UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering
ECE 417 Multimedia Signal Processing

Lecture 21 Sample Problems

Problem 21.1

The reason that sinc-squared interpolation is sometimes better than sinc-interpolation: natural images tend to have $1/f$ spectra. This means that the spectrum of a natural image is often of the form $X(e^{j\omega}) = \frac{1}{|\omega|}$ over a wide range of frequencies, from a low frequency equal to the low-frequency cutoff of the recording microphone (call that ω_L, maybe) up to Nyquist.

Suppose that $u[n]$ is a signal with a $1/f$ spectrum. Suppose you lowpass filter with an ideal $\pi/2$ lowpass filter to produce $v[n]$, then downsample by a factor of 2 to produce $x[n]$, then upsample by 2 to produce $y[n]$, then filter with some interpolating filter $h[n]$ to produce the output $z[n]$.

1. Suppose that $h[n]$ is an ideal lowpass filter,

$$h_a[n] = \frac{\sin(\pi n/2)}{\pi n/2}$$

   What is the spectrum of $z[n]$? How does it compare to the spectrum of $u[n]$?

2. Now suppose that $h[n]$ is a sinc-squared,

$$h_b[n] = \left(\frac{\sin(\pi n/2)}{\pi n/2}\right)^2$$

   What is the spectrum of $z[n]$? How does it compare to the spectrum of $u[n]$?