UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Signal and Image Analysis

Homework 7

Fall 2014

Assigned: Thursday, 3/28/2017 Due: Thursday, 3/28/2017

Reading: 194–230

Problem 7.1

A periodic continuous-time signal has the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kt/T_0}$$

Suppose that $T_0 = 0.01$s. Suppose that $x(t)$ is lowpass filtered by an ideal anti-aliasing filter with a cutoff of 5kHz, then sampled at $F_s = 10$kHz to create $x[n]$. $x[n]$ is then passed through a 50-sample averager to create $y[n]$:

$$y[n] = \frac{1}{50} \sum_{m=0}^{49} x[n-m]$$

The signal $y[n]$ is sent through an ideal D/A with the same sampling frequency, $F_s = 10$kHz, to create the signal $y(t)$, which can be written as

$$x(t) = \sum_{k=-\infty}^{\infty} Y_k e^{j2\pi kt/T_0}$$

(a) $Y_k = 0$ for $|k| \geq 50$ because of the anti-aliasing filter, and for $k = 2\ell$ because of the discrete-time averaging.

(b) The frequency of the first null is $\omega_c = 2\pi/50$ radians/second. In Hertz, this is

$$\left[\frac{2\pi}{50}\frac{\text{radians}}{\text{sample}}\right] \times \left[10,000\frac{\text{samples}}{\text{second}}\right] \times \left[\frac{1}{2\pi}\frac{\text{cycles}}{\text{radian}}\right] = 200\frac{\text{cycles}}{\text{second}}$$

(c)

$$|Y_k| = \begin{cases} |X_k| & k = 0 \\ 0 & |k| \geq 50 \\ 0 & k = 2\ell,\ \text{integer } \ell \\ \left|\frac{\sin(\pi k/2)}{50\sin(\pi k/50)} X_k\right| & \text{otherwise} \end{cases}$$

Problem 7.2

A periodic continuous-time signal has the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kt/T_0}$$

Suppose that $T_0 = 0.01$s. Suppose that $x(t)$ is lowpass filtered by an ideal anti-aliasing filter with a cutoff of 4kHz, then sampled at $F_s = 8$kHz to create $x[n]$. $x[n]$ is then passed through a 40-sample averager to create $y[n]$:

$$y[n] = \frac{1}{40} \sum_{m=0}^{39} x[n-m]$$

The signal $y[n]$ is sent through an ideal D/A with the same sampling frequency, $F_s = 8$kHz, to create the signal $y(t)$, which can be written as

$$x(t) = \sum_{k=-\infty}^{\infty} Y_k e^{j2\pi kt/T_0}$$

(a) $Y_k = 0$ for $|k| \geq 40$ because of the anti-aliasing filter, and for $k = 2\ell$ because of the discrete-time averaging.

(b) The frequency of the first null is $\omega_c = 2\pi/40$ radians/second. In Hertz, this is

$$\left[\frac{2\pi}{40}\frac{\text{radians}}{\text{sample}}\right] \times \left[8000\frac{\text{samples}}{\text{second}}\right] \times \left[\frac{1}{2\pi}\frac{\text{cycles}}{\text{radian}}\right] = 200\frac{\text{cycles}}{\text{second}}$$

(c)

$$|Y_k| = \begin{cases} |X_k| & k = 0 \\ 0 & |k| \geq 40 \\ 0 & k = 2\ell,\ \text{integer } \ell \\ \left|\frac{\sin(\pi k/2)}{40\sin(\pi k/40)} X_k\right| & \text{otherwise} \end{cases}$$