UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering
ECE 417 Multimedia Signal Processing

Lecture 11 Sample Problem Solutions

Problem 11.1

$$\Delta = \frac{1}{33}\left(m(8000) - m(0)\right) = \frac{2595}{33}\log_{10}\left(\frac{87}{7}\right)$$

$$f_m = 700\left(10^{m\Delta/2595} - 1\right) = 700\left(\left(\frac{87}{7}\right)^{\frac{m}{33}} - 1\right)$$

$$k_m = \frac{Nf_m}{F_s} = \left(\frac{7\times 1024}{160}\right)\left(\left(\frac{87}{7}\right)^{\frac{m}{33}} - 1\right)$$

Problem 11.2

The DCT is equivalent to the real-symmetric DFT of a real-symmetric function. So, under the assumption that $c[0] = 0$, computing the DCT of a 12-sample rectangular window is equivalent to computing the DFT of a $(2\times 12+1)$-sample, zero-centered window,

$$w[n] = \begin{cases} 1 & -12 \le n \le 12 \\ 0 & \text{otherwise} \end{cases}$$

…under the assumption that $c[0] = 0$. So if that assumption is valid, then

$$W\left(e^{j\omega}\right) = \frac{\sin(\omega 25/2)}{25\sin(\omega/2)}$$

which has its first null at $\omega = \frac{2\pi}{25}$, corresponding to $f = \frac{16000}{25} = 640$ Hertz.

Problem 11.3

W is the DFT matrix, so

$$w_{mn} = e^{-\frac{j2\pi mn}{N}}$$

V is the mel-frequency filterbank matrix, so

$$v_{lm} = \begin{cases} \frac{m-m_{l-1}}{m_l - m_{l-1}} & m_l \ge m \ge m_{l-1} \\ \frac{m_{l+1}-m}{m_{l+1}-m} & m_{l+1} \ge m \ge m_l \\ 0 & \text{otherwise} \end{cases}$$

with band edges m_l as given in problem 1.

U is the (inverse) discrete cosine transform matrix, which is the symmetric real IDFT of a symmetric real sequence. It's not totally clear what should be the length of the DFT, here, but a reasonable choice is $2L+1$, which would give:

$$u_{kl} = \frac{2}{2L+1}\cos\left(\frac{2\pi kl}{2L+1}\right)$$