Lecture 8: Impulse Response

ECE 401: Signal and Image Analysis

University of Illinois

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Outline

Interpolation Review

The following signal is passed through two different D/A circuits, both with a sampling frequency of $F_s = \frac{1}{T} = 10,000\text{Hz}$. The first circuit is a piece-wise-constant (PWC) interpolator, and constructs a signal $x_{PWC}(t)$. The second is a piece-wise-linear (PWL) interpolator, and constructs a signal $x_{PWL}(t)$.

$$x[n] = \begin{cases} 0.7 & n = -1 \\ 1.0 & n = 0 \\ 0.7 & n = -1 \\ 0 & \text{otherwise} \end{cases}$$

Draw $x_{PWC}(t)$ and $x_{PWL}(t)$.

Outline

Discrete-Time System

- A discrete-time system is anything (software, hardware, wetware, or vaporware) that accepts one signal $x[n]$ as input, and generates another signal $y[n]$ as output.
- In this class we'll assume that the behavior of the system is predictable, so we can write a **system equation** specifying the relationship between input and output.

Examples: Averaging

- Here's a system that takes the average of two consecutive input samples:

$$y[n] = \frac{1}{2}\left(x[n] + x[n-1]\right)$$

- Here's a system that takes the average of three consecutive input samples:

$$y[n] = \frac{1}{3}\left(x[n+1] + x[n] + x[n-1]\right)$$

- Here's a system that takes a weighted average of five consecutive input samples:

$$y[n] = 0.1x[n-2] + 0.2x[n-1] + 0.4x[n] + 0.2x[n+1] + 0.1x[n+2]$$

Examples: Differencing

- Here's a system that estimates $y[n] \approx \frac{dx}{dt}$ using the forward-Euler method:

$$y[n] = \frac{1}{T}\left(x[n+1] - x[n]\right)$$

- Here's a system that estimates $y[n] \approx \frac{dx}{dt}$ using the backward-Euler method:

$$y[n] = \frac{1}{T}\left(x[n] - x[n-1]\right)$$

- Here's a system that estimates $y[n] \approx \frac{dx}{dt}$ using the central-Euler method:

$$y[n] = \frac{1}{2T}\left(x[n+1] - x[n-1]\right)$$

Other Examples

- Here's a system that estimates $y[n] \approx \frac{d^2x}{dt^2}$:

$$y[n] = \frac{1}{T^2}\left(x[n+1] - 2x[n] + x[n-1]\right)$$

- Here's a system that estimates the degree to which the most recent 20 samples of $x[n]$ resemble $\cos(\pi n/10)$:

$$y[n] = \sum_{m=0}^{19} \cos\left(\frac{\pi m}{10}\right) x[n-m]$$

Other Examples

- Here's a system that acts kind of like an integral:

$$y[n] = \sum_{m=0}^{\infty} x[n-m]$$

- Here's a system that just delays the input:

$$y[n] = x[n-3]$$

Outline

Special signals that you need to know

- The unit impulse is

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

- The unit step is

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Impulse Response

- The "impulse response" of a system, $h[n]$, is the output that it produces in response to an impulse input.

  Definition: if and only if $x[n] = \delta[n]$ then $y[n] = h[n]$

- Given the system equation, you can find the impulse response just by feeding $x[n] = \delta[n]$ into the system.
- If the system is linear and time-invariant (terms we'll define later), then you can use the impulse response to find the output for **any** input, using a method called **convolution** that we'll learn in two weeks.
- For today, let's get some practice at finding the impulse response from the system equation.

Example: Averaging

Consider the system

$$y[n] = \frac{1}{2}\left(x[n] + x[n-1]\right)$$

Suppose we insert an impulse:

$$x[n] = \delta[n]$$

Then whatever we get at the output, by definition, is the impulse response. In this case it is

$$h[n] = \frac{1}{2}\left(\delta[n] + \delta[n-1]\right) = \begin{cases} 0.5 & n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

Example: Forward Euler

Consider the system

$$y[n] = x[n+1] - x[n]$$

Suppose we insert an impulse:

$$x[n] = \delta[n]$$

Then whatever we get at the output, by definition, is the impulse response. In this case it is

$$h[n] = (\delta[n+1] - \delta[n]) = \begin{cases} 1 & n = -1 \\ -1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$$

Example: Second Difference

Consider the system

$$y[n] = x[n+1] - 2x[n] + x[n-1]$$

Suppose we insert an impulse:

$$x[n] = \delta[n]$$

Then whatever we get at the output, by definition, is the impulse response. In this case it is

$$h[n] = (\delta[n+1] - 2\delta[n] + \delta[n-1]) = \begin{cases} 1 & n = -1, 1 \\ -2 & n = 0 \\ 0 & \text{otherwise} \end{cases}$$

Example: Cosine Matcher

Consider the system

$$y[n] = \sum_{m=0}^{19} \cos\left(\frac{\pi m}{10}\right) x[n-m]$$

Suppose we insert an impulse:

$$x[n] = \delta[n]$$

Then whatever we get at the output, by definition, is the impulse response. In this case it is

$$h[n] = \begin{cases} \cos\left(\frac{\pi n}{10}\right) & 0 \le n \le 19 \\ 0 & \text{otherwise} \end{cases}$$

Example: Integrator

Consider the system

$$y[n] = \sum_{m=0}^{\infty} x[n-m]$$

Suppose we insert an impulse:

$$x[n] = \delta[n]$$

Then whatever we get at the output, by definition, is the impulse response. In this case it is

$$h[n] = u[n]$$

Example: Delay

Consider the system

$$y[n] = x[n-3]$$

Suppose we insert an impulse:

$$x[n] = \delta[n]$$

Then whatever we get at the output, by definition, is the impulse response. In this case it is

$$h[n] = \delta[n-3] = \begin{cases} 1 & n = 3 \\ 0 & n \neq 3 \end{cases}$$