UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering
ECE 417 Multimedia Signal Processing

Lecture 7 Sample Problems

Problem 7.1

Linear Regression: Suppose you have two vectors, $\vec{y} = [y_0, \ldots, y_{N-1}]^T$ and $\vec{x} = [x_0, \ldots, x_{N-1}]^T$. For now, suppose that $\vec{x}$ and $\vec{y}$ are both real; problem 2 will address complex scalars. Your goal is to find a coefficient, a, that minimizes

$$\varepsilon = \sum_{k=0}^{N-1} (y_k - a x_k)^2$$

Differentiate ε, and set $d\varepsilon/da = 0$, in order to find the value of a that minimizes ε.

Problem 7.2

Suppose that x, y, and a are all complex scalars, i.e., $x = x_R + jx_I$, $y = y_R + jy_I$, and $a = a_R + ja_I$. Suppose $\varepsilon(a) = \frac{1}{2}|y - ax|^2$. The function $\varepsilon(a)$ is actually not complex-differentiable in the normal sense ($d\varepsilon/da$ is undefined, in the normal sense), but there is a useful special kind of derivative called the $\mathbb{CR}$-derivative that gives us the following very useful definitions:

$$\frac{\partial\varepsilon}{\partial a} = \frac{1}{2}\left(\frac{\partial\varepsilon}{\partial a_R} - j\frac{\partial\varepsilon}{\partial a_I}\right), \quad \frac{\partial\varepsilon}{\partial a^*} = \frac{1}{2}\left(\frac{\partial\varepsilon}{\partial a_R} + j\frac{\partial\varepsilon}{\partial a_I}\right)$$

Using the definitions above, find $\frac{\partial\varepsilon}{\partial a^*}$, and set it to zero in order to find the value of a that minimizes ε (since y and x are scalars, it is possible to achieve $\varepsilon = 0$).

Problem 7.3

Suppose you have a family of real-valued vectors, $\vec{x}(P) = [x_0(P), \ldots, x_{N-1}(P)]^T$ (for example, P might be the pitch period). For each one, you find

$$\varepsilon(P) = \min_a \sum_{k=0}^{N-1} (y_k - a x_k(P))^2$$

where y_k and a are also real-valued. Now suppose you want to find the pitch period that minimizes $\varepsilon(P)$,

$$\hat{P} = \arg\min \varepsilon(P)$$

Show that $\hat{P}$ can also be written as

$$\hat{P} = \arg\max \frac{\left(\sum_{k=0}^{N-1} y_k x_k(P)\right)^2}{\sum_{k=0}^{N-1} x_k^2(P)}$$