8}\,F/cm^2$$

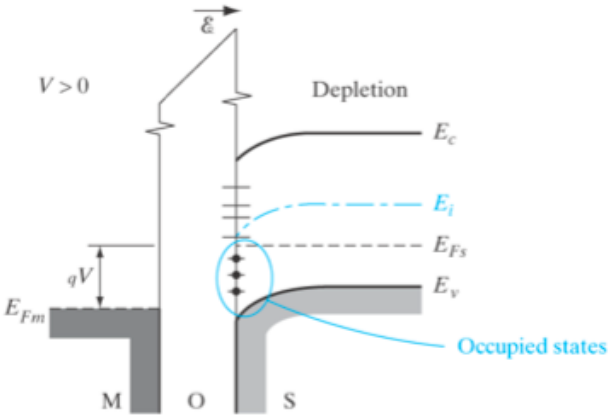


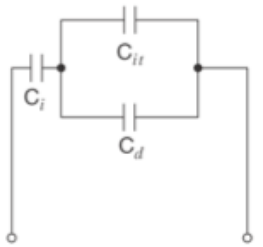
MOS C-V Analysis

MOS Device Parameter Extraction

- Insulator thickness, substrate doping, and threshold voltage can be determined by examining the C-V curve

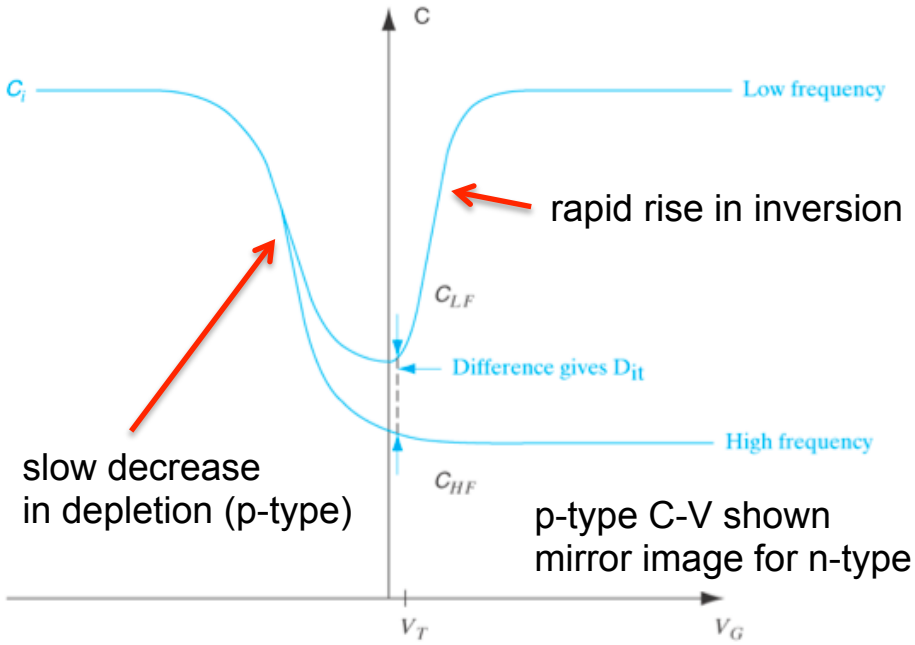


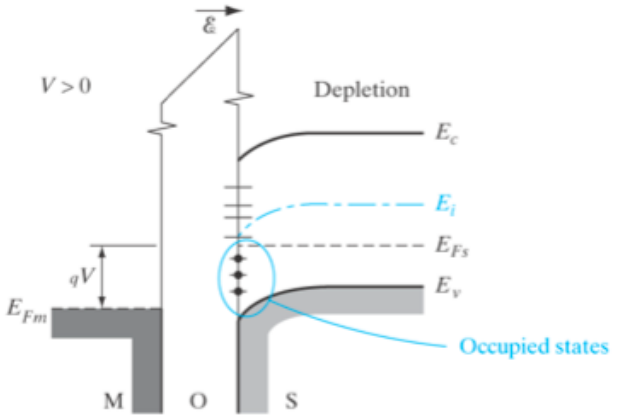


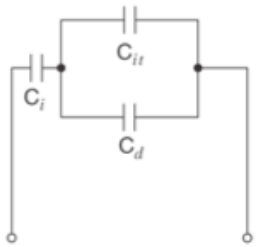


Substrate Doping Type

- p-type substrate: high frequency capacitance large for negative gate biases and small for positive biases
- n-type substrate: high frequency capacitance small for negative gate biases and high for positive biases

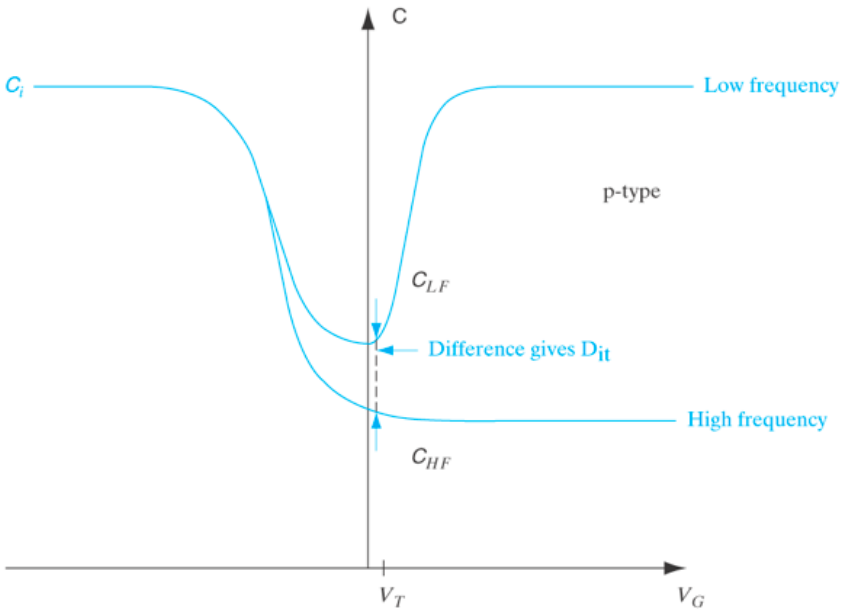






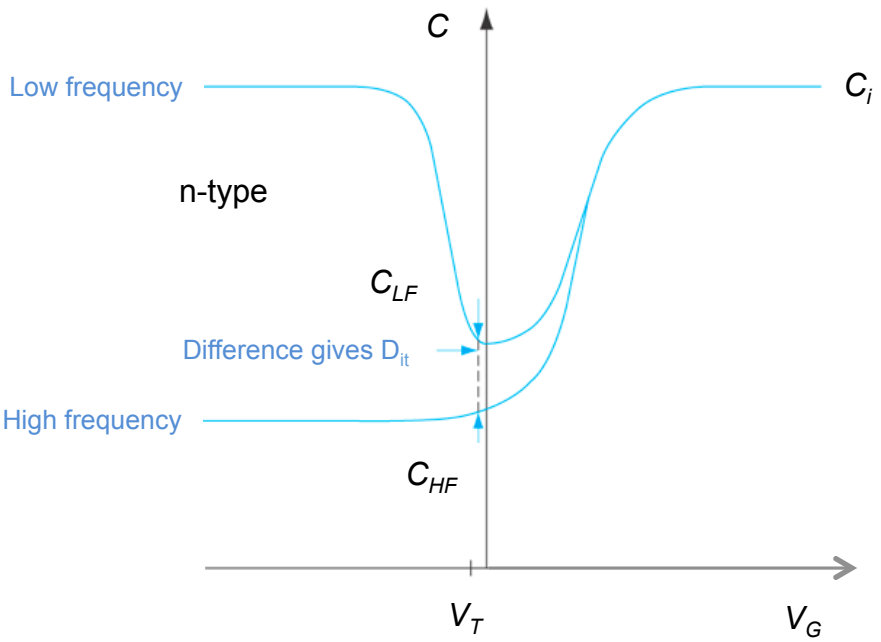
Substrate/Well Doping Type

p-Type Substrate (Well)



n-Channel Device

n-Type Substrate (Well)



p-Channel Device

Insulator Thickness and Substrate Doping Density

- The minimum capacitance C_{min} is the series combination of the insulator capacitance C_i and the minimum depletion capacitance C_{dmin} corresponding to the maximum depletion width W_m ($C_{dmin}=\varepsilon_s/W_m$)
- The expression for W_m can be solved numerically to give substrate doping density

$$C_i = \frac{\varepsilon_i}{d}$$

$d \equiv$ oxide thickness

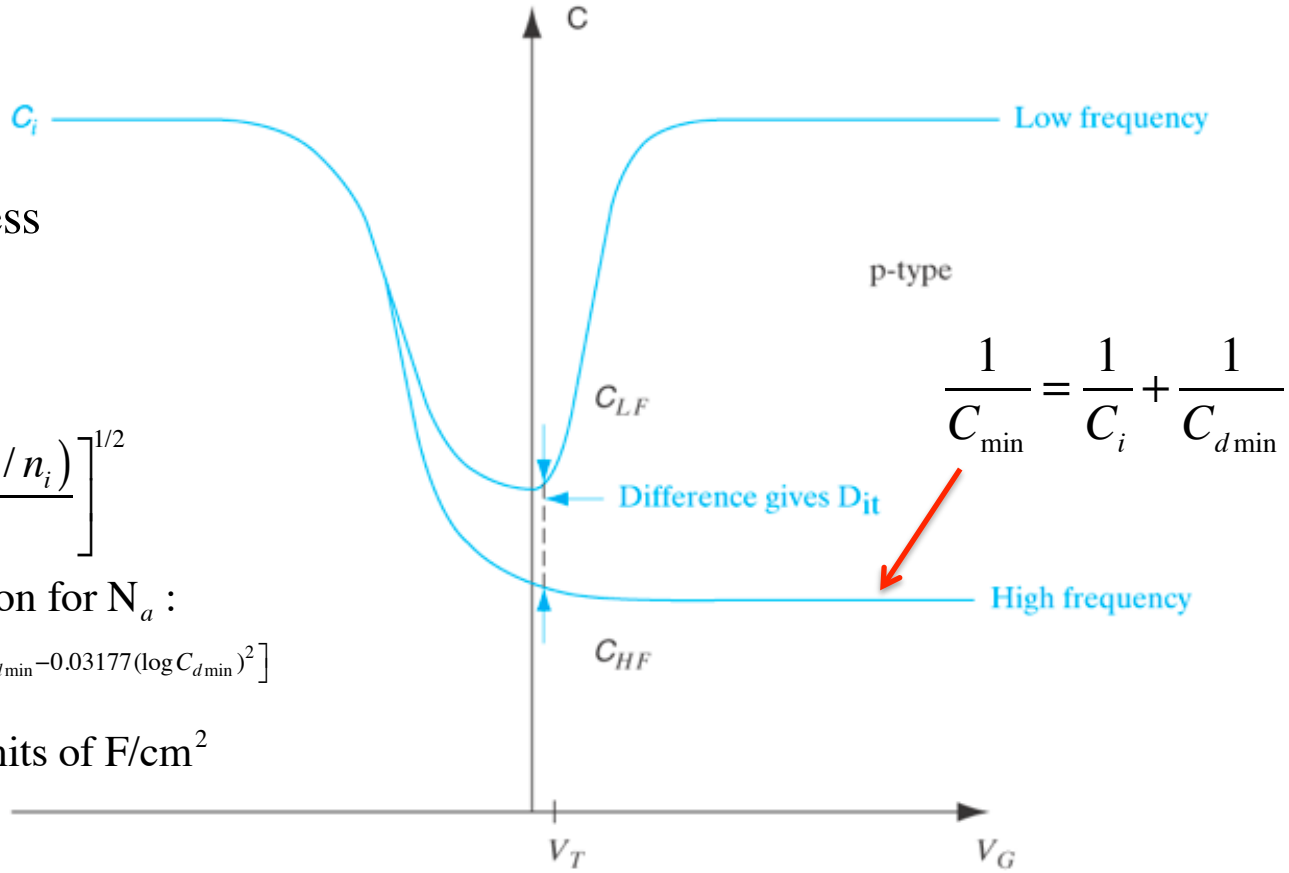
$$C_{d\min} = \frac{\varepsilon_s}{W_m}$$

$$W_m = 2\left[\frac{\varepsilon_s kT \ln\left(N_a / n_i\right)}{q^2 N_a}\right]^{1/2}$$

Approximate Solution for N_a :

$$N_a \approx 10^{\left[30.388+1.638\log C_{d\min}-0.03177(\log C_{d\min})^2\right]}$$

Where $C_{d\min}$ is in units of F/cm^2



$$\frac{1}{C_{\min}} = \frac{1}{C_i} + \frac{1}{C_{d\min}}$$

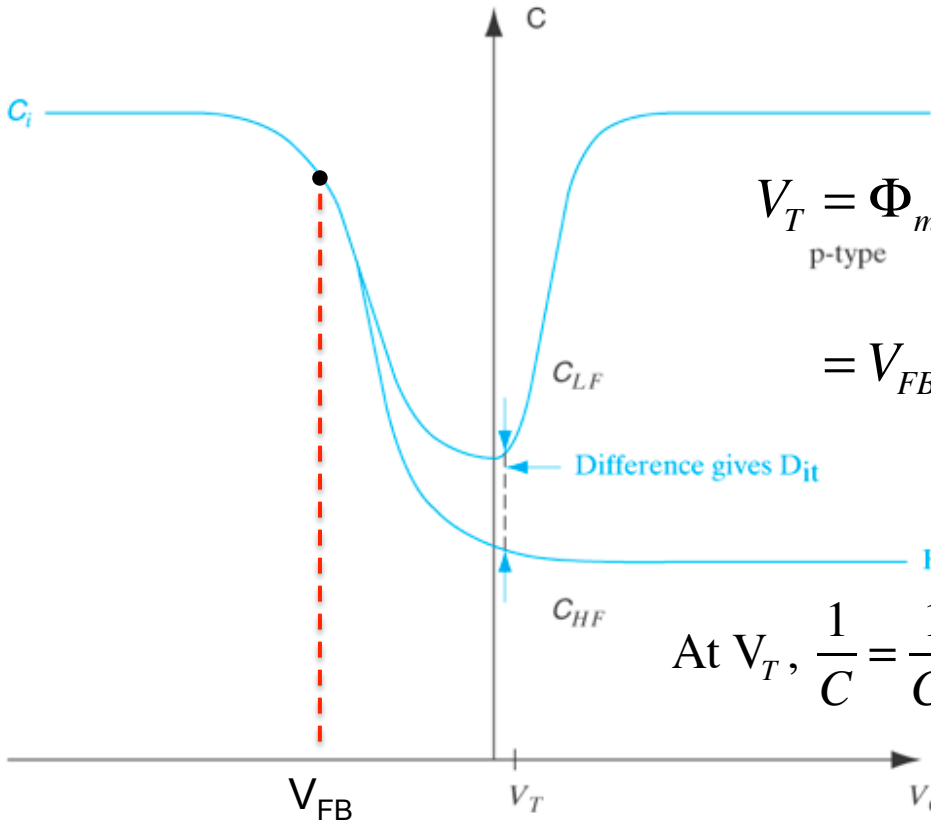
Flat Band Capacitance and Threshold Voltage

- The flat-band capacitance C_{FB} is the series combination of the Debye length capacitance and the insulator capacitance
- The measurement voltage that gives a value of C_{FB} is the flat-band voltage V_{FB}
- With this, all of the parameters have been determined that are needed to calculate V_T
- At the onset of strong inversion, the change of charge is the sum of the change in depletion charge and inversion charge – leading to the $2C_{dmin}$ term for the capacitance at V_T

$$\frac{1}{C_{FB}} = \frac{1}{C_i} + \frac{1}{C_{debye}}$$

$$C_{debye} = \frac{\varepsilon_s}{L_D}$$

$$L_D \equiv \sqrt{\frac{\varepsilon_s kT}{q^2 p_o}}$$



$$V_T = \underset{\text{p-type}}{\Phi_{ms}} - \frac{Q_i}{C_i} - \frac{Q_d}{C_i} + 2\phi_F$$

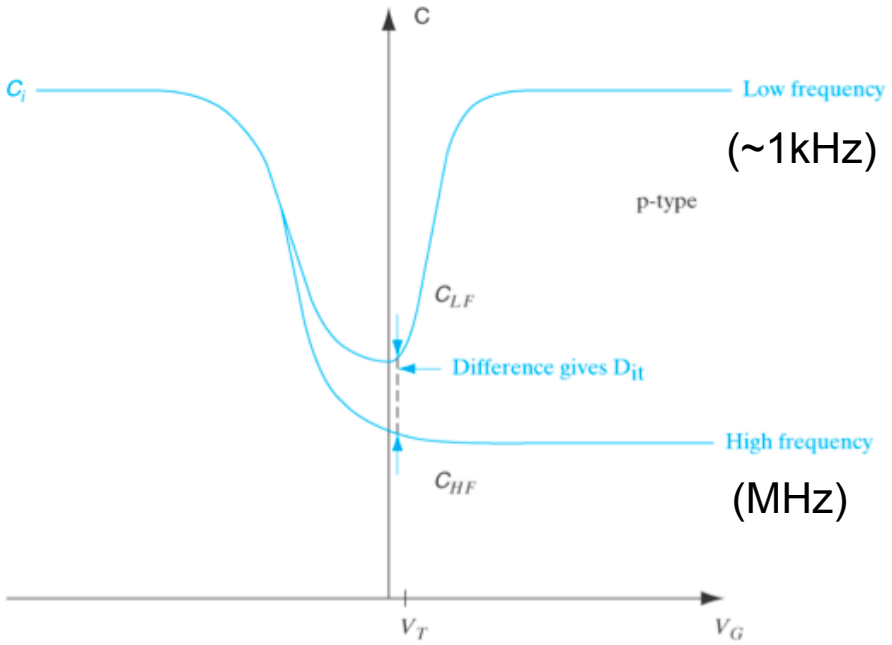
$$= V_{FB} + \frac{qN_A W_m}{C_i} + 2\frac{kT}{q}\ln\frac{N_A}{n_i}$$

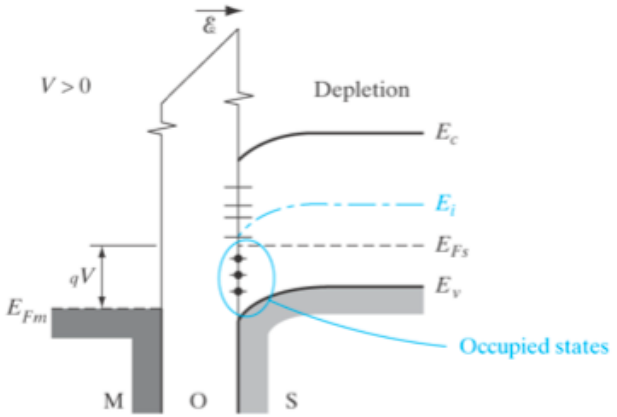
$$\text{At } V_T, \quad \frac{1}{C} = \frac{1}{C_i} + \frac{1}{2C_{d\min}}$$

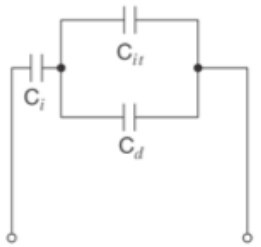
Fast Interface State Density D_{it}

- Fast interface state density D_{it} can also be determined from the C-V curve
- Variation in the surface potential moves the Fermi level, causing states to become occupied or unoccupied
- Since charge is stored or released in response to an applied potential, this is a capacitance in parallel with the channel depletion capacitance (additive relationship) and in series with the insulator capacitance
- The interface states can respond to low frequency changes but not to high frequency changes (they are fast but not "fast enough")

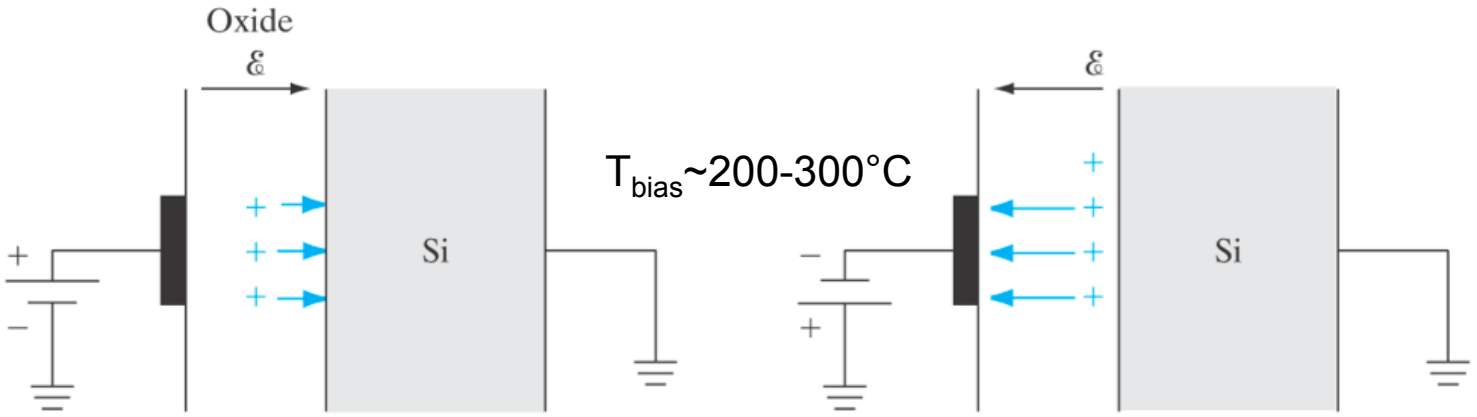
$$D_{it} = \frac{1}{q}\left(\frac{C_i C_{LF}}{C_i - C_{LF}} - \frac{C_i C_{HF}}{C_i - C_{HF}}\right) cm^{-2}eV^{-1}$$



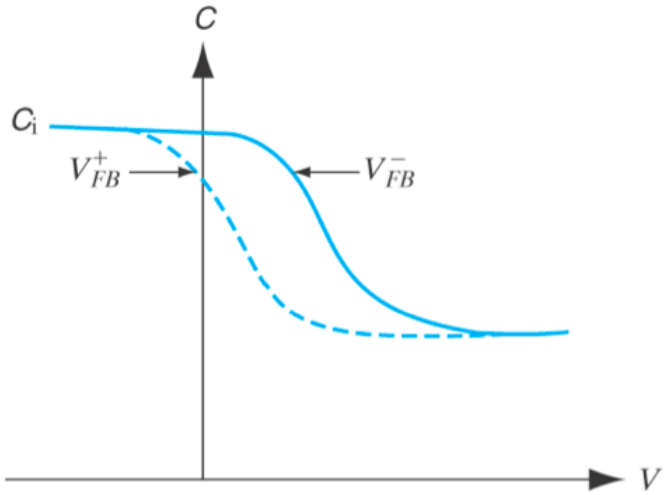




Fixed and Mobile Oxide Charge



- Fixed charge Q_f modifies the flat band and threshold voltage
- Mobile charge Q_m can be measured using an bias-temperature stress test
  - MOS capacitor heated to ~200-300° C with a bias applied to the gate (~1 MV/cm)
  - Mobile charge will move under the influence of the applied bias
- When the charge is next to the oxide-semiconductor interface, negative charge is induced in the semiconductor and a more negative gate voltage is required to achieve the flat band condition
- When the charge is next to the metal, a negative image charge is induced in the metal, and a less negative voltage needs to be applied to the gate to achieve the flat band condition



Since $V_{FB} = \Phi_{ms} - \frac{Q_i}{C_i}$, $Q_m = C_i\left(V_{FB}^- - V_{FB}^+\right)$

MOSFETs

Output Characteristics

Gate Voltage & Charge

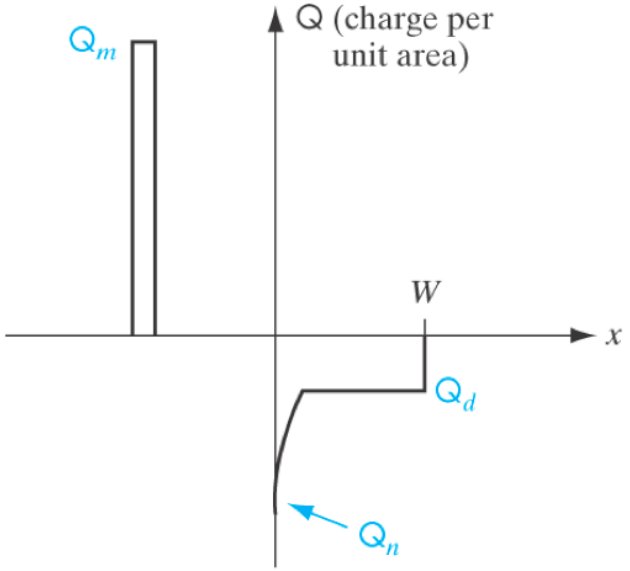
In general, the gate voltage can be broken down into the following:

$$V_G = \left(\Phi_{ms} - \frac{Q_i}{C_i} \right) - \frac{Q_s}{C_i} + \phi_s = V_{FB} - \frac{Q_s}{C_i} + \phi_s$$

Potential Across Semiconductor

Voltage to Achieve Flat Band Condition

Voltage Due to Charge Across Insulator



Q_s ← induced charge

$Q_s = Q_d + Q_n$

Q_d ← Fixed charge (depletion region)

Q_n ← Mobile charge (channel)

ϕ_s ← potential across semiconductor (band bending)

$$V_G = V_{FB} - \frac{(Q_d + Q_n)}{C_i} + \phi_s \quad \rightarrow \quad Q_n = -C_i \left[V_G - \left(V_{FB} + \phi_s - \frac{Q_d}{C_i} \right) \right]$$

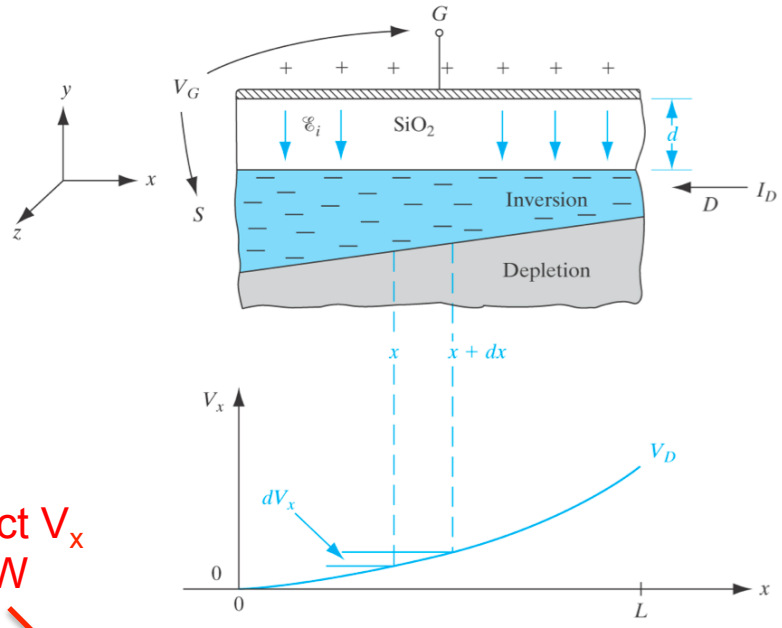
Above Threshold ($V_G > V_T$): $Q_n = -C_i \left[V_G - (V_T) \right]$ ← Does not hold at or below threshold

Charge With Applied Drain Voltage

With a voltage V_D applied to the drain and the source grounded:

1) There is a voltage rise along the channel from the source to drain with any point along the channel x having a voltage $V_x(x)$
2) The potential $\phi_s(x)$ along the channel is:

$$\phi_s(x) = 2\phi_F + V_x(x)$$



Therefore:

Neglect V_x in W

$$Q_n(x) = -C_i\left[V_G - V_{FB} - 2\phi_F - V_x(x) - \frac{1}{C_i}\sqrt{2q\varepsilon_s N_a\left(2\phi_F + V_x\right)}\right]$$

x

Since $V_T = V_{FB} - \frac{Q_d}{C_i} + 2\phi_F$ and neglecting the variation in Q_d :

$$Q_n(x) = -C_i\left(V_G - V_T - V_x(x)\right)$$

Mobile Charge in Channel at "x" (low V_D)

Note:

$$Q_d = -qN_a^- W(x)$$

$$W(x) = \left[\frac{2\varepsilon_s(2\varphi_F + V_x)}{qN_a}\right]^{1/2}$$

Output Characteristics: General Case

For larger drain currents, the variation in $Q_n(x)$ cannot be neglected, and the full expression is integrated:

$$\int_0^L I_D\,dx = \bar{\mu}_n Z C_i \int_0^{V_D}\left[V_G - V_{FB} - 2\phi_F - V_x(x) - \frac{1}{C_i}\sqrt{2q\varepsilon_s N_a\left(2\phi_F + V_x(x)\right)}\right]dV_x \text{ so:}$$

$$I_D = \frac{\bar{\mu}_n Z C_i}{L}\left\{\left(V_G - V_{FB} - 2\phi_F - \frac{1}{2}V_D\right)V_D - \frac{2}{3}\frac{\sqrt{2q\varepsilon_s N_a}}{C_i}\left[\left(V_D + 2\phi_F\right)^{3/2} - \left(2\phi_F\right)^{3/2}\right]\right\}$$

Low Drain Bias:

$$I_D = \frac{\bar{\mu}_n Z C_i}{L}\left[(V_G - V_T)V_D - \frac{1}{2}V_D^2\right]$$

General Bias Condition:

$$I_D = \frac{\bar{\mu}_n Z C_i}{L}\left\{\left(V_G - V_{FB} - 2\phi_F - \frac{1}{2}V_D\right)V_D - \frac{2}{3}\frac{\sqrt{2q\varepsilon_s N_a}}{C_i}\left[\left(V_D + 2\phi_F\right)^{3/2} - \left(2\phi_F\right)^{3/2}\right]\right\}$$

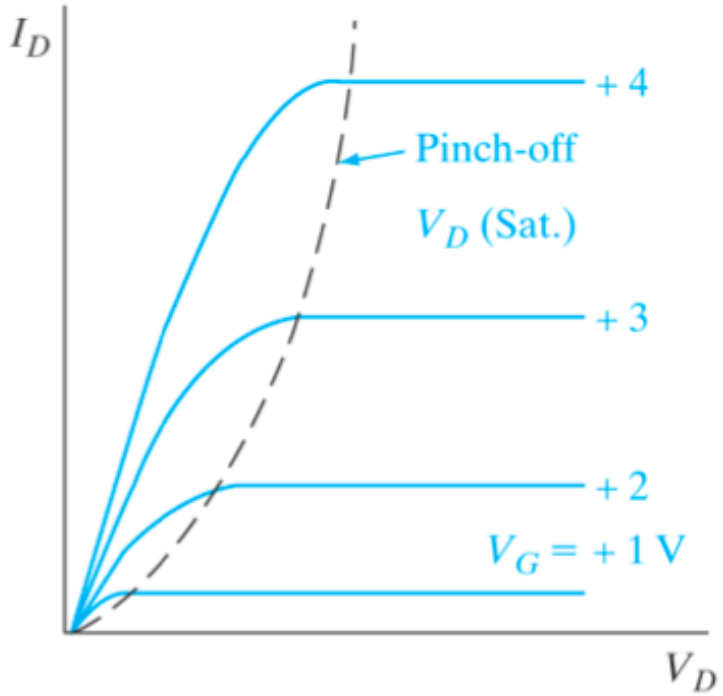
Above $V_D \simeq V_G - V_T$, use $V_D \simeq V_D(sat.) = V_G - V_T$

Output Characteristics: Conductance

Under low drain voltage and $V_G > V_T$, the channel resembles a linear resistor:

$$g = \frac{\partial I_D}{\partial V_D} = \frac{\bar{\mu}_n Z C_i}{L} \frac{\partial}{\partial V_D}\left[(V_G - V_T)V_D - \frac{1}{2}V_D^2\right]$$

$$= \frac{\bar{\mu}_n Z C_i}{L}\left[(V_G - V_T) - V_D\right] \simeq \frac{\bar{\mu}_n Z C_i}{L}(V_G - V_T) \text{ for } V_D \ll V_G - V_T$$



As V_D is increased, ΔV between the oxide and semiconductor near the drain decreases. When ΔV becomes lower than V_T, the channel becomes pinched off:

$$V_D(sat.) \simeq V_G - V_T$$

The current then becomes constant after entering saturation:

$$I_D(sat.) \simeq \frac{1}{2}\frac{\bar{\mu}_n Z C_i}{L}(V_G - V_T)^2 = \frac{1}{2}\frac{\bar{\mu}_n Z C_i}{L}V_D^2(sat.)$$

The **transconductance** at saturation is:

$$g_m(sat.) = \frac{\partial I_D(sat.)}{\partial V_G} \simeq \frac{\bar{\mu}_n Z C_i}{L}(V_G - V_T)$$

For p-channel devices➔ V_G V_D V_T and I_D are negative

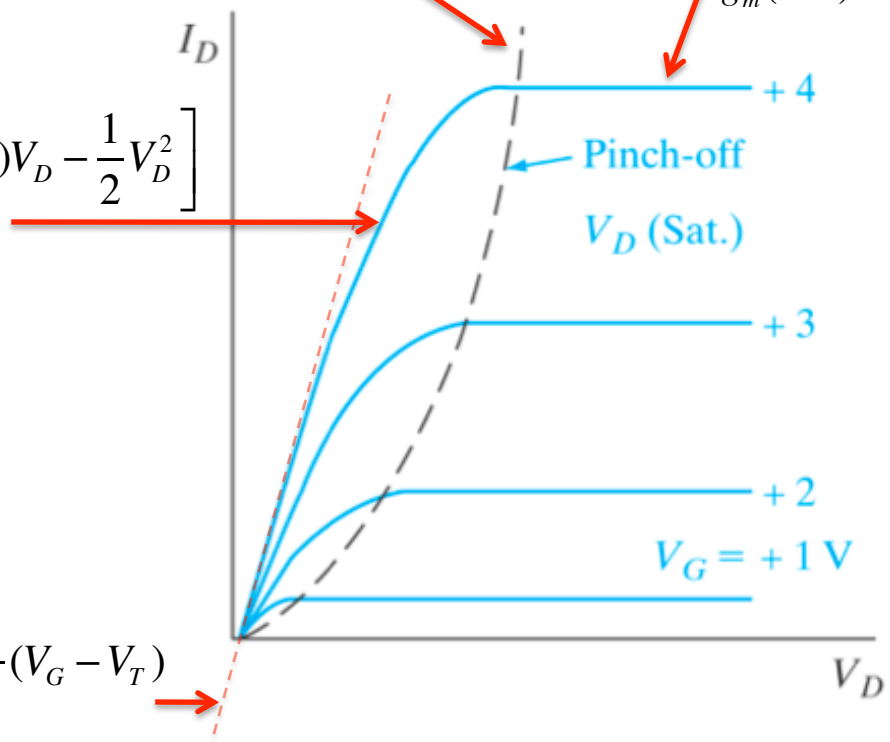
Summary: Output Characteristics

$$I_D(sat.) \simeq \frac{1}{2}\frac{\bar{\mu}_n Z C_i}{L}(V_G - V_T)^2 = \frac{1}{2}\frac{\bar{\mu}_n Z C_i}{L}V_D^2(sat.)$$

$$V_D(sat.) \simeq V_G - V_T$$

$$g_m(sat.) \simeq \frac{\bar{\mu}_n Z C_i}{L}(V_G - V_T)$$

$$I_D = \frac{\bar{\mu}_n Z C_i}{L}\left[(V_G - V_T)V_D - \frac{1}{2}V_D^2\right]$$



$$g = \frac{\bar{\mu}_n Z C_i}{L}(V_G - V_T)$$

General Bias Condition:

$$I_D = \frac{\bar{\mu}_n Z C_i}{L}\left\{\left(V_G - V_{FB} - 2\phi_F - \frac{1}{2}V_D\right)V_D - \frac{2}{3}\frac{\sqrt{2q\varepsilon_s N_a}}{C_i}\left[(V_D + 2\phi_F)^{3/2} - (2\phi_F)^{3/2}\right]\right\}$$

Above $V_D \simeq V_G - V_T$,
use $V_D \simeq V_D(sat.) = V_G - V_T$

Characteristics for n-channel and p-channel MOSFETs

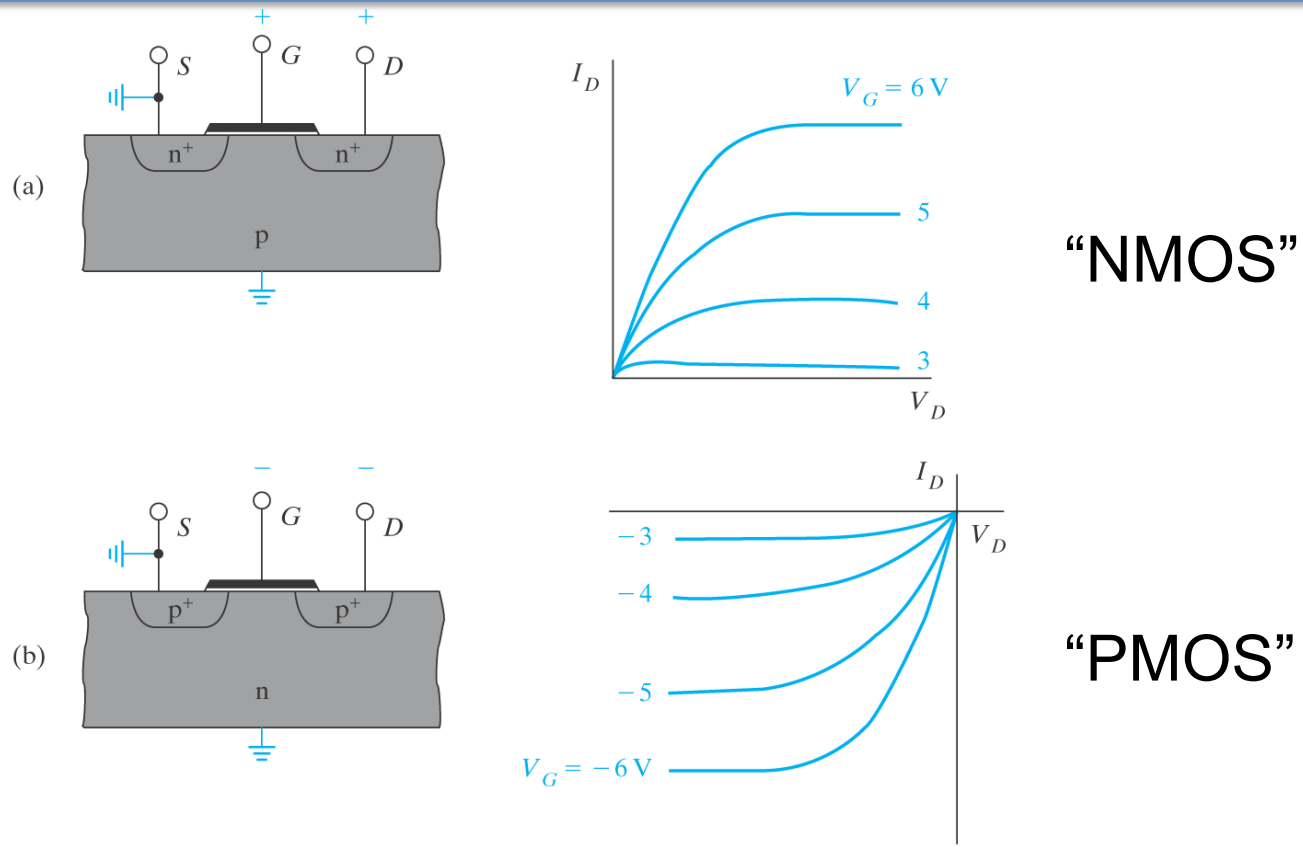


Figure 6.27

Drain current–voltage characteristics for enhancement transistors: (a) for n-channel V_D, V_G, V_T, and I_D are positive; (b) for p-channel all these quantities are negative.

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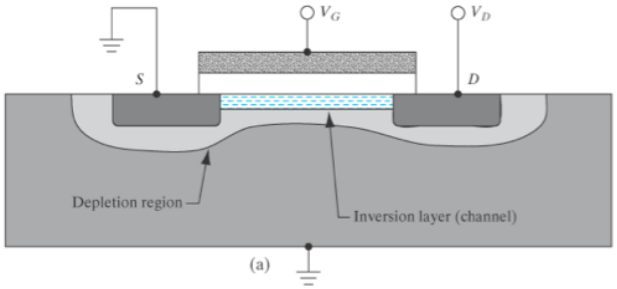
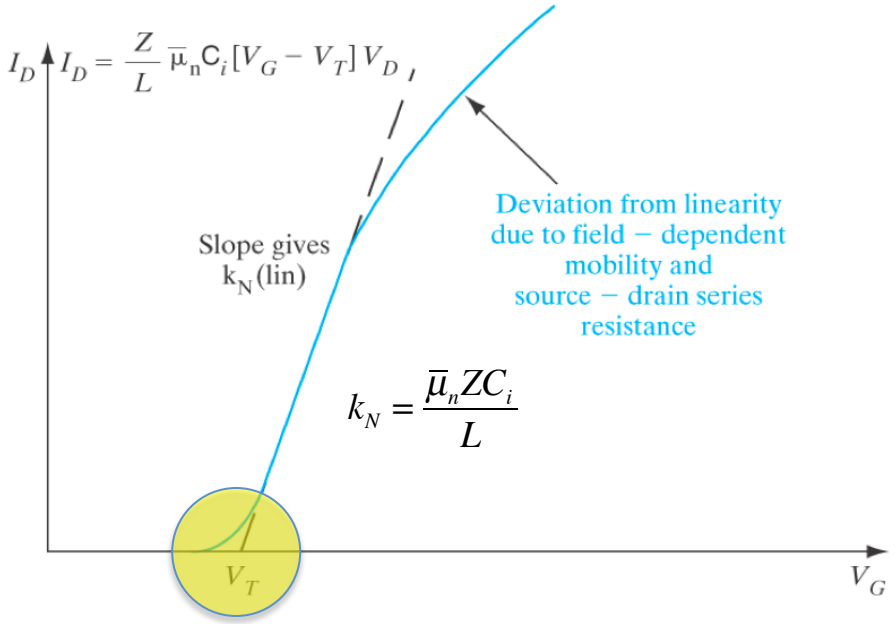
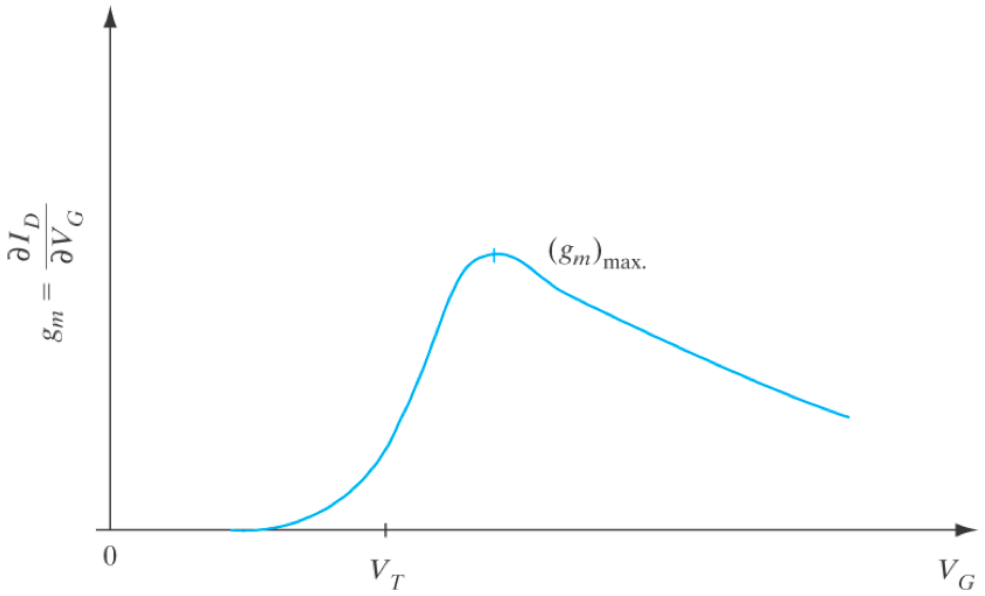
Transfer Characteristics

Transfer Characteristic: Small V_D

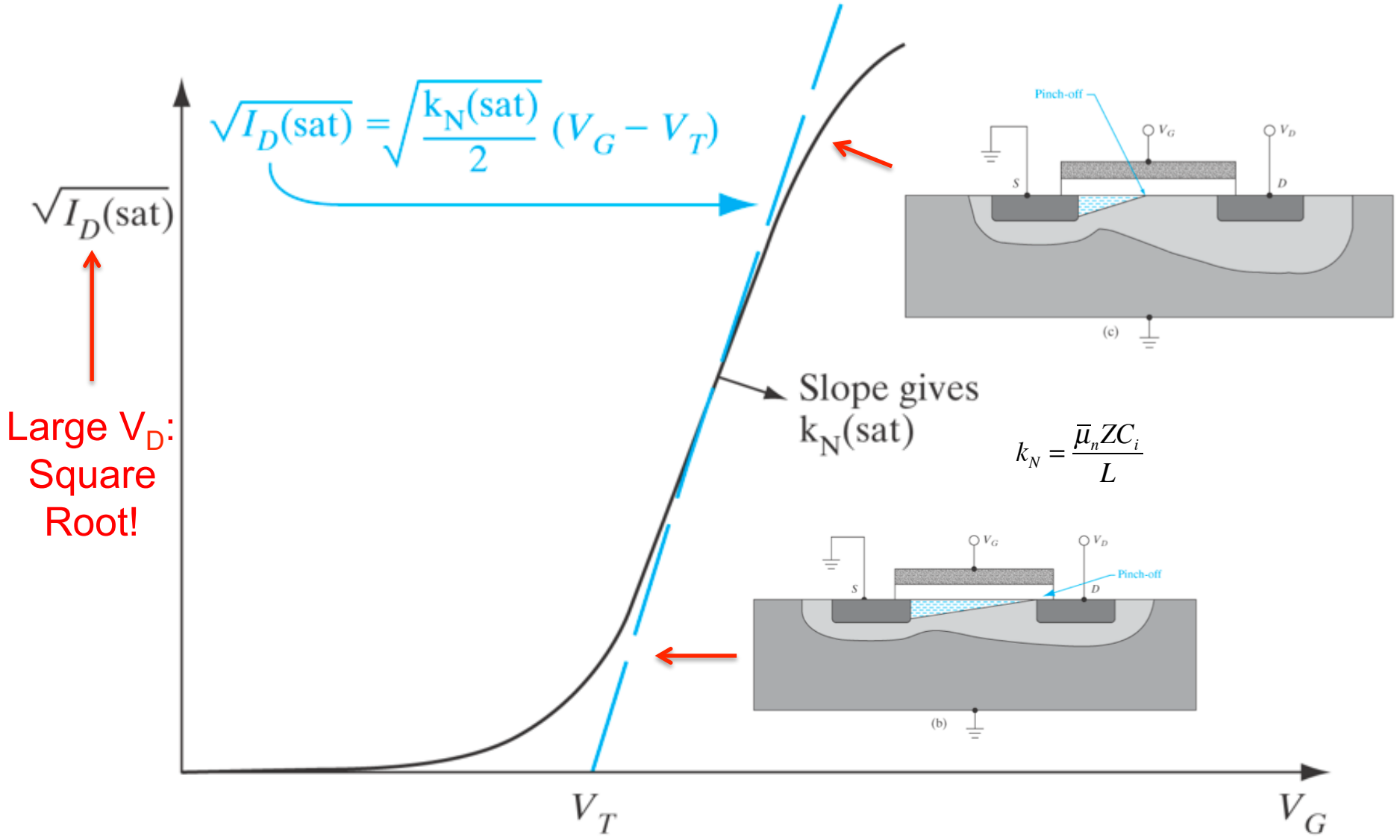






$Q_n = -C_i [V_G - V_T]$ is not valid near $V_G = V_T$

Transfer Characteristic: Large V_D



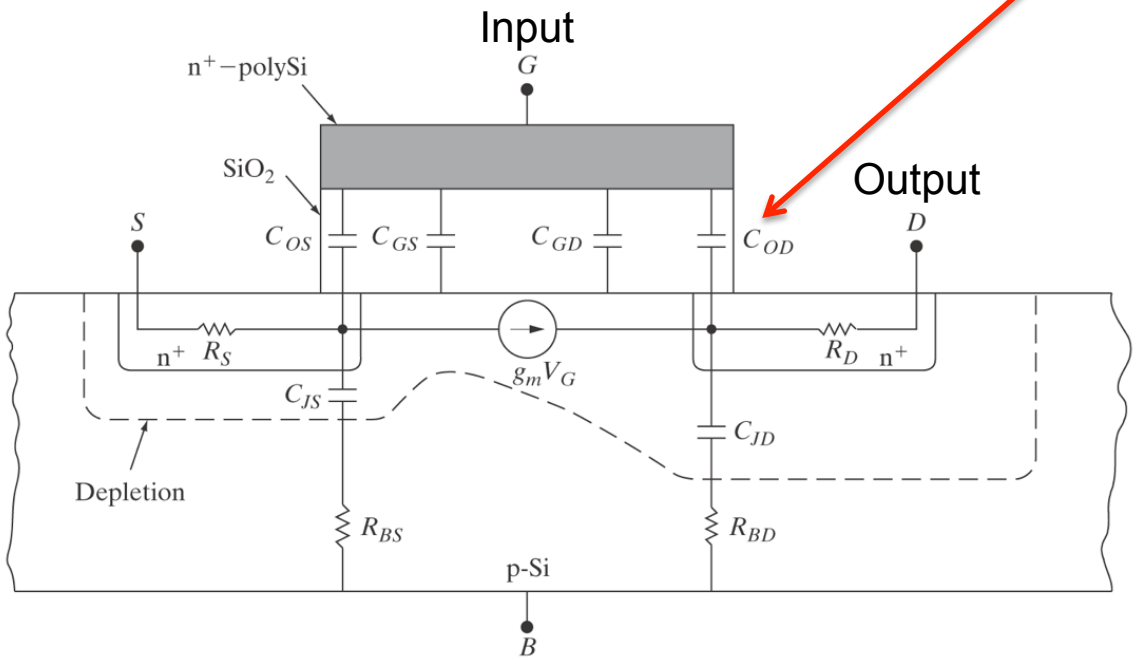
Small Signal Analysis

MOSFET Equivalent Circuit

Gate Insulator Capacitance

Contributions to C_i:

- C_{OS}: Gate to source overlap capacitance
- C_{GS}: Gate to source distributed capacitance
- C_{OD}: Gate to drain overlap capacitance ("Miller overlap capacitance") – potential source of feedback
  - Minimize with self-aligned gate, measured with V_G=0
- C_{GD}: Gate to drain distributed capacitance



Other Components

Other Components:

- C_{JS}: Source p-n junction capacitance
- C_{JD}: Drain p-n junction capacitance
- R_S: Source parasitic series resistance
- R_D: Drain parasitic series resistance
- R_{BS}, R_{BD}: Source and Drain parasitic bulk resistance
- $g_m V_G$: Voltage-controlled current source

$$\frac{V_D}{I_D} = R_{Ch} + R_{SD} = \frac{L - \Delta L}{Z - \Delta Z} \frac{1}{\bar{\mu}_n C_i (V_G - V_T)} + R_{SD}$$

Effective Channel Width:

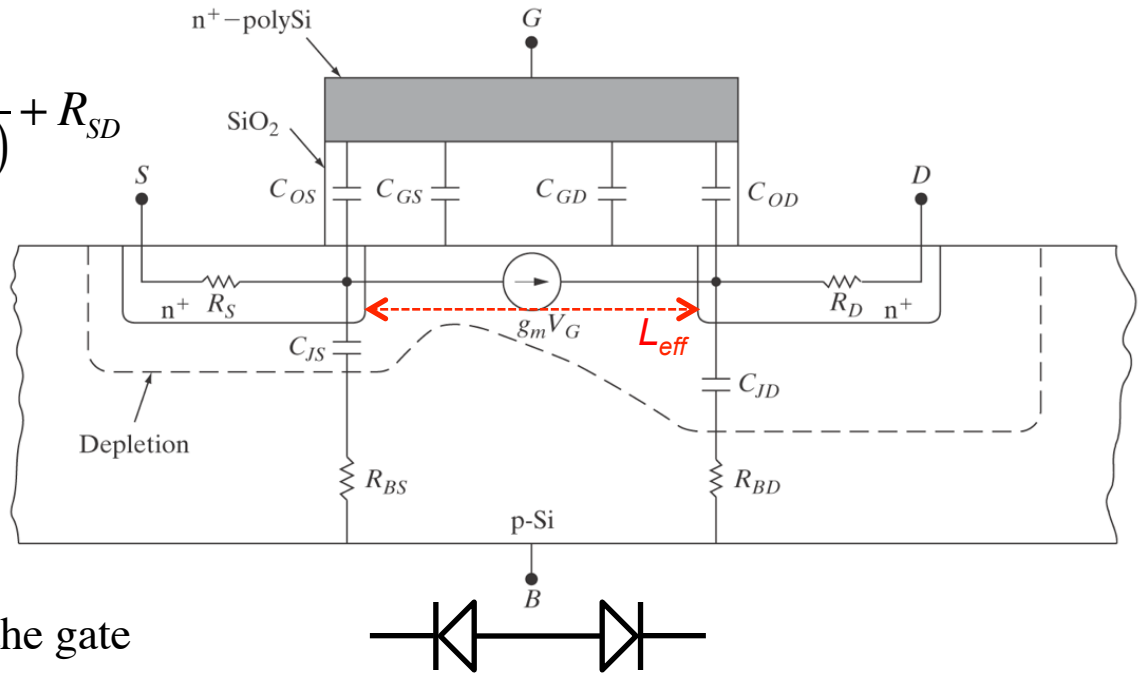
$Z_{eff} = Z - \Delta Z$

ΔZ is the side spread under the gate

Effective Channel Length:

$L_{eff} = L - \Delta L_R$

ΔL_R is the spread of source/drain under the gate

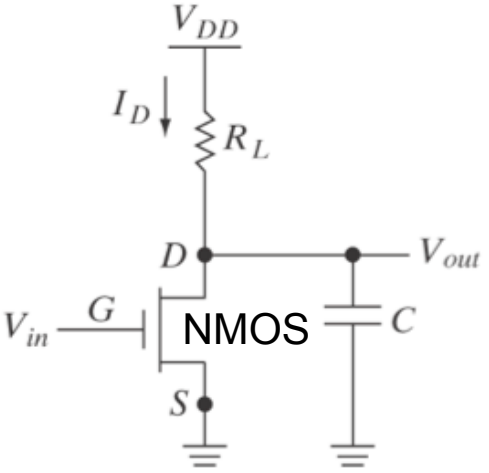


Logic Devices

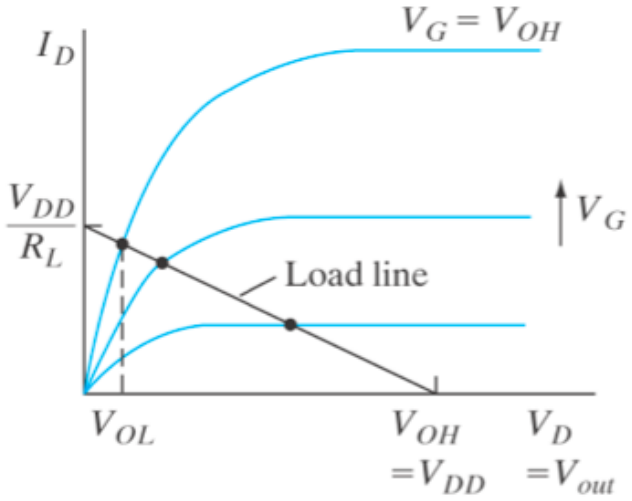
Inverter

MOSFET Inverter (n-Channel)

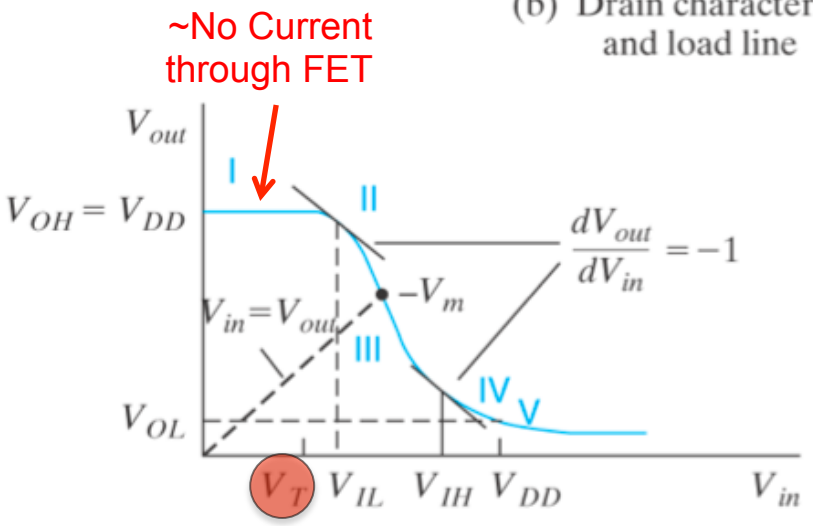
- Voltage Transfer Characteristic (VTC): output voltage as a function of input bias
- V_{OH}: logic high level
- V_{OL}: logic low level
- V_{IL} and V_{IH}: unity gain points (between V_{IL} and V_{IH} the input is amplified)
- V_m: logic threshold, point where output equals input
- Load line used to create VTC (resistor current equals FET current)



(a) Inverter



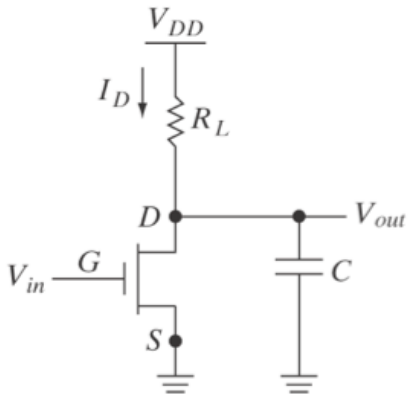
(b) Drain characteristics and load line



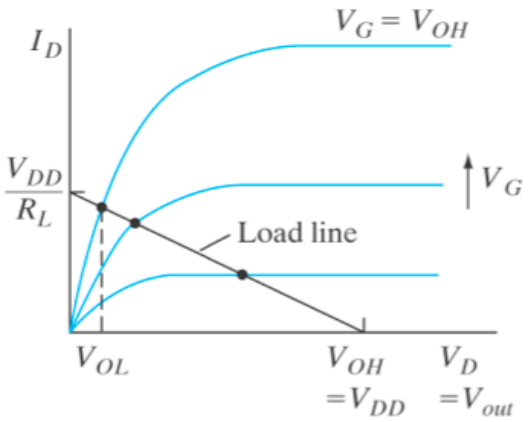
(c) Voltage transfer characteristic

NMOS: n-channel

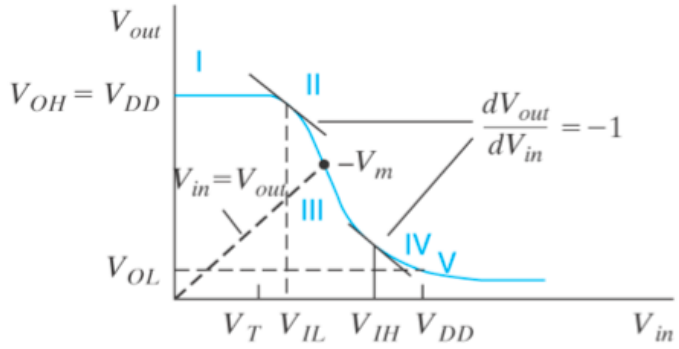
MOSFET Inverter (n-Channel)



(a) Inverter



(b) Drain characteristics and load line



(c) Voltage transfer characteristic

- Gate voltage low – no current flow, no voltage drop across R_L and V_{out} is high
- Gate voltage high – current flow, large voltage drop across R_L and V_{out} is low

MOSFET Drain Current (linear region):

$$I_D = k\left[V_G - V_T - \frac{V_D}{2}\right]V_D = k\left[V_{DD} - V_T - \frac{V_{OL}}{2}\right]V_{OL}$$

MOSFET Drain Current (saturation region):

$$I_D = \frac{1}{2}k\left[V_G - V_T\right]^2$$

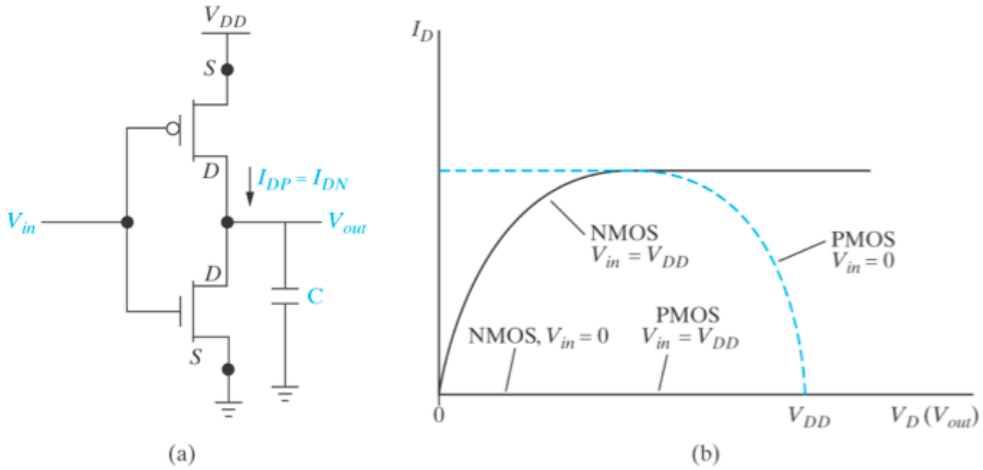
Resistor Current (equal to MOSFET current):

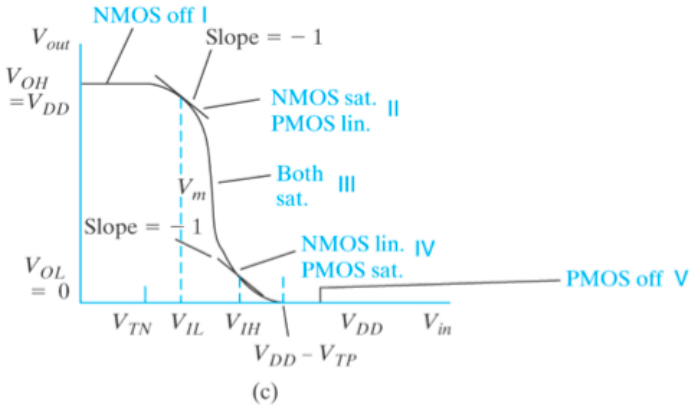
$$I_R = I_D = \frac{V_{DD} - V_{OL}}{R_L}$$

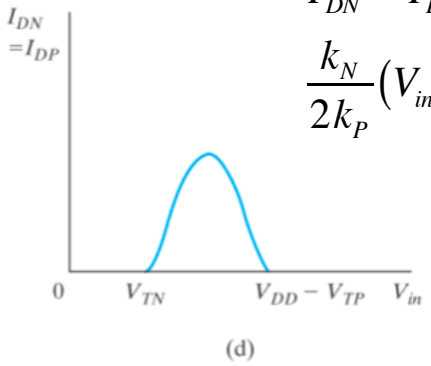
Logic Devices

CMOS VTC

CMOS Current-Voltage Characteristic







Region II Current Calculation

NMOSFET Drain Current (Saturation):

$$I_{DN} = \frac{k_N}{2}\left(V_{in} - V_{TN}\right)^2$$

PMOSFET Drain Current (Linear):

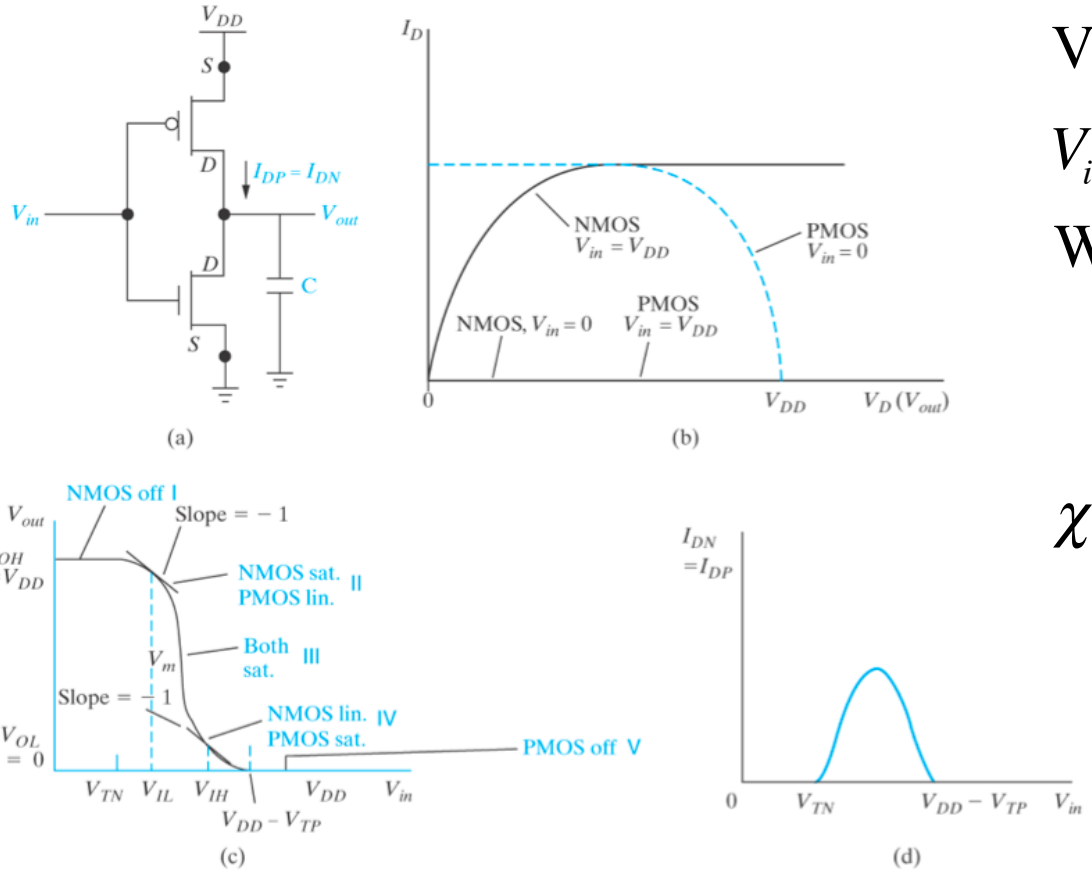
$$I_{DP} = k_P\left[\left(V_{DD} - V_{in}\right) + V_{TP} - \frac{\left(V_{DD} - V_{out}\right)}{2}\right]\left(V_{DD} - V_{out}\right)$$

To determine operation point (VTC):

$I_{DN} = I_{DP}$ So:

$$\frac{k_N}{2k_P}\left(V_{in} - V_{TN}\right)^2 = \left[\frac{V_{DD}}{2} - V_{in} + V_{TP} + \frac{V_{out}}{2}\right]\left(V_{DD} - V_{out}\right)$$

CMOS Transition Voltage



VTC Transition Occurs At:

$$V_{in} = \left(V_{DD} + \chi V_{TN} + V_{TP}\right) / \left(1 + \chi\right)$$

Where:

$$\chi = \left(\frac{k_N}{k_P}\right)^{\frac{1}{2}} = \frac{\left[\bar{\mu}_n C_i \left(\frac{Z}{L}\right)_N\right]^{\frac{1}{2}}}{\left[\bar{\mu}_p C_i \left(\frac{Z}{L}\right)_P\right]^{\frac{1}{2}}}$$

Want to have $V_{in} \sim \frac{V_{DD}}{2}$

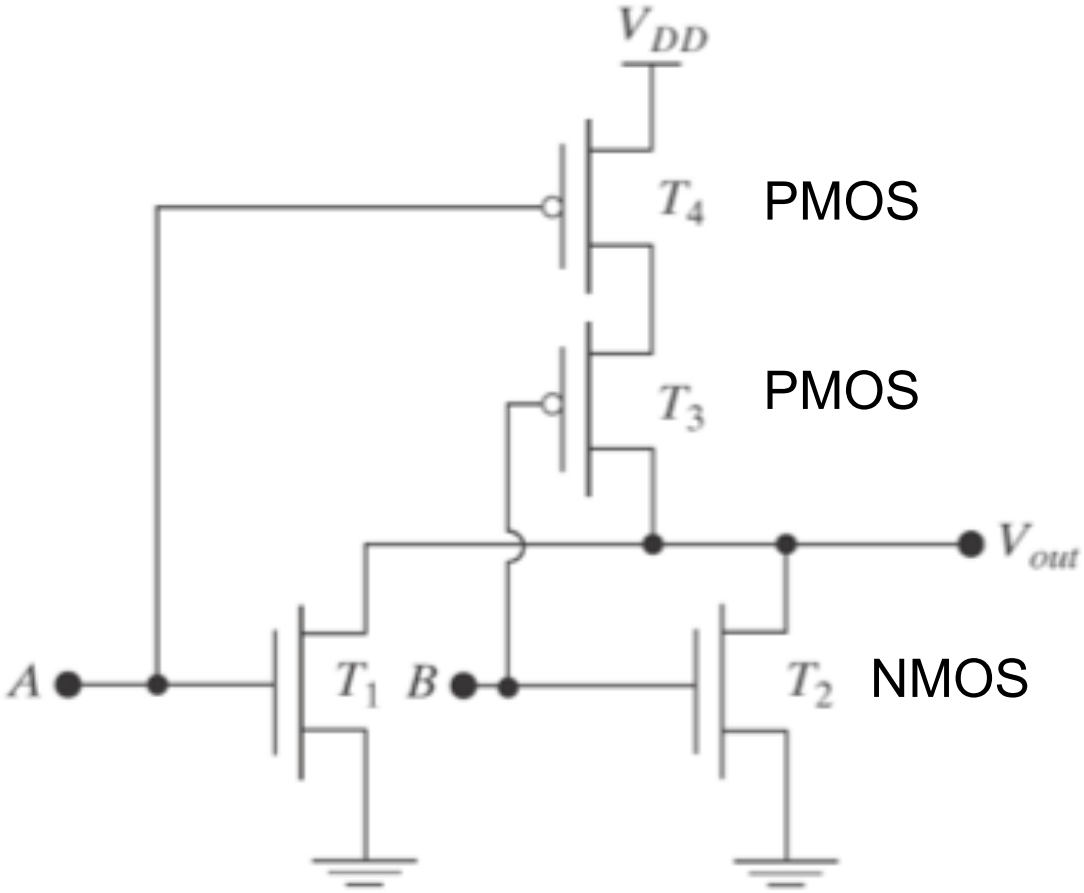
To get $V_{in}=V_{DD}/2$ at the switching point, design device such that $V_{TN}=-V_{TP}$ and $\chi=-1$:
For Si, with an electron mobility roughly 2X hole mobility, $(Z/L)_P=2(Z/L)_N$

Logic Devices

NOR, NAND

CMOS NOR



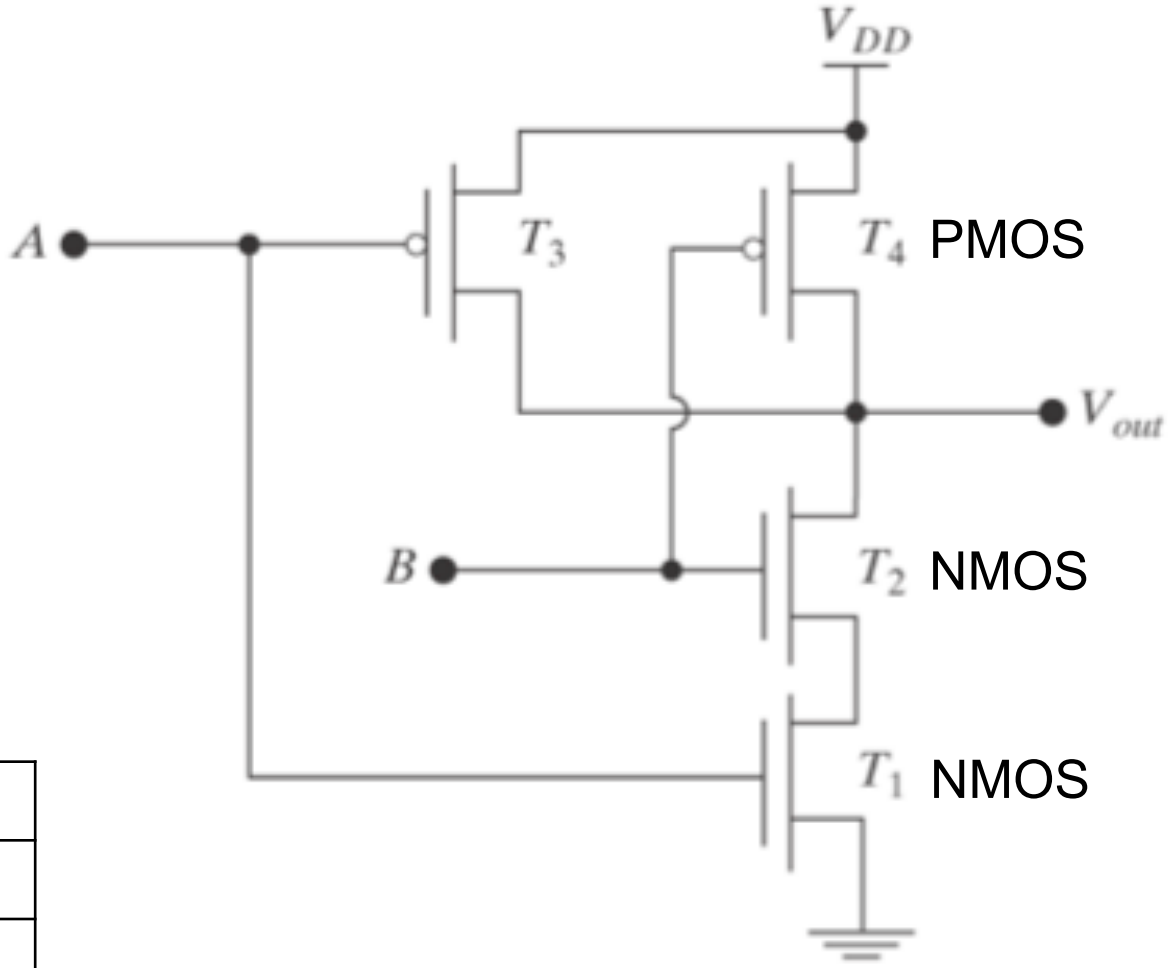


A	B	Out
0	0	1
0	1	0
1	0	0
1	1	0

PMOS in series (slower)

CMOS NAND





A	B	Out
0	0	1
0	1	1
1	0	1
1	1	0

NMOS in series (faster)

Power Dissipation

The input capacitance of the inverter is the parallel combination of the NMOSFET and PMOSFET:

$$C_{inv} = C_i\left\{(ZL)_N + (ZL)_P\right\}$$

Multiplying this term by the fan-out gives a total load capacitance of "C"

The energy expended in charging "C" is given by integrating the product of the time-dependent voltage and the time-dependent current:

Energy to Charge $\rightarrow$ $E_C = \int i_p(t)\left[V_{DD} - v(t)\right]dt = V_{DD}\int i_p(t)\,dt - \int i_p(t)v(t)\,dt$

but $i_p(t) = C\dfrac{dv(t)}{dt}$ so:

$$E_C = V_{DD}\int C\frac{dv(t)}{dt}dt - \int Cv(t)\frac{dv(t)}{dt}dt = CV_{DD}\int_0^{V_{DD}} dv - C\int_0^{V_{DD}} v\,dv = CV_{DD}^2 - \frac{1}{2}CV_{DD}^2 = \frac{1}{2}CV_{DD}^2$$

During a discharge cycle:

Energy to Discharge $\rightarrow$ $E_d = \int i_n(t)v(t)\,dt = -\int_{V_{DD}}^{0} Cv\,dv = \dfrac{1}{2}CV_{DD}^2$

For a charge/discharge frequency "*f*":

$$P = CV_{DD}^2 f$$

Switching Delays

The propagation delay time t_P is a metric for the switching speed of the gate.

$t_{PHL} \equiv$ the time for the output to transition from V_{OH} to $V_{OH}/2$

$t_{PLH} \equiv$ the time for the output to transition from V_{OL} to $V_{OH}/2$

Also, $V_{OL} = 0$ and $V_{OH} = V_{DD}$

The time needed to reach $V_{OH}/2$ is found by dividing charge stored in the capacitor by the charge or discharge current. Since the transistor providing the current is in saturation, $I(sat). = \frac{k}{2}\left(V_{DD} - V_T\right)^2$:

Using $Q = CV$:

$$t_{PHL} = \frac{\frac{1}{2}Q}{I} = \frac{\frac{1}{2}CV_{OH}}{I} = \frac{\frac{1}{2}CV_{DD}}{I_{DN}} = \frac{\frac{1}{2}CV_{DD}}{\frac{k_N}{2}\left(V_{DD} - V_{TN}\right)^2}$$

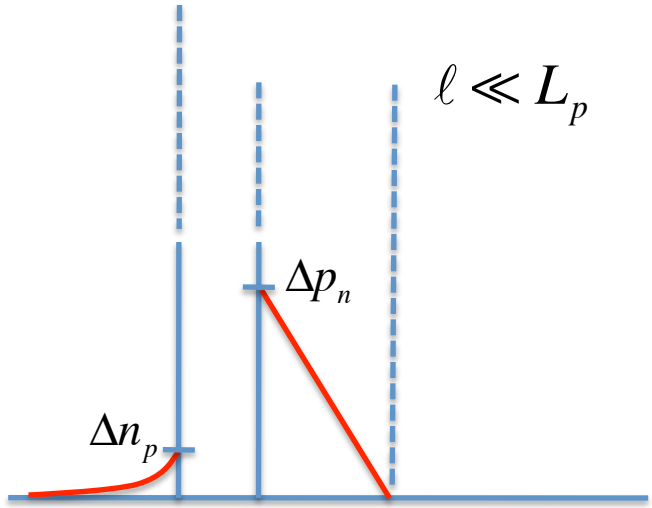
$$t_{PLH} = \frac{\frac{1}{2}CV_{DD}}{I_{DP}} = \frac{\frac{1}{2}CV_{DD}}{\frac{k_P}{2}\left(V_{DD} + V_{TP}\right)^2}$$

Narrow Base Diode

(Class Website Material)

The Narrow-Base Junction

- The narrow-base junction is a p^+-n-n^+ structure
- Minority carrier holes in the n region are the dominant current carriers (large relative to minority carrier electrons in p^+ region)
- Holes entering the n^+ region are assumed to recombine instantly
  - τ is much shorter in the n^+ region
- The "n" region is much smaller than the minority carrier diffusion length L_p so the minority carrier concentration in the n region can be approximated by a straight line



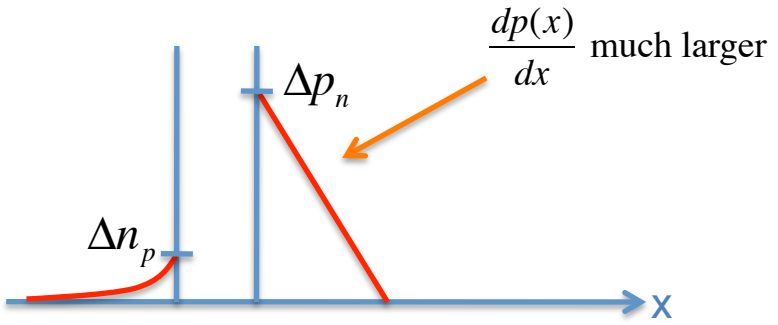
Boundary Condition for Narrow-Base Diode:

$$\delta p(x_n = 0) = \Delta p_n = p_n(e^{qV/kT} - 1)$$

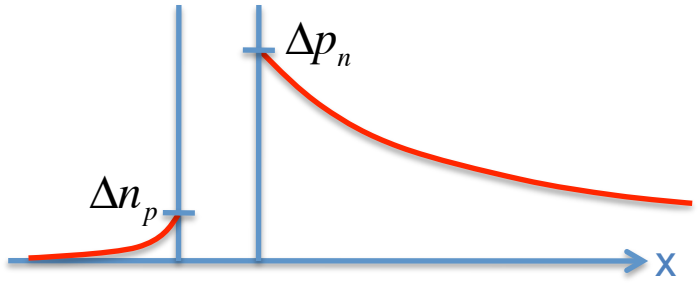
$$\delta p(x_n = \ell) = 0$$

Current Density Impact

Narrow Base Diode



Conventional p^+n Junction



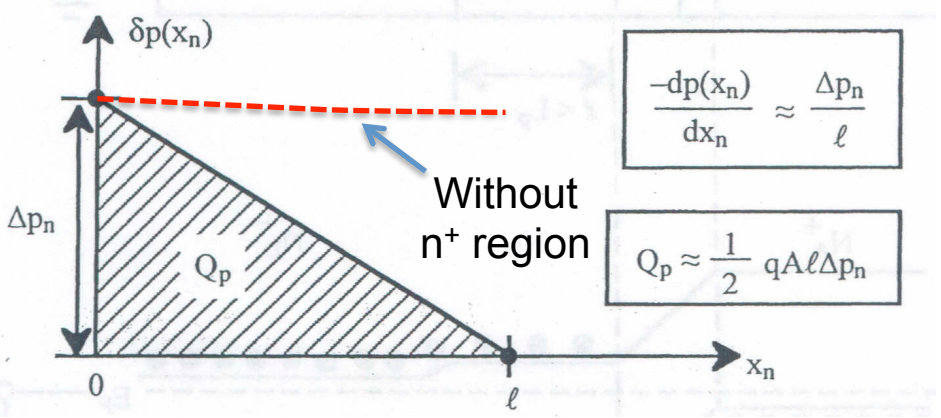
Higher Current Density!

$$J_{p,diff}(x) = -qD_p \frac{dp(x)}{dx}$$

- The slope of the excess carrier concentration is much higher for the narrow base diode than the conventional junction
- The current density in the narrow base diode is therefore much higher (same voltage produces more current)

Straight-Line Approximation

- If $\ell \ll L_p$ the hole diffusion current at the "contact" $x_n=\ell$ is almost as large as at the point of minority carrier injection $x_n=0$
- Most carriers diffuse across the base without recombining
- Key point: the diffusion current in the narrow-base diode is is much larger than that in a standard p^+n diode since ℓ replaces L_p in the denominator of the diode equation
- Key point: from a physical perspective, any hole that random walks across the line $x_n=\ell$ does not "come back" and reduce net current



The hole current in the n-region is approximately:

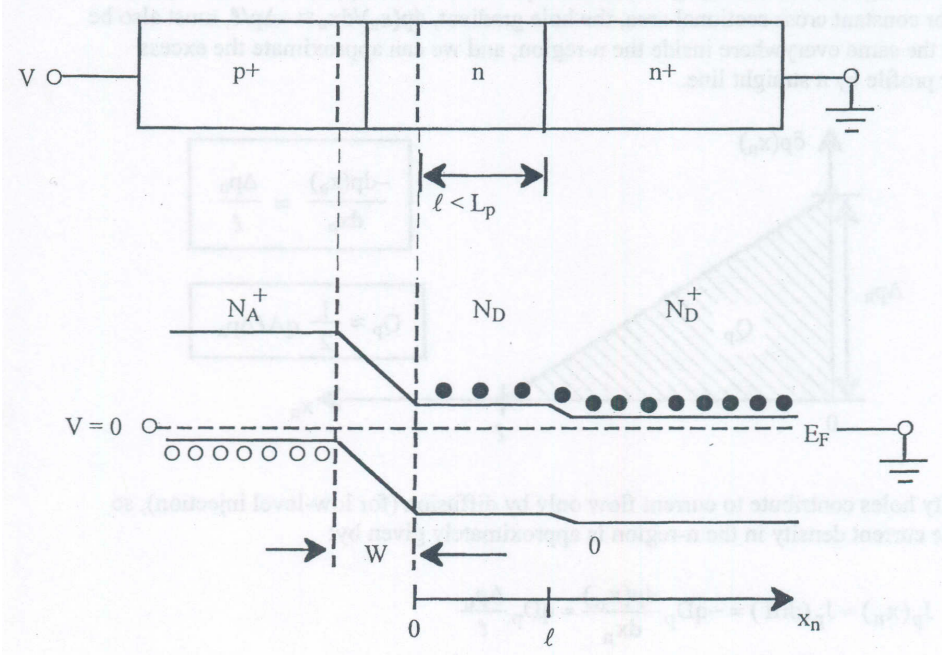
$$J_p(x_n) \approx J_p(diff) = -qD_p \frac{dp(x_n)}{dx_n} \approx qD_p \frac{\Delta p_n}{\ell}$$

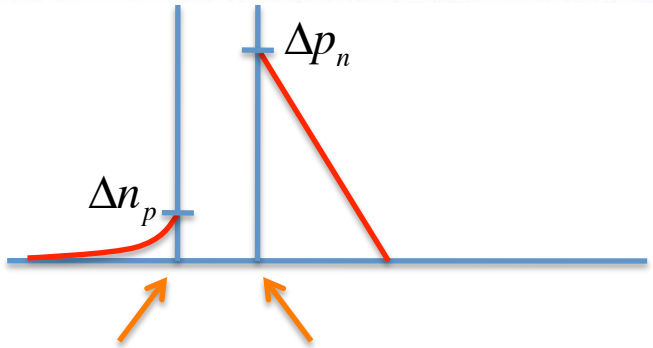
The total hole current diffusing across the base is then:

$$I_p(x_n) \approx AJ_p(diff) \approx qA \frac{D_p}{\ell} p_n \left(e^{qV/kT} - 1\right)$$

ℓ typically much smaller than L_p

Total Current: Straight Line Model

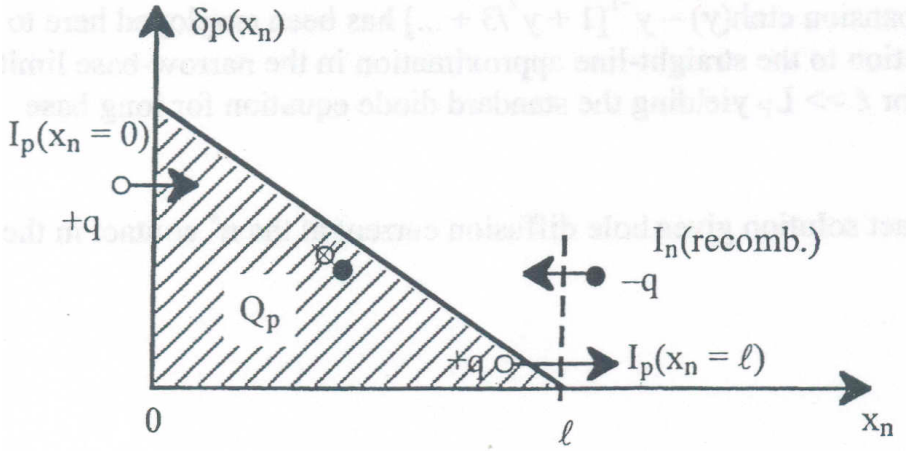




$$I = qA\left(\frac{D_n}{L_n}n_p + \frac{D_p}{l}p_n\right)\left(e^{qV/kT} - 1\right)$$

Space Charge Neutrality

- When recombination occurs, an electron must flow in from the n^+ contact to preserve space-charge neutrality



$$I_p(x_n = 0) = I_p(x_n = \ell) + I_n(recomb.)$$

$$I_n(recomb.) = \frac{Q_p}{\tau_p} \approx \frac{\frac{1}{2} qA\ell\Delta p_n}{\tau_p}$$

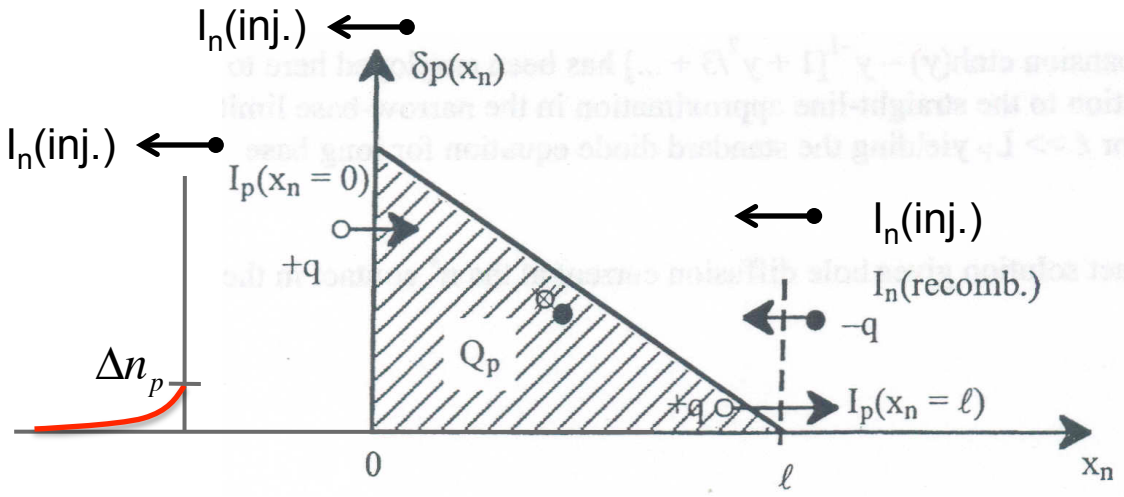
$$\approx \frac{qA\ell}{2\tau_p} p_n \left(e^{qV/kT} - 1\right)$$

The majority electron current flowing into the n-region at $x_n = \ell$ compensates the small decrease in hole diffusion current due to recombination within the "base" region:

$$I_n(recomb.) = I_p(x_n = 0) - I_p(x_n = \ell)$$

$$\approx I_p(x_n = 0)\left[\frac{\ell^2}{2L_p^2}\right]$$

Narrow Base Diode Total Current



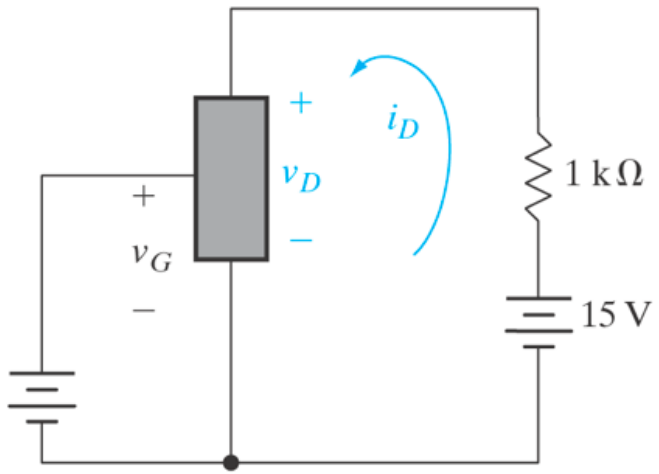
$$I_{tot} = I_p(x_n = \ell) + I_n(recomb.) + I_n(inj.)$$

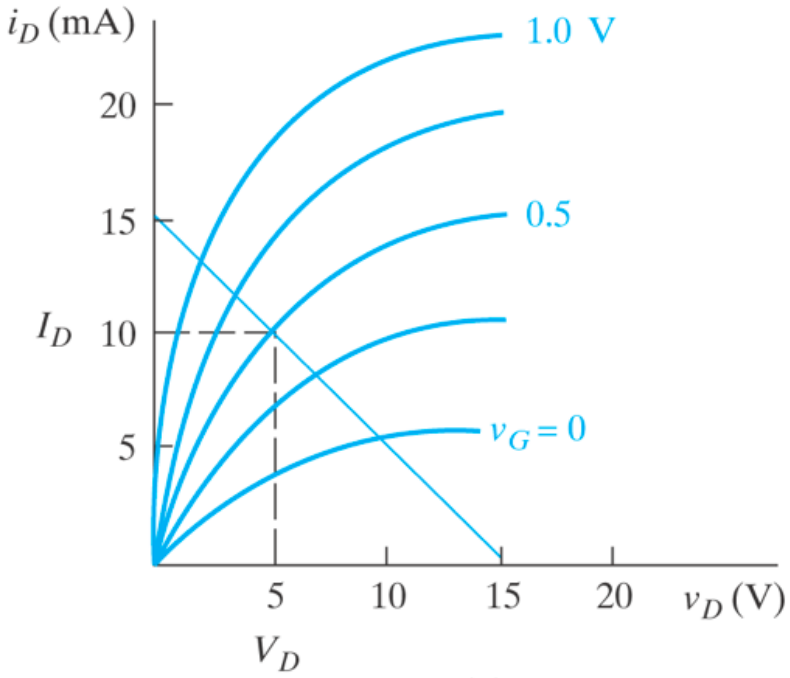
$$= qA\frac{D_p}{L_p}\Delta p_n \operatorname{csch}\left(\frac{\ell}{L_p}\right) + qA\frac{D_p}{L_p}\Delta p_n \tanh\left(\frac{\ell}{2L_p}\right) + qA\frac{D_n}{L_n}\Delta n_p$$

$$= \left(qA\frac{D_p}{L_p}p_n \operatorname{csch}\left(\frac{\ell}{L_p}\right) + qA\frac{D_p}{L_p}p_n \tanh\left(\frac{\ell}{2L_p}\right) + qA\frac{D_n}{L_n}n_p\right)\left(e^{qV/kT} - 1\right)$$

BJT Fundamentals

3-Terminal Load Line Analysis



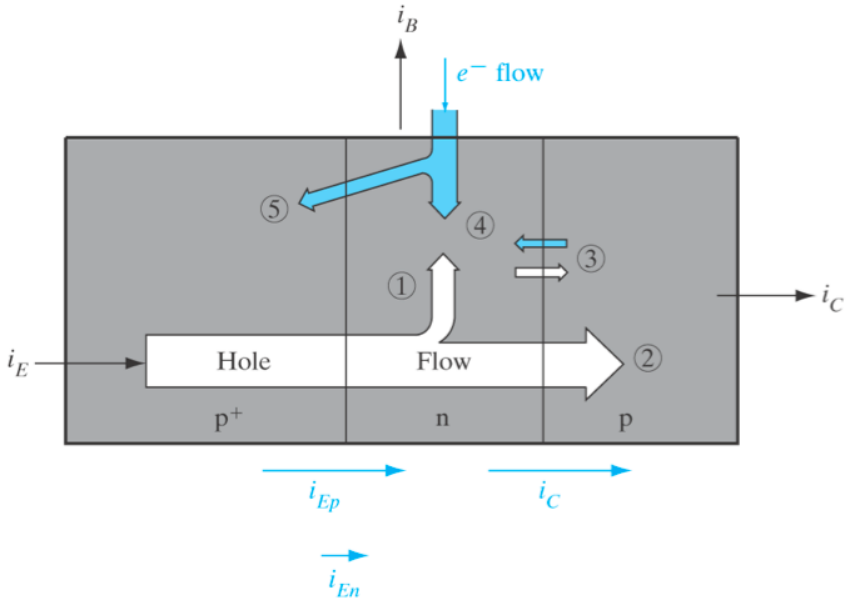


- Loop equation: $E=Ri_D+v_D$
- Device i-v relationship: $i_D=f(v_D,v_G)$ or $i_D=f(v_D,i_B)$
- The load line is used to find the simultaneous solution to both equations

Current Flow in a pnp Transistor

Transistor Current Components

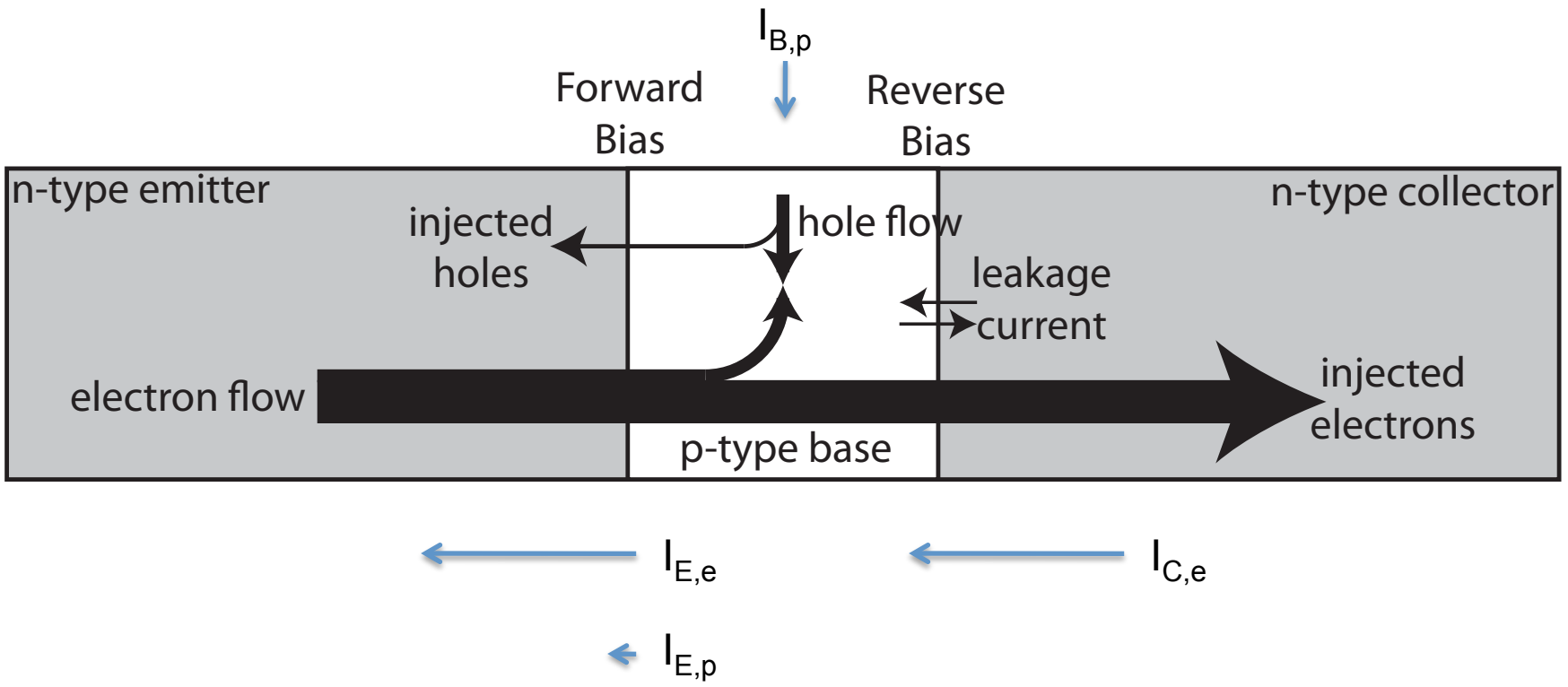
- Hole recombination in base (1)
- Holes injected into reverse-biased base-collector junction (2)
- Thermal generation-recombination current in reverse-biased base-collector junction (3)
- Electron flow into base to compensate for electrons lost due to recombination with injected holes (4)
- Electron flow from n-type base to p^+ emitter under forward bias (5)



Components of electron (particle) flow into base:

- Recombination (-)
- Forward injection into p^+ region (-)
- Thermal generation flow from collector (+)

npn Transistor



Key Definitions

$I_C = Bi_{E_p}$

$B \equiv$ fraction of injected holes that make it to the collector

[Note: $(1-B)$ is the fraction of holes that recombine in the base]

B is called the "**Base Transport Factor**"

Emitter injection efficiency : $\gamma = \dfrac{i_{Ep}}{i_{En} + i_{Ep}}$

Ideally, B and γ approach unity for an efficient transistor.

Note also: $\dfrac{i_C}{i_E} = \dfrac{Bi_{Ep}}{i_{En} + i_{Ep}} = B\gamma \equiv \alpha$

α is the "**current transfer ratio**"

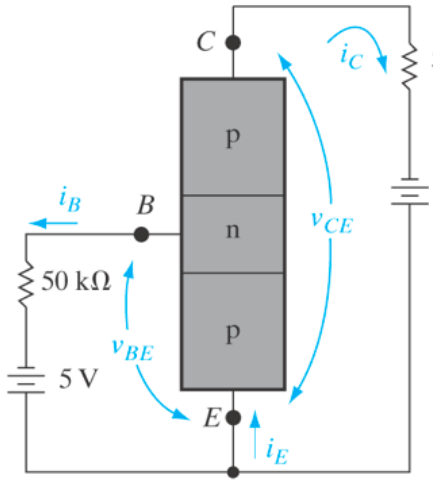
Base Current: $i_B = i_{En} + (1-B)i_{Ep}$

and so, neglecting I_o in the base-collector junction:

$$\frac{i_C}{i_B} = \frac{Bi_{Ep}}{i_{En} + (1-B)i_{Ep}} = \frac{B\left[i_{Ep}/\left(i_{En}+i_{Ep}\right)\right]}{1 - B\left[i_{Ep}/\left(i_{En}+i_{Ep}\right)\right]} = \frac{B\gamma}{1-B\gamma} = \frac{\alpha}{1-\alpha} \equiv \beta$$

β is the "**base to collector current amplification factor**"

Typical values of α are ~0.99 and β are ~100



$\tau_p = 10\ \mu s$

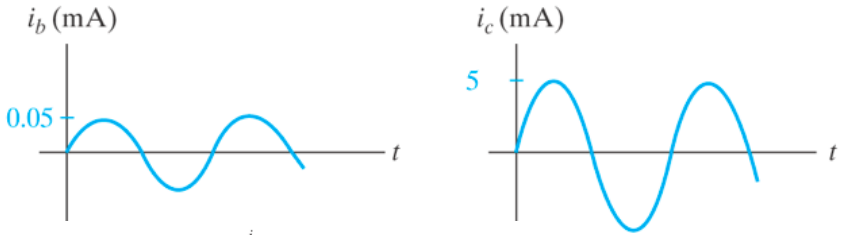
$\tau_t = 0.1\ \mu s$

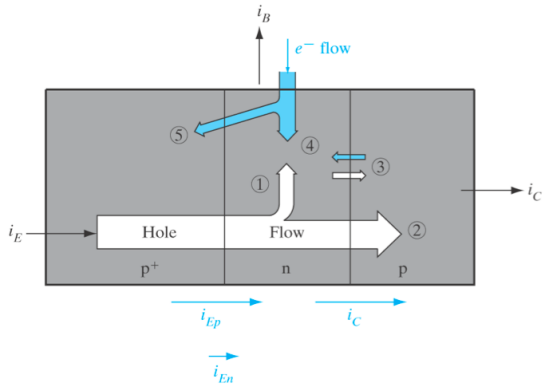
$\dfrac{i_C}{i_B} = \beta = \dfrac{\tau_p}{\tau_t} = 100$

Neglecting v_{BE}

$I_B = \dfrac{5\text{ V}}{50\text{ k}\Omega} = 0.1\text{ mA}$

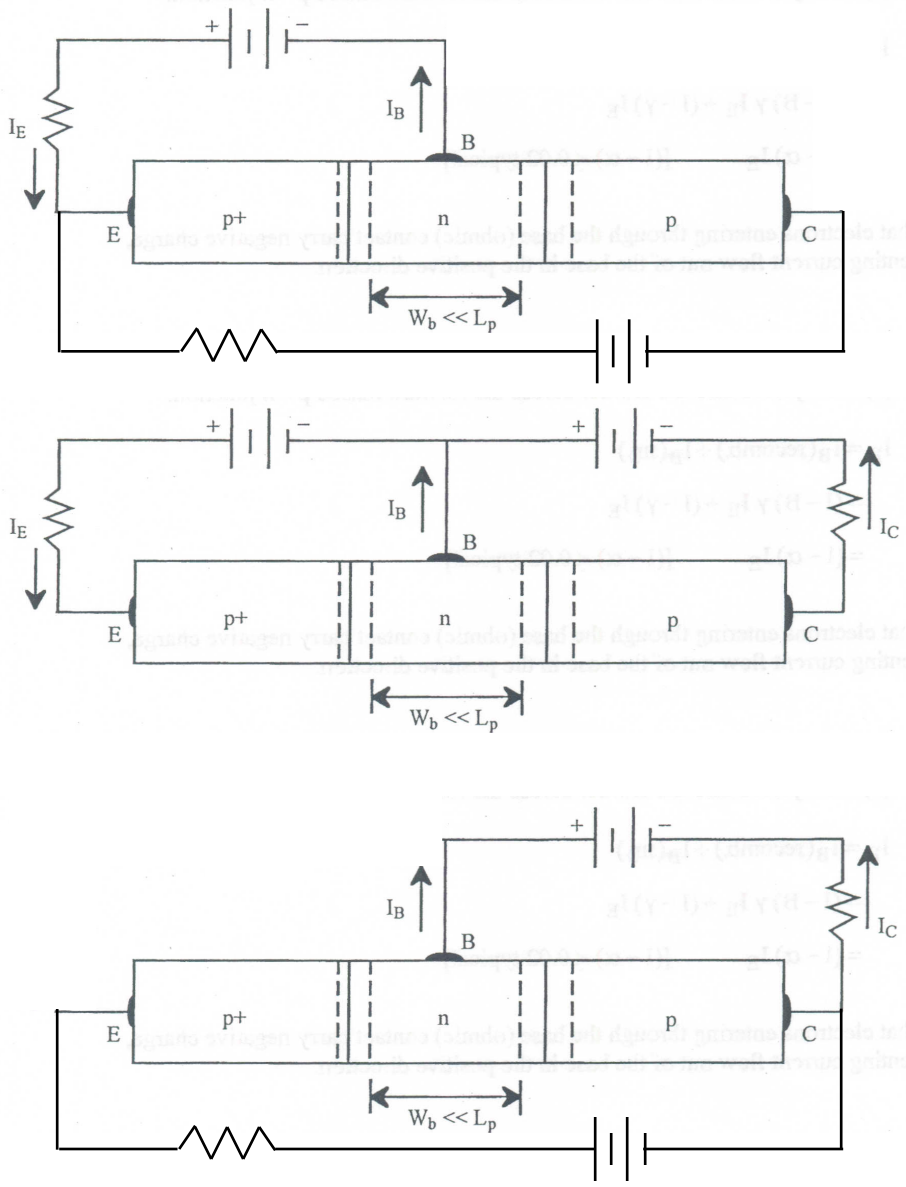
$I_C = \beta I_B = 10\text{ mA}$





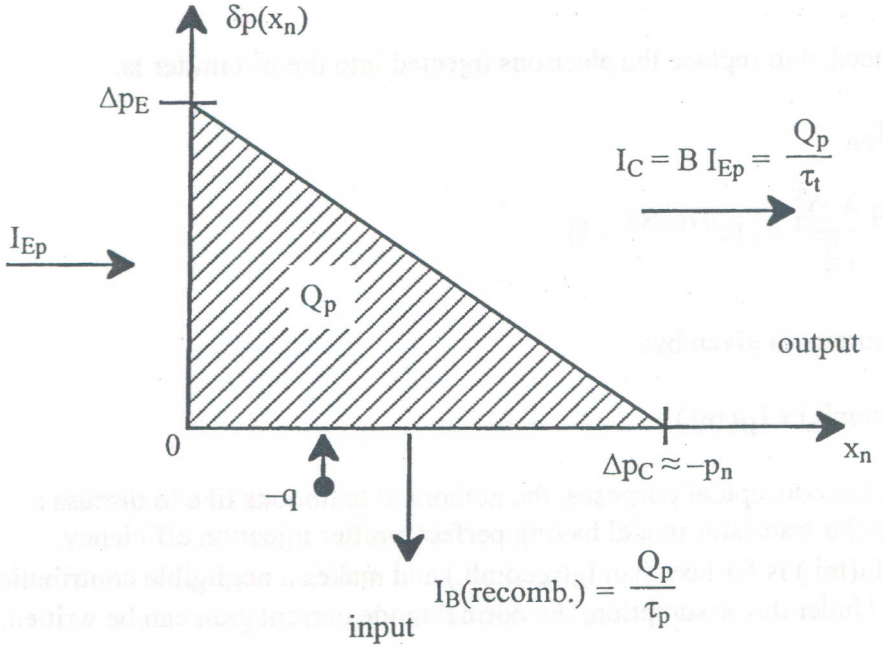
Biasing Configurations

- Various BJT biasing methods are possible
  - "Common emitter"
  - "Common base"
  - "Common collector"
- Preferred biasing method depends upon intended application
  - Input and output impedance
  - Voltage/current/power gain
  - Input/output phase relationship



Space-Charge Neutrality in the Base

- The base region between the emitter-base and base-collector depletion regions is electrostatically neutral
- In an n-type base, electrons enter from the base contact and the reverse-biased base-collector junction
- In an n-type base, electrons leave through recombination with injected minority carrier holes and through injection into the p-type emitter across the forward-biased emitter-base junction
- In a p-n-p device holes enter the base through injection by the p-type emitter
- In a p-n-p device holes leave through recombination with electrons in the base or by diffusing into the field region of the reverse-biased base-collector junction



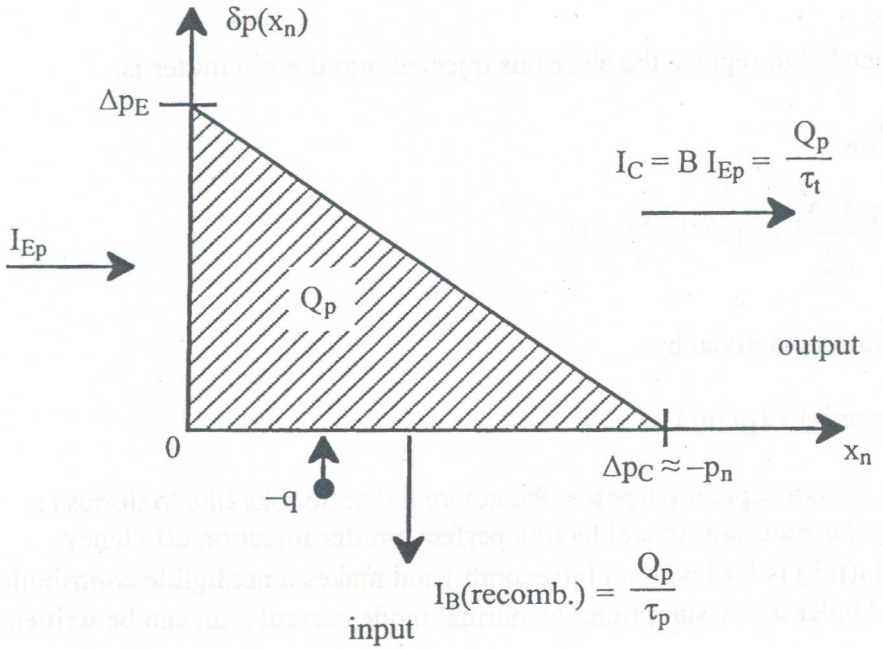
$$\tau_p \gg \tau_t$$

$$I_{E_p}(x=0) = \frac{Q_p}{\tau_p} + \frac{Q_p}{\tau_t} \approx \frac{Q_p}{\tau_t} = \frac{qAW_B\Delta p_n}{2\tau_t}$$

$$\approx \frac{qAW_B p_n e^{qV_{EB}/kT}}{2\tau_t} \approx I_o e^{qV_{EB}/kT} \approx I_C$$

Space-Charge Neutrality in the Base

- The average hole spends a time equal to the transit time τ_t in the base, and τ_t is significantly less than the carrier lifetime τ_p
- The average electron spends a time equal to the hole minority carrier lifetime τ_p waiting to recombine in the base
- Each electron can provide space charge compensation for (τ_p/τ_t) holes!
- The rate of flow of electrons (base current) determines how many holes can flow (emitter/ collector current)
  - A small base current can control much larger currents in the emitter and collector



$$\beta = \frac{i_C}{i_B} = \frac{\left(\frac{Q_p}{\tau_t}\right)}{\left(\frac{Q_p}{\tau_p}\right)} = \frac{\tau_p}{\tau_t}$$

Diffusion Equation in the Base

Diffusion Equation in the Base

Apply Boundary Conditions:

$\delta p(x_n = 0) = C_1 + C_2 = \Delta p_E$ and

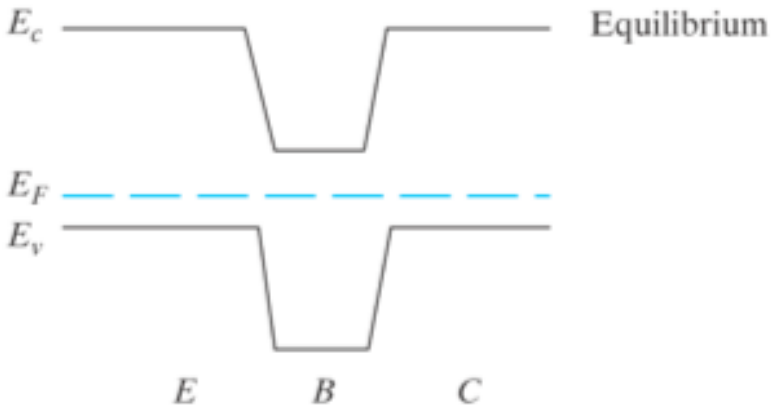
$\delta p(x_n = W_b) = C_1 e^{W_b/L_p} + C_2 e^{-W_b/L_p} = \Delta p_C$

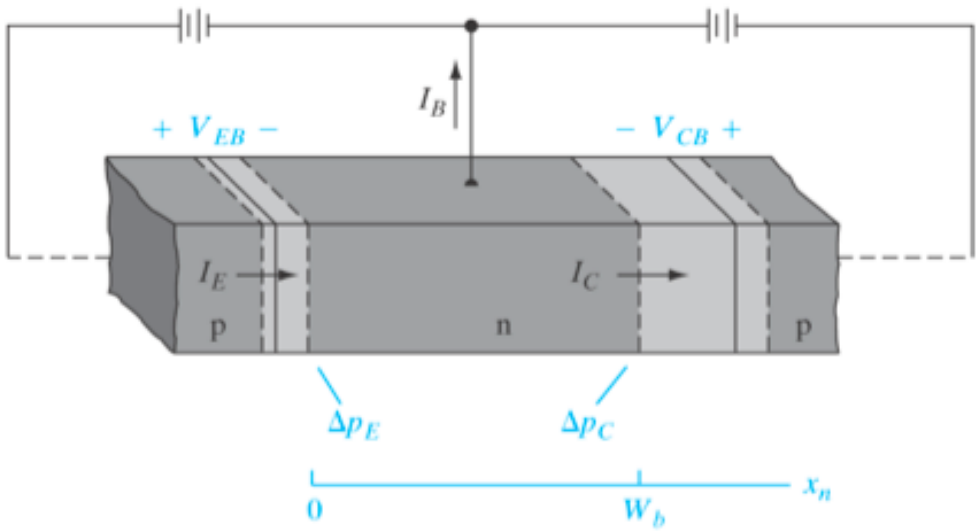
Solving for C_1 and C_2:

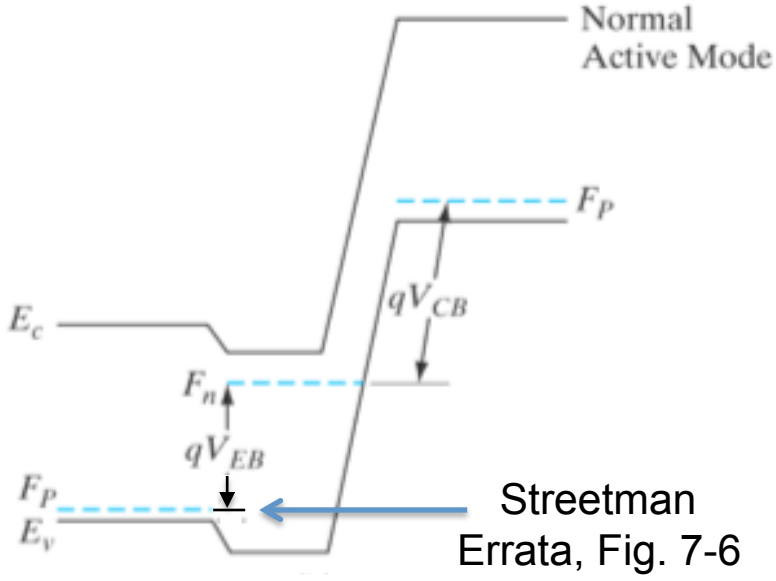
$$C_1 = \frac{\Delta p_C - \Delta p_E e^{-W_b/L_p}}{e^{W_b/L_p} - e^{-W_b/L_p}} \text{ and } C_2 = \frac{\Delta p_E e^{W_b/L_p} - \Delta p_C}{e^{W_b/L_p} - e^{-W_b/L_p}}$$

For a strongly reverse-biased collector and $\Delta p_E \gg p_n$:

$$\delta p(x_n) = C_1 e^{x_n/L_p} + C_2 e^{-x_n/L_p} = \Delta p_E \frac{e^{W_b/L_p} e^{-x_n/L_p} - e^{-W_b/L_p} e^{x_n/L_p}}{e^{W_b/L_p} - e^{-W_b/L_p}} \text{ for } \Delta p_C \simeq 0$$

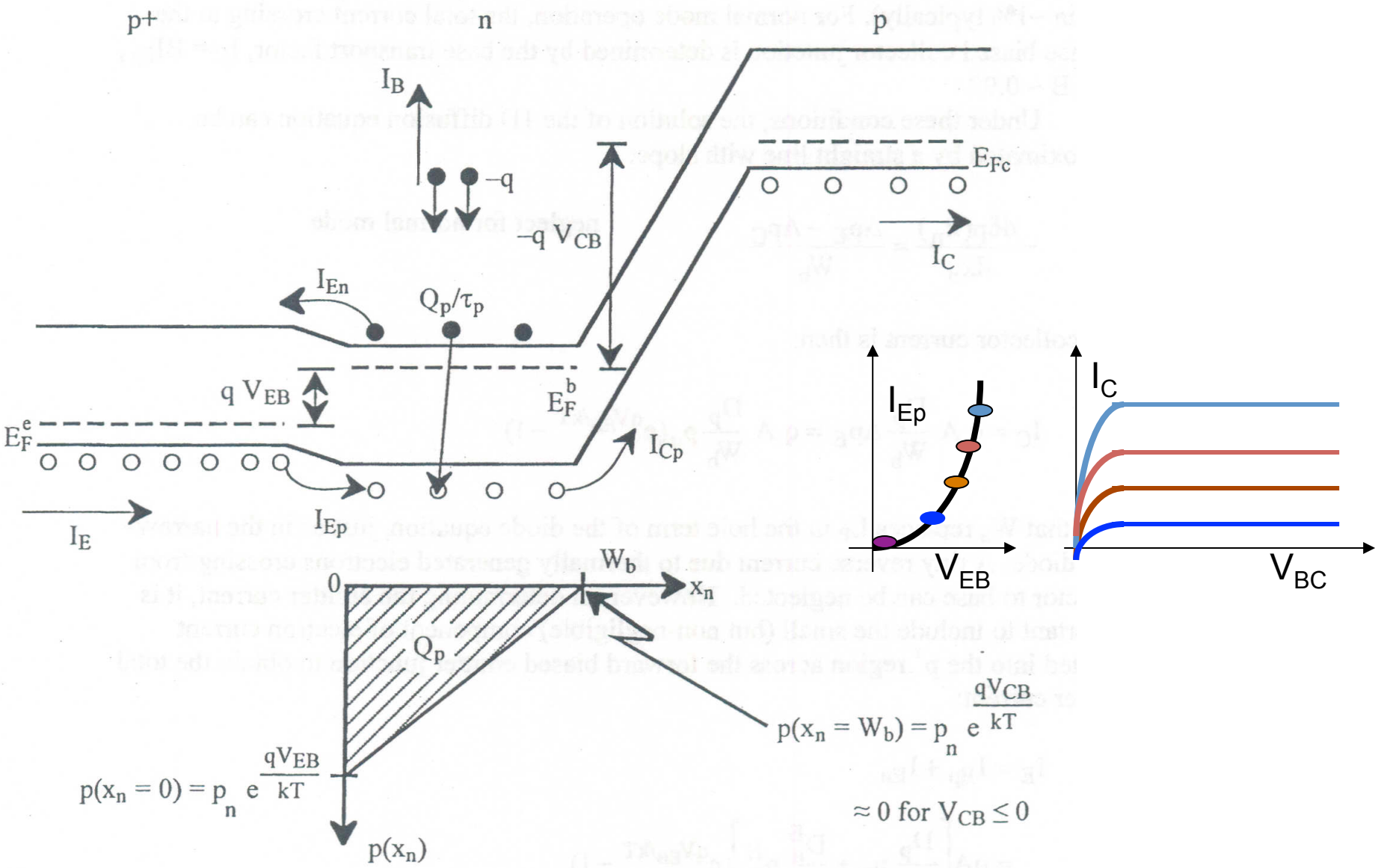






Streetman Errata, Fig. 7-6

Band Diagram Under Normal Bias



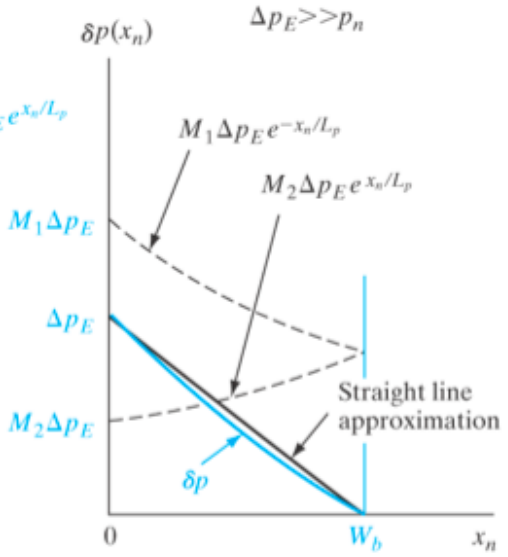
Minority Carrier Distributions

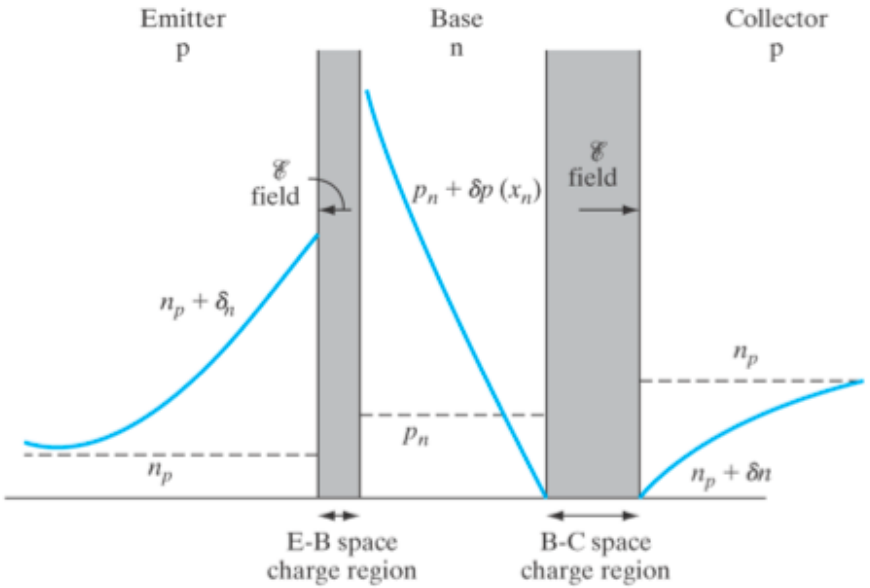
$$\delta p(x_n) = M_1 \Delta p_E e^{-x_n/L_p} - M_2 \Delta p_E e^{x_n/L_p}$$

where

$$M_1 = \frac{e^{W_b/L_p}}{e^{W_b/L_p} - e^{-W_b/L_p}}$$

$$M_2 = \frac{e^{-W_b/L_p}}{e^{W_b/L_p} - e^{-W_b/L_p}}$$





$$\delta p(x_n = 0) = C_1 + C_2 = \Delta p_E \text{ and}$$

$$\delta p(x_n = W_b) = C_1 e^{W_b/L_p} + C_2 e^{-W_b/L_p} = \Delta p_C$$

So:

$$C_1 = \frac{\Delta p_C - \Delta p_E e^{-W_b/L_p}}{e^{W_b/L_p} - e^{-W_b/L_p}} \text{ and } C_2 = \frac{\Delta p_E e^{W_b/L_p} - \Delta p_C}{e^{W_b/L_p} - e^{-W_b/L_p}}$$

For General Biasing: $\delta p(x_n) = C_1 e^{x_n/L_p} + C_2 e^{-x_n/L_p}$

For a strongly reverse-biased collector and $\Delta p_E \gg p_n$:

$$\delta p(x_n) = \Delta p_E \frac{e^{W_b/L_p} e^{-x_n/L_p} - e^{-W_b/L_p} e^{x_n/L_p}}{e^{W_b/L_p} - e^{-W_b/L_p}} \text{ for } \Delta p_C \simeq 0$$

Diffusion Equation: General Solution

Excess Hole Concentration:

$$\Delta p_E = p_n\left(e^{qV_{EB}/kT} - 1\right) \text{ and } \Delta p_C = p_n\left(e^{qV_{CB}/kT} - 1\right)$$

Using the diffusion equation: $\dfrac{d^2\delta p(x_n)}{dx_n^2} = \dfrac{\delta p(x_n)}{L_p^2}$

Solution: $\delta p(x_n) = C_1 e^{x_n/L_p} + C_2 e^{-x_n/L_p}$

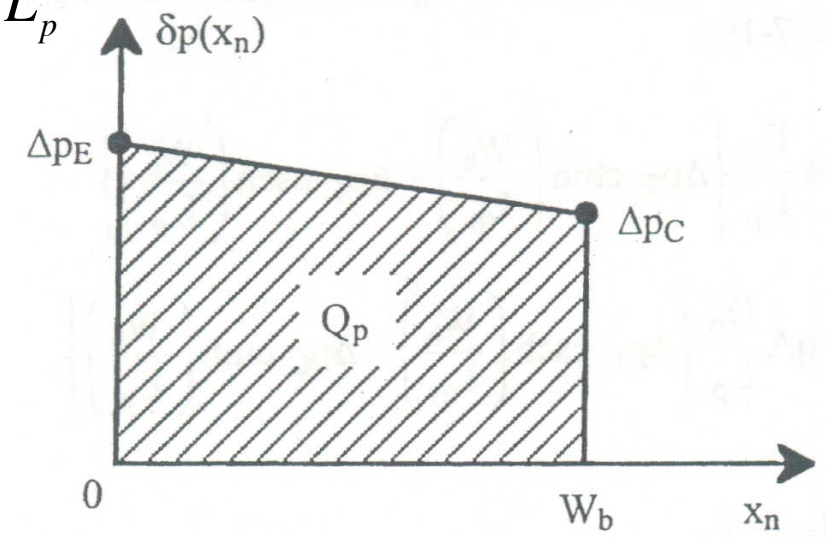
Apply Boundary Conditions:

$\delta p(x_n = 0) = C_1 + C_2 = \Delta p_E$ and

$\delta p(x_n = W_b) = C_1 e^{W_b/L_p} + C_2 e^{-W_b/L_p} = \Delta p_C$

Solving for C_1 and C_2:

$$C_1 = \frac{\Delta p_C - \Delta p_E e^{-W_b/L_p}}{e^{W_b/L_p} - e^{-W_b/L_p}} \text{ and } C_2 = \frac{\Delta p_E e^{W_b/L_p} - \Delta p_C}{e^{W_b/L_p} - e^{-W_b/L_p}}$$



Evaluation of the Terminal Currents

Terminal Currents

Emitter and collector currents are calculated at the edge of each depletion region from the minority carrier (hole) concentration gradient:

$$I_p(x_n) = -qAD_p \frac{d\delta p(x_n)}{dx_n} = -qAD_p \frac{d}{dx_n}\left[C_1 e^{x_n/L_p} + C_2 e^{-x_n/L_p}\right]$$ therefore, at $x_n = 0$:

$$I_{Ep} = I_p(x_n = 0) = qA\frac{D_p}{L_p}\left(C_2 - C_1\right)$$

Neglecting collector reverse saturation current, I_C is dominated by holes:

$$I_C = I_P(x_n = W_b) = qA\frac{D_p}{L_p}\left(C_2 e^{-W_b/L_p} - C_1 e^{W_b/L_p}\right)$$

Substituting in the previously determined values for C_1 and C_2:

$$I_{Ep} = qA\frac{D_p}{L_p}\left[\frac{\Delta p_E\left(e^{W_b/L_p} + e^{-W_b/L_p}\right) - 2\Delta p_C}{e^{W_b/L_p} - e^{-W_b/L_p}}\right]$$ or, expressing as hyperbolic functions:

$$I_{Ep} = qA\frac{D_p}{L_p}\left(\Delta p_E \operatorname{ctnh}\frac{W_b}{L_p} - \Delta p_C \operatorname{csch}\frac{W_b}{L_p}\right) \text{ and } I_C = qA\frac{D_p}{L_p}\left(\Delta p_E \operatorname{csch}\frac{W_b}{L_p} - \Delta p_C \operatorname{ctnh}\frac{W_b}{L_p}\right)$$

The value of I_B is obtained via node analysis: $I_B = I_E - I_C$

$$I_B = qA\frac{D_p}{L_p}\left[\left(\Delta p_E + \Delta p_C\right)\left(\operatorname{ctnh}\frac{W_b}{L_p} - \operatorname{csch}\frac{W_b}{L_p}\right)\right] = qA\frac{D_p}{L_p}\left[\left(\Delta p_E + \Delta p_C\right)\tanh\frac{W_b}{2L_p}\right]$$

Terminal Currents: General Case

Terminal Currents for Arbitrary Transistor Biasing:

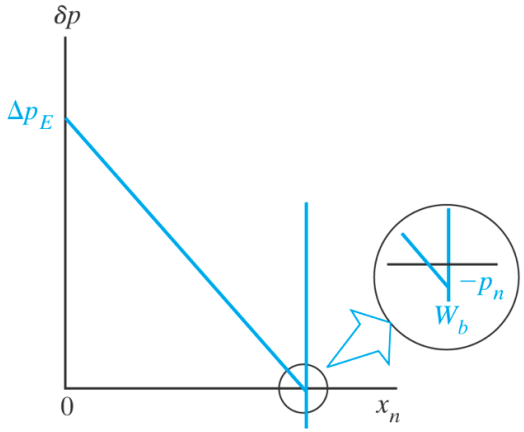
$$I_{Ep} = qA\frac{D_p}{L_p}\left(\Delta p_E \text{ctnh}\frac{W_b}{L_p} - \Delta p_C \text{csch}\frac{W_b}{L_p}\right)$$

$$I_{Cp} = qA\frac{D_p}{L_p}\left(\Delta p_E \text{csch}\frac{W_b}{L_p} - \Delta p_C \text{ctnh}\frac{W_b}{L_p}\right)$$

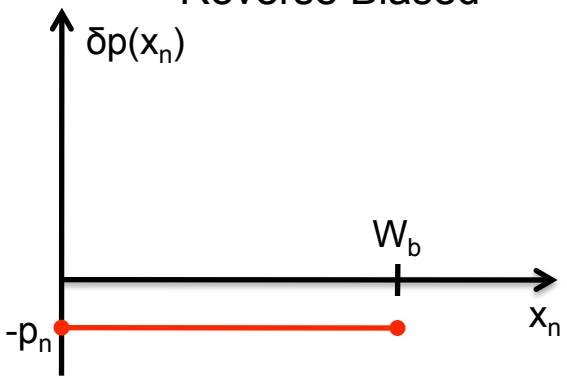
$$I_B = qA\frac{D_p}{L_p}\left[\left(\Delta p_E + \Delta p_C\right)\tanh\frac{W_b}{2L_p}\right]$$

Note: These do not include the electron component across the emitter and collector junctions.

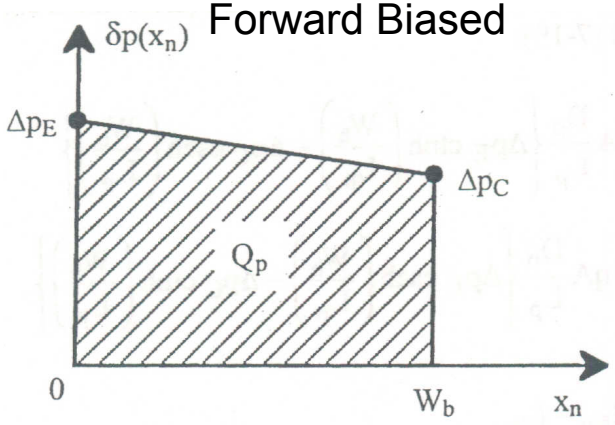
Normal Bias Condition



Both Junctions Reverse Biased



Both Junctions Forward Biased



Terminal Currents: Normal Biasing

General Expressions:

$$I_{Ep} = qA\frac{D_p}{L_p}\left(\Delta p_E \operatorname{ctnh}\frac{W_b}{L_p} - \Delta p_C \operatorname{csch}\frac{W_b}{L_p}\right)$$

$$I_{Cp} = qA\frac{D_p}{L_p}\left(\Delta p_E \operatorname{csch}\frac{W_b}{L_p} - \Delta p_C \operatorname{ctnh}\frac{W_b}{L_p}\right)$$

$$I_B = qA\frac{D_p}{L_p}\left[\left(\Delta p_E + \Delta p_C\right)\tanh\frac{W_b}{2L_p}\right]$$

If the collector is reverse biased, $\Delta p_C = -p_n$

If p_n is small, Δp_C can be neglected

If $\gamma \simeq 1$, $I_E \simeq I_{Ep}$ therefore:

$$I_{Ep} \simeq qA\frac{D_p}{L_p}\Delta p_E \operatorname{ctnh}\frac{W_b}{L_p}$$

$$I_{Cp} \simeq qA\frac{D_p}{L_p}\Delta p_E \operatorname{csch}\frac{W_b}{L_p}$$

$$I_B \simeq qA\frac{D_p}{L_p}\Delta p_E \tanh\frac{W_b}{2L_p}$$

Hyperbolic Function Expansion:

$$\operatorname{sech} y = 1 - \frac{y^2}{2} + \frac{5y^4}{24} - \ldots$$

$$\operatorname{ctnh} y = \frac{1}{y} + \frac{y}{3} - \frac{y^3}{45} + \ldots$$

$$\operatorname{csch} y = \frac{1}{y} - \frac{y}{6} + \frac{7y^3}{360} - \ldots$$

$$\tanh y = y - \frac{y^3}{3} + \ldots$$

$$I_{Ep} = qA\frac{D_p}{L_p}\Delta p_E\left(\frac{1}{W_b/L_p} + \frac{W_b/L_p}{3}\right)$$

$$I_{Cp} = qA\frac{D_p}{L_p}\Delta p_E\left(\frac{1}{W_b/L_p} - \frac{W_b/L_p}{6}\right)$$

$$I_B \simeq qA\frac{D_p}{L_p}\Delta p_E\frac{W_b}{2L_p} = \frac{qAD_p\Delta p_E W_b}{2L_p^2} = \frac{qAW_b\Delta p_E}{2\tau_p}$$

using $L_p = \sqrt{D_p\tau_p}$

Emitter Injection Efficiency, Base Transport Factor, and Current Transfer Ratio

Analytical Expressions

Emitter Injection Efficiency ($\gamma < 1$)

In the case where $\gamma < 1$, we need to include I_{En} in the total emitter current. Assuming the emitter region is long compared to the electron diffusion length, normal biasing, and $V_{EB} \gg kT/q$:

$$I_E = I_{Ep} + I_{En} \text{ using } I_{En} = \frac{qAD_n^p}{L_n^p} n_p e^{qV_{EB}/kT} \text{ and } I_{Ep} = \frac{qAD_p^n}{L_p^n}\left(p_n e^{qV_{EB}/kT}\right)\text{ctnh}\frac{W_b}{L_p^n}$$

$$I_E = qA\left[\frac{D_p^n}{L_p^n} p_n \text{ctnh}\frac{W_b}{L_p^n} + \frac{D_n^p}{L_n^p} n_p\right] e^{qV_{EB}/kT} \text{ and so:}$$

$$\gamma = \frac{I_{Ep}}{I_E} = \left[\frac{I_{Ep} + I_{En}}{I_{Ep}}\right]^{-1} = \left[1 + \frac{I_{En}}{I_{Ep}}\right]^{-1} = \left[1 + \frac{\frac{D_n^p}{L_n^p}\frac{n_p}{p_n}}{\frac{D_p^n}{L_p^n}} \tanh\frac{W_b}{L_p^n}\right]^{-1}$$

But $\frac{n_p}{p_n} = \frac{n_n}{p_p}$, $\frac{D_n^p}{D_p^n} = \frac{\mu_n^p}{\mu_p^n}$, and $\frac{D}{\mu} = \frac{kT}{q}$. Therefore:

$$\gamma = \left[1 + \frac{L_p^n n_n \mu_n^p}{L_n^p p_p \mu_p^n} \tanh\frac{W_b}{L_p^n}\right]^{-1} \simeq \left[1 + \frac{W_b n_n \mu_n^p}{L_n^p p_p \mu_p^n}\right]^{-1}$$

Base Transport Factor and Current Transfer Ratio

Assuming the emitter region is long compared to the electron diffusion length, normal biasing, and $V_{EB} \gg kT/q$.

The base transport factor B is:

$$B = \frac{I_C}{I_{Ep}} = \frac{qA\dfrac{D_p}{L_p}\Delta p_E \operatorname{csch}\dfrac{W_b}{L_p}}{qA\dfrac{D_p}{L_p}\Delta p_E \operatorname{ctnh}\dfrac{W_b}{L_p}} = \frac{\operatorname{csch}\dfrac{W_b}{L_p}}{\operatorname{ctnh}\dfrac{W_b}{L_p}} = \operatorname{sech}\frac{W_b}{L_p}$$

The current transfer ratio α is the product of B and γ:

$$\alpha = \gamma B \cong \left[1 + \frac{W_b n_n \mu_n^p}{L_n^p p_p \mu_p^n}\right]^{-1} \operatorname{sech}\frac{W_b}{L_p}$$

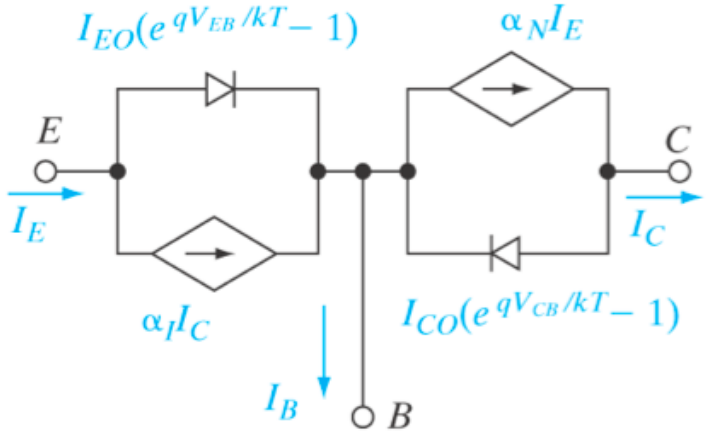
Recall Also:

$$\beta = \frac{\alpha}{1-\alpha}$$

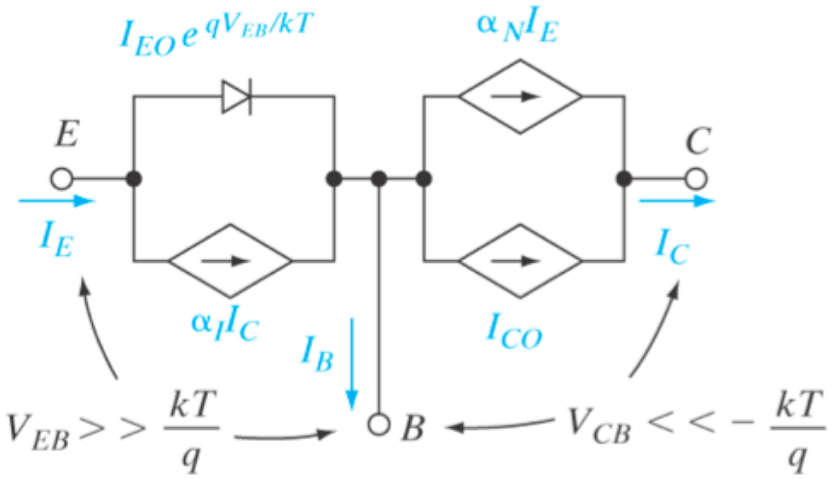
Ebers-Moll Equations

A Few Comments

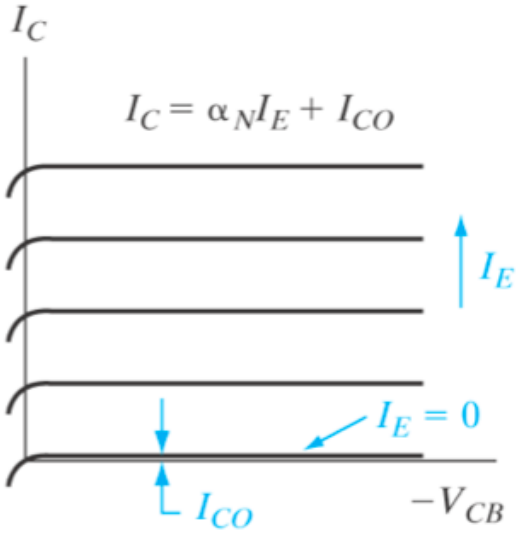
Coupled Diode Model



(a)



(b)



(c)

Ebers-Moll Equations

It can be shown that:

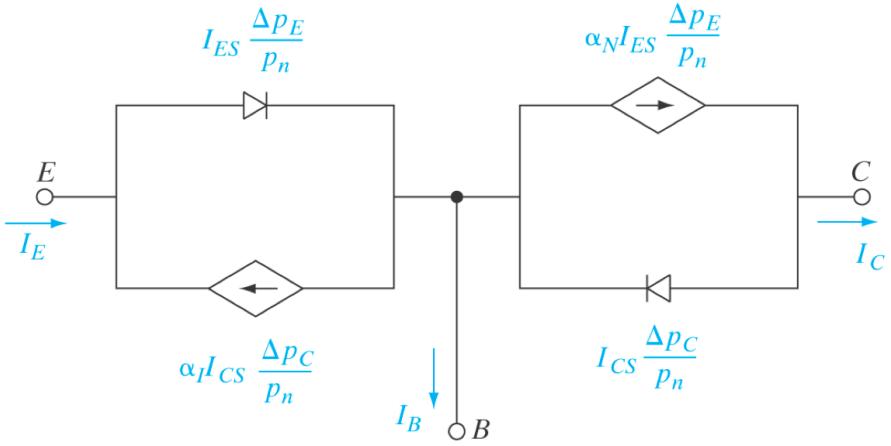
$$I_E = I_{ES}\left(e^{qV_{EB}/kT} - 1\right) - \alpha_I I_{CS}\left(e^{qV_{CB}/kT} - 1\right)$$

$$I_C = \alpha_N I_{ES}\left(e^{qV_{EB}/kT} - 1\right) - I_{CS}\left(e^{qV_{CB}/kT} - 1\right)$$

where

α_N and α_I are the normal and inverted current transfer ratios

I_{ES} and I_{CS} are the diode saturation current with the other junction shorted



$$I_B = (1 - \alpha_N)\, I_{ES}\frac{\Delta p_E}{p_n} + (1 - \alpha_I)\, I_{CS}\frac{\Delta p_C}{p_n}$$

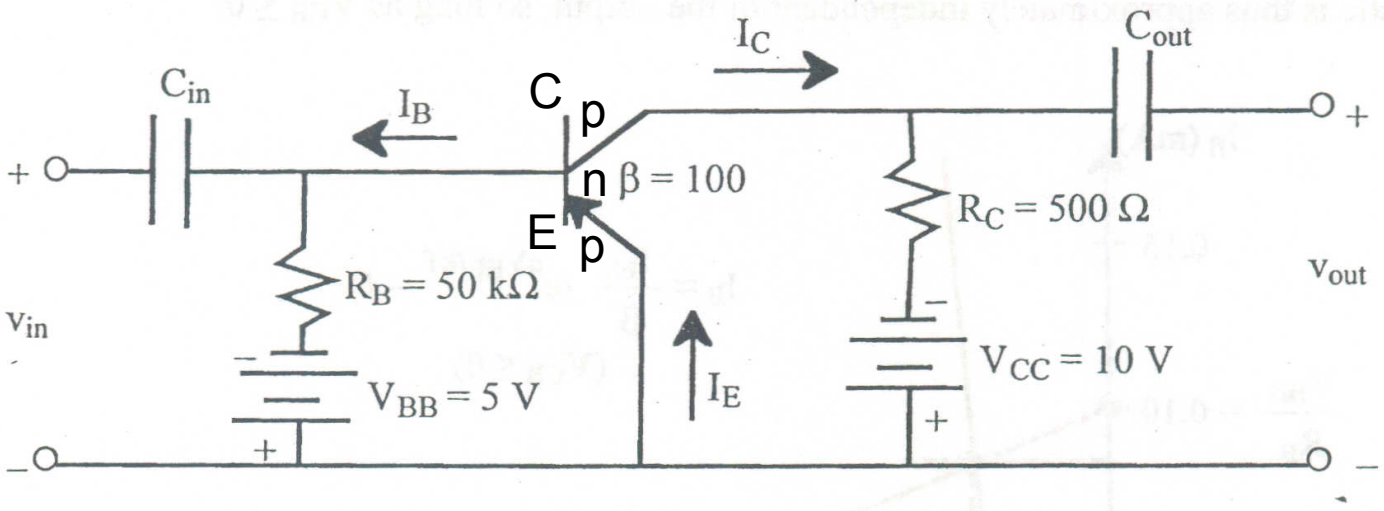
Note For Example:

$$I_{ES} \approx qA\left(\frac{D_p}{W_b}p_n + \frac{D_n^E}{L_n^E}n_p^E\right) \quad \text{for} \quad W_b \ll L_p$$

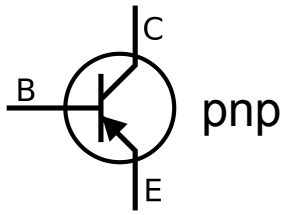
Common Emitter Amplifier

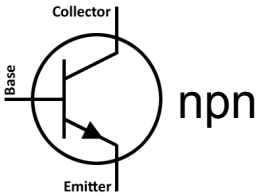
Large Signal Model

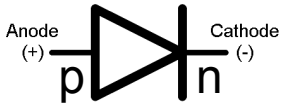
Common-Emitter Amplifier



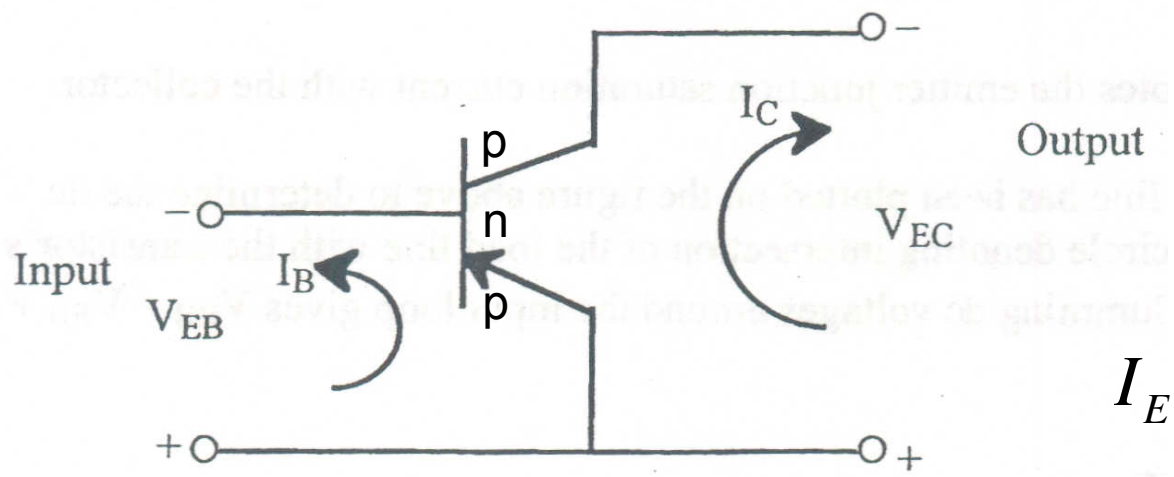
- Coupling capacitors on the input and output
  - Block DC, pass AC
- Emitter identified by an arrow that points in the direction of emitter **<u>current</u>** flow
  - **pnp**, arrow points <u>into</u> base, holes injected into base
  - **npn**, arrow points <u>away from</u> base, electrons injected into base (but current is in the opposite direction)







Current Loops: Common-Emitter Amplifier



$$I_E = I_B + I_C$$

- For the common-emitter amplifier, input is applied to the base and taken from the collector

Common-Emitter Input Characteristic

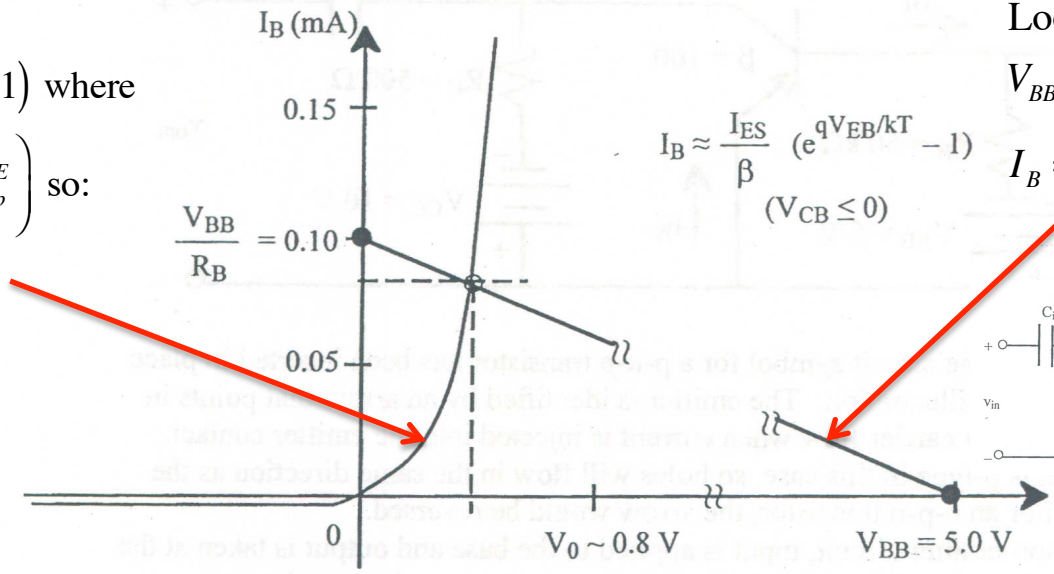
- Transistor biased in normal mode:
  - Collector and base currents independent of the reverse bias voltage across the collector junction
- Operating point determined using load line

Ideal Diode Equation:

$$I_C = \beta I_B \approx I_{ES}\left(e^{qV_{EB}/kT} - 1\right) \text{ where}$$

$$I_{ES} \approx qA\left(\frac{D_p}{W_b}p_n + \frac{D_n^E}{L_n^E}n_p^E\right) \text{ so:}$$

$$I_B \approx \frac{I_{ES}}{\beta}\left(e^{qV_{EB}/kT} - 1\right)$$

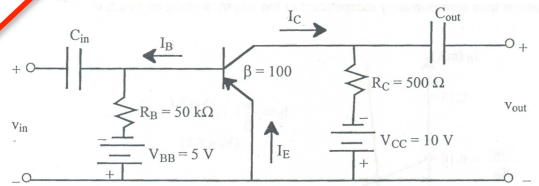


Input Characteristic

Loop Analysis on Input Loop:

$$V_{BB} = V_{EB} + I_B R_B \text{ so:}$$

$$I_B = \frac{V_{BB} - V_{EB}}{R_B}$$

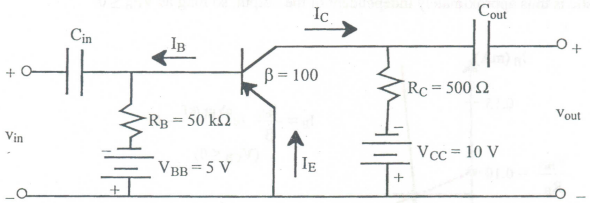


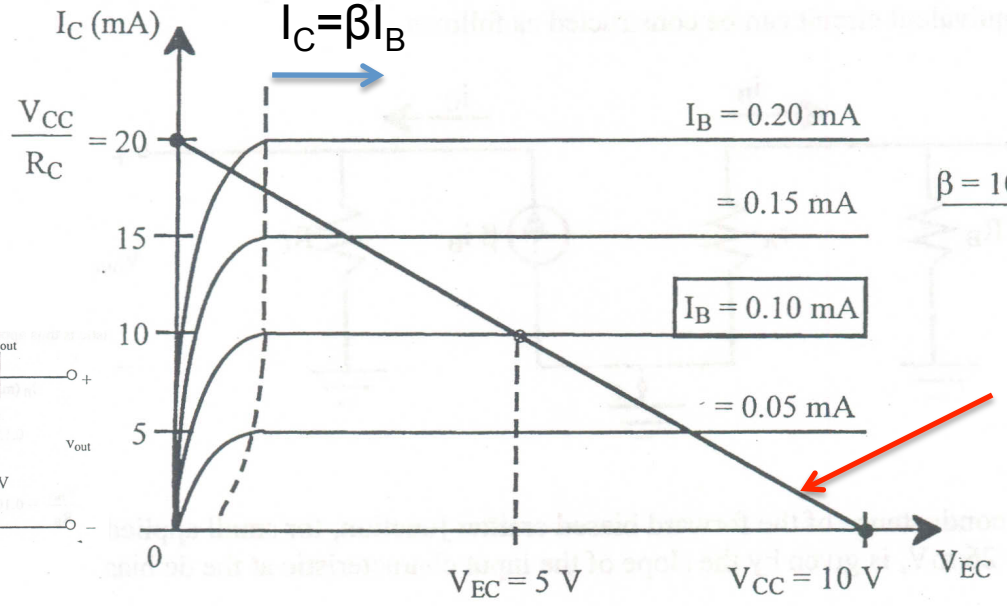
Since V_{EB} is small, $I_B \approx \frac{V_{BB}}{R_B} = \frac{5V}{50k\Omega} = 0.10$ mA

Common-Emitter Output Characteristic

- I_C is plotted as a function of V_{EC} for increasing base current I_B
- The collector load line is plotted to determine operating points
- Except for V_{EC}<~1V, $I_C=\beta I_B$
  - As V_{EC} approaches 0, V_{CB} approaches V_{EB} and reverse bias on the collector is lost and I_C falls toward zero
- Positioning the operating point midway on the family of curves help to preserve normal mode operation over a larger range

$$V_{EC} = V_{EB} + V_{BC}$$
$$= V_{EB} - V_{CB}$$





Output Characteristics

Load Line:

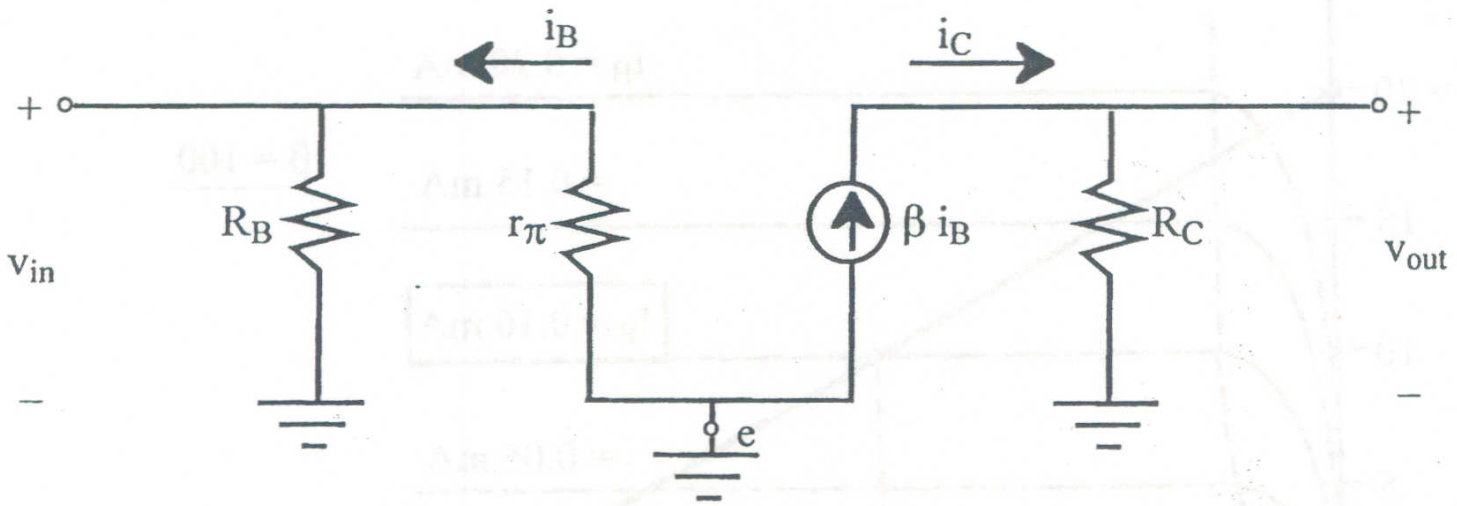
$$I_C R_C - V_{CC} + V_{EC} = 0$$

$$\text{so } I_C = \frac{V_{CC} - V_{EC}}{R_C}$$

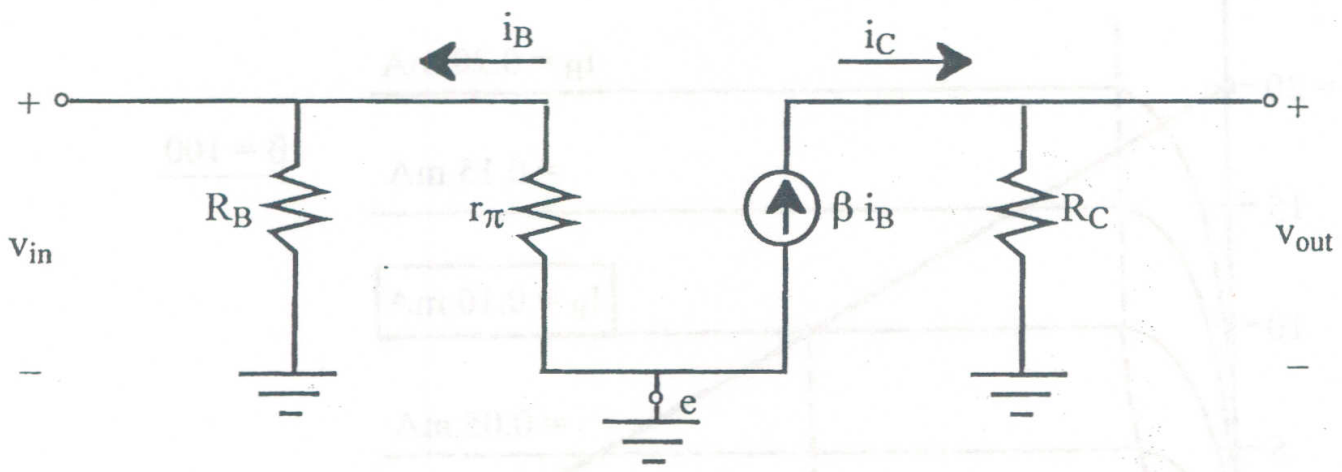
Small Signal Current Gain

Common-Emitter AC Equivalent Circuit

- AC equivalent circuit model treats coupling capacitor and DC voltage source as short circuits
- A positive voltage applied to the input v_{in} opposes the bias voltage (output phase shift)
- Junction resistance r_π is differential resistance of forward-biased junction



Common-Emitter AC Equivalent Circuit



Differential conductance for small AC signal $v_{in} < kT/q \approx 26mV$:

$$\frac{1}{r_\pi} = \frac{dI_B}{dV_{EB}} \approx \frac{q}{kT} I_B$$

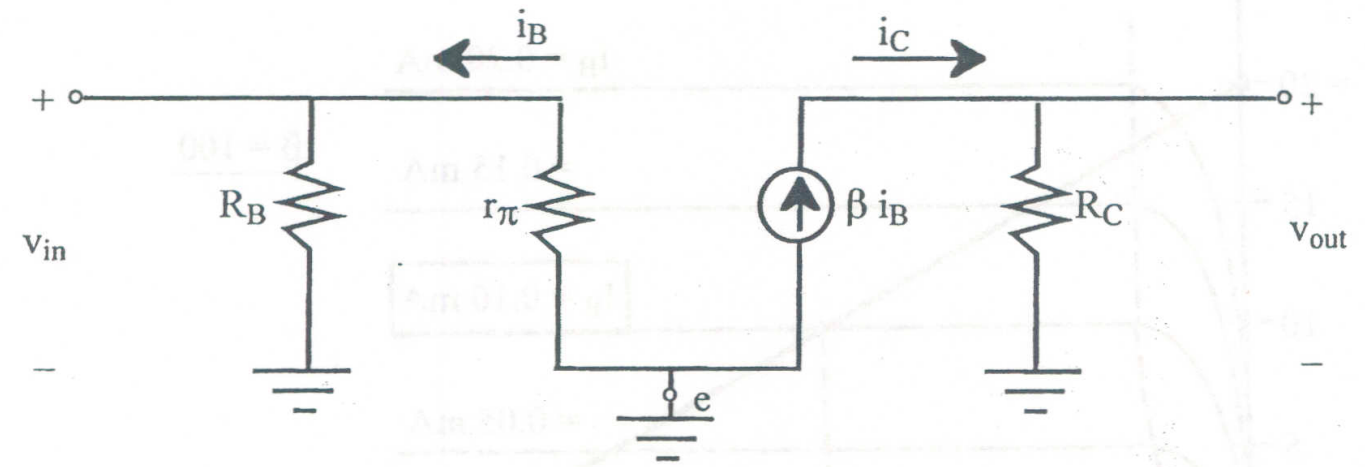
so, an input voltage v_{in} produces a small AC modulation on the base current:

$$i_B = -\frac{v_{in}}{r_\pi}$$

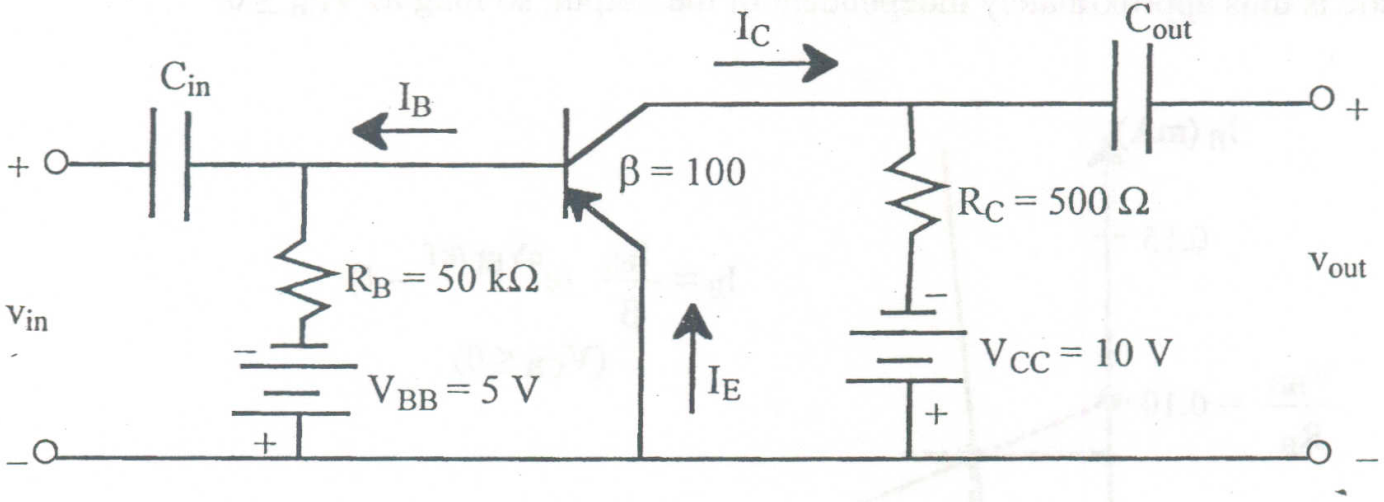
Why "-"?

For the example in the handout: $r_\pi = \frac{kT/q}{I_B} \approx \frac{0.026V}{0.1mA} \approx 260\Omega$

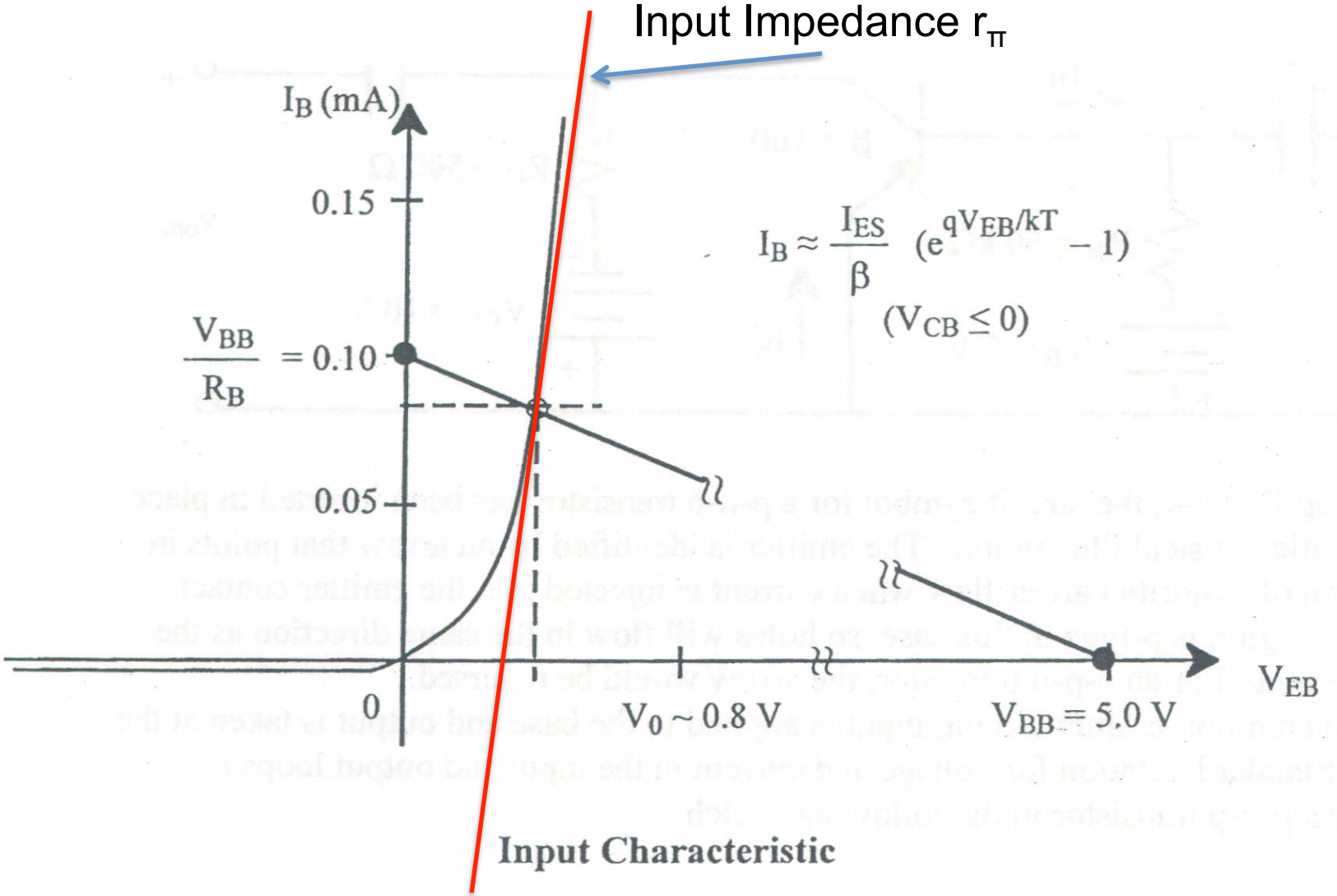
Why "-"?



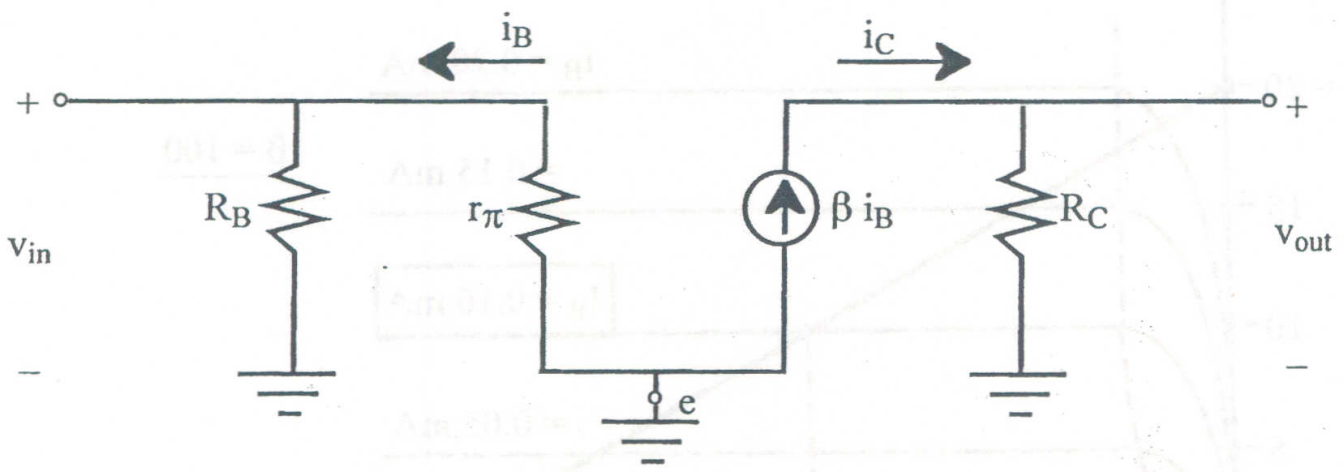
$$i_B = -\frac{v_{in}}{r_\pi}$$



Input Impedance



Common-Emitter AC Equivalent Circuit



For this input current i_B, the resultant AC component of the collector current is:

$$i_C = \beta i_B = -\beta \frac{v_{in}}{r_\pi}$$

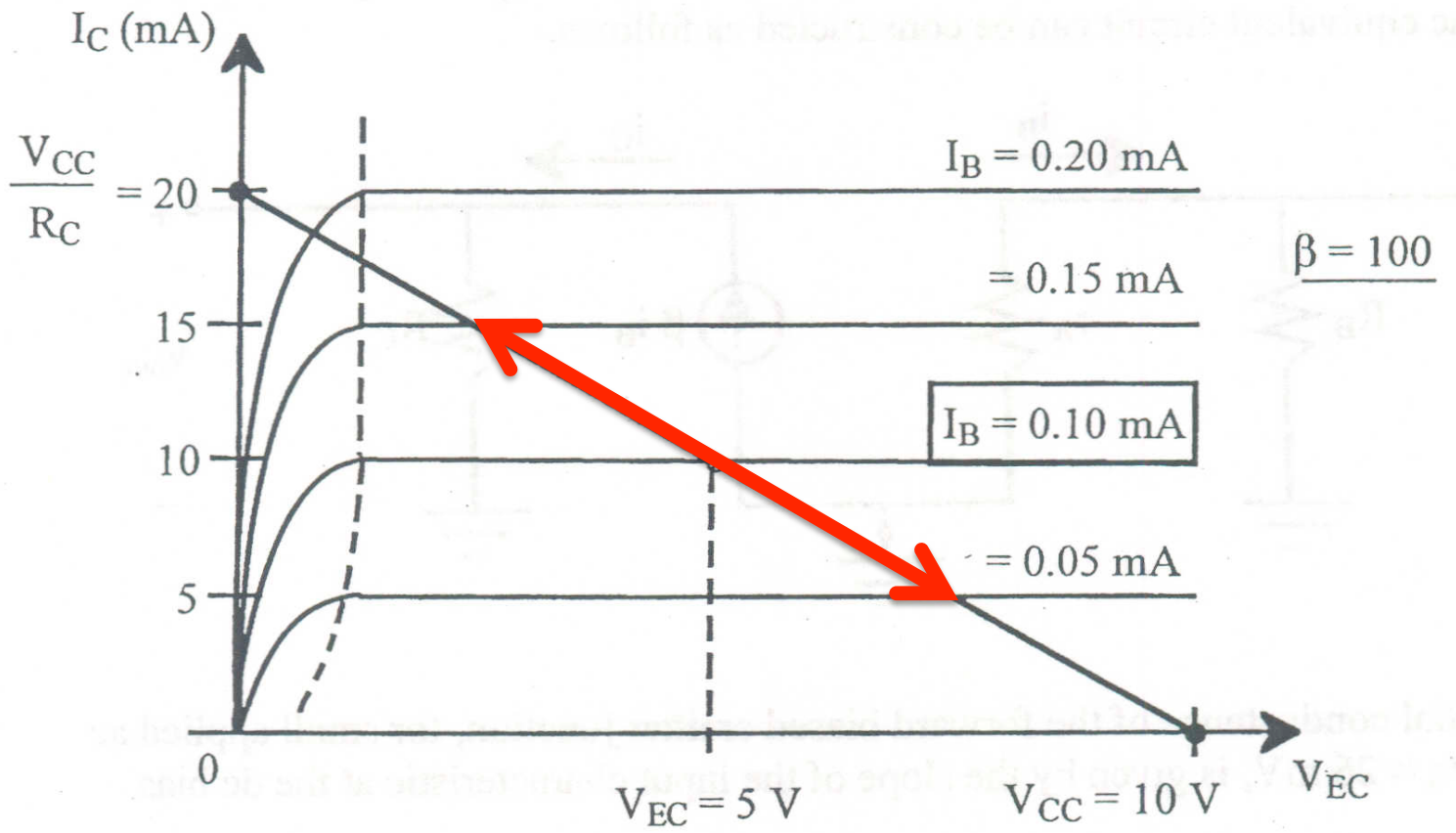
For the simplified case of no load resistance, the output voltage is therefore:

$$v_{out} = i_C R_C = -\beta \frac{R_C}{r_\pi} v_{in}$$

The **open - circuit voltage gain** is therefore:

$$\frac{v_{out}}{v_{in}} = -\beta \frac{R_C}{r_\pi} \approx -100 \times \frac{500\Omega}{260\Omega} \approx -192$$

Output Modulation



Output Characteristics

For an I_B swing of ± 0.05 mA, there is an I_C swing of ± 5 mA

Assignments

Assignments

- Homework assigned every Friday, due following Friday
- Reading from Streetman's book:
  - Mon 4/16: §'s 6.5.1, 6.5.2
  - Wed 4/18: §'s 6.5.8, 9.3.1, 9.5.1
  - Fri 4/20: Handout on BJT (Posted on Website)
  - Mon 4/23: §'s 6.1.1, 6.1.2, 7.1, 7.2, 7.3, and handout
  - Wed 4/25: §'s 7.3, 7.4.1, 7.4.2, 7.4.3, 7.4.4, and handout
  - Fri 4/27: §'s 7.3, 7.4.1, 7.4.2, 7.4.3, 7.4.4, and handout
  - Mon 4/30: §'s 7.4.1, 7.4.2, 7.4.3, 7.4.4, and handout
- Chapters 16, 17, 18, 10, 11, and 12 in Pierret cover similar material

Thank You for Listening!

Tentative Schedule [1]

	JAN 17 Course overview	JAN 19 Intro to semiconductor electronics
JAN 22 Materials and crystal structures	JAN 24 Bonding forces and energy bands in solids	JAN 26 Metals, semiconductors, insulators, electrons, holes
JAN 29 Intrinsic and extrinsic material	JAN 31 Distribution functions and carrier concentrations	FEB 2 Distribution functions and carrier concentrations
FEB 5 Temperature dependence, compensation	FEB 7 Conductivity and mobility	FEB 9 Resistance, temperature, impurity concentration
FEB 12 Invariance of Fermi level at equilibrium	FEB 14 Optical absorption and luminescence	FEB 16 Generation and recombination

Subject to Change

Tentative Schedule [2]

FEB 19 Quasi-Fermi levels and photoconductive devices	FEB 21 Carrier diffusion	FEB 23 Built-in fields, diffusion and recombination
Feb 26 Review, discussion, problems (2/27 exam)	FEB 28 Steady state carrier injection, diffusion length	MAR 2 p-n junctions in equilibrium & contact potential
MAR 5 p-n junction Fermi levels and space charge	MAR 7 Continue p-n junction space charge	MAR 9 NO CLASS (EOH)
MAR 12 p-n junction current flow	MAR 14 Carrier injection and the diode equation	MAR 16 Minority and majority carrier currents
3/19-3/23 Spring Break MAR 26 Reverse-bias breakdown	MAR 28 Stored charge, diffusion and junction capacitance	MAR 30 Photodiodes, I-V under illumination

Subject to Change

Schedule & Policies

Important Information

- Course Website:
  - http://courses.engr.illinois.edu/ece340/
- Download and Review Syllabus / Course Information from Website!
- Course Coordinator: Prof. John Dallesasse
  - jdallesa@illinois.edu
  - Coordinates schedule, policies, absence issues, homework, quizzes, exams, etc.
- Contact Information and Office Hours for All ECE340 Professors & TAs in Syllabus
- Lecture Slides: Click on "(Sec. X)" next to my name in instructor list
- DRES Students: Contact Prof. Dallesasse ASAP
- Textbook:
  - "Solid State Electronic Devices," Streetman & Banerjee, 7th Edition
  - Supplemental: "Semiconductor Device Fundamentals," Pierret
  - Additional reference texts listed in syllabus

Key Points

- Attend Class!
  - 3 unannounced quizzes, each worth 5% of your grade
  - **You must take the quiz in your section**
  - Excused absences must be **pre-arranged** with the course director
  - Absences for illness, etc. need a note from the Dean
    - See policy on absences in the syllabus
- No Late Homework
  - Homework due on the date of an excused absence must be turned in ahead of time
  - You must turn in homework in your section
  - No excused absences for homework assignments
  - Top 10 of 11 homework assignments used in calculation of course grade
    - Do all of them to best prepare for the exams!
- No Cheating
  - Penalties are severe and will be enforced
- Turn Off Your Phone
  - No video recording, audio recording, or photography

Homework

- Assigned Friday, Due Following Friday
  - Due dates shown in syllabus
- Due at Start of Class
- Follow Guidelines in Syllabus
- Peer Discussions Related to Homework are Acceptable and Encouraged
- Directly Copying Someone Else's Homework is Not Acceptable
  - Graders have been instructed to watch for evidence of plagiarism
  - Both parties will receive a "0" on the problem or assignment

Absences

- The absence policy in the syllabus will be strictly enforced
- To receive an excused absence (quiz), you must:
  - Pre-arrange the absence with the course director (valid reason and proof required)
  - Complete an Excused Absence Form at the Undergraduate College Office, Room 207 Engineering Hall (333-0050)
    - The form must be signed by a physician, medical official, or the Emergency Dean (Office of the Dean of Students)
    - The Dean's Office has recently put a strict policy in place (3 documented days of illness)
  - Excused quiz score will be prorated based upon average of completed scores
  - No excused absences are given for homework, but only the best 10 of 11 are used to calculate your final grade
  - Excused absences are not given for exams, except in accordance with the UIUC Student Code
  - Unexcused work will receive a "0"
- Failure to take the final will result in an "incomplete" grade (if excused) or a "0" (if unexcused)

Exams

- Exam I: Tuesday February 27th, 7:30-8:30 pm
- Exam II: Thursday April 12th, 7:30-8:30 pm
- Final Exam: Friday, May 4th, 1:30-4:30 pm

Grading

Grading Criterion

Homework	10 %
Quizzes	15 %
Hour Exam I	20 %
Hour Exam II	20 %
Final Exam	35 %
Total	100 %

Historical Grade Trends*

	Spring 2016	Fall 2016	Spring 2017
A's	27 %	28 %	27 %
B's	37 %	26 %	38 %
C's	27 %	25 %	27%
D's	6 %	16 %	4 %
F's	3 %	5 %	4 %

*Past performance is not necessarily indicative of future results

My Recommendations

- Read the syllabus and information posted on the course website
- **Attend class** & participate
- Attend office hours (TA and Professors)
- **Read the book**
- Re-read the book
- Look at and read selected portions of the supplemental texts
- Form study groups to review concepts and discuss high-level approaches for solving homework problems
  - Don't form study groups to copy homework solutions
- **Don't miss any homework, quizzes, or exams**
  - It's hard to overcome a zero
- Ask questions in class!

Instructional Objectives (1)

By the time of exam No. 1 (after 17 lectures), the students should be able to do the following:

✔1. Outline the classification of solids as metals, semiconductors, and insulators and distinguish direct and indirect semiconductors.

✔2. Determine relative magnitudes of the effective mass of electrons and holes from an E(k) diagram.

✔3. Calculate the carrier concentration in intrinsic semiconductors.

✔4. Apply the Fermi-Dirac distribution function to determine the occupation of electron and hole states in a semiconductor.

✔5. Calculate the electron and hole concentrations if the Fermi level is given; determine the Fermi level in a semiconductor if the carrier concentration is given.

✔6. Determine the variation of electron and hole mobility in a semiconductor with temperature, impurity concentration, and electrical field.

✔7. Apply the concept of compensation and space charge neutrality to calculate the electron and hole concentrations in compensated semiconductor samples.

✔8. Determine the current density and resistivity from given carrier densities and mobilities.

✔9. Calculate the recombination characteristics and excess carrier concentrations as a function of time for both low level and high level injection conditions in a semiconductor.

✔10. Use quasi-Fermi levels to calculate the non-equilibrium concentrations of electrons and holes in a semiconductor under uniform photoexcitation.

✔11. Calculate the drift and diffusion components of electron and hole currents.

✔12. Calculate the diffusion coefficients from given values of carrier mobility through the Einstein's relationship and determine the built-in field in a non-uniformly doped sample.

Plus continuity equation, steady-state carrier injection, and diffusion length

https://my.ece.illinois.edu/courses/description.asp?ECE340

Quiz 1 Statistics

- Average: 8.65
- Standard Deviation: 1.49

Quiz 2 Statistics

- Average: 4.59
- Standard Deviation: 1.86

Quiz 3 Statistics

- Average: 7.04
- Standard Deviation: 2.30

Exam I Statistics

- Average (All Sections): 71.88
- Standard Deviation (All Sections): 15.44

Exam II Statistics

- Average (All Sections): 61.85
- Standard Deviation (All Sections): 14.18

Streetman Errata (6th Edition)

- Equation 4-30: “ΔxA” not δxA
- Equation 4-33b: “τ_p” not “τ_n”

Streetman Errata

- Equation 6-35 in Streetman's "Solid State Electronic Devices" 6th edition has a typo:

Equation in Streetman:

$$C_i = \frac{\varepsilon_s}{W}$$

Corrected Version:

$$C_d = \frac{\varepsilon_s}{W}$$