UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Signal and Image Analysis

Homework 6

Fall 2014

Assigned: Thursday, 3/14/2017 Due: Thursday, 3/14/2017

Reading: 1–40

Problem 6.1

(a) Yes, it is linear. If you add two inputs, the result is the same thing you'd get by adding the outputs.

$$\begin{aligned} ay_1[n] + by_2[n] &= a\cos(\alpha n)x_1[2n] + b\cos(\alpha n)x_2[2n] \\ y_3[n] &= \cos(\alpha n)x_3[2n] \\ &= \cos(\alpha n)(ax_1[2n] + bx_2[2n]) \end{aligned}$$

These are the same thing, so the system is linear.

(b) No, it's not time-invariant. Delaying the input changes which samples are chosen by the $2n$ operator, and also changes the cosine-multiplier for each sample.

$$\begin{aligned} y_1[n-m] &= \cos(\alpha(n-m))x_1[2(n-m)] \\ y_2[n] &= \cos(\alpha n)x_2[2n] \\ &= \cos(\alpha n)x_1[2n-m] \end{aligned}$$

Problem 6.2

(a) No, it's not linear. If you add two outputs, then the constant term gets added twice; if you add the inputs and then put it through the system, the constant term shows up only once.

$$\begin{aligned} ay_1[n] + by_2[n] &= a(x_1[n] + 127) + b(x_2[n] + 127) \\ y_3[n] &= x_3[n] + 127 \\ &= (ax_1[n] + bx_2[n]) + 127 \end{aligned}$$

(b) Yes, it's time-invariant. Shifting the constant term, in time, doesn't change its value.

$$\begin{aligned} y_1[n-m] &= x_1[n-m] + 127 \\ y_2[n] &= x_2[n] + 127 \\ &= x_1[n-m] + 127 \end{aligned}$$