UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering
ECE 417 MULTIMEDIA SIGNAL PROCESSING

Lecture 8 Sample Problems

Problem 8.1

Consider a nearest neighbors classifier with just two training tokens:

$$\vec{x}_0 = \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}, \quad y_0 = -1, \quad \vec{x}_1 = \begin{bmatrix} u_1 \\ v_1 \end{bmatrix}, \quad y_1 = +1$$

Prove that a nearest neighbor classifier, constructed from this training dataset, gives you a linear dichotomizer. Find the vector $\vec{w}$ and the offset b in terms of the variables u_0, v_0, u_1, v_1.

Problem 8.2

Consider the following Bayesian classifier:

$$p_Y(0) = \pi_0, \quad p_Y(1) = 1 - \pi_0$$

Suppose that $\mathcal{X} = (\Re_+)^d$, that is, $\vec{x}$ is a d-dimensional vector, all of whose elements are non-negative. Within this domain, the likelihoods are determined by the parameter vectors $\vec{u} = [u_1, \ldots, u_d]^T$ and $\vec{v} = [v_1, \ldots, v_d]^T$ as

$$p_{X|Y}(\vec{x}|0) = c_0 e^{-\vec{u}^T \vec{x}}, \quad p_{X|Y}(\vec{x}|1) = c_1 e^{-\vec{v}^T \vec{x}}$$

where $c_0 = \prod_{k=1}^d u_k$ and $c_1 = \prod_{k=1}^d v_k$ are normalizing constants. Show that this Bayesian classifier is actually a linear dichotomizer. Find $\vec{w}$ and b in terms of $\vec{u}$, $\vec{v}$, and π_0.

Problem 8.3

A **minimum-risk** classifier is a generalized Bayesian classifier which, instead of minimizing the probability of error, minimizes some other type of expected loss function ("risk" means "expected loss"). For example, consider the following loss function:

$$\mathcal{L}(y, \hat{y}) = \begin{cases} 0 & y = \hat{y} \\ 1 & y = -1 \text{ but } \hat{y} = +1 \quad \textbf{(false alarm)} \\ C & y = +1 \text{ but } \hat{y} = -1 \quad \textbf{(miss)} \end{cases}$$

The **minimum-risk** classifier is defined by

$$h(x) = \arg\min E\left[\mathcal{L}(y, h(x))\right]$$

Consider the table $p_{X,Y}(x, y)$ on lecture slide 21. Depending on the value of C, the minimum-risk classification rule might result in up to five different classification functions $h(x)$. List all five of the classification functions, and say for what values of C each one is a minimum-risk classifier.

Problem 8.4

The **Bayes risk** is defined by

$$\mathcal{R}_{Bayes} = \min E\left[\mathcal{L}(y, h(x))\right]$$

Find the Bayes risk, as a function of C, for each of the five classifiers from problem 3.