

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN  
Department of Electrical and Computer Engineering

ECE 401 SIGNAL AND IMAGE ANALYSIS  
Spring 2017

**EXAM 2**

Tuesday, March 28, 2017

- This is a **CLOSED BOOK** exam. You may use one sheet (front and back) of hand-written notes.
- No calculators are permitted. You need not simplify explicit numerical expressions.
- There are a total of 40 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must **SHOW YOUR WORK** to get full credit.

Problem	Score
1	
2	
3	
4	
Total	

Name: \_\_\_\_\_

**Problem 1 (10 points)**

A particular system adds together a signal, 90% of its own echo 3 samples later, and 80% of its own echo 10 samples later:

$$y[n] = x[n] + 0.9x[n - 3] + 0.8x[n - 10]$$

What is the impulse response,  $h[n]$ , of this system?

**Problem 2 (10 points)**

Short-time energy,  $y[n]$ , is computed by averaging  $N$  consecutive squared samples of a signal  $x[n]$ , where  $N$  is some arbitrary constant:

$$y[n] = \frac{1}{N} \sum_{m=0}^{N-1} x^2[n-m]$$

- (a) Is this system linear? Prove your answer, by determining whether or not  $x_3[n] = ax_1[n] + bx_2[n]$  produces the output  $y_3[n] = ay_1[n] + by_2[n]$ .

- (b) Is this system time-invariant? Prove your answer, by determining whether or not  $x_2[n] = x_1[n - n_0]$  produces the output  $y_2[n] = y_1[n - n_0]$ .

**Problem 3 (10 points)**

A periodic continuous-time signal has the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kt/T_0}$$

Suppose that  $T_0 = 0.001$ s. Suppose that  $x(t)$  is lowpass filtered by an ideal anti-aliasing filter with a cutoff of 3kHz, then sampled at  $F_s = 6$ kHz to create  $x[n]$ .  $x[n]$  is then passed through a 3-sample averager to create  $y[n]$ :

$$y[n] = \frac{1}{3} \sum_{m=0}^2 x[n-m]$$

The signal  $y[n]$  is sent through an ideal D/A with the same sampling frequency,  $F_s = 10$ kHz, to create the signal  $y(t)$ , which can be written as

$$x(t) = \sum_{k=-\infty}^{\infty} Y_k e^{j2\pi kt/T_0}$$

- (a) For which values of  $k$  does  $Y_k = 0$ , either because of the anti-aliasing filter or because of the digital filter?

- (b) Find the amplitudes of  $Y_k$  in terms of  $X_k$  for all  $k$ , including  $k = 0$ . You don't need to worry about phase.

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**Problem 4 (10 points)**

Consider the following ideal bandpass filter:

$$H(\omega) = \begin{cases} 1 & \frac{\pi}{6} < |\omega| < \frac{\pi}{3} \\ 0 & \text{otherwise} \end{cases}$$

Find the impulse response,  $h[n]$ .