UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Signal and Image Analysis

Homework 7

Fall 2014

Assigned: Thursday, 3/2/2017 Due: Thursday, 3/9/2017

Reading: 1–40

Do **one** of the following two problems, and submit by 11:59pm 3/9/2017 (on Compass, if you don't hand it in during class). Homework will be returned on 3/14/2017. If you don't like your grade, then you can hand in the **other** problem for a grade, no later than 3/28/2017.

Problem 7.1

A periodic continuous-time signal has the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kt/T_0}$$

Suppose that $T_0 = 0.01$s. Suppose that $x(t)$ is lowpass filtered by an ideal anti-aliasing filter with a cutoff of 5kHz, then sampled at $F_s = 10$kHz to create $x[n]$. $x[n]$ is then passed through a 50-sample averager to create $y[n]$:

$$y[n] = \frac{1}{50}\sum_{m=0}^{49} x[n-m]$$

The signal $y[n]$ is sent through an ideal D/A with the same sampling frequency, $F_s = 10$kHz, to create the signal $y(t)$, which can be written as

$$x(t) = \sum_{k=-\infty}^{\infty} Y_k e^{j2\pi kt/T_0}$$

(a) For which values of k does $Y_k = 0$, either because of the anti-aliasing filter or because of the digital filter?

(b) An averager is a sort of lowpass filter, albeit not a very good one. Suppose we say that the cutoff frequency is equal to the frequency of the first null. What is the cutoff frequency of this averager, in Hertz?

(c) Find the amplitudes of Y_k in terms of X_k for all k, including $k = 0$. You don't need to worry about phase.

Problem 7.2

A periodic continuous-time signal has the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kt/T_0}$$

Suppose that $T_0 = 0.01$s. Suppose that $x(t)$ is lowpass filtered by an ideal anti-aliasing filter with a cutoff of 4kHz, then sampled at $F_s = 8$kHz to create $x[n]$. $x[n]$ is then passed through a 40-sample averager to create $y[n]$:

$$y[n] = \frac{1}{40} \sum_{m=0}^{39} x[n-m]$$

The signal $y[n]$ is sent through an ideal D/A with the same sampling frequency, $F_s = 8$kHz, to create the signal $y(t)$, which can be written as

$$x(t) = \sum_{k=-\infty}^{\infty} Y_k e^{j2\pi kt/T_0}$$

(a) For which values of k does $Y_k = 0$, either because of the anti-aliasing filter or because of the digital filter?

(b) An averager is a sort of lowpass filter, albeit not a very good one. Suppose we say that the cutoff frequency is equal to the frequency of the first null. What is the cutoff frequency of this averager, in Hertz?

(c) Find the amplitudes of Y_k in terms of X_k for all k, including $k = 0$. You don't need to worry about phase.