UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering
ECE 417 Multimedia Signal Processing

Lecture 21 Sample Problem Solutions

Problem 21.1

Let's assume that the spectrum of $u[n]$ is

$$U(\omega) = \frac{1}{|\omega|}$$

for $|\omega| < \pi$. After filtering with an ideal anti-aliasing filter, it is

$$V(\omega) = \begin{cases} \frac{1}{|\omega|} & |\omega| \le \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

After downsampling, we get

$$X(\omega) = \frac{1}{2}\sum_{k=0}^{1} V(\frac{\omega - 2\pi k}{2}) = \frac{1}{2|\omega/2|} = \frac{1}{|\omega|}$$

After upsampling,

$$Y(\omega) = X(2\omega) = \begin{cases} \frac{1}{2|\omega|} & |\omega| \le \frac{\pi}{2} \\ \frac{1}{2(\pi-|\omega|)} & \frac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

1. If $h_a[n] = \frac{\sin(\pi n/2)}{\pi n/2}$, then

   $$H_a(\omega) = \begin{cases} 2 & |\omega| \le \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

   So

   $$Z(\omega) = H_a(\omega)Y(\omega) = \begin{cases} \frac{1}{|\omega|} & |\omega| \le \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

   Which is exactly equal to $U(\omega)$ for $|\omega| \le \pi/2$, and exactly equal to zero at higher frequencies.

2. If $h_b[n] = (h_a[n])^2$, then

   $$H_b(\omega) = \frac{1}{2\pi} H_a(\omega) \circledast H_a(\omega) = 2\left(\frac{\pi - |\omega|}{\pi}\right)$$

   and therefore

   $$Z(\omega) = H_b(\omega)Y(\omega) = \begin{cases} \frac{(\pi-|\omega|)/\pi}{|\omega|} & |\omega| \le \frac{\pi}{2} \\ \frac{1}{\pi} & \frac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

   which is slightly less than $U(\omega)$ at low frequencies ($(\pi - |\omega|)/pi$ times less), but then slightly higher than $U(\omega)$ at high frequencies (π/ω times higher). Actually, the spectral shape in general is closer to the spectral shape of $U(\omega)$, so that, even though the high frequencies are entirely constructed from aliasing, it is still often true that this version of $Z(\omega)$ is a better-looking image than the ideal band-limited version.

   An even better interpolating filter can be constructed using a two-part interpolator, $H_c(\omega)$, which has one type of spectrum below $\pi/2$, and a different nonzero type of spectrum at high frequencies.

   An even better solution is to estimate the high frequencies using a nonlinear regression algorithm, e.g., a neural net trained to estimate the high-frequency component of $U(\omega)$ given its low-frequency component.