UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Signal and Image Analysis

Homework 2

Fall 2014

Assigned: Thursday, 1/26/2017 Due: Thursday, 2/2/2017

Reading: 1–40

Do **one** of the following three problems, and submit by 12:30pm 2/2/2017. Homework will be handed back on 2/7/2017. If you don't like your grade, then you can hand in either of the **other** problems for a grade, no later than 2/14/2017.

Problem 2.1

Consider the signal

$$x(t) = |\cos(2\pi t)|$$

(a) Sketch $x(t)$.

(b) What is its period, T_0? What is its fundamental frequency, Ω_0?

(c) Find the Fourier series coefficients.

- Hint #1: notice that $|\cos(2\pi t)|$ is sometimes equal to $\cos(2\pi t)$, and sometimes equal to $-\cos(2\pi t)$, so if you choose the right period of time over which to integrate, you might be able to get rid of the absolute value signs.
- Hint #2: use the relationship $\cos(2\pi t) = \frac{1}{2}(e^{j2\pi t} + e^{-j2\pi t})$ so that you can integrate exponentials instead of integrating cosines.
- Hint #3:

$$\begin{aligned}\int_c^d (e^{at} + e^{bt})dt &= \left[\frac{1}{a}e^{at} + \frac{1}{b}e^{bt}\right]_c^d \\ &= \left(\frac{1}{a}e^{ad} + \frac{1}{b}e^{bd}\right) - \left(\frac{1}{a}e^{ac} + \frac{1}{b}e^{bc}\right)\end{aligned}$$

Problem 2.2

Consider the signal

$$x(t) = 1 - |\sin(200\pi t)|$$

(a) Sketch $x(t)$.

(b) What is its period, T_0? What is its fundamental frequency, Ω_0?

(c) Find the Fourier series coefficients.

- Hint #1: notice that $|\sin(200\pi t)|$ is sometimes equal to $\cos(2\pi t)$, and sometimes equal to $-\sin(200\pi t)$, so if you choose the right period of time over which to integrate, you might be able to get rid of the absolute value signs.
- Hint #2: use the relationship $\sin(200\pi t) = \frac{1}{2j}(e^{j200\pi t} - e^{-j200\pi t})$ so that you can integrate exponentials instead of integrating cosines.
- Hint #3:

$$\begin{aligned}\int_c^d (e^{at} + e^{bt})dt &= \left[\frac{1}{a}e^{at} + \frac{1}{b}e^{bt}\right]_c^d \\ &= \left(\frac{1}{a}e^{ad} + \frac{1}{b}e^{bd}\right) - \left(\frac{1}{a}e^{ac} + \frac{1}{b}e^{bc}\right)\end{aligned}$$

Problem 2.3

Consider the signal

$$x(t) = \begin{cases} 1 - e^{-100|t|} & -0.01 \le t \le 0.01 \\ x(t - 0.02) & \text{otherwise} \end{cases}$$

(a) Sketch $x(t)$.

(b) What is its period, T_0? What is its fundamental frequency, Ω_0?

(c) Find the Fourier series coefficients.

- Hint #1: for any time points $a \le b \le c$,

$$\int_a^c x(t)dt = \int_a^b x(t)dt + \int_b^c x(t)dt$$

- Hint #2: notice that $e^{-100|t|}$ is sometimes equal to e^{-100t}, and sometimes equal to e^{100t}, so if you divide the integral as shown in Hint #1, then you won't have to use the absolute value sign any more. (And notice that $e^{-100t}e^{jk\omega_0 t} = e^{(-100+jk\omega_0)t}$).
- Hint #3:

$$\begin{aligned}\int_c^d (e^{at} + e^{bt})dt &= \left[\frac{1}{a}e^{at} + \frac{1}{b}e^{bt}\right]_c^d \\ &= \left(\frac{1}{a}e^{ad} + \frac{1}{b}e^{bd}\right) - \left(\frac{1}{a}e^{ac} + \frac{1}{b}e^{bc}\right)\end{aligned}$$