Lecture 1: Cosines

ECE 401: Signal and Image Analysis

University of Illinois

1/19/2017

Outline

Who should take this course?

- Grads. Need DSP for your research. Don't know anything about DSP. You want to.
- Grads. Plan to take ECE 551 (Advanced DSP) or 537 (Speech Processing), but your undergrad DSP wasn't enough to prepare you; you need a course that will teach you what you're missing.
- Undergrads. Not in ECE. Your startup company or research project needs DSP. Don't know anything about DSP. You want to.

Who should NOT take this course?

- ECE undergrads who hope to take ECE 418 (Image Processing) or ECE 420 (DSP Lab). This class is not adequate pre-req for those classes.
- ECE undergrads who've had ECE 310. Credit is not given for both ECE 401 and ECE 310.

What's the workload?

- Lots of little assignments
  - Every week, one lab and one written assignment
  - Three in-class exams
- *Very* little
  - Each written assignment: solve *one* problem in *two* different ways, check your answer.
  - Grade replacement policy: If you don't like your grade on any given homework, there is a second problem available. Submit the second problem within one week after receiving your first grade. Your second grade, if better than the first, replaces the first.
  - Each lab: create three signals: one abstract, one audio, and one image.
  - Each exam: four problems, nearly identical to homework questions.

Grading

- HW: 25%, Labs: 25%, Project: 20%, Exams: 30%
- A-: 90%+, B-: 80%+, C-: 70%+
- Thresholds adjusted downward if necessary to ensure at least 1/3 of the class have A- or better, at least 2/3 have B- or better (but it's rarely necessary to adjust thresholds downward to make sure this happens).
- Extra credit for in-class participation: 0.5% each time you solve a problem in class on the blackboard, up to a total of 2.5%.

Project

Make a movie, using digital effects based on lab exercises in class. I will spend the last week of class (after the last exam) showing you techniques you can use to do this. Movie is due by the end of finals week. Examples produced by students in previous semesters are available on the course web page.

Outline

Cosines: the three critical numbers

$$x(t) = A\cos\left(2\pi\left(F(t-t_0)\right)\right)$$

- A = magnitude
  - Units, e.g.: Pascals (Pa) for sound pressure, volts (V) for voltage
- F=frequency, Units: Hertz (Hz) = cycles/second
  - $\Omega = 2\pi F$=radial frequency, Units = radians/second (rad/s)
  - $T_0 = \frac{1}{F} = \frac{2\pi}{\Omega}$ =period, Units=seconds (s)
- t_0=time shift, Units: seconds (s)
  - $\theta_0 = 2\pi t_0$=phase shift, Units: radians (rad)

Three Angles, Three Identities

θ	$\cos\theta$	$\sin\theta$
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}\approx 0.87$	$\frac{1}{2}=0.5$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}\approx 0.71$	$\frac{\sqrt{2}}{2}\approx 0.71$
$\frac{\pi}{3}$	$\frac{1}{2}=0.5$	$\frac{\sqrt{3}}{2}\approx 0.87$

$$\begin{aligned}
\cos\left(\frac{\pi}{2}-\theta\right) &= \sin\theta \\
\cos\left(\pi-\theta\right) &= -\cos\theta \\
\cos\left(2\pi\pm\theta\right) &= \cos\theta
\end{aligned}$$

Outline

Sampled Signals

$$x[n] = x(n/F_S) = x(nT)$$

- n = sample index (integer).
- F_s = sampling rate. Units: samples/second (Hz)
- T = sampling period. Units: seconds/sample (s)

Sampled Cosines

$$x[n] = A\cos\left(2\pi\left(f\left(n - n_0\right)\right)\right)$$

- A = magnitude
- $f = \frac{F}{F_s}$=digital frequency, Units: cycles/sample
    - $\omega = 2\pi f = \frac{\Omega}{F_s}$=radial digital frequency, Units: radians/sample (rad/s)
- $n_0 = t_0 F_s$=digital time shift, Units: samples
    - $\theta_0 = 2\pi n_0$=phase shift, Units: radians

Nyquist's Theorem

- Nyquist's theorem: $F_s \geq 2F$
- Meaning: you need at least two samples per period
- Reason: $\cos(\omega n) = \cos((2\pi + \omega)n)$. A cosine at frequency ω (radians/sample) and a cosine at frequency $2\pi + \omega$ (radians/sample) have exactly the same values at integer sample points, n. There's no way to tell them apart! Since there's no way to tell them apart, the sound card on your PC needs to figure out which one you mean. By default, it assumes that you mean the lowest possible frequency that matches the digital input. Other choices are possible, but they require hardware that you can somehow modify to pick one of the higher frequencies. Such hardware exists (e.g., for fiber optic communication), but a regular PC sound card doesn't have such hardware, so to get perfect reconstruction from a PC sound card, you need to make sure that $F_s \geq 2F$.

Sinusoidal Images

$$x[m,n] = B + A\cos\left(2\pi\left(f_1(m-m_0) + f_2(n-n_0)\right)\right)$$

- B = constant offset, $B \geq A/2$
- A = magnitude, either $A \leq \frac{1}{2}$ or $A \leq \frac{255}{2}$
- m = row index, f_1=row frequency (units: cycles/row)
  - To get frequency in cycles/image, multiply $M \times f_1$ where M = number of rows in the image.
- n = column index, f_2=column frequency (units: cycles/column)
  - To get frequency in cycles/image, multiply $N \times f_2$ where N = number of columns in the image.
- $\theta_0 = 2\pi(f_1 m_0 + f_2 n_0)$ = phase shift

On-Board Exercise

Plot the first twelve samples of the following signal:

$$x[n] = 6\cos\left(2\pi\left(\frac{n}{12} + \frac{7}{12}\right)\right)$$